Introduction to the concepts of stress, strain and elasticity in tension

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This chapter introduces the three basic equations of strength of materials: stress equals force on area; strain equals deformation on length; and stiffness equals stress on strain. These equations are simple but powerful.

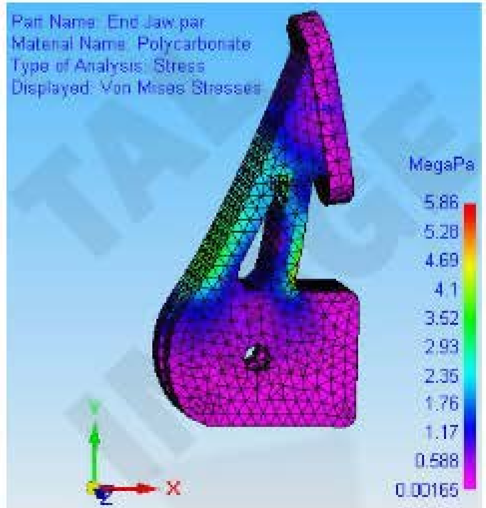
There are three fundamental equations used in strength of materials, with seven terms:

Stress: $\sigma = \dfrac{F}{A}$ Strain: $\varepsilon = \dfrac{e}{l}$

Modulus: $E = \dfrac{\sigma}{\varepsilon}$

By combining these equations we have the capacity to solve a wide range of problems.

They look simple, but they form the foundation of all the later work in this subject, and they are the core equations behind complex computerised stress analysis known as finite element analysis (FEA).



Computerised stress analysis using FEA

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Strength of materials includes the fundamental concepts of stress, strain and stiffness for the elastic behaviour of materials in tension. It should not be confused with metallurgy, which deals with the chemistry of metals to improve manufacture, economy and performance.

Strength of materials

Engineers need to predict how a new design will work. They can do that by estimating what a material will do when subjected to tension, compression, torsion and bending. Will it do the job or will it deform too much? What if it breaks completely?

Rail car crushed under the 35W bridge in Minneapolis USA, 2007

Engineering science deals with the strength and stability of structures and mechanical components. More specifically, the study of how materials respond under load is known as strength of materials.

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OK

What is meant by the term 'strength of materials'?

Click the correct answer.

- Determining how a material responds under load
- Determining the stability of structures and mechanical components
- Determining the loads and forces on structures and mechanical components
- Determining the chemical composition and processing parameters of an engineering component

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Common terms used in strength of materials

Words like strength, elasticity and toughness have strict meanings in engineering, sometimes quite different to their ordinary usage. Take toughness and strength for example. In everyday language they mean much the same thing but in engineering they are almost opposites. Heat-treated steel is made stronger but loses toughness.

Common terms used in strength of materials

- **Strength** – the ability of a material to withstand high forces compared to its size
- **Elasticity** – the ability of a material to return to its original dimensions after the deforming force has been removed
- **Plasticity** – the ability of a material to undergo permanent deformation without failure
- **Ductility** – tensile plasticity
- **Malleability** – compressive plasticity
- **Stiffness** – the ability of a material to resist deformation under load
- **Toughness** – the ability of a material to absorb energy until it breaks
- **Resilience** – the ability of a material to absorb energy before it deforms permanently
- **Hardness** – the ability of a material to resist surface scratching, abrasion or indentation

GIVE FEEDBACK

OK

Match each property with its correct definition.

Drag statements on the right to match the left.

Hardness	The ability of a material to resist surface scratching, abrasion or indentation
Impact strength	The ability of a material to withstand sudden application of a load
Fatigue	The tendency of a material to fail by cracking under repeated application of a relatively small cyclic load
Creep	The tendency of a material to slowly deform under stress, especially at elevated temperature
Resilience	The ability of a material to absorb energy until it deforms permanently

Do you know the answer?

Match these common mechanical properties with their correct definitions.

Drag statements on the right to match the left.

Elasticity	The ability of a material to return to its original dimensions after having been deformed, upon removal of the deforming force
Plasticity	The ability of a material to undergo permanent deformation without failure
Ductility	The ability of a material to be permanently deformed by predominantly tensile forces
Malleability	The ability of a material to be permanently deformed by predominantly compressive forces
Stiffness	The ability of a material to resist deformation under load
Toughness	The ability of a material to absorb energy when being deformed and therefore resist deformation and failure

Strength is the ability of a material to withstand high forces compared to its size.

This is one of the most important mechanical properties of engineering materials.

'Withstand' can mean many things. The two most common meanings are to withstand the load until the material:

- Breaks (ultimate strength)
- Starts to deform permanently (yield strength)

GIVE FEEDBACK

CONTINUE >

There three basic types of strength:

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- Tensile (pulling)
- Compressive (squashing)
- Shear (sliding)

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A material breaks if the load is increased until it reaches ______.

Click the correct answer.

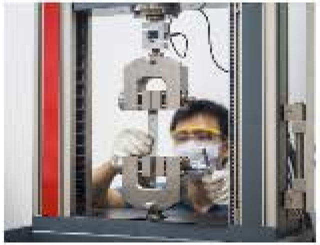
- Ultimate strength
- Permanent strength
- Yield strength
- All three—tensile, compression and shear strength

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The tensile test

The **tensile strength** of a material is determined by the tensile test under a gradual extension. The force increases as the specimen stretches and it eventually breaks.



An engineering test for tensile strength; the machine is called a tensometer.

Tensile test specimens are deliberately made narrow in the middle to control where the elongation and fracture occur.

GIVE FEEDBACK

OK

Match the terms to the blanks to describe conditions for a standard tensile test for engineering materials (e.g. metals like steel, aluminium, etc.).

Drag statements on the right to match the left.

A machine that measures tensile strength is called a ______.	tensometer
The test specimen is loaded by a ______ increase in force.	gradual
To make sure the specimen breaks in the right place, it is made ______ in the middle.	narrow
To measure the ultimate tensile strength, the tensile test is not completed until the specimen is ______.	broken

Do you know the answer?

Ultimate tensile strength describes the maximum tensile force a specimen can endure before it breaks, divided by its initial cross-sectional area.

Ultimate tensile strength

Engineering design is mostly based on the tensile strength, especially the **ultimate tensile strength** (UTS).

UTS is defined as the maximum tensile force applied before fracture occurs, divided by the initial cross-sectional area of the test specimen.

$$UTS = \frac{\text{maximum tensile force}}{\text{initial cross-sectional area}}$$

Other names for **UTS** include:

- **Ultimate tensile stress** – the words strength and stress are closely related. Strength is the maximum stress a material can take before something happens
- **Tensile strength** – a common shorthand
- **Strength** – not a good name but it would most likely mean UTS

GIVE FEEDBACK

OK

Which of the following terms usually refers to the UTS of a material?

Check all that apply.

- [ ] Ultimate tensile stress
- [ ] Ultimate tensile strength
- [ ] Tensile strength
- [ ] Tensile stress

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

There are many types of strength but they are all a measure of ______.

Click the correct answer.

- Stress
- Force
- Mass
- Size

Do you know the answer?

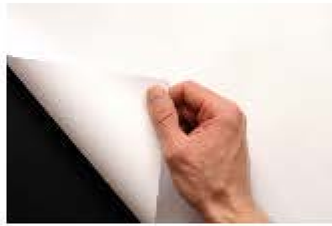
I KNOW IT | THINK SO | UNSURE | NO IDEA

The standard unit for stress is the pascal but it is too small for measuring stress in metals, ceramics, polymers and composites. Instead, engineers measure stress using the megapascal, which is a million pascals.

The SI unit of stress is the **pascal** (**Pa**). A pascal is a newton per square metre:

$$Pa = \frac{N}{m^2}$$

The pascal is used for stress (in a solid) and also for pressure (in a fluid). One pascal is a minute unit, about as much pressure as a single piece of paper applies to your desk.



For 100 gsm paper, the pressure applied to a table is about one pascal

GIVE FEEDBACK

CONTINUE >

The pascal is too small to measure stress. For example, some steels can handle a billion (1000 million) pascals. Instead engineers prefer to use the **megapascal** (MPa) where:

1 megapascal = 1 MPa = 1 million pascals = 1,000,000 Pa

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A megapascal is equivalent to a:

Check all that apply.

- [ ] Newton per square millimetre
- [ ] Million pascals
- [ ] Newton per cubic millimetre
- [ ] Million newtons per square millimetre
- [ ] Million newtons per square metre

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A pascal is defined as a:

Click the correct answer.

- Newton per square metre
- Newton per square millimetre
- Square newton per millimetre
- Newton per metre

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

It is convenient to calculate in newtons and millimetres, which gives the ultimate tensile stress in newtons per square millimetre, i.e. megapascals.

Formula for ultimate tensile strength

It is convenient to calculate the ultimate tensile strength (UTS) in newtons and millimetres, which gives the stress in newtons per square millimetre (N/mm^2):

$$\text{UTS} = \frac{\text{maximum tensile force (newtons)}}{\text{initial cross-sectional area (mm}^2\text{)}}$$

Since there are a million square millimetres in a square metre, it turns out that a N/mm^2 is a million pascals, which is a megapascal (MPa):

$$1 \text{ MPa} = \frac{1 \text{ newton}}{1 \text{ mm}^2}$$

Engineers usually work in megapascals and dimensions are usually in millimetres, which suits this formula (where force is in newtons).

GIVE FEEDBACK

OK

A newton per square millimetre is also called a ______.

Click the correct answer.

- Megapascal
- Pascal
- Square newton per millimetre
- Newton
- Square millimetre

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Fill in the correct units for calculating MPa:

MPa = ____ / ____ ^{2}

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

From the definition of ultimate tensile strength, match each term to its correct unit.

Drag statements on the right to match the left.

Ultimate tensile strength	Megapascal
Maximum tensile force	Newton
Original cross-sectional area	Square millimetre

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

From the definition of ultimate tensile strength, match each symbol to its correct unit.

Drag statements on the right to match the left.

UTS	MPa
F	N
A	mm^2

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

A steel test specimen, $10\ \mathrm{mm}$ in diameter, ruptures under a tensile load of $37\ \mathrm{kN}$. What was the ultimate tensile strength (UTS) of the steel?

GIVE FEEDBACK

CONTINUE >

Example

A steel test specimen, $10\ \mathrm{mm}$ in diameter, ruptures under a tensile load of $37\ \mathrm{kN}$. What was the ultimate tensile strength (UTS) of the steel?

Solution

$$\text{Cross-sectional area} = \frac{\pi \times 10^2}{4}$$
$$= 78.54\ \mathrm{mm}^2$$

$$\text{Tensile force} = 37 \times 1{,}000 = 37{,}000\ \mathrm{N}$$

$$\therefore \text{UTS} = \frac{37{,}000\ \mathrm{N}}{78.54\ \mathrm{mm}^2}$$
$$= 471\ \mathrm{N/mm}^2$$
$$= 471\ \mathrm{MPa}$$

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Type your answer in the box.

A square steel bar is $10\ \mathrm{mm}$ by $10\ \mathrm{mm}$ thick and $1{,}350\ \mathrm{mm}$ long.
If it ruptures under a tensile load of $37\ \mathrm{kN}$, enter the correct numbers to calculate the UTS:

UTS = [ ] / [ ]

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A 4mm thick steel plate is cut into a strip 18 mm wide. if there is a tensile force of 12 kN applied, calculate the stress. (Round off to nearest integer, type units as MPa.)

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Average ultimate tensile strengths for common materials

Typical average values of ultimate tensile strengths of some materials are given in this table.

Note that wood strength varies considerably with the direction of application of the load. The strength given here is for parallel (i.e. along the grain) direction of the force. The transverse (i.e. perpendicular to the grain) strength for pine timber is only $3\ \mathrm{N/mm^2}$.

Ultimate tensile strength

Material	UTS (N/mm^2) or megapascals (MPa)
tool steel	1000
high-tensile steel	590
mild steel	470
copper wire	415
copper sheet	210
brass	190
cast iron	180
aluminium	150
timber (pine)	150*
nylon	70

GIVE FEEDBACK

OK

Match the typical ultimate tensile stresses for each common material.

Drag statements on the right to match the left.

Mild steel	470 MPa
Nylon	70 MPa
Tool steel	1000 MPa
Aluminium	150 MPa
Cast iron	180 MPa

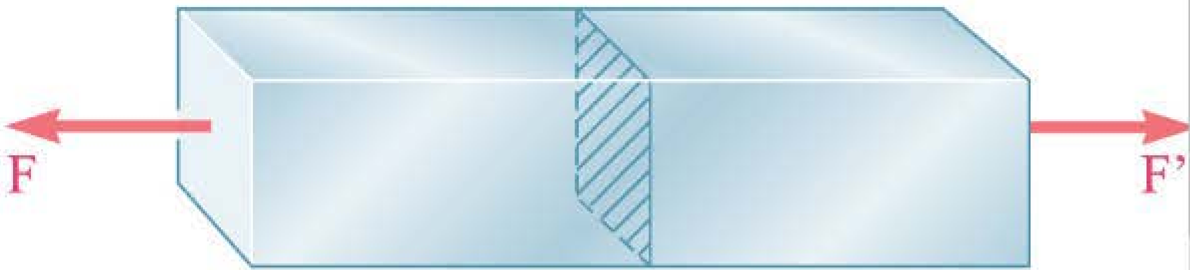
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Direct axial load

To calculate the allowable load on a structure or a component which will ensure a sufficient degree of safety, we need to consider the axial load and calculate the axial stress.

Consider a bar of solid material, forming a component of a machine or structure, subjected to an axial pull, i.e. tension. The force acting on the bar is acting along the axis and is called the **direct axial load**, F.



GIVE FEEDBACK

OK

Check your understanding of the direct axial load by matching the answers to the following questions.

Drag statements on the right to match the left.

What is the definition of axial load?	The force acting along the axis
What is the definition of tensile load?	The force trying to pull apart the component
What is the definition of compressive load?	The force trying to squash the component

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The direct stress is the average force distributed over the cross-sectional area of the object in tension. Direct stress can be thought of as the share of total axial load being carried by each unit of a cross-sectional area.

The **direct stress** is the intensity of force distribution over the cross-sectional area of the material subjected to direct load. Direct stress can be defined as the share of the total axial load carried by each unit of cross-sectional area:

$$\sigma = \frac{F}{A}$$

where:
σ = direct axial stress
A = cross-sectional area
F = the axial force.

The axial force may be assumed to be distributed uniformly over the cross-section when the object is relatively long and uniformly shaped, and loaded in pure tension.

GIVE FEEDBACK

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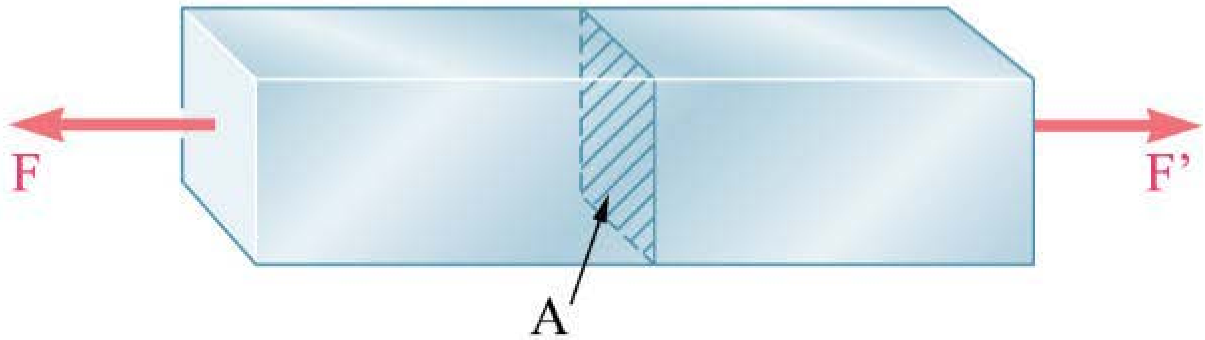
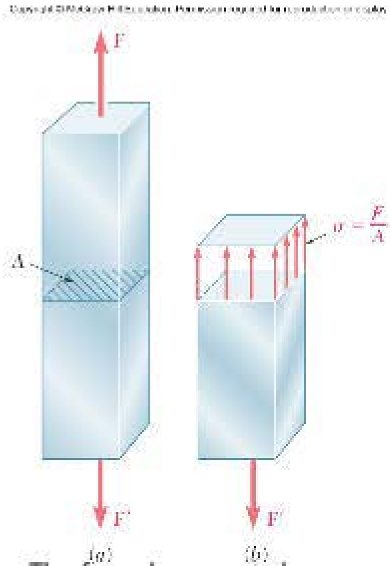


Stress is the intensity of force through a cross-section



The force is averaged over the cross-section

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For direct axial stress, it may be assumed that the force ______.

Click the correct answer.

- is distributed uniformly over the cross-section
- is too small to measure
- is high enough to break the specimen
- is twice as high as the load on the specimen

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

If a bar of mild steel, $20\ \text{mm} \times 10\ \text{mm}$ in cross-section, is subjected to a tensile force of $18.8\ \text{kN}$, determine the stress in the material.

Use the standard measurements, N and mm, to give MPa:

$$18.8\ \text{kN} = 18.8 \times 1{,}000 = 18{,}800\ \text{N}$$

GIVE FEEDBACK

CONTINUE >

Determine the direct stress to a material—Example

Example

If a bar of mild steel, $20\ \text{mm} \times 10\ \text{mm}$ in cross-section, is subjected to a tensile force of $18.8\ \text{kN}$, determine the stress in the material.

Use the standard measurements, N and mm, to give MPa:

$$18.8\ \text{kN} = 18.8 \times 1{,}000 = 18{,}800\ \text{N}$$

Solution

$$\text{Stress:}\quad \sigma = \frac{F}{A}$$

$$= \frac{18{,}800\ \text{N}}{20\ \text{mm} \times 10\ \text{mm}}$$

$$= 94\ \text{MPa}$$

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Type your answer in the box.

A rectangular steel bar is $8\ \mathrm{mm}$ by $10\ \mathrm{mm}$ thick and $1{,}350\ \mathrm{mm}$ long.
What is the stress in the bar when the tensile load is $17\ \mathrm{kN}$?

Stress = ______ / ______

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Find the stress (in MPa) of a round steel bar of diameter 38 mm with a tensile force of 115 kN applied. (Include units, use two decimal places.)

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
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Hint

Each hint will reduce the credit received for this question

Factor of safety

We often need to compare the direct stress we have calculated (e.g. 94 MPa) with the ultimate tensile strength of the material (e.g. the UTS of mild steel is 470 MPa).

This ratio is an indication of the degree of safety built into the design.

Factor of safety (FS) works exactly the same way whether the stress is tensile, compressive or shear.

Find the Factor of Safety (FS) when the ultimate tensile strength (UTS) is 470 MPa and the actual working stress σ is 94 MPa.

GIVE FEEDBACK

OK

Which of the following statements about factor of safety (FS) are correct?

Check all that apply.

- [ ] The higher the FS, the safer the design
- [ ] The lower the FS, the safer the design
- [ ] FS = UTS/working stress
- [ ] The equation for FS works the same for tensile, compressive and shear stresses

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Factors of safety for common materials

The factor of safety (FS) is a dimensionless number always greater than one.

The FS needs to be higher (safer) if the design includes:

- possible defects in materials or workmanship
- loads that are not known with accuracy
- shock or impact loads
- risks to human life or property
- allowance for decay, wear, corrosion, etc.
- a long service life

The FS tends to be lower when the product is designed to be:

- as light as possible (sports and racing equipment, aircraft)
- as cheap as possible, usually saving on material costs (mass-produced items)
- carefully maintained or with a short life expentancy
- very large or expensive and so must be carefully analysed (large ship, bridge)

GIVE FEEDBACK

CONTINUE >

Typical factors of safety used in design practice are given in the simplified table below:

Material	FS (static loading)	FS (cyclic loading)
Steel and ductile materials	3 to 4	8
Cast iron and brittle materials	5 to 6	10 to 12
Timber	7	15

< BACK GIVE FEEDBACK OK

Match the following design requirements against whether they tend to increase or lower the possible factor of safety (FS).

Drag statements on the right to match the left.

As light as possible (sports and racing equipment, aircraft)	Tend to raise the FS
Possible defects in materials or quality of work	Tend to raise the FS
Possibility of shock or impact loads	Tend to raise the FS
Risks to human life or property	Tend to lower the FS
Long service life	Tend to raise the FS
Carefully maintained or with a short life expectancy	Tend to lower the FS
Large or expensive and so carefully analysed	Tend to lower the FS
As lightweight as possible	Tend to lower the FS

Design stress or allowable stress

Working stress is the stress calculated in any given situation. The allowable stress is the maximum working stress the designer will permit the component to undergo. Allowable stress is also called design stress.

When designing a component, the question arises: what is a suitable cross-section to handle a certain load at the recommended safety factor? We need to know how much stress to allow in the component. This stress is called the **design stress** or **allowable stress**:

$$\sigma_a = \frac{UTS}{FS}$$

This is the maximum stress the designer will allow the component to endure.

Working stress: $\sigma = \frac{F}{A}$

This is the stress calculated in any given condition (not necessarily the worst case). There's nothing particular about this type of stress, so no subscript is required.

GIVE FEEDBACK

OK

Match the definitions of each of the following types of stress.

Drag statements on the right to match the left.

Allowable stress	$\sigma = \frac{UTS}{FS}$: The maximum stress the designer will allow the component to endure
Working stress	$\sigma = \frac{F}{A}$: The stress calculated in any given condition
Ultimate stress	$\sigma = \frac{F_{max}}{A}$: The highest stress the material can withstand before breaking

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Another name for allowable stress is:

Click the correct answer.

- Design stress
- Ultimate stress
- Working stress

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

Determine the minimum required diameter of a high-tensile steel rod to carry a tensile load of $26\ \mathrm{kN}$ with a safety factor of 3.5.

Example

Determine the minimum required diameter of a high-tensile steel rod to carry a tensile load of 26 kN with a safety factor of 3.5.

Solution

From $FS = \frac{UTS}{\sigma_a}$, allowable stress is:

$$\sigma_a = \frac{UTS}{FS}$$

$$= \frac{590\ N/mm^2}{3.5}$$

$$= 168.6\ MPa$$

< BACK GIVE FEEDBACK CONTINUE >

Now, from $\sigma_a = \frac{F}{A}$, the area required is:

$$A = \frac{F}{\sigma_a}$$
$$= \frac{26{,}000\ \text{N}}{168.6\ \text{MPa}}$$
$$= 154.2\ \text{mm}^2$$

But area A = $\frac{\pi D^2}{4}$ = 154.2 mm^2, hence:

$$D = \sqrt{\frac{154.2 \times 4}{\pi}}$$
$$= 14\ \text{mm}$$

< BACK GIVE FEEDBACK OK

Find the allowable stress for a component where ultimate tensile stress is 350 MPa and the factor of safety is 4. (Include units, use 2 decimal places.)

± ▾ $\frac{\square}{\square}$ $1\frac{2}{3}$ $\square^2$ $\sqrt{\square}$ $(\square)$ ▾ Clear

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≤ ▾ π MPa f(x) ▾ σ ↵ ? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
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Hint

Each hint will reduce the credit received for this question

Type your answer in the box.

A steel bar must have a factor of safety of 5. If the UTS is 600 MPa, find the allowable stress. Fill in the correct values (numbers) in the equation below:

Allowable stress = ____ / ____

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Find the necessary UTS for a component that reaches a maximum stress of 100 MPa and must have a factor of safety of 4. Fill in the correct values (numbers) in the equation below.

UTS = [ ] x [ ]

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Did you know that you can bend a bridge by walking on it? Every object deforms under stress, even by a small amount. To find out how serious the deformation is, engineers need to compare it to the original length. This ratio is called strain.

Define axial strain

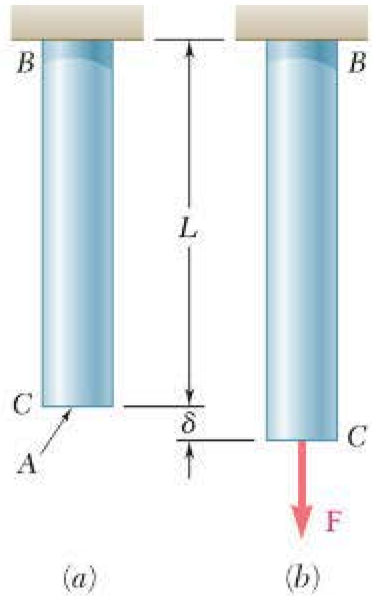
Every material under stress will **deform.**

The proportion of this deformation is called **strain**, ε, which is defined as:

$$\varepsilon = \frac{\delta}{L}$$

where:

ε = axial strain (no units)
δ = elongation or deformation (mm)
L = original length (mm) in fig (a)



Axial strain is the proportion of stretch

GIVE FEEDBACK

OK

Match the terms used to calculate axial strain to their correct units.

Drag statements on the right to match the left.

Axial strain	mm/mm
Elongation or deformation	mm
Original length	mm

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the variables used to calculate axial strain.

Drag statements on the right to match the left.

Axial strain	ε
Elongation or deformation	δ
Original length	L

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Since strain is the ratio between two lengths, it is dimensionless, provided both are the same unit, typically mm. Sometimes it is quoted as mm per mm, which is still dimensionless, but it does show how it is calculated. Since metals deform only a very small proportion, the value for strain will be very small, a fraction of 1%. Strain is often quoted as a percentage.

Identify the formula for axial strain

If the total elongation is δ, and the original length is L, then the axial strain ε is:

$$\varepsilon = \frac{\delta}{L}$$

- Since strain is the ratio between two lengths, it is **dimensionless**, provided that both δ and L are in the same units (usually millimetres). It can also be written as *mm/mm*.
- Most engineering materials deform only a very small proportion. Typical maximum strains, before damage occurs, would be: steel ε = 0.002; ceramic ε = 0.001; nylon ε = 0.05.
- For a larger strain, like ε = 2, the object will be three times as long as it was to start with. This would have to be something like rubber.
- Strain is often quoted as a percentage. To convert to percentage, simply multiply by 100.

GIVE FEEDBACK

OK

The units for strain can be written as:

Check all that apply.

- [ ] Dimensionless, i.e. there are no units
- [ ] mm/mm
- [ ] m/m
- [ ] N/mm
- [ ] MPa/mm
- [ ] MPa

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match typical maximum strains for the following materials:

Drag statements on the right to match the left.

Steel	$\varepsilon = 0.002$
Rubber	$\varepsilon = 2$
Ceramic	$\varepsilon = 0.001$
Nylon	$\varepsilon = 0.05$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate axial strain—Example

Example

If a mild steel bar is 2.7 m long and extends 1.27 mm under the load, what is the axial strain?

GIVE FEEDBACK

CONTINUE >

Calculate axial strain—Example

Example

If a mild steel bar is $2.7\ \mathrm{m}$ long and extends $1.27\ \mathrm{mm}$ under the load, what is the axial strain?

Solution

$$\text{Strain } \varepsilon = \frac{\delta}{L}$$

$$= \frac{1.27\ \mathrm{mm}}{2{,}700\ \mathrm{mm}}$$

$$= 0.47 \times 10^{-3} \text{ (or 0.47 per 1000)}$$

$$= 0.00047 \text{ (or 0.047\%)}$$

< BACK GIVE FEEDBACK OK

A nitrile rubber tube is 3.5 m long and stretches 400 mm under load. Calculate the axial strain. (Round to minimum 3 decimal places.)

Clear

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Type your answer in the box.

A mild steel bar is 2.7 m long and extends 1.27 mm under the load. What is the axial strain? Fill in the correct values (numbers) in the equation below:

Axial strain = [ ] / [ ]

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The load–extension diagram recorded during tensile testing shows that whenever a force is applied, there will be some elongation. If we convert the load to stress, and extension to strain, we can create the stress–strain diagram, which is one of the most important diagrams for an engineering material.

The elastic limit

All solid materials exhibit some degree of **stiffness**, which is the ability to resist deformation under load.

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GIVE FEEDBACK

CONTINUE >

Every load-extension diagram recorded during tensile testing shows that whenever a force is applied, there will be some elongation:

- The greater the force, the greater the elongation
- Stiffness is only measured where the material behaves elastically
- Up to a certain point, known as the elastic limit, the material will return to its original size after the force has been removed (elastic behaviour)

< BACK GIVE FEEDBACK OK

An object will begin to stretch when:

Check all that apply.

- [ ] Any tensile force is applied, no matter how small
- [ ] A tensile force is applied that takes it over the elastic limit
- [ ] Any tensile stress is applied
- [ ] A tensile stress over the elastic limit is applied

Do you know the answer?

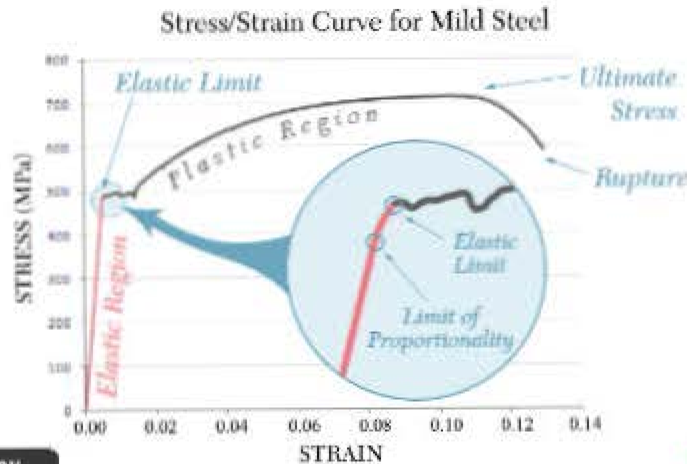
I KNOW IT | THINK SO | UNSURE | NO IDEA

Plasticity is the ability of a material to stay deformed after the force has been removed. A plastic material is one that does not spring back after the load is removed. Soft clay behaves as an almost perfectly plastic material. Soft copper wire is also very plastic. If you bend copper wire then let it go, it will stay bent in almost the same position, with only a small amount of of springback (elasticity).

Define plasticity

Plasticity is the ability of a material to stay deformed after the force has been removed.

- Once the material is stretched to its elastic limit, further stretching will cause permanent deformation (plastic region).
- The force may continue to increase, but at a reduced rate.
- Eventually it will reach the ultimate stress, after which rupture will occur.



GIVE FEEDBACK

OK

For a ductile material like mild steel, which of the following options correctly finishes the sentence? Once it is stretched to its elastic limit, the next thing to happen is that ______.

Click the correct answer.

- further stretching will cause permanent deformation
- rupture will occur
- the force will halve
- the force will increase more rapidly

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The limit of proportionality

Besides the elastic limit, there is another limit called the limit of proportionality. It is the end of the straight line of the elastic region. For metals, the limit of proportionality is usually so close to the elastic limit that we treat them as the same thing.

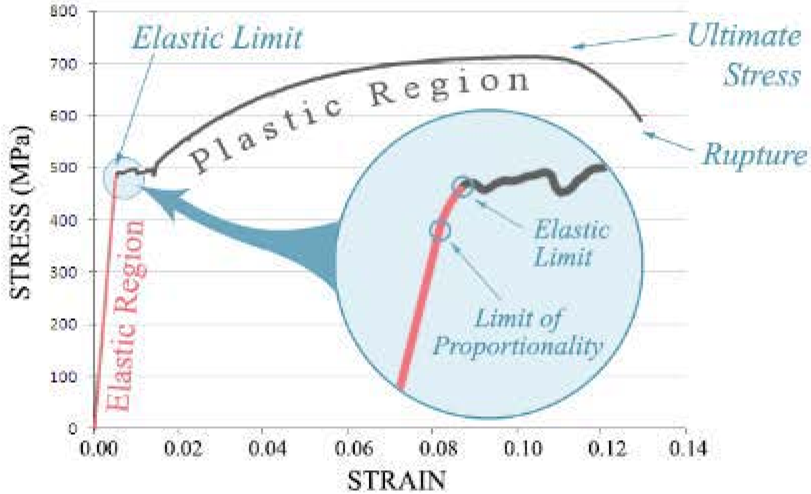
With closer observation we find there is another limit, known as the **limit of proportionality**, up to which the elongation is directly proportional to the force applied.

- For most materials, particularly metals, the elastic limit and the limit of proportionality almost coincide, often known as the 'proportional elastic limit'.
- For other materials, especially polymers, the straight line (proportionality) begins to curve well before the elastic limit is reached.
- For some materials, notably rubbers, there is hardly any straight line at all, even though it is elastic. So the limit of proportionality is far from the elastic limit.

GIVE FEEDBACK

CONTINUE >

Stress/Strain Curve for Mild Steel



Proportionality limit and elastic limit for mild steel

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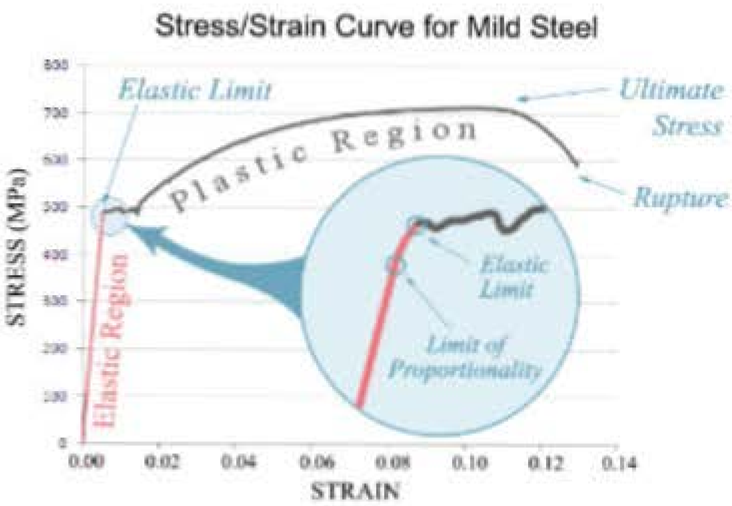
Examine the graph and identify which of the following statements are true.



Check all that apply.

- [ ] The end of the straight line is called the limit of proportionality
- [ ] The end of the straight line is called the elastic limit
- [ ] For metals, the proportionality and elastic limits are regarded as the same
- [ ] For polymers and rubbers, the proportionality and elastic limits are regarded as the

Hooke's law states that for an elastic material, the elongation is directly proportional to the deforming force. Double the force and you double the stretch.

The formula for Hooke's law

Hooke's law states that for a bar of elastic material, the elongation is directly proportional to the deforming force. This is only true up to the limit of proportionality (straight line elasticity).

Mathematically, Hooke's law says:

$$k = \frac{F}{\delta}$$

where:

k = stiffness (N/mm)
F = force (N)
δ = elongation (mm)

GIVE FEEDBACK

OK

Match the units used in the following formula:

$$k = \frac{F}{\delta}$$

Drag statements on the right to match the left.

k	(N/mm)
δ	(mm)
F	N
Not used	(GPa)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

What is represented by each of the symbols in the formula:

$$k = \frac{F}{\delta}$$

Match the symbols to their definitions.

Drag statements on the right to match the left.

k	Stiffness
F	Force
δ	Elongation

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The modulus of elasticity

The modulus of elasticity is the ratio of stress divided by strain. It is a measure of the stiffness of a material, determined in the elastic range.

Hooke's law can be generalised for all situations by using strain instead of elongation and stress instead of force.

This is called the modulus of elasticity: $E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$

where:
E = modulus of elasticty E (MPa)
σ = stress (MPa)
ε = strain ( )

GIVE FEEDBACK

CONTINUE >

The slope of a line is vertical rise/horizontal run. On the stress-strain diagram, the slope of the straight-line elastic region is vertical (axial stress)/horizontal (axial strain), which equals E.

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Match the units used in the following formula:

$$E = \frac{\sigma}{\varepsilon}$$

Drag statements on the right to match the left.

E	(MPa)
σ	(MPa)
ε	(no units)
Not used in this formula	(mm)
Not used in this formula	(GPa)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the terms used in the following formula:

$$E = \frac{\sigma}{\varepsilon}$$

Drag statements on the right to match the left.

E	Modulus of elasticity
σ	Stress
ε	Strain

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Fill in the correct terms (in words) for the equation definition of the modulus of elasticty:

E = ________ / ________

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The modulus of elasticity has lots of other names: Young's modulus, stiffness modulus or just modulus. But it always has the symbol E, so we will use modulus of elasticity. This also reminds us that we only calculate it in the elastic region of the stress–strain diagram.

Other names for modulus of elasticity

The modulus of elasticity (E) has lots of names:

- Modulus of elasticity—we will mostly use this name because it reminds us that it only applies in the elastic region, as well as starting with the letter E
- Young's modulus—after Thomas Young (1773–1829), who came up with the concept of using the stress-strain ratio as a measure of elastic stiffness for many materials
- Modulus of stiffness—another good name because it directly relates to Hooke's law where stiffness equals force proportional to extension
- Modulus—almost exclusively refers to modulus of elasticity

GIVE FEEDBACK

OK

The property E has lots of names. What are they?

Check all that apply.

- [ ] Modulus of elasticity
- [ ] Young's modulus
- [ ] Modulus of stiffness
- [ ] Modulus
- [ ] Hooke's modulus
- [ ] Elastomer

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The modulus of elasticity for common materials

The modulus of elasticity can be determined experimentally. Typical values for common engineering materials are given below. Note that E is usually given in GPa.

Modulus of elasticity (E)

Material	E (MPa)	E (GPa)
Aluminium	70 000	70
Brass	90 000	90
Bronze	105 000	105
Cast iron	120 000	120
Copper	112 000	112
Steel	200 000	200
Timber	12 000	12

GIVE FEEDBACK

OK

Match the modulus of elasticity (in GPa) for each material below.

Drag statements on the right to match the left.

Aluminium	70 GPa
Brass	90 GPa
Cast iron	120 GPa
Steel	200 GPa
Timber	12 GPa

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the modulus of elasticity—Example

Example

The stress in a mild steel bar, 20 mm x 10 mm, is $94\ \mathrm{MPa}$ and the corresponding strain is 0.47×10^{-3}. Calculate the modulus of elasticity. Check the answer against Modulus values.

GIVE FEEDBACK

CONTINUE >

Calculate the modulus of elasticity—Example

Example

The stress in a mild steel bar, 20 mm x 10 mm, is $94\ \mathrm{MPa}$ and the corresponding strain is 0.47×10^{-3}. Calculate the modulus of elasticity. Check the answer against Modulus values.

Solution

$$
\begin{aligned}
E &= \frac{\sigma}{\varepsilon} \\
&= \frac{94\ \mathrm{MPa}}{0.47 \times 10^{-3}} \\
&= 200{,}000\ \mathrm{MPa} \\
&= 200\ \mathrm{GPa}
\end{aligned}
$$

< BACK GIVE FEEDBACK OK

The stress in a mild steel bar, 20 mm x 10 mm, is 85 MPa and the corresponding strain is 0.4×10^{-3}. Calculate the modulus of elasticity in MPa. (Round off to nearest integer.)

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(□) ≤ π MPa f(x) ↵ Clear line ? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
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Hint

Each hint will reduce the credit received for this question

Type your answer in the box.

The stress in a nylon cable is 40 MPa. This stretches it to produce a strain of 13 $\times 10^{-3}$. Enter the correct numbers in the equation below to calculate the modulus of elasticity:

E = ______ / ______

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

We can calculate the modulus of elasticity directly from the specimen measurements of force, length, elongation and cross-sectional area.

The formula for direct calculation of E

Combining the formulas for stress and strain, we can find E directly:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\delta/L} = \frac{F \cdot L}{A \cdot \delta}$$

where:

E = Young's modulus of elasticity (MPa)
F = axial force (N)
A = cross-sectional area (mm^2)
δ = elongation (mm)
L = original length (mm)
σ = stress (MPa)
ε = strain, dimensionless ( ), or can be written as (mm/mm)

GIVE FEEDBACK

OK

Match the names of the terms used in the combined equation shown:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\delta/L} = \frac{F \cdot L}{A \cdot \delta}$$

Drag statements on the right to match the left.

E	Modulus of elasticity
F	Axial force
A	Cross-sectional area
δ	Elongation
L	Original length
σ	Stress
ε	Strain

Do you know the answer?

Match the units used in the combined equation shown:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\delta/L} = \frac{F \cdot L}{A \cdot \delta}$$

Drag statements on the right to match the left.

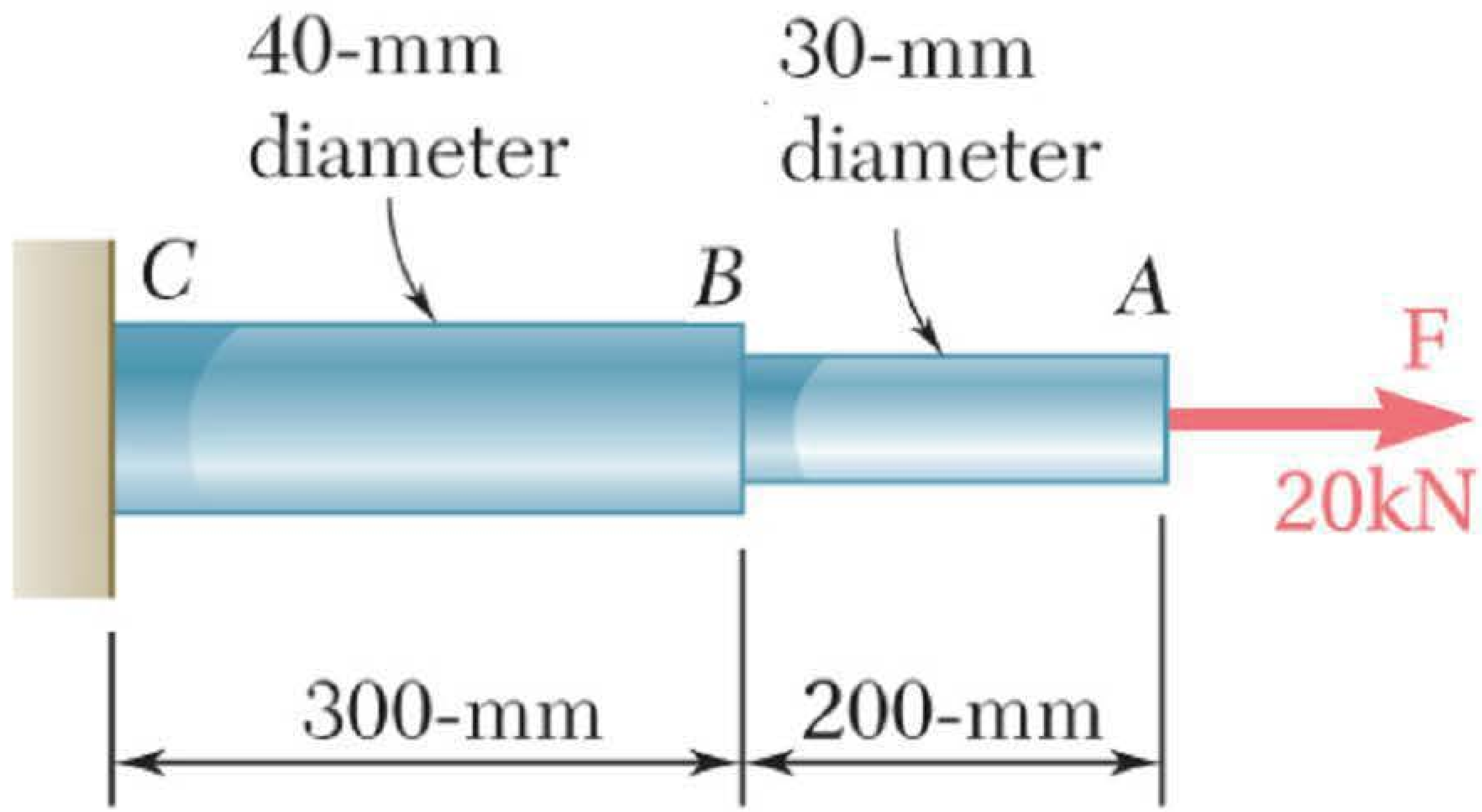
E	(MPa)
F	(N)
A	(mm^2)
δ	(mm)
L	(mm)
σ	(MPa)
ε	(no units)

Determine total extension and safety factor

A 40 mm diameter aluminium rod, 500 mm long, is turned down to a diameter of 30 mm over 200 mm of its length. The rod is then subjected to a 20 kN axial pull.

The ultimate tensile strength of aluminium is 150 MPa and Young's modulus is 70 GPa.

Determine the total amount of elongation and the safety factor.



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

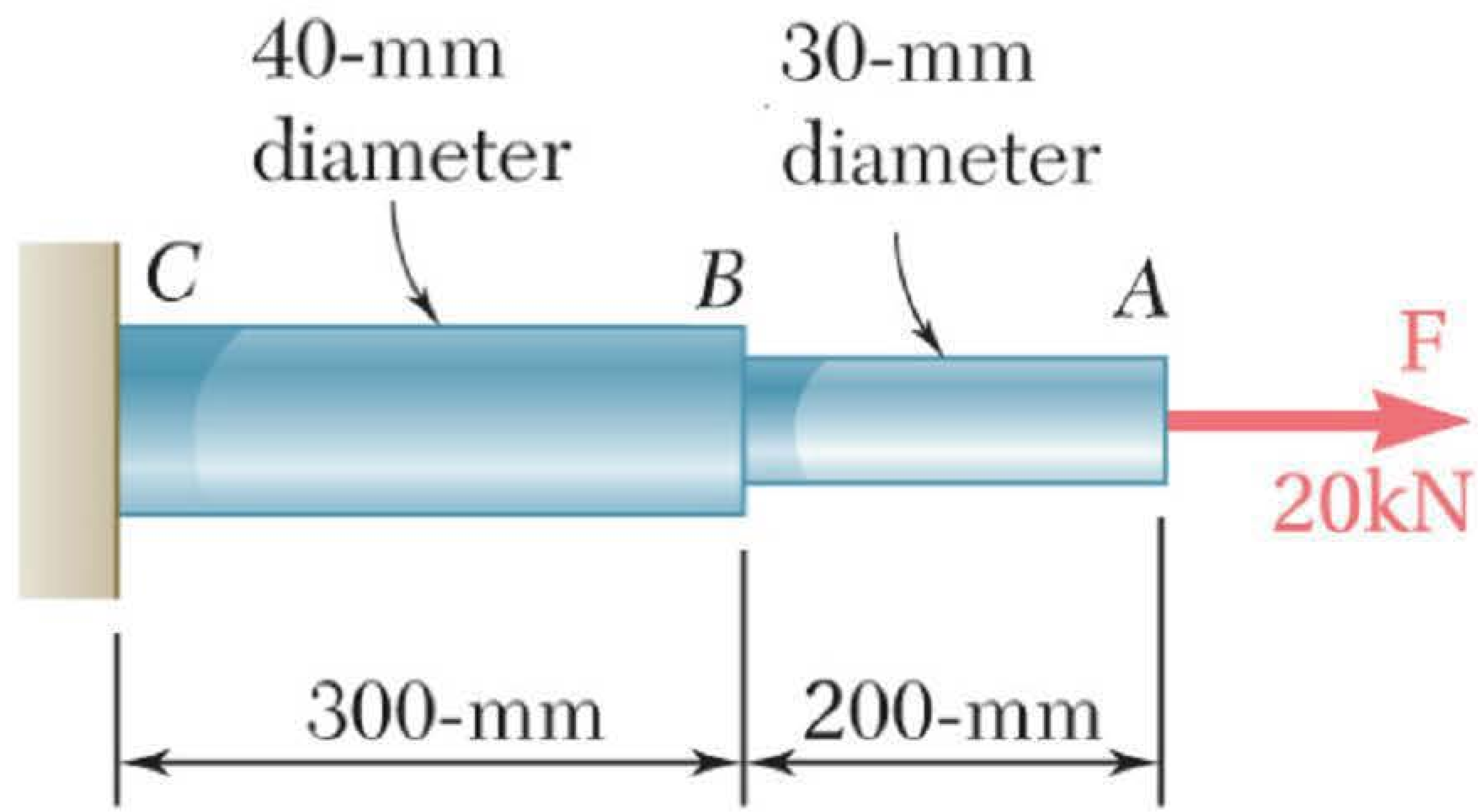
OK

Determine total extension and safety factor

Calculate stress in the Φ40 mm x 300 mm section:

Stress:

$$\sigma_1 = \frac{F}{A_1} = \frac{20{,}000\ \text{N}}{\frac{\pi \cdot 40^2}{4}\ \text{mm}^2} = 15.92\ \text{MPa}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

OK

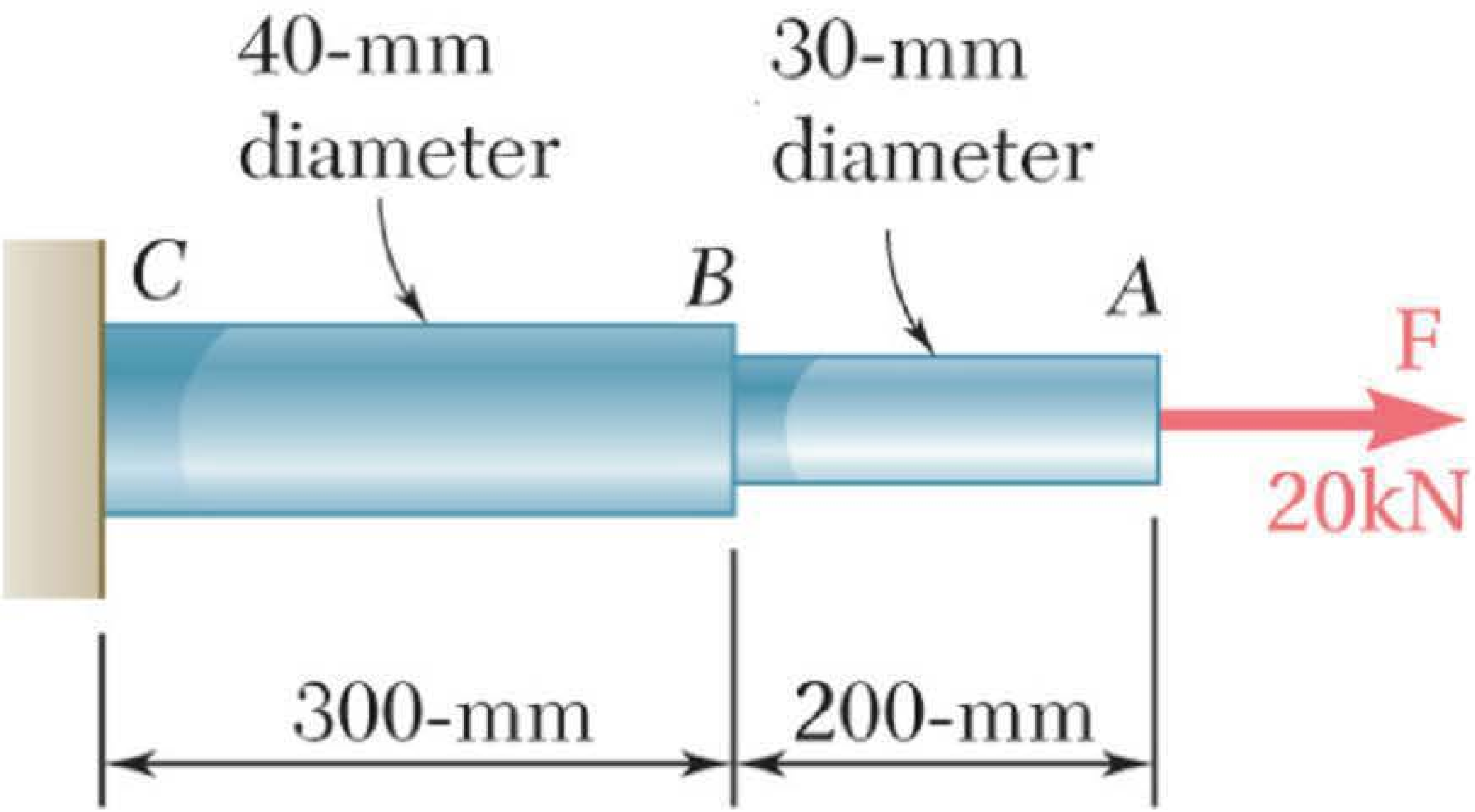
Determine total extension and safety factor

Calculate strain for the Φ40 mm x 300 mm section:

$$\varepsilon_1 = \frac{f_1}{E}$$

$$= \frac{15.92}{70{,}000}$$

$$= 0.227 \times 10^{-3}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

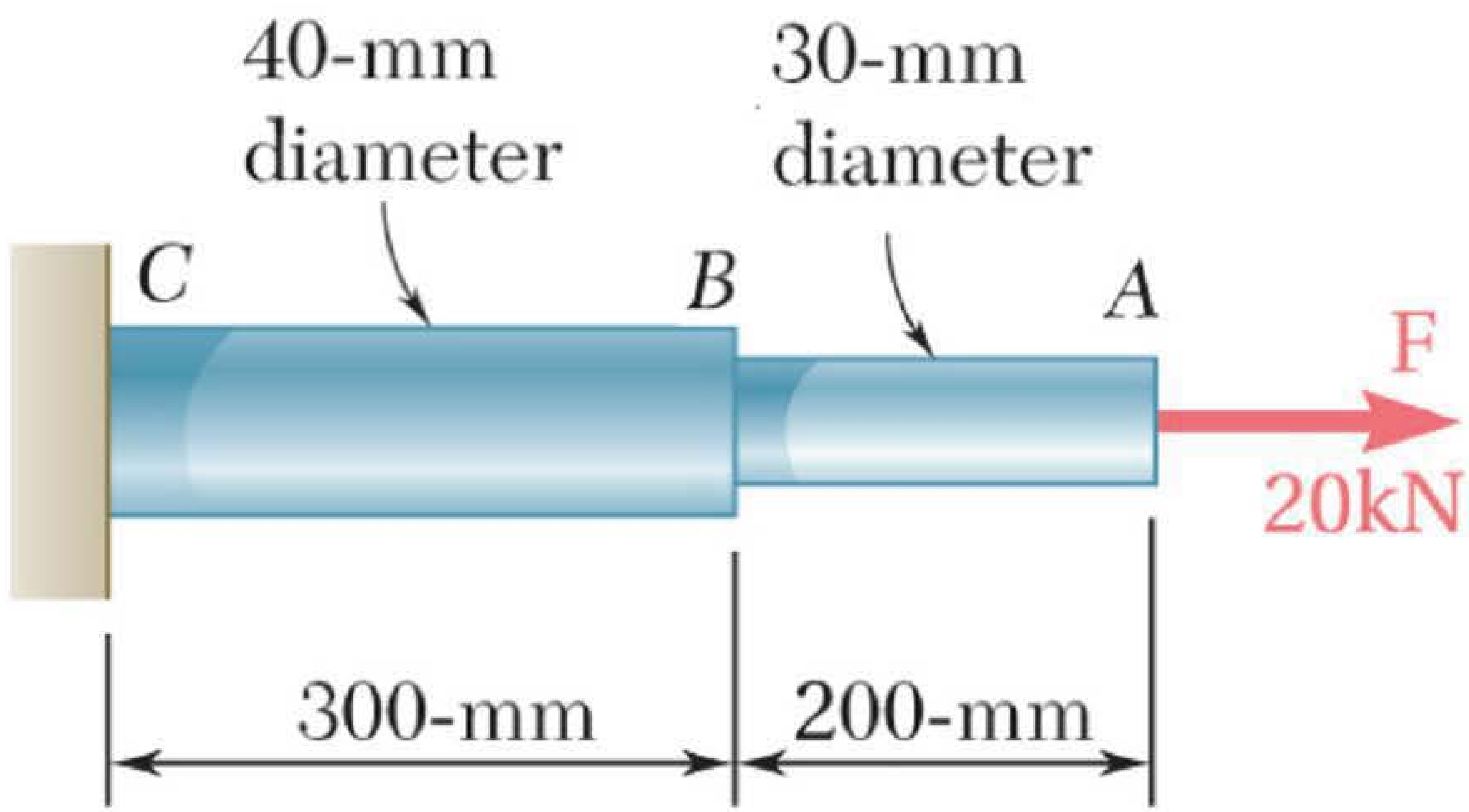
GIVE FEEDBACK

OK

Determine total extension and safety factor

Calculate the elongation for the Φ40 x 300 mm section:

$$\delta_1 = \varepsilon_1 L_1$$
$$= 0.227 \times 10^{-3} \cdot 300 \text{ mm}$$
$$= 0.0682 \text{ mm}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

OK

Determine total extension and safety factor

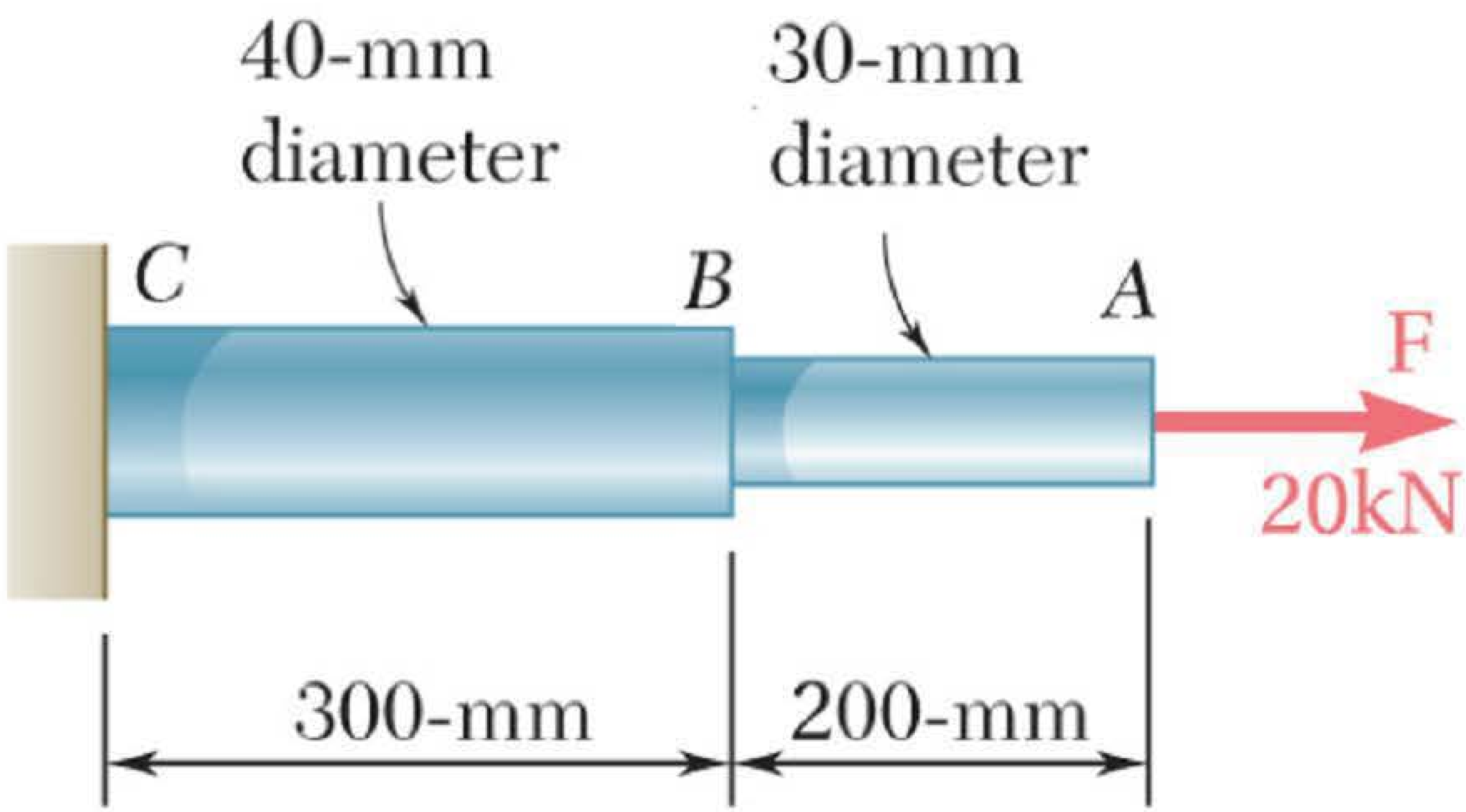
Find stress in the 30 mm diameter section of length 200 mm:

Stress:

$$\sigma_2 = \frac{F}{A_2}$$

$$= \frac{20{,}000\ \text{N}}{\frac{\pi \cdot 30^2}{4}\ \text{mm}^2}$$

$$= 28.29\ \text{MPa}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

OK

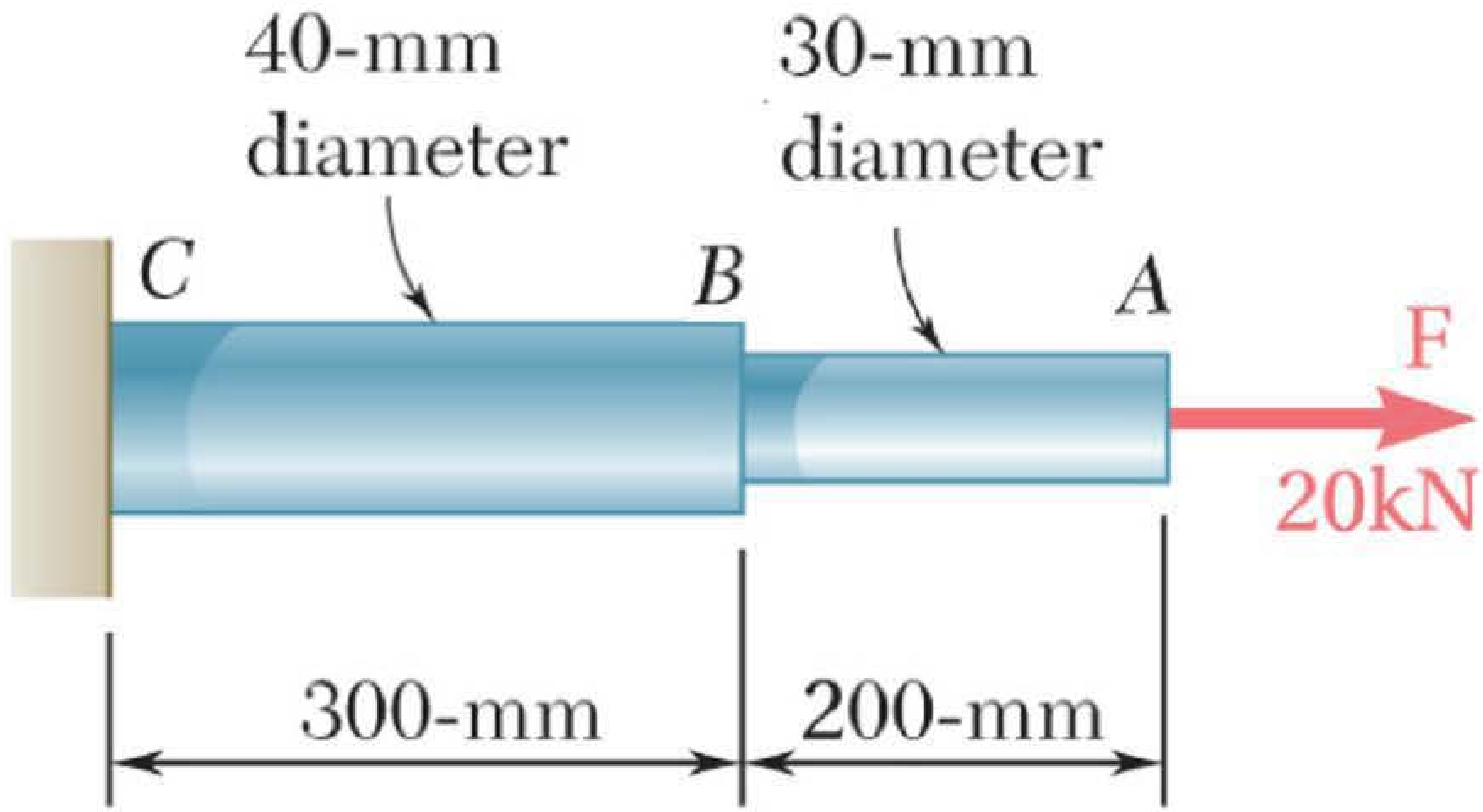
Determine total extension and safety factor

Now find strain in the Φ30 x 200 mm section:

$$\varepsilon_2 = \frac{\sigma_2}{E}$$

$$= \frac{28.29}{70{,}000}$$

$$= 0.404 \times 10^{-3}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

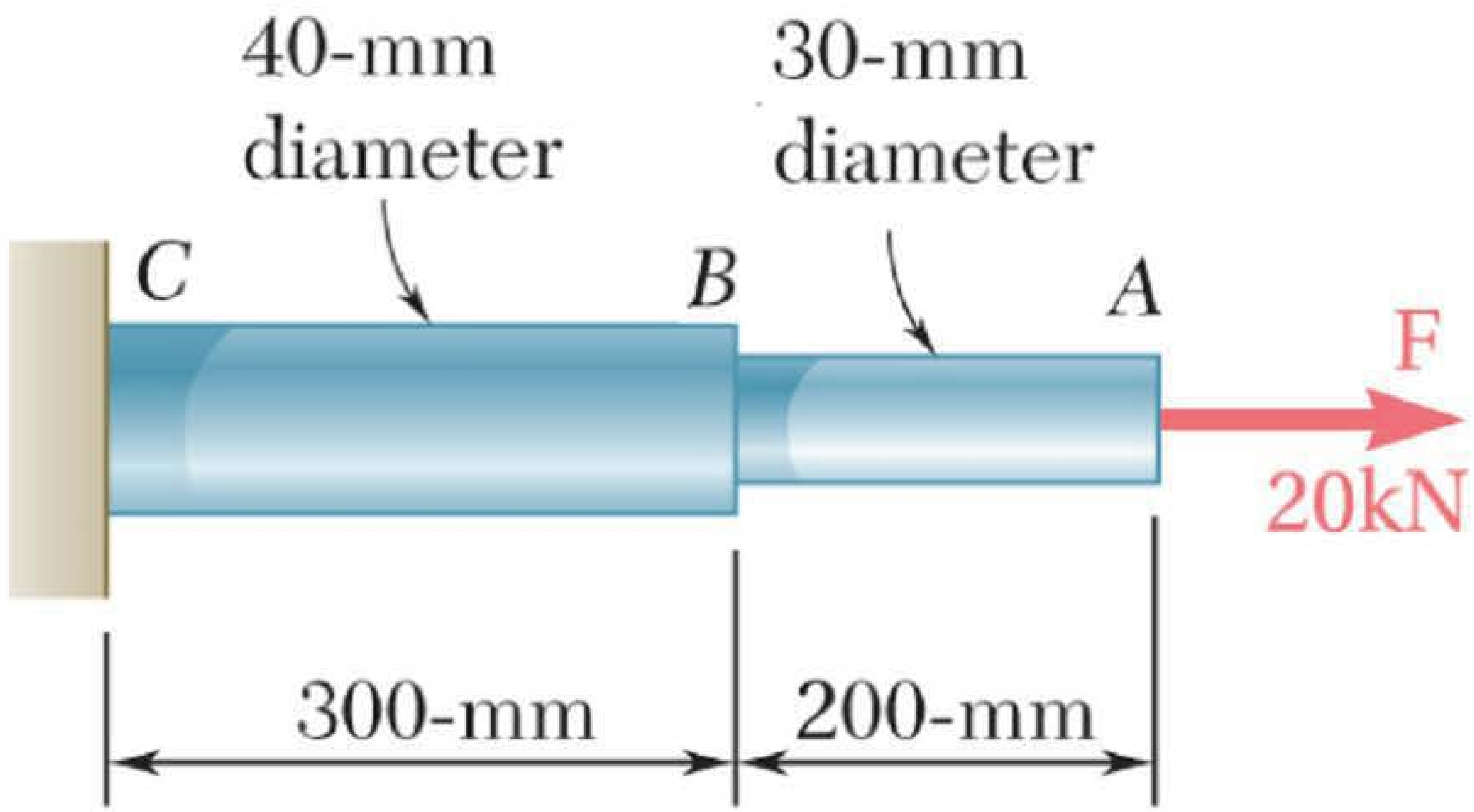
OK

Determine total extension and safety factor

The total elongation of the rod is made up of the extensions of the two parts.

Total elongation:

$$\delta = \delta_1 + \delta_2$$
$$= 0.0682\ \text{mm} + 0.0808\ \text{mm}$$
$$= 0.149\ \text{mm}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

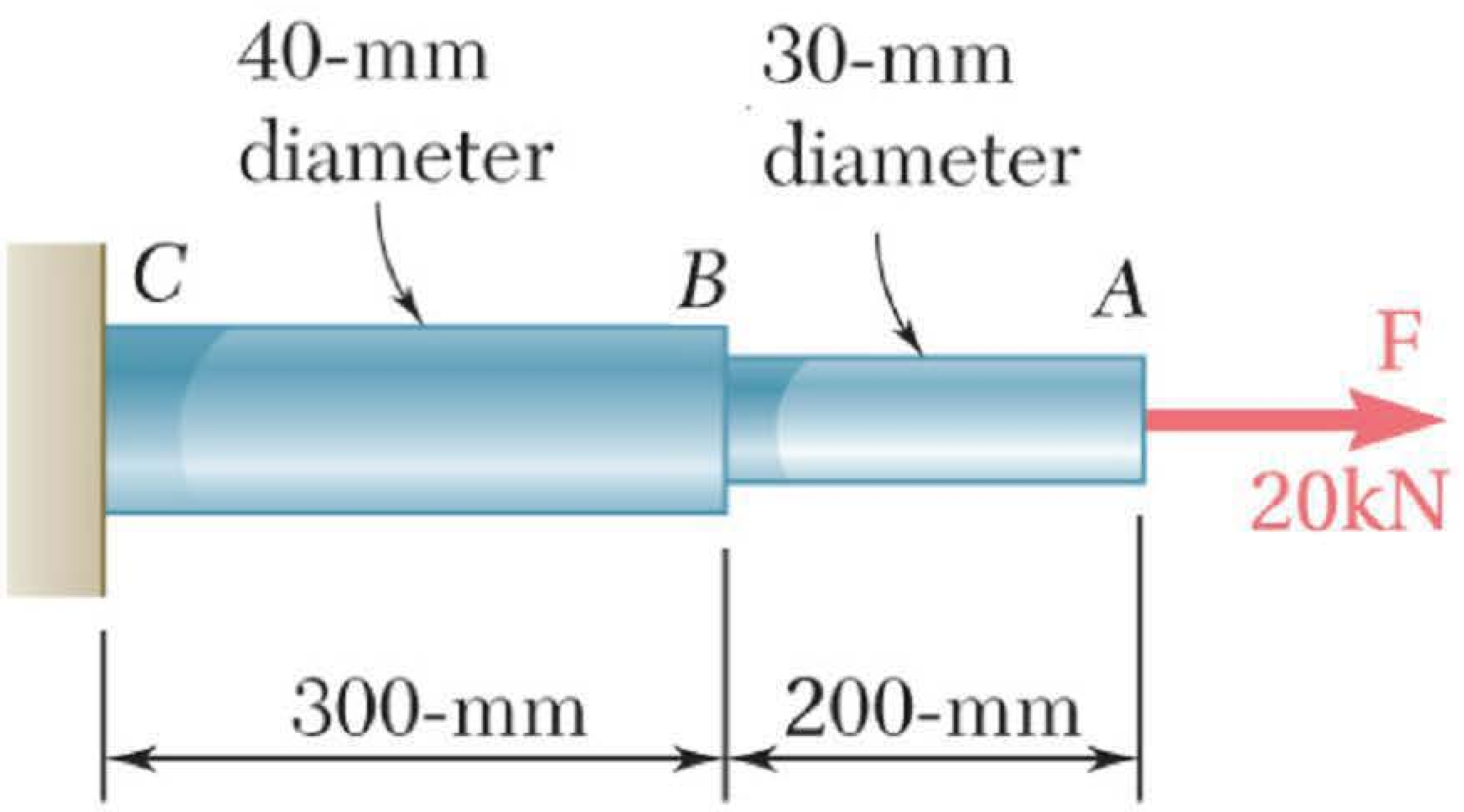
GIVE FEEDBACK

OK

Determine total extension and safety factor

Calculate elongation in the Φ30 mm section:

$$\delta_2 = \varepsilon_2 L_2$$
$$= 0.404 \times 10^{-3} \cdot 200\ \text{mm}$$
$$= 0.0808\ \text{mm}$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

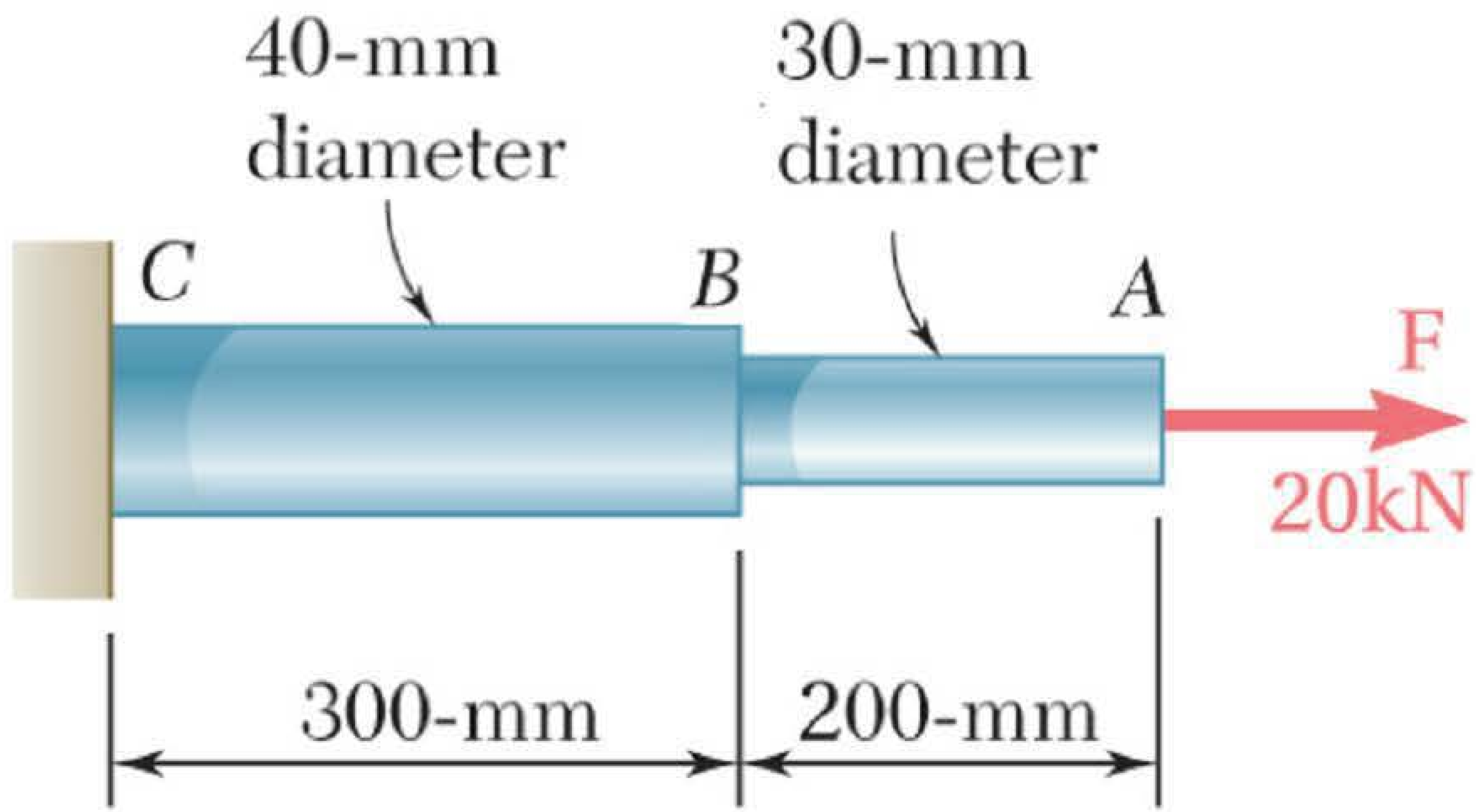
OK

Determine total extension and safety factor

The safety factor must be based on the maximum stress in the material, which occurs in the Φ30 mm section and is equal to 28.29 MPa.

Safety factor:

$$FS = \frac{UTS}{\sigma} = \frac{150}{28.29} = 5.3$$



Determining safety factor—Example	Stress in the 40 mm section	Strain in 40 mm section	Elongation of 40 mm section	Stress in the 30 mm section	Strain in the 30 mm rod	Total elongation	Elongation of the 30 mm rod	Safety factor

GIVE FEEDBACK

OK

A steel rod has a diameter of 35 mm and length 4000 mm. It is under a tensile force of 150 kN. By how much will it stretch (in mm)? Use E = 200 GPa. *(Minimum 2 decimal places, include units)*

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Any problem involving stress and strain can be solved using this simple method. Write down the three equations for stress, strain and modulus. Next, fill in every variable that is given in the question. Now solve whichever equation has two of the three variables. Use this information to solve the next equation and again for the last of the three equations. It works every time.

Solve any problem using the three equations for stress, strain and modulus

There are three fundamental equations used in strength of materials, with seven terms:

Stress: $\sigma = \frac{F}{A}$ Strain: $\varepsilon = \frac{e}{l}$ Modulus: $E = \frac{\sigma}{\varepsilon}$

To solve any problem involving stress and strain on an object:

1. Determine (from the question) as many of these seven terms as possible
2. Once you have any two terms in any equation, that equation can be solved
3. Use the new term from this equation to help solve another one

Repeat this process until you find the term you are after (or you find all seven terms).

This method will work for any solvable problem.

Hint: In most cases the material is known, e.g. steel. This means the modulus is known, e.g. E = 200 GPa.

GIVE FEEDBACK

OK

A stainless steel (E = 195 GPa) guitar string of diameter 0.3 mm and length 855 mm is tensioned to 48 N. Calculate the amount of stretch (elongation). Sort the following formulas into the order in which they must be calculated while solving this question.

Stress: $\sigma = \frac{F}{A}$ Strain: $\varepsilon = \frac{\delta}{L}$ Modulus: $E = \frac{\sigma}{\varepsilon}$ Area $A = \frac{\pi \cdot D^2}{4}$

↑↓ Place these in the proper order.

- Find area $A = \frac{\pi \cdot D^2}{4}$
- Find stress: $\sigma = \frac{F}{A}$
- Find strain using: $E = \frac{\sigma}{\varepsilon}$
- Find elongation using: $\varepsilon = \frac{\delta}{L}$

Do you know the answer?

An aluminium bolt is 12 mm diameter and 250 mm long. If it is stretched by 0.6 mm, how much force was on the bolt? Sort the following formulas into the order in which they must be calculated while solving this question.

Stress: $\sigma = \frac{F}{A}$ Strain: $\varepsilon = \frac{\delta}{L}$ Modulus: $E = \frac{\sigma}{\varepsilon}$ Area $A = \frac{\pi \cdot D^2}{4}$

Hint: Find the area first, then sort the three equations for stress, strain and modulus.

↑↓ **Place these in the proper order.**

Find Area $A = \frac{\pi \cdot D^2}{4}$

Find strain using: $\varepsilon = \frac{\delta}{L}$

Find stress using: $E = \frac{\sigma}{\varepsilon}$

Find force using: $\sigma = \frac{F}{A}$

Do you know the answer?

A stainless steel (E = 195 GPa) guitar string of diameter 0.3 mm and length 840 mm is tensioned to 45 N. Calculate the amount of stretch. Which variables can be derived from the question before using any of the three formulas?

Stress: $\sigma = \frac{F}{A}$ Strain: $\varepsilon = \frac{\delta}{L}$ Modulus: $E = \frac{\sigma}{\varepsilon}$

Check all that apply.

- [ ] E
- [ ] L
- [ ] A
- [ ] F
- [ ] σ
- [ ] ε
- [ ] δ

Do you know the answer?