

GIVE FEEDBACK

CONTINUE >

Circular motion covers the motion of bodies in a circular path.

Circular motion is related to both rotational and linear motion. The body moves at the end of a radius, which rotates around the centre of the circular path and the path along which the body moves is a line, albeit a curved one.

Examples of circular motion include a car travelling around a bend in the road, a person running around a circular track and the earth moving around the sun.

An understanding of circular motion is necessary for the solution of problems such as the stability of vehicles on curved roads, the banking of roads and railway tracks and compensation for out-of-balance components.

< BACK

GIVE FEEDBACK

OK

Rotary motion and circular motion

In **rotary motion**, an object simply spins, usually around its own geometrical axis.
Notice that this is different to the linear motion of a point travelling in a circular path.



In **circular motion**, a point or an object moves along a circumference of a circle at a fixed distance r from the centre of its circular path, such as a car driven around a curve of a certain radius.



GIVE FEEDBACK



OK

Type your answer in the box.

In motion, a point or an object moves along the circumference of a circle at a fixed distance r from the centre of its circular path.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Type your answer in the box.

In **rotary motion**, an object simply spins, usually around its own

.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

The relationship between rotation and circular motion

There is a relationship between the angular terms that describe the rotary motion, i.e. angular displacement, angular velocity and angular acceleration, and their linear counterparts measured along the circumference.

This relationship stems from the definition of the **radian** as the angle subtended at the centre of a circle by an arc equal in length to the radius.



GIVE FEEDBACK

OK

What is the correct definition of the **radian**?

Click the correct answer.

The angle subtended at the centre of a circle by an arc equal in length to the radius

The arc length of a circle produced by an angle subtended at the centre by two radii

The angle subtended at the centre of a semi-circle

The arc length produced by a semi-circle with a radius equal to π

Do you know the answer?

I KNOW IT

THINK SO

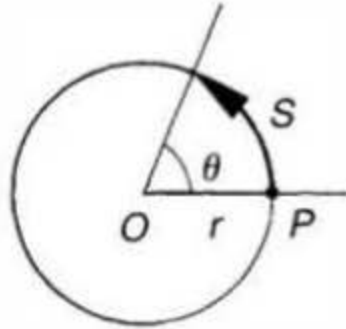
UNSURE

NO IDEA

Relating angular displacement and linear displacement along a circular path

If we consider the circular motion of a point P on a rotating disc at a radius r from the centre, as in the figure below, it can be seen that:

$$S = r \cdot \theta$$



Where:

S is the linear displacement along the circumference

r is the radius, in the same units as displacement

θ is the corresponding angular displacement in radians

GIVE FEEDBACK

OK

Match the equation symbol with the correct definition when calculating angular displacement and linear displacement along a circular path using the equation $S = r \theta$.



Drag statements on the right to match the left.

S



The linear displacement along the circumference



r



The radius of the circular path



θ



The angular displacement in radians



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Relating instantaneous angular velocity and linear velocity along a circular path

Dividing the equation $S = r \cdot \theta$ by time produces the equation relating instantaneous angular velocity to linear velocity along the circular path.

$$v = r \omega$$

Where:

v is the linear velocity along the circular path

r is the radius of the circular path

ω is the instantaneous angular velocity

GIVE FEEDBACK

OK

Match the equation symbol with the correct definition when relating instantaneous angular velocity and linear velocity along a circular path using the equation $v = r \omega$.



Drag statements on the right to match the left.

v



The linear velocity along the circular path



r



The radius of the circular path



ω



The angular velocity in radians per second



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Relating angular acceleration and linear acceleration along a circular path

Dividing the equation $v = r \omega$ by time produces the equation relating angular and linear accelerations.

$$a = r \alpha$$

Where:

a is the linear acceleration along the circular path

r is the radius of the circular path

α is the angular acceleration

GIVE FEEDBACK

OK

Match the equation symbols with the correct description when relating angular acceleration and linear acceleration along a circular path using the equation $a = r \alpha$.



Drag statements on the right to match the left.

a



The linear acceleration



r



The radius of the circular motion



α



The angular acceleration



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Convert angular displacement, velocity and acceleration to corresponding linear terms

On the former British passenger liner, the *Queen Mary*, there were four gearboxes transmitting power from turbines to a propeller shaft rotating at 180 revolutions per minute at normal cruising speed.

Each large driven gear was approximately 4 metres in pitch circle diameter.

For a point on the pitch circle of the gear, determine the linear velocity and the distance travelled along the circumference for each revolution.

Example	Determine circular motion characteristics	Determine linear circumferential velocity	Determine linear displacement per revolution	Application
---------	---	---	--	-------------

Convert angular displacement, velocity and acceleration to corresponding linear terms

Solution

Radius of circular motion: $r = 2 \text{ m}$

Angular velocity:

$$\begin{aligned}\omega &= 180 \cdot \frac{2\pi}{60} \\ &= 18.85 \text{ rad/s}\end{aligned}$$

Angular displacement:

$$\begin{aligned}\theta &= 1 \text{ revolution} \\ &= 2\pi \text{ rad}\end{aligned}$$

Substitute these values into the appropriate formulae and solve.

Example	Determine circular motion characteristics	Determine linear circumferential velocity	Determine linear displacement per revolution	Application
---------	---	---	--	-------------

Convert angular displacement, velocity and acceleration to corresponding linear terms

$$\begin{aligned}v &= r \cdot \omega \\&= 2 \text{ m} \cdot 18.85 \text{ rad/s} \\&= 37.7 \text{ m/s}\end{aligned}$$

Example	Determine circular motion characteristics	Determine linear circumferential velocity	Determine linear displacement per revolution	Application
---------	---	---	--	-------------

Convert angular displacement, velocity and acceleration to corresponding linear terms

$$\begin{aligned} S &= r \cdot \theta \\ &= 2 \text{ m} \cdot 2 \pi \text{ rad} \\ &= 12.57 \text{ m} \end{aligned}$$

Example	Determine circular motion characteristics	Determine linear circumferential velocity	Determine linear displacement per revolution	Application
---------	---	---	--	-------------

Convert angular displacement, velocity and acceleration to corresponding linear terms

The comparison between linear motion of a vehicle and the rotation of its wheels also depends on the relationships used in this example.

When a wheel rolls on a road surface, its axis is actually moving forward relative to the road.

One can also visualise this situation from the driver's point of view in terms of a 'fixed' axis of rotation, in relation to which the road surface is 'moving backwards' under the wheel.

In either case, the point of contact between the wheel and the road has momentarily a linear velocity v and an angular velocity ω , which are related by $v = r \cdot \omega$.

Example	Determine circular motion characteristics	Determine linear circumferential velocity	Determine linear displacement per revolution	Application
---------	---	---	--	-------------

At a certain speed the road wheel of a car rotates at 900 rpm.

The wheel has a diameter of 60 cm.



SMALL

MEDIUM

LARGE



Determine the angular velocity of the road wheel in radians per second.

(Answer in rad/s correct to three decimal places.)

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

At a certain speed the road wheel of a car rotates at 900 rpm.

The wheel has a diameter of 60 cm.



SMALL

MEDIUM

LARGE



Determine the linear displacement of the road wheel per revolution.

(Answer correct to three decimal places.)

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

Determine the angular velocity of the road wheel in radians per second.

(Answer in rad/s correct to three decimal places.)

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Clear

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

Now that we know the angular velocity is 94.248 rad/s determine the linear circumferential velocity.

(Answer in metres per second correct to two decimal places.)

\pm $\frac{\square}{\square}$ $1\frac{2}{3}$ \square^2 $\sqrt{\square}$ (\square)

\leq π $\square \times 10 \square$ lb \square \leftarrow ? Clear Clear line Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Determine the linear displacement of the road wheel per revolution.

(Answer correct to three decimal places.)

+	-	.	÷	$\frac{\square}{\square}$	\square^2	$\sqrt{\square}$	Clear		
(\square)	▼	≤	▼	π	m	▼	$\overline{\square}$?	Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

Convert linear displacement, velocity and acceleration to corresponding angular terms

We know that when a wheel rolls on a road surface, its axis is actually moving forward relative to the road.

You can also visualise this situation from the driver's point of view in terms of a 'fixed' axis of rotation in relation to which the road surface is 'moving backwards' under the wheel.

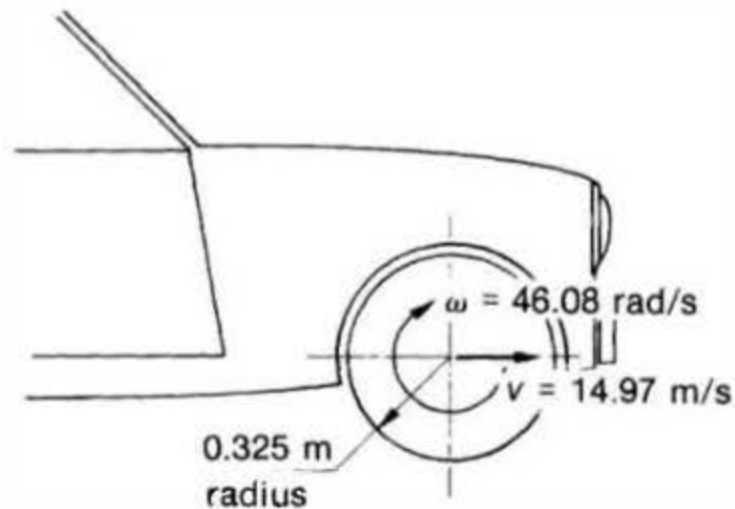
In either case, the point of contact between the wheel and the road has momentarily a linear velocity v and an angular velocity ω , which are related by $v = r \cdot \omega$.

Introduction	Example	Solution Determine the angular velocity	Determine the radius of rotation	Determine the linear velocity
--------------	---------	---	--	----------------------------------

Convert linear displacement, velocity and acceleration to corresponding angular terms

Example

Determine the speed of a car in kilometres per hour when its wheels, which are 650 mm in diameter, are rotating at 440 rpm.



Introduction	Example	Solution Determine the angular velocity	Determine the radius of rotation	Determine the linear velocity
--------------	---------	--	----------------------------------	-------------------------------

Convert linear displacement, velocity and acceleration to corresponding angular terms

Solution

Angular velocity:

$$\begin{aligned}\omega &= 440 \cdot \frac{2\pi}{60} \\ &= 46.08 \text{ rad/s}\end{aligned}$$

Note that since the radian is dimensionless it may be used to compose units involving angular measure and left out when transition to linear units is involved, as above, without disturbing dimensional homogeneity of an equation.

Introduction	Example	Solution Determine the angular velocity	Determine the radius of rotation	Determine the linear velocity
--------------	---------	--	--	----------------------------------

Convert linear displacement, velocity and acceleration to corresponding angular terms

Radius of rotation:

$$\begin{aligned} r &= \frac{0.65 \text{ m}}{2} \\ &= 0.325 \text{ m} \end{aligned}$$

Introduction

Example

Solution
Determine the
angular velocity

Determine the
radius of
rotation

Determine the
linear velocity

GIVE FEEDBACK

OK

Convert linear displacement, velocity and acceleration to corresponding angular terms

Linear velocity:

$$\begin{aligned}v &= r \cdot \omega \\&= 0.325 \text{ m} \cdot 46.08 \text{ rad/s} \\&= 14.97 \text{ m/s}\end{aligned}$$

Converting to kilometres per hour:

$$\begin{aligned}v &= 14.97 \cdot \frac{3,600}{1,000} \\&= 53.9 \text{ km/h}\end{aligned}$$

Introduction	Example	Solution Determine the angular velocity	Determine the radius of rotation	Determine the linear velocity
--------------	---------	---	--	----------------------------------

A Ferris wheel at a carnival has a diameter of 30 metres.

When operating at its maximum speed it completes two revolutions per minute.

SMALL

MEDIUM

LARGE



Type your answer in the box.

The angular velocity of the Ferris wheel is radians per second. (Answer correct to three decimal places.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A Ferris wheel at a carnival has a diameter of 30 metres.

When operating at its maximum speed it completes two revolutions per minute.

SMALL

MEDIUM

LARGE



Type your answer in the box.

Now that we know the angular velocity is 0.209 rad/s , the linear circumferential velocity is

metres per second, which is equal to km/h.

(Answer correct to two decimal places.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



Delta is the fourth letter of the Greek alphabet. It is used as a symbol in mathematics and science to represent a difference or a change in some quantity.



Equations for velocity and acceleration for uniformly accelerated linear motion

When we consider uniformly accelerated linear motion, two defining equations could be said to apply.

The change of velocity is given by:

$$\Delta v = v_2 - v_1$$

and acceleration, as the measure of change of velocity over time, is given by:

$$a = \frac{\Delta v}{t}$$

These equations work in a purely algebraic sense, when all we do is substitute numerical values and calculate, provided the motion under investigation is linear. This is clearly so when motion is in a straight line (rectilinear motion).

GIVE FEEDBACK

OK

Match the symbol with the correct description for the equations $\Delta v = v_2 - v_1$ and $a = \frac{\Delta v}{t}$.

 Drag statements on the right to match the left.

t



The time taken for the change to occur



Δv



Change in velocity



v_2



The velocity at time 2



v_1



The velocity at time 1



a



The acceleration



Do you know the answer?

Tangential acceleration

Acceleration measured along the circumferential direction of motion (i.e. in the direction tangential to the circular path) is usually called **tangential acceleration**.



Necessary tangential acceleration can be distinguished from any other kind of acceleration by the symbol subscript t , as in a_t .



GIVE FEEDBACK



OK

Type your answer in the box.

Acceleration measured along the circumferential direction of motion (i.e. in the direction tangential to the circular path) is known as .

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

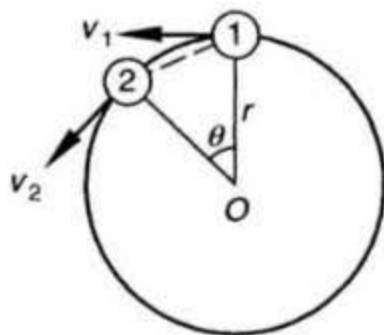
NO IDEA

Consider the case of **uniform circular motion**, i.e. a body moving in a circle of radius r with constant speed.

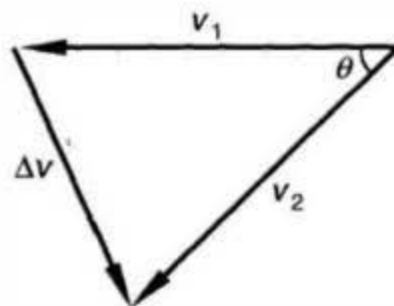
Since the magnitude of the velocity remains the same, then $v_2 = v_1 = v$, and the value of tangential acceleration a_t must be equal to zero.

As the body moves in a circular path from position 1 to 2 as shown in Figure (a) on the following slide, the *direction* of its velocity is constantly changing. Therefore, there is some change in velocity, even if only in terms of direction, and when there is change in velocity there must be some kind of acceleration involved.

[GIVE FEEDBACK](#)[CONTINUE >](#)



(a)



(b)

Velocity, like force, is a vector quantity. This means that it has magnitude and direction, and for this reason, addition of velocities that involve changes of direction must always be done vectorially.

The velocity equation ($v_2 = v_1 + \Delta v$) can be represented graphically as a triangle of velocity vectors, as shown in Figure (b) on the previous slide.

This is just a graphical way of showing that velocity at point 1 (v_1) plus the amount of directional change that occurs between points 1 and 2 results in a new velocity at point 2 (v_2). This is so, even if the magnitudes of the two velocities remain the same.

The velocity triangle in Figure (b) is similar to the physical triangle $O - 1 - 2$ in Figure (a), as both have two equal sides and the same included angle θ . Therefore, it follows that:

$$\frac{\Delta v}{v} = \frac{\text{length of chord } 1 - 2}{r}$$

When the angle θ is very small, the length of the chord 1 - 2 is very nearly equal to the length of the arc 1 - 2, i.e. to the amount of linear displacement S of the body along the circumference, which takes place over the correspondingly short interval of time t .

Therefore:

$$\frac{\Delta v}{v} = \frac{S}{r}$$

We also know that when the velocity along the path is uniform, then:

$$S = v t$$

Hence:

$$\frac{\Delta v}{v} = \frac{v t}{r}$$

Also:

$$\Delta v = \frac{v^2 t}{r}$$

Since acceleration is defined as the change of velocity over time, substitution yields:

$$\text{Acceleration} = \frac{\Delta v}{t} = \frac{v^2 t}{r} t = \frac{v^2}{r}$$

< BACK

GIVE FEEDBACK

OK

Type your answer in the box.

In uniform circular motion, there is an acceleration acting, since the of the velocity is constantly changing.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



It is possible to demonstrate mathematically that the exact instantaneous value of centripetal acceleration holds true for variable conditions as well as for uniform conditions of circular motion.

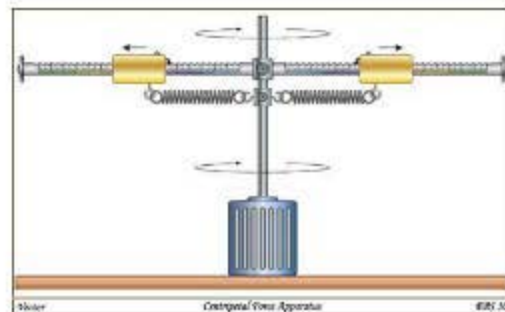


Centripetal acceleration

In uniform circular motion the *magnitude* of the velocity does not change, therefore the acceleration that causes the change in direction must be perpendicular to the direction of the velocity.

This can also be seen from the triangle of velocities since the angle is very small. In fact, this acceleration is directed along the radius towards the centre of the circular path.

Therefore, it is called **normal**, **radial** or **centripetal acceleration**, a_c .



GIVE FEEDBACK

OK

Type your answer in the box.

In uniform circular motion the magnitude of the velocity does not change, therefore the acceleration that causes the change in direction must be to the direction of the velocity.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal acceleration for a body moving along a circular path

It can be said that at each instant, when a body is moving with velocity v along a circular path of radius r , it experiences a centripetal acceleration, i.e. an acceleration directed radially towards the centre of the circle, which is given by:

$$a_c = \frac{v^2}{r}$$

a_c is the centripetal acceleration in m/s^2



v is the linear velocity along the circumference in m/s



r is the radius of the circular path in m



GIVE FEEDBACK



OK

Match the symbol from the equation $a_c = \frac{v^2}{r}$ with the correct description.



Drag statements on the right to match the left.

a_c



The centripetal acceleration in m/s^2



v



The linear velocity along the circumference in m/s



r



The radius of the circular path in m



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal acceleration

Calculate the **centripetal acceleration** of a car which is travelling at 108 km/h around a curve of radius 200 m.

Example

Convert velocity
to m/s

Solution

GIVE FEEDBACK

OK

Calculating centripetal acceleration

Convert velocity to metres per second:

$$\begin{aligned}v &= 108 \text{ km/h} \\&= \frac{108}{3.6} \text{ m/s} \\&= 30 \text{ m/s}\end{aligned}$$

Example

Convert velocity
to m/s

Solution

Calculating centripetal acceleration

Substitution gives:

$$\begin{aligned}a_c &= \frac{v^2}{r} \\&= \frac{30^2}{200} \\&= 4.5 \text{ m/s}^2\end{aligned}$$

Example

Convert velocity
to m/s

Solution

Calculate the centripetal acceleration of a car which is travelling at 72 km/h around a curve of radius 50 m.



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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Centripetal force

According to Newton's laws of motion, every moving body would persevere in its state of uniform motion in a straight line unless it is compelled to change that state by an external unbalanced force impressed on it.

In the case of circular motion, the path is not a straight line, and there is a change of direction of velocity, as we have already discussed. Therefore there must exist an unbalanced force acting on the body, which is the cause of this change.

The rate of change of velocity in this case is centripetal acceleration, a_c .

The force which is responsible for this acceleration is referred to as the **centripetal force**, F_c .

GIVE FEEDBACK

RM
RM

OK

Type your answer in the box.

The force which is responsible for centripetal acceleration is known as the force.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal force using linear velocity for a body moving along a circular path

Applying Newton's second law results in an expression which can be used for calculating the centripetal force acting on a body.

$$F_c = m a_c = \frac{m v^2}{r}$$

where:

m is the mass of the object, in kg

a_c is the centripetal acceleration, in m/s^2

v is the linear velocity, in m/s

r is the radius of the circular motion, in m

GIVE FEEDBACK



OK

Match the symbols from the equation $F_c = m a_c = \frac{m v^2}{r}$ with the correct description.



Drag statements on the right to match the left.

F_c



The centripetal force



a_c



The centripetal acceleration



v



The linear velocity of the object



r



The radius of the circular motion



m



The mass of the object



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal force using centripetal acceleration

What is the magnitude of the centripetal force which causes a car to travel in a circular path if the mass of the car is 1.2 t and the required centripetal acceleration is 4.5 m/s^2 ?

Example

Convert mass
to kg

Solution

GIVE FEEDBACK

OK

Calculating centripetal force using centripetal acceleration

Mass in kilograms:

$$\begin{aligned}m &= 1.2 \cdot 1,000 \text{ kg} \\ &= 1,200 \text{ kg}\end{aligned}$$

Example

Convert mass
to kg

Solution

GIVE FEEDBACK

OK

Calculating centripetal force using centripetal acceleration

Hence centripetal force:

$$\begin{aligned} F_c &= m a_c \\ &= 1,200 \text{ kg} \cdot 4.5 \text{ m/s}^2 \\ &= 5,400 \text{ N} \\ &= 5.4 \text{ kN} \end{aligned}$$

Example

Convert mass
to kg

Solution

Type your answer in the box.

The magnitude of the centripetal force which causes a 1400 kg car to travel in a circular path when it experiences a centripetal acceleration of 5 m/s^2 is kN.



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

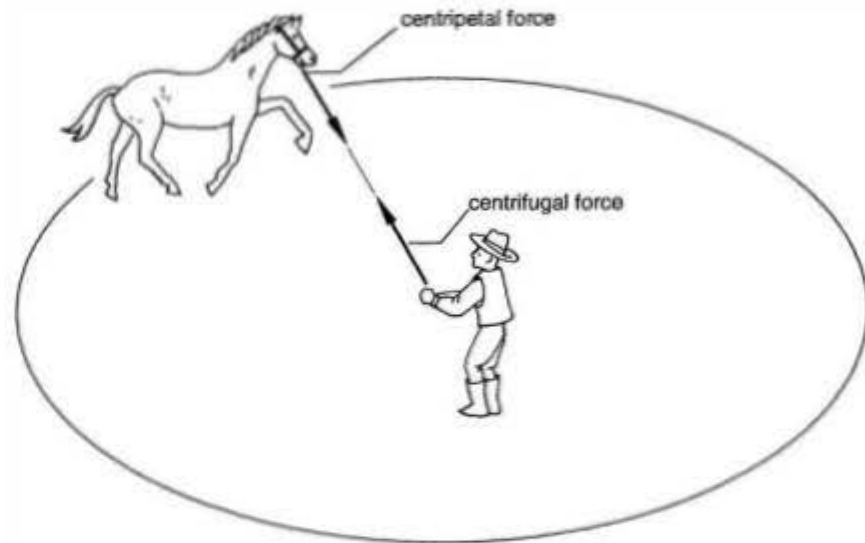
NO IDEA



The word 'centrifugal' comes from 'centrum', meaning 'centre', and 'fugere', meaning 'to fly from'.



Below is an example of the nature of the force we call centripetal force F_c .



GIVE FEEDBACK

CONTINUE >



If a ball on a string is spun in a horizontal circle, a constant centripetal force must be applied to maintain the circular motion of the ball.

While the centripetal force acts on the ball there is a constant equal and opposite centrifugal reaction force pulling on the string.



When a racehorse on a tether is exercised in a small circle, it is prevented from galloping in a straight line by the pull of the rope held by its trainer. Every time the horse tries to extend the range of its movement, the rope tightens and the horse experiences a centripetal pull telling it to stay on the circular track. At the same time the trainer experiences an equal but opposite pull of the rope on his hands.

This can be described as the centrifugal reaction to the centripetal pull, as shown in the figure on the previous slide.

There is one very important lesson to be learned from this. The centripetal pull is experienced by the horse, i.e. the moving object, and is directed radially towards the centre of the circular path. The centrifugal pull is experienced by the trainer, i.e. the object which provides the controlling force, and is directed radially away from the centre.

< BACK

GIVE FEEDBACK

OK

Type your answer in the box.

The pull is experienced by an object in circular motion and is directed radially towards the centre of the circular path. The pull is experienced by the object which provides the controlling force and is directed radially away from the centre.

Do you know the answer?

I KNOW IT

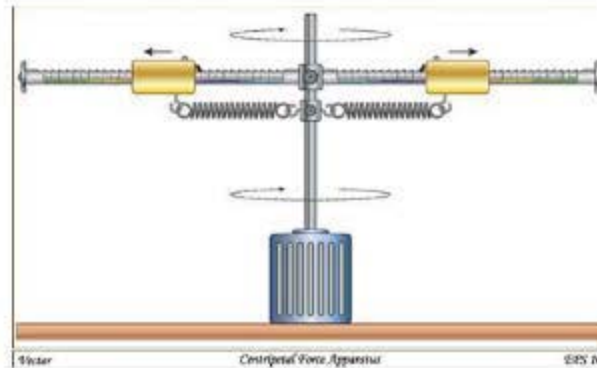
THINK SO

UNSURE

NO IDEA

The centripetal pull is experienced by the moving object and is directed radially towards the centre of the circular path. The centrifugal pull is experienced by the object which provides the controlling force and is directed radially away from the centre.

In the world of inanimate objects, a constant centripetal force must be applied continuously in order to maintain circular motion.



GIVE FEEDBACK

CONTINUE >

For instance, if you make a simple model aeroplane fly in a horizontal circle at the end of a string which is attached to the aeroplane at one end and held in your hand at the other, the string will pull the plane into the circular path all the time. Otherwise the plane would move off in a straight line.

While the string is continuously pulling the model plane towards you as the centre of the circle, and the radial distance between you and the plane remains the same, it is also true that there is a constant equal and opposite centrifugal reaction on your hand.

< BACK

GIVE FEEDBACK

CONTINUE >

In the case of a car turning a corner, the moving object is the car, and the centripetal force experienced by it is generated by road surface friction acting on its turned front wheels.

It is because of the existence of this constant centripetal force, directed towards the centre of curvature of the road, that the car is impelled to follow the circular path.

It can also be said that a centrifugal reaction force is experienced by the road surface.

< BACK

GIVE FEEDBACK

OK

Select examples of centripetal force from the given list.

Check **all** that apply.

- ☐ The force exerted on a model aeroplane by a string as it flies in a circle around its stationary pilot
- ☐ The force exerted on a stationary pilot by a string attached to a model aeroplane as it flies in a circle around its stationary pilot
- ☐ The force generated by road surface friction acting on the turned front wheels of a car as it turns a corner
- ☐ The force acting on the road surface due to the turned front wheels of a car as it turns a corner
- ☐ The force acting on a person riding a carousel as the carousel spins
- ☐ The force acting on a carousel caused by a person riding it, as the carousel spins.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Select examples of centrifugal force from the given list.

Check **all** that apply.

- ☐ The force exerted on a stationary pilot by a string attached to a model aeroplane as it flies in a circle around its stationary pilot
- ☐ The force exerted on a model aeroplane by a string as it flies in a circle around its stationary pilot
- ☐ The force acting on a carousel caused by a person riding it, as the carousel spins
- ☐ The force acting on a person riding a carousel as the carousel spins
- ☐ The force acting on the road surface due to the turned front wheels of a car as it turns a corner
- ☐ The force generated by road surface friction acting on the turned front wheels of a car as it turns a corner

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal force using angular velocity for a body moving along a circular path

Occasionally it is necessary to solve problems in which circular motion of a mass is closely related to the rotation of some physical component, such as a shaft, a flywheel or a washing machine drum, whose speed of rotation is given in revolutions per minute or in radians per second.



Alternatively, the formula for centripetal force can be modified in terms of angular velocity ω .



Hence centripetal force acting on the mass is given by:

$$F_c = m \omega^2 r$$



where:

F_c is the centripetal force

m is the mass of the object

ω is the angular velocity

r is the radius of the circular motion



GIVE FEEDBACK



OK

Calculating centripetal force using angular velocity for a body moving along a circular path

Remembering that circumferential velocity v of a point P moving in a circle of radius r about the centre point O is related to the angular velocity ω of the radial line OP by $v = r \omega$; we can easily convert angular velocity ω of the rotating component to the corresponding linear velocity v of the mass along its circular path.



We can determine, by substitution, that centripetal acceleration of a mass located at point P is:

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$



Hence centripetal force acting on the mass is given by:

$$F_c = m \omega^2 r$$



where:

F_c is the centripetal force


m is the mass of the object

ω is the angular velocity

r is the radius of the circular motion



Match the symbol from the equation $F_c = m \omega^2 r$ with the correct description.

 Drag statements on the right to match the left.

F_c



The centripetal force



m



The mass of the object



ω



The angular velocity



r



The radius of the circular motion



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating centripetal force using angular acceleration—Example

Example

Calculate the centripetal force acting on a 150 g balancing mass attached to a car wheel, at a radius of 200 mm, when the wheel rotates at 955 rpm.

GIVE FEEDBACK

CONTINUE >

Calculating centripetal force using angular acceleration—Example

Example

Calculate the centripetal force acting on a 150 g balancing mass attached to a car wheel, at a radius of 200 mm, when the wheel rotates at 955 rpm.

Solution

Angular velocity:

$$\begin{aligned}\omega &= 955 \cdot \frac{2\pi}{60} \\ &= 100 \text{ rad/s}\end{aligned}$$

Hence centripetal force can be calculated using the modified formula:

$$\begin{aligned}F_c &= m \omega^2 r \\ &= 0.15 \cdot 100^2 \cdot 0.2 \\ &= 300 \text{ N}\end{aligned}$$

< BACK

GIVE FEEDBACK

OK

Calculate the centripetal force acting on a 40 kg child on a carousel, at a radius of 6 metres, when the carousel rotates at 10 rpm.

(Answer to the nearest newton.)



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CHALLENGE

SUBMIT

SHOW ANSWER



Like any other force, centripetal force is always caused by some form of interaction between physical objects.

One of the most common forms of interaction between two surfaces in contact is due to dry sliding friction, such as between the road surface and the tyres of a moving vehicle.



How centripetal force is related to friction for a vehicle travel in a circular path around a horizontal curve on a level road

In order to make a vehicle travel in a circular path around a horizontal curve on a level road, it is necessary to have a centripetal force of sufficient magnitude act on the vehicle through the points of contact between the wheels and the road surface.

The magnitude of the required force is dependent on the mass m of the vehicle, the radius r of road curvature and the circumferential speed v .



GIVE FEEDBACK

OK

Type your answer in the box.

In order to make a vehicle travel in a circular path around a horizontal curve on a level road, it is necessary to have a force of sufficient magnitude act on the vehicle through the points of contact between the wheels and the road surface.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

The relationship between centripetal force, friction, mass of vehicle and coefficient of static friction

The centripetal force acting on a vehicle through the points of contact between the wheels and the road surface is generated by friction.



It can be seen that frictional force is also dependent on the mass of the vehicle, since on a horizontal surface the normal reaction F_n is equal to the weight of the vehicle $F_w = m g$.



Therefore:

$$F_c = F_r = \mu F_n = \mu m g$$



where:

F_c is the centripetal force

F_r is the reaction force due to friction



GIVE FEEDBACK



OK

The relationship between centripetal force, friction, mass of vehicle and coefficient of static friction

Therefore its maximum possible value is limited by the coefficient of static friction μ between the road and the tyres.



It can be seen that frictional force is also dependent on the mass of the vehicle, since on a horizontal surface the normal reaction F_n is equal to the weight of the vehicle $F_w = m g$.



Therefore:

$$F_c = F_r = \mu F_n = \mu m g$$



μ is the coefficient of friction

F_n is the normal reaction force

m is the mass of the vehicle

g is the acceleration due to gravity



Match each of the symbols from the equations $F_c = F_r = \mu F_n = \mu m g$ with the correct description.



Drag statements on the right to match the left.

F_c



The centripetal force



F_r



The reaction force due to friction



μ



The coefficient of static friction



F_n



The normal reaction force



m



The mass of the vehicle



g



Acceleration due to gravity



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Give two examples of instability of vehicles on level curved roads

Experience suggests that, for a given set of road and tyre conditions, the ability of a vehicle to successfully negotiate a curve on a level road appears to depend primarily on the speed of travel.

Two undesirable alternatives occur: the vehicle skidding or overturning.



GIVE FEEDBACK

OK

Type your answer in the box.

Experience suggests that, for a given set of road and tyre conditions, the ability of a vehicle to successfully negotiate a curve on a level road appears to depend primarily on the of travel.

Two undesirable alternatives occur: the vehicle or .

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



The centre of mass is also known as the centre of inertia or the centre of gravity, since both the linear inertia of the entire mass and the force of gravity (weight) acting on it can be considered to be located at this point.



Define centre of mass or centre of gravity

The analysis of the tendency for a vehicle to overturn as it attempts to negotiate a curve requires consideration of the vehicle's speed and the dimensions of the vehicle.

Experience teaches that stability against overturning has a lot to do with the speed of the vehicle travelling in a curved path, but also depends on the dimensions of the vehicle.

In particular, it depends on the relationship between the width between the wheels of the vehicle and the height of its **centre of mass** above the ground.



GIVE FEEDBACK

OK

Select the correct terms to describe the location on a body where the linear inertia of the entire mass and the force of gravity (weight) acting on it are located.

Check **all** that apply.

- ☐ Centre of mass
- ☐ Centre of inertia
- ☐ Centre of force
- ☐ Centre of gravity
- ☐ Centre of action
- ☐ Centre of reaction

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

The formula for limit of safe velocity to avoid vehicle skidding

In order to determine the maximum safe speed for a given set of conditions, compare the two separate, relatively independent criteria, one for skidding and the other for overturning.

Comparison of the speed that would cause skidding and the speed that would cause overturning allows the maximum speed limit for safety to be determined.

It is the lower of the two speeds.



The limiting criteria for vehicle stability is found by direct substitution of given data, followed by simple calculation.

Limit of safe velocity in km/h to avoid skidding:

$$v = 3.6 \sqrt{\mu g r}$$



GIVE FEEDBACK



OK

Match the symbol from the equation $v = 3.6 \sqrt{\mu g r}$ (the equation to determine the limit of safe velocity to avoid skidding) with the correct description.



Drag statements on the right to match the left.

v



The velocity of the vehicle



μ



The coefficient of friction between the road and the tyres



g



Acceleration due to gravity



r



The radius of the curve in the road



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



Since all of the forces acting on a vehicle in this situation are proportional to the mass of the vehicle, the mass is eventually cancelled out from the calculations, rendering the criteria for safe velocity independent of mass.



The formula for limit of safe velocity to avoid vehicle overturning

Limit of safe velocity in km/h to avoid overturning:

$$v = 3.6 \sqrt{\frac{b g r}{2 h}}$$

where:

b is the vehicle track width

g is acceleration due to gravity

r is the radius of the corner


h is the height of the vehicle's centre of gravity above the road



GIVE FEEDBACK

OK

Match the symbol from the equation $v = 3.6 \sqrt{\frac{b g r}{2 h}}$ with the correct definition.

 Drag statements on the right to match the left.

v



The limit of safe velocity to avoid overturning



b



The vehicle's track width



g



Acceleration due to gravity



r



The radius of the curve



h



The height of the vehicle's centre of gravity above the road



Do you know the answer?

Determining the speed that may cause a vehicle to skid

Example

A car of mass 1.35 t is travelling around a horizontal curve of radius 115 m. If the coefficient of static friction between the road surface and the tyres is 0.5, determine the minimum speed at which the vehicle may skid.

Example	Determine the maximum centripetal force	Solution	Discussion
---------	---	----------	------------

Determining the speed that may cause a vehicle to skid

Solution

The maximum centripetal force that can be provided by frictional contact is:

$$\begin{aligned} F_c &= \mu m g \\ &= 0.5 \cdot 1,350 \text{ kg} \cdot 9.81 \text{ N/kg} \\ &= 6,622 \text{ N} \end{aligned}$$

Example	Determine the maximum centripetal force	Solution	Discussion
---------	---	----------	------------

Determining the speed that may cause a vehicle to skid

Solution

Substitute into the formula for centripetal force:

$$F_c = \frac{m v^2}{r}$$
$$6,622 = \frac{1,350 \cdot v^2}{115}$$
$$\therefore v = 23.75 \text{ m/s}$$
$$= 85.5 \text{ km/h}$$

Example	Determine the maximum centripetal force	Solution	Discussion
---------	---	----------	------------

Determining the speed that may cause a vehicle to skid

Friction has the uncanny ability to progressively build up resistance to sliding motion to match the physical demands of the situation.

Therefore, if the car in the example is travelling at slower speeds below 85.5 km/h, just sufficient friction would be generated between the wheels and the road surface to provide the necessary centripetal force, corresponding exactly to the lower speed of travel, to enable the car to go around the curve without sliding.

However, friction cannot increase indefinitely. In this example, it cannot increase beyond its limiting value of 6622 N, and any attempt to travel faster than 85.5 km/h would result in a skid, i.e. the vehicle would continue in a straight line instead of moving in the circular path.

Example	Determine the maximum centripetal force	Solution	Discussion
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Calculate the maximum centripetal force that can be provided by frictional contact.

(Answer to the nearest newton.)



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CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

A car of mass 1200 kg is travelling around a horizontal curve of radius 60 m.

The coefficient of static friction between the road surface and the tyres is 0.45.



SMALL

MEDIUM

LARGE



Calculate the minimum centripetal force that can be provided by frictional contact.

(Answer to the second question.)



Only and type your answer here

CHALLENGE

INSTRUCTIONS

- No intermediate steps are required.
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

800-441-2344

A car of mass 1200 kg is travelling around a horizontal curve of radius 60 m.

The coefficient of static friction between the road surface and the tyres is 0.45.



SMALL

MEDIUM

LARGE



Now that we know the maximum centripetal force that can be provided by frictional contact is 5297 N, determine the minimum speed that may cause the vehicle to skid.

(Answer to the nearest km/h.)



Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required.
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

1/10 Your work will receive the credit awarded for this question



Now that we know the maximum centripetal force that can be provided by frictional contact is 5,297 N. Determine the minimum speed that may cause the vehicle to skid.

(Answer to the nearest km/h.)

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question



Determining the speed that may cause a vehicle to overturn

Example

A car of mass 1.35 t is travelling around a horizontal curve of radius 115 m.

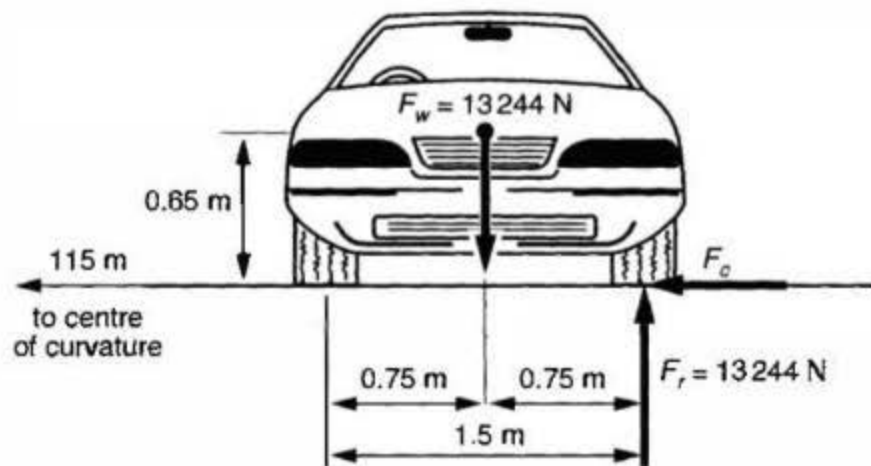
The coefficient of static friction between the road surface and the tyres is 0.5.

The car's centre of mass is 0.65 m above the ground and the track width is 1.5 m.

Determine the minimum speed at which the vehicle may overturn.

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
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Determining the speed that may cause a vehicle to overturn



Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
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Determining the speed that may cause a vehicle to overturn

Solution

At the critical instant when the car is about to overturn, its entire weight rests on the outer wheels. Therefore, while the reaction on the inner wheels is zero, the total vertical reaction force at the outer wheels is equal to the weight of the car.

$$\begin{aligned}F_r &= F_w \\&= m g \\&= 1,350 \text{ kg} \cdot 9.81 \text{ N/kg} \\&= 13,244 \text{ N}\end{aligned}$$

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
---------	---------	--------------------	--------------------	--------------------	--------------------	------------

Determining the speed that may cause a vehicle to overturn

Weight F_w , which is acting through the centre of mass, together with reaction force F_r constitute a couple which has a perpendicular distance between the pair of its forces equal to one-half of the wheel base width. The moment due to this couple is:

$$\begin{aligned}M &= F \frac{w}{2} \\&= 13,244 \text{ N} \cdot \frac{1.5 \text{ m}}{2} \\&= 9,933 \text{ N.m}\end{aligned}$$

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
---------	---------	--------------------	--------------------	--------------------	--------------------	------------

Determining the speed that may cause a vehicle to overturn

In order to make the car follow the circular path, a horizontal centripetal force F_c must exist, acting through the points of effective contact between the tyres and the road surface. The moment of the centripetal force about the centre of mass depends on the height of the centre of mass above the road surface:

$$\begin{aligned}M &= F_c h \\ &= F_c \cdot 0.65\end{aligned}$$

The critical condition occurs when the two moments are just balanced against each other:

$$\begin{aligned}F_c \cdot 0.65 &= 9,933 \\ \text{therefore } F_c &= \frac{9,933 \text{ N.m}}{0.65 \text{ m}} \\ &= 15,282 \text{ N}\end{aligned}$$

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
---------	---------	--------------------	--------------------	--------------------	--------------------	------------

Determining the speed that may cause a vehicle to overturn

Substitute into the formula for centripetal force:

$$F_c = \frac{m v^2}{r}$$
$$15,282 = \frac{1,350 \cdot v^2}{115}$$

therefore $v = 36.08 \text{ m/s}$
 $= 130 \text{ km/h}$

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
---------	---------	--------------------	--------------------	--------------------	--------------------	------------

Determining the speed that may cause a vehicle to overturn

We can conclude that, provided there is adequate friction to generate the required centripetal force without sliding, the car will start overturning, i.e. rotating about its centre of mass, as soon as its velocity exceeds 130 km/h.

The flanged wheels on railway locomotives and carriages make it practically impossible for a train to skid off the rails. However, overturning may occur at excessive speeds, as for any vehicle moving in a curved path.

Example	Diagram	Solution—Part 1	Solution—Part 2	Solution—Part 3	Solution—Part 4	Conclusion
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A car of mass 1200 kg is travelling around a horizontal curve of radius 60 m.

The coefficient of static friction between the road surface and the tyres is 0.45.

The car has its centre of mass 600 mm above the ground and the wheel base is 1480 mm wide.

SMALL

MEDIUM

LARGE



Type your answer in the box.

At the critical instant when the car is about to overturn, its entire weight rests on the outer wheels. Therefore, while the reaction on the inner wheels is zero, the total vertical reaction force at the outer wheels is equal to the weight of the car (N).

Weight F_w , which is acting through the centre of mass, together with reaction force F_r , constitute a couple which has a perpendicular distance between the pair of its forces equal to one-half of the wheel base width. The moment due to this couple is Nm.



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A car of mass 1200 kg is travelling around a horizontal curve of radius 60 m.

The coefficient of static friction between the road surface and the tyres is 0.45.

The car has its centre of mass 600 mm above the ground and the wheel base is 1480 mm wide.

SMALL

MEDIUM

LARGE



Type your answer in the box.

In order to make the car follow the circular path, a horizontal centripetal force F_c must exist, acting through the points of effective contact between the tyres and the road surface. The moment of the centripetal force about the centre of mass depends on the height of the centre of mass above the road surface ($M = F_c h$).

The critical condition occurs when the two moments are just balanced against each other ($F_c \cdot h = P_r \cdot \frac{w}{2}$),

therefore $F_c =$ N.



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A car of mass 1200 kg is travelling around a horizontal curve of radius 60 m.

The coefficient of static friction between the road surface and the tyres is 0.45.

The car has its centre of mass 600 mm above the ground and the wheel base is 1480 mm wide.

SMALL

MEDIUM

LARGE



Type your answer in the box.

Substitution into the formula for centripetal force ($F_c = \frac{m v^2}{r}$) allows the calculation of $v =$

m/s = km/h (to the nearest whole number).

Therefore we may conclude that, provided there is adequate friction to generate the required centripetal force without sliding, the car will start overturning, i.e. rotating about its centre of mass, as soon as its velocity exceeds

km/h.



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

The reason for the banking of roads and railway tracks

In order to counteract the tendency for skidding and overturning when vehicles round a curve at high speeds, it is common practice to bank highways, railway tracks and velodrome surfaces by giving them a transverse slope suitable for the estimated speed of travel.



GIVE FEEDBACK

OK

Type your answer in the box.

In order to counteract the tendency for skidding and overturning when vehicles round a curve at high speeds, it is common practice to highways, railway tracks and velodrome surfaces by giving them a transverse slope suitable for the estimated normal speed of travel.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating the correct angle of inclination for the expected normal speed of travel for a vehicle on a banked curve 1/4

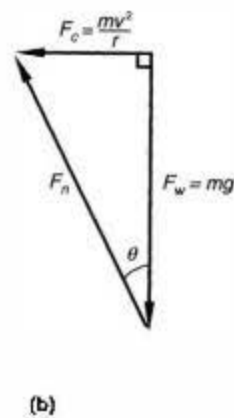
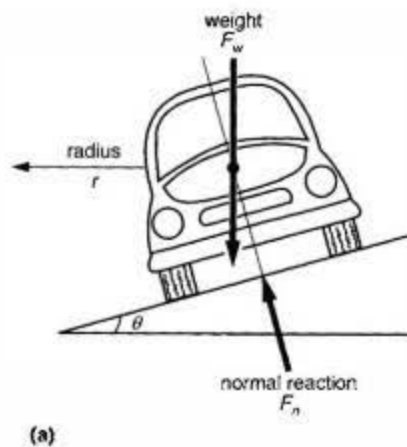
When a vehicle travels around a banked curve at the exact speed v for which the curve is designed, there is no side thrust on the wheels, the total reaction being perpendicular to the inclined surface, as illustrated in Figure (a) on the next slide.

The centripetal force F_c is the horizontal resultant of two forces acting on the vehicle: the weight F_w and the normal reaction F_n . This resultant, which is directed towards the centre of road curvature, can be found by constructing a triangle of forces as shown in Figure (b) on the next slide.

GIVE FEEDBACK

CONTINUE >

Calculating the correct angle of inclination for the expected normal speed of travel for a vehicle on a banked curve 2/4



< BACK

GIVE FEEDBACK

CONTINUE >

Calculating the correct angle of inclination for the expected normal speed of travel for a vehicle on a banked curve 2/4

Since the angle θ within the triangle of forces is equal to the angle of inclination of the road surface to the horizontal, we can also write:

$$\tan \theta = \frac{F_c}{F_w} = \frac{m v^2 / r}{m g}$$

< BACK

GIVE FEEDBACK

CONTINUE >

Calculating the correct angle of inclination for the expected normal speed of travel for a vehicle on a banked curve 4/4

After the mass m is cancelled out, we obtain the following expression, which can be used to calculate the correct angle of inclination for the expected normal speed of travel:

$$\tan \theta = \frac{v^2}{g r}$$

where:

θ is the angle between the road surface and the horizontal

v is the designed speed

g is acceleration due to gravity

r is the radius of the curve

< BACK

GIVE FEEDBACK

OK

Match each symbol from the equation $v = \sqrt{g r \tan \theta}$ with the correct description.



Drag statements on the right to match the left.

θ



The angle between the road surface and the horizontal



v



The designed speed



g



Acceleration due to gravity



r



The radius of the curve



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Superelevation defined

In railroad engineering, the difference in height between the outer and inner rails on a banked railway curve is usually referred to as the **superelevation**.

Over 60 per cent of the world's railroads are built to a standard gauge width of 1.435 metres.

Standard gauge was originally defined as 4 feet 8.5 inches between rail centres, and is used on all major railway lines in Australia, Great Britain, USA and Europe.



GIVE FEEDBACK



OK

Type your answer in the box.

In railroad engineering, the difference in height between the outer and inner rails on a banked railway curve is usually referred to as the .

Do you know the answer?

I KNOW IT

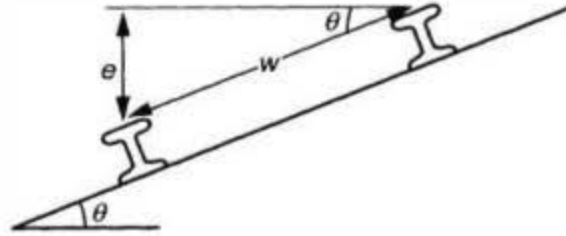
THINK SO

UNSURE

NO IDEA

The formula for calculating superelevation

If we let w represent the width between the rail centres, as shown in the figure below, the superelevation e will be given by:



$$e = w \sin \theta$$

The superelevation (e) of a railway track

where:

e is the superelevation

w is the distance between the rail centres (gauge)

θ is the banking angle

GIVE FEEDBACK

OK

Match each symbol from the equation $e = w \sin \theta$ with the correct description.



Drag statements on the right to match the left.

e



The superelevation



w



The distance between the rail centres
(gauge)



θ



The banking angle



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Calculating the required angle of inclination for a given speed

A highway curve with a radius of 355 m is to be banked at an angle to suit an estimated normal speed of 110 km/h.

What is the required angle of inclination?

The speed in metres per second is: $v = 30.56 \text{ m/s}$

The tangent of the required angle is calculated from:

$$\begin{aligned}\tan \theta &= \frac{v^2}{g r} \\ &= \frac{30.56^2}{9.81 \cdot 355} \\ &= 0.2681\end{aligned}$$

Hence the required angle of banking to suit the specified speed is:

$$\begin{aligned}\theta &= \tan^{-1} 0.2681 \\ &= 15^\circ\end{aligned}$$

The expression used to solve this example can be transposed into another form for calculating the correct speed of travel for a curve banked at a known angle θ :

$$v = \sqrt{g r \tan \theta}$$

where:

v is the designed speed for cornering

g is the acceleration due to gravity

r is the radius of the curve

θ is the banking angle

A roadway curve with a radius of 100 m is to be banked at an angle to suit an estimated normal speed of 60 km/h.

What is the required angle of inclination?

(Answer in degrees correct to one decimal place.)



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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Calculating the required speed for a given angle of inclination

What is the most comfortable speed for a car when travelling on a 300 m radius highway curve banked at 12° ?

For this road the comfortable speed of travel is found from:

$$\begin{aligned} v &= \sqrt{g r \tan \theta} \\ &= \sqrt{9.81 \cdot 300 \cdot \tan 12^\circ} \\ &= 25 \text{ m/s} \\ &= 90 \text{ km/h} \end{aligned}$$

GIVE FEEDBACK



OK

What is the most comfortable speed for a car when travelling on a 100 m radius roadway curve banked at an angle of 8 degrees? (Answer to the nearest km/h.)



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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

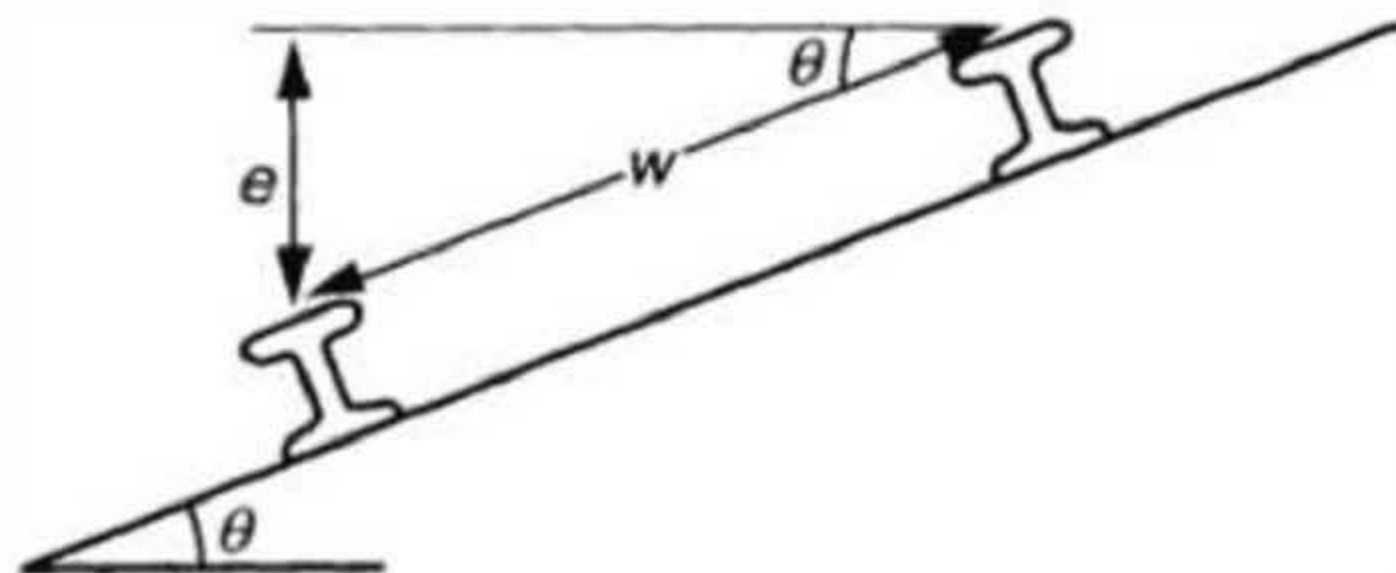
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Calculating the superelevation for a given speed or angle of inclination

Calculate the superelevation for a standard gauge railway track, on a curve of radius 1 km, if there is no side thrust on the wheels travelling at 150 km/h.



The speed in metres per second is: $v = 41.67 \text{ m/s}$.

The angle of banking corresponding to this speed is calculated from:

$$\begin{aligned}\tan \theta &= \frac{v^2}{g r} \\ &= \frac{41.67^2}{9.81 \cdot 1,000} \\ &= 0.177\end{aligned}$$

$$\begin{aligned}\text{therefore } \theta &= \tan^{-1} 0.177 \\ &= 10^\circ\end{aligned}$$

Hence the required superelevation is:

$$\begin{aligned}e &= w \sin \theta \\ &= 1.435 \cdot \sin 10^\circ \\ &= 0.25 \text{ m}\end{aligned}$$



A standard gauge railway track, on a curve of radius 600 m, is to be designed for no side thrust on the wheels when the train is travelling at 100 km/h.

SMALL

MEDIUM

LARGE



Type your answer in the box.

A speed of 100 km/h corresponds to m/s. (Answer correct to three decimal places.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A standard gauge railway track, on a curve of radius 600 m, is to be designed for no side thrust on the wheels when the train is travelling at 100 km/h.

SMALL

MEDIUM

LARGE



Type your answer in the box.

The appropriate angle of superelevation for this track design is degrees. (Answer correct to two decimal places.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A standard gauge railway track, on a curve of radius 600 m, is to be designed for no side thrust on the wheels when the train is travelling at 100 km/h.

SMALL

MEDIUM

LARGE



Type your answer in the box.

Knowing the appropriate angle of superelevation is 7.47° , the required superelevation is

metres. (Answer correct to three decimal places.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

The differences between superelevation and transverse inclination

The term 'superelevation' can also be used to describe the amount by which the outer edge of a highway of known width is elevated above its inner edge.

However, unlike the standard gauge railway track, with its definite and unchanging distance between the two rails, highways do not necessarily have a standard uniform width.

Therefore, it is more common to state the **transverse inclination** of a road surface, expressed as either an angle from the horizontal or a percentage gradient.



GIVE FEEDBACK

OK

Type your answer in the box.

Highways do not necessarily have a standard uniform width. Therefore it is more common to state the

of a road surface, expressed as either an angle from the

horizontal or a percentage gradient.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



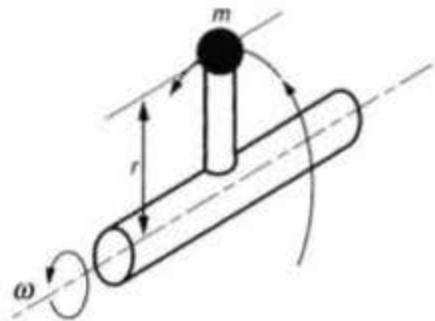
Centripetal force is the force applied to a moving mass which causes the mass to travel in a circular path.

Whenever a centripetal force exists, there must always be an equal and opposite centrifugal reaction force, which is directed radially outwards from the centre of the circular path and experienced by another component in the interacting system of physical objects.



How the out-of-balance centrifugal force affects the efficient running of rotating machinery

If a mass m is attached to a rotating shaft at a distance r from the axis of rotation, as shown in the figure below, there will be an out-of-balance centrifugal force acting on the shaft away from its axis. Such an out-of-balance effect can be severely detrimental to the efficient running of rotating machinery, resulting in excessive vibrations and wear.



GIVE FEEDBACK

OK

Which of the following is a typical effect of a mass m attached to a rotating shaft at a distance r from the axis of rotation?

Check **all** that apply.

- ☐ An out-of-balance centrifugal force acting on the shaft away from its axis
- ☐ Severe detriment to the efficient running of rotating machinery
- ☐ Excessive vibrations and wear
- ☐ An out-of-balance centripetal force acting on the shaft away from its axis
- ☐ Improved efficiency in the running of rotating machinery
- ☐ Minimal change to the long-term operation of the rotating machine

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

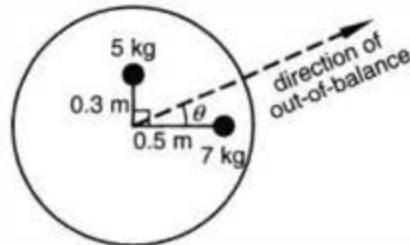
When several masses are made to revolve in one plane about a common axis of rotation, the resultant pull on the axis is found by vectorial addition of their individual centrifugal effects, as illustrated by the following example.

[GIVE FEEDBACK](#)[CONTINUE >](#)

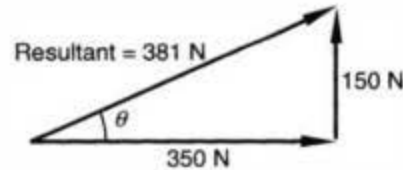
Example

A turntable revolving about its centre at 95.5 rpm has a metal object of mass 5 kg attached at a radius of 0.3 m, and another object of mass 7 kg attached at a radius of 0.5 m. The angle between the objects is 90° , as shown in the figure below.

Determine the magnitude and direction of the resultant force due to the out-of-balance centrifugal forces.



(a)



(b)

Solution

$$\begin{aligned}\text{Angular velocity} &= 95.5 \cdot \frac{2\pi}{60} \\ &= 10 \text{ rad/s}\end{aligned}$$

Hence the individual centrifugal forces are as follows:

(i) For the 5 kg mass:

$$\begin{aligned}F_c &= m \omega^2 r \\ &= 5 \cdot 10^2 \cdot 0.3 \\ &= 150 \text{ N}\end{aligned}$$

(ii) For the 7 kg mass:

$$\begin{aligned}F_c &= m \omega^2 r \\ &= 7 \cdot 10^2 \cdot 0.5 \\ &= 350 \text{ N}\end{aligned}$$

With the two forces acting at 90° to each other, the resultant centrifugal force is found by constructing the right-angled triangle to scale (figure (b)) or solving it by Pythagoras's rule.

Magnitude:

$$\sqrt{150^2 + 350^2} = 381 \text{ N}$$

Direction:

$$\begin{aligned}\tan \theta &= \frac{150}{350} \\ &= 0.4286 \\ \text{therefore } \theta &= \tan^{-1} 0.4286 \\ &= 23.2^\circ\end{aligned}$$

Therefore the resultant out-of-balance force of 381 N makes an angle of 23.2° with the radial position of the 7 kg mass.

It is possible to balance any arrangement of out-of-balance masses revolving in the same plane by a single balancing mass, provided it is positioned in the direction exactly opposite to that of the resultant out-of-balance force.

The size of the balancing mass and its distance from the axis of rotation must be selected so that the centrifugal force due to the balancing mass would be equal in magnitude to the resultant out-of-balance force.

< BACK

GIVE FEEDBACK

OK

Calculating the out-of-balance centrifugal force—Example 2

Calculate the out-of-balance centrifugal force acting on a shaft which is rotating at 700 rpm and carries a 0.6 kg mass at a radial distance of 250 mm from its axis.

In the case of a single mass, the centrifugal reaction force is exactly equal in magnitude to the centripetal force acting on the out-of-balance mass.

$$\begin{aligned}\text{Angular velocity} &= 700 \cdot \frac{2\pi}{60} \\ &= 73.3 \text{ rad/s}\end{aligned}$$

Hence the centrifugal force is:

$$\begin{aligned}F_c &= m \omega^2 r \\ &= 0.6 \cdot 73.3^2 \cdot 0.25 \\ &= 806 \text{ N}\end{aligned}$$



GIVE FEEDBACK



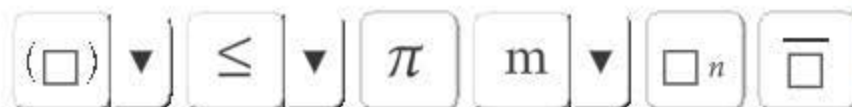
OK

Calculate the out-of-balance centrifugal force acting on a shaft that is rotating at 1000 rpm and carries a 300 g mass at a radial distance of 40 cm from its axis.

(Answer to the nearest newton.)



Clear



Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

A turntable revolving around its centre at 60 rpm has a metal object of mass 8 kg attached at a radius of 25 cm, and another object of mass 4 kg attached at a radius of 40 cm. The angle between the objects is 90° .

SMALL

MEDIUM

LARGE



Type your answer in the box.

The angular speed is equal to radians per second. (Answer correct to three decimal places.)

The centrifugal force acting on the 8 kg mass is newtons. (Answer to the nearest newton.)

The centrifugal force acting on the 4 kg mass is newtons. (Answer to the nearest newton.)



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A turntable revolving around its centre at 60 rpm has a metal object of mass 8 kg attached at a radius of 25 cm, and another object of mass 4 kg attached at a radius of 40 cm. The angle between the objects is 90° .

SMALL

MEDIUM

LARGE



Type your answer in the box.

The resultant out-of-balance force of N (to the nearest newton) makes an angle of $^\circ$ (to one decimal place) with the radial position of the 4 kg mass.



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

It is possible to balance any arrangement of out-of-balance masses revolving in the same plane by a single balancing mass, provided it is positioned in the direction exactly opposite to that of the resultant out-of-balance force.

The size of the balancing mass and its distance from the axis of rotation must be selected so that the centrifugal force due to the balancing mass would be equal in magnitude to the resultant out-of-balance force.

Example

Determine the required magnitude of the balancing mass which is to be placed at the rim of a turntable at a radial distance from its centre equal to 1.19 m. The turntable has a resultant out-of-balance force of 381 N and an angular speed of 10.0 radians per second.

[GIVE FEEDBACK](#)[CONTINUE >](#)

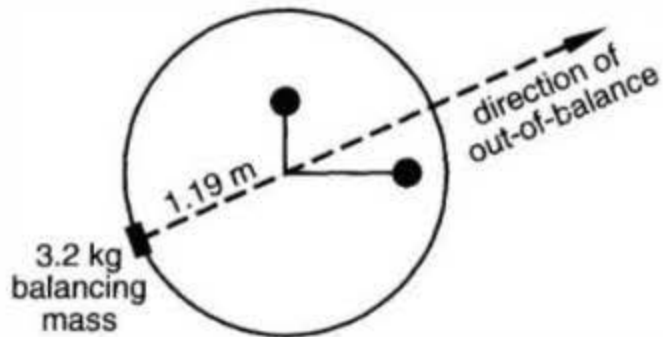
Solution

The resultant out-of-balance force is 381 N. Therefore for the balancing mass:

$$F_c = m \omega^2 r$$

$$381 = m \cdot 10^2 \cdot 1.19$$

$$\text{therefore } m = 3.2 \text{ kg}$$



< BACK

GIVE FEEDBACK

OK

Determine the required magnitude of the balancing mass that is to be placed at the rim of a turntable at a radial distance from its centre equal to 55 cm.

The turntable has a resultant out-of-balance force of 220 N and an angular speed of 120 rpm.

(Answer in kg correct to four significant figures).

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(\square)	▼	≤	▼	π	m	▼	\square_n	$\overline{\square}$?	Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER