GIVE FEEDBACK

CONTINUE >

If you already have an understanding of linear motion, you should notice some similarities in the equations and laws of motion.

Flywheels, pulleys, shafts and gears are some examples of rotary motion.

Rotation or rotary motion involves the movement of a body around a fixed axis.

The body as a whole does not move from one place to another, but every particle of the body, except the axis, travels along a circular path.

Rotational motion can be described by suitable application of the equations and laws of motion.

< BACK

GIVE FEEDBACK

OK

Linear motion can be described as motion in which the body moves from one place to another. Compare this to the definition of rotation shown here.

Define rotation

Rotation, or rotary motion, is motion during which a body turns around a fixed axis in such a way that every particle of the body, except the axis, travels along a circular path.

In rotary motion, the body as a whole doesn't move from one place to another.

Common examples of rotation are flywheels, pulleys, shafts, gears and turbine rotors.

GIVE FEEDBACK

OK

Which of the following are examples of rotational motion?

Check all that apply.

- [ ] A bicycle wheel rotating
- [ ] A spinning top
- [ ] A car turning a corner
- [ ] A runner running on an oval track
- [ ] A ceiling fan in operation

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Highlight examples of rotary motion in the following list.

Click to highlight.

A person running in a park; a car driving around a corner; a clothesline spinning in the wind; a bicycle being ridden down a hill; a motorcycle wheel spinning as the motorcycle is ridden.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Rotational motion of an object, such as a flywheel, can be described in terms of its orientation in space and time.

More specifically, rotation is usually described in terms of angular displacement, angular velocity and angular acceleration.

Angular displacement

When a body undergoes rotational motion its orientation in space changes.

Angular displacement of a rotating object is a measure of its change of orientation with respect to a fixed radius as an arbitrary datum. This is usually denoted by the Greek letter θ (theta).

GIVE FEEDBACK

OK

Type your answer in the box.

Angular displacement of a rotating object is a measure of its change of ______ with respect to a fixed radius as an arbitrary ______.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The components of angular displacement

Angular displacement is a rotational quantity which has direction as well as magnitude.

It is usually convenient to measure angular displacement from the initial position of the object, in the direction of rotation which is assumed to be positive. In that case, angular displacement is simply the angle through which the object turns.

The magnitude of angular displacement can be measured in revolutions, degrees or radians.

Revolutions are the most convenient units for measuring angular displacement of mechanical components such as shafts and flywheels.

Radians are the units that must be used for angular displacement when calculations are performed.

GIVE FEEDBACK

OK

Type your answer in the box.

Angular displacement is a rotational quantity, which has ________ as well as magnitude.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In what units can the magnitude of angular displacement be measured?

Check all that apply.

- [ ] Revolutions
- [ ] Theta
- [ ] Degrees
- [ ] Metres per second
- [ ] Seconds
- [ ] Metres
- [ ] Radians

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Symbols and units used for angular displacement

When relating angular displacement to torque, work and power, it's necessary to convert to base units (radians) before any further calculations are performed.

Remember that one revolution contains 360 degrees or 2 π radians.

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ rad}$$

GIVE FEEDBACK

OK

Type your answer in the box.

Complete the following statement using the correct numbers.

2 revolutions = [ ] degrees = [ ] π radians

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The components of angular velocity

The rate at which a body changes its angular position is called its **angular velocity.** This is usually denoted by the symbol ω (omega, the last letter of the Greek alphabet).

Angular velocity is a quantity, with a magnitude and direction.

For rotational motion in a particular direction, the directional sense of angular velocity coincides with that of displacement and can be assumed to be positive.

The magnitude of angular velocity at any instant of time can be described as the angle turned through per unit time.

GIVE FEEDBACK

OK

The magnitude of angular velocity at any instant of time can be described as ______.

Click the correct answer.

- the angle turned through per unit time
- the distance turned through per unit time
- the angle turned through per unit distance
- the distance turned through per unit angle

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

The rate at which a body changes its angular position is called its ______ ______.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

It is normally necessary to convert angular velocity from revolutions per minute to radians per second, particularly if calculations involve other related concepts, such as angular acceleration, energy or power.

The symbols and units used for angular velocity

The base unit of angular velocity is **radian per second** (rad/s).

However, the most common practical unit of measurement used to express rotational speeds of mechanical components is **revolution per minute** (rpm).

> To convert revolutions per minute to radians per second, multiply speed in rpm by $2\pi/60$.

For example:

$$
\begin{aligned}
955\ \text{rpm} &= 955 \times \frac{2\pi}{60} \\
&= 100\ \text{rad/s}
\end{aligned}
$$

GIVE FEEDBACK

OK

What is the conversion factor for converting from revolutions per minute (rpm) to radians per second (rad/s)?

Click the correct answer.

- $2\pi/60$
- $2\pi/120$
- $60/2\pi$
- $120/2\pi$
- 120π

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

477 rpm = ______ rad/s (answer to the nearest whole number).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If angular velocity isn't constant, but is increasing gradually at a uniform rate, an object is said to be rotating with a **uniformly accelerated motion.**

The rate at which angular velocity is changing with time is called **angular acceleration**. The symbol used is α (alpha, the first letter of the Greek alphabet).

GIVE FEEDBACK CONTINUE >

This equation is similar to the equation for linear acceleration.

If, over a period of time equal to t seconds, angular velocity of an object changes from its initial value ω_0 to a final value ω, it follows from the definition that angular acceleration is the quotient of the increment of angular velocity $(\omega - \omega_0)$ and the time t.

This is shown mathematically as:

$$\alpha = \frac{\omega - \omega_0}{t}$$

Calculate the angular acceleration if a bicycle wheel's rotational speed changes from rest to 45 rad/s in 9 seconds.

(Answer in rad/s^2, type units).

Clear

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

Type your answer in the box.

The rate at which angular velocity is changing with time is called ________ ________ .

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The symbols and units used for angular acceleration

The unit of angular acceleration is the unit of angular velocity, rad/s, divided by the unit of time, s. In other words, it is radians per second squared: rad/s^2.

GIVE FEEDBACK

OK

What is the SI unit for angular acceleration?

Click the correct answer.

- rad/s^2
- rad/s
- rpm
- degrees/minute
- degrees/s

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If the angle turned through in each successive interval of time is the same, the angular velocity is said to be constant. Otherwise, average angular velocity can be calculated using total angular displacement and the time in which it occurs.

Example

A cam in a mechanism makes 500 revolutions in 2 minutes 37 seconds.

What is its average angular velocity in revolutions per minute and in radians per second?

GIVE FEEDBACK

CONTINUE >

Solution

$$\text{Total angular displacement: } \theta = 500 \text{ revolutions}$$
$$\text{Time: } t = 157 \text{ s}$$
$$= 2.617 \text{ min}$$

Average angular velocity:

$$\omega = \frac{\theta}{t}$$
$$= \frac{500 \text{ revolutions}}{2.617 \text{ min}}$$
$$= 191 \text{ rpm}$$

< BACK GIVE FEEDBACK CONTINUE >

Also equal to:

$$\omega = \frac{500 \times 2\,\pi \text{ rad}}{157 \text{ s}}$$
$$= 20 \text{ rad/s}$$

< BACK GIVE FEEDBACK OK

Type your answer in the box.

An automobile wheel completes 2000 revolutions in 5 minutes. Its average angular velocity is ______ radians per second? (Answer to two decimal places.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

An engine flywheel completes 5000 revolutions in 4 minutes 10 seconds. What is its average angular velocity in radians per second?

(Answer in rad/s^2 correct to two decimal places and include units in the answer).

+ − · ÷ $\frac{\square}{\square}$ $1\frac{2}{3}$ $\square^2$ $\sqrt{\square}$ Clear

$(\square)$ ▼ $\leq$ ▼ π ▼ $\square \times 10^{\square}$ s ▼ ? Undo

$\overline{\square}$

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

The relationship between the initial and final angular velocities, time and angular acceleration for uniformly accelerated rotational motion is usually stated as a formula in which final angular velocity is the subject.

The relationship between initial and final angular velocities, time and angular acceleration

$$\omega = \omega_0 + \alpha t$$

where:

ω is the final angular velocity in rad/s
ω_0 is the initial angular velocity in rad/s
α is the angular acceleration in rad/s^2
t is the time taken in s

Note the similarity between this relationship and that for linear velocity and acceleration: $v = v_0 + a t$.

GIVE FEEDBACK

OK

In the equation $\alpha = \frac{(\omega - \omega_0)}{t}$, the symbol ω represents ______.

Click the correct answer.

- angular velocity
- initial angular acceleration
- time
- angular acceleration

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match each of the quantities with its symbol in the equation $\alpha = \frac{(\omega - \omega_0)}{t}$.

Drag statements on the right to match the left.

Angular velocity	ω
Initial angular velocity	ω_0
Angular acceleration	α
Time	t

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Differences between uniformly accelerated rotation and uniformly decelerated rotation

If, instead of increasing, angular velocity is gradually decreasing, the rotation can be described as uniformly decelerated.

Angular deceleration (or retardation) is regarded as negative acceleration, i.e. angular acceleration acting in the direction opposite to angular velocity.

GIVE FEEDBACK

OK

Type your answer in the box.

Angular deceleration or retardation is regarded as ______ angular acceleration.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

A flywheel starts from rest and is accelerated at the rate of 2.4 rad/s^2 for 30 s. Determine the angular velocity reached after 30 s.

GIVE FEEDBACK

CONTINUE >

Calculate the angular acceleration of a moving body—Example 1

Example

A flywheel starts from rest and is accelerated at the rate of 2.4 rad/s^2 for 30 s. Determine the angular velocity reached after 30 s.

Solution

$$\text{Initial angular velocity: } \omega_0 = 0$$
$$\text{Time: } t = 30 \text{ s}$$
$$\text{Angular acceleration: } \alpha = 2.4 \text{ rad/s}^2$$

Substitute into $\omega = \omega_0 + \alpha t$:

$$\begin{aligned}\omega &= 0 + 2.4 \text{ rad/s}^2 \times 30 \text{ s} \\ &= 72 \text{ rad/s}\end{aligned}$$

Therefore, angular velocity after 30 s is 72 rad/s (= 687.5 rpm).

< BACK GIVE FEEDBACK OK

Example

If, after rotating for some time at a constant angular velocity of 72 rad/s, brakes are applied to the flywheel producing a retardation of 4 rad/s^2, determine the time taken to reduce its angular velocity to 40 rad/s (= 382 rpm).

GIVE FEEDBACK

CONTINUE >

Solution

$$\text{Initial angular velocity: } \omega_0 = 72 \text{ rad/s}$$
$$\text{Final angular velocity: } \omega = 40 \text{ rad/s}$$
$$\text{Angular acceleration: } \alpha = -4 \text{ rad/s}^2$$

Substitute into $\omega = \omega_0 + \alpha t$:

$$40 = 72 - 4t$$

Hence, time taken:

$$\begin{aligned} t &= \frac{72 - 40}{4} \\ &= 8 \text{ s} \end{aligned}$$

< BACK GIVE FEEDBACK OK

A bicycle wheel starts from rest and is accelerated at the rate of 0.4 rad/s/s for 2.5 minutes.

Determine the angular velocity, in radians per second, reached after 2.5 minutes.

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

If, after rotating for some time at a constant angular velocity of 60 rad/s, brakes are applied to the bicycle wheel producing a retardation of 1.2 rad/s/s, determine the time taken to bring the wheel to rest.

Clear

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

The four equations of rotational motion

Note very carefully how these equations compare with those for linear motion.

Any problem involving kinematics of rotational motion can be solved by using these equations. However, the use of appropriate units is extremely important. It's usually better to make all necessary unit conversions before substituting them into the appropriate equations.

$$\theta = t\left(\frac{\omega_0 + \omega}{2}\right)$$

$$\theta = \omega_0 t + \frac{\alpha t^2}{2}$$

$$\omega = \omega_0 + \alpha t$$

$$2\alpha\theta = \omega^2 - \omega_0^2$$

GIVE FEEDBACK

OK

Match the correct parts to show four equations of rotational motion.

Drag statements on the right to match the left.

2θ	$= t(\omega_0 + \omega)$
ω	$= \omega_0 + \alpha t$
θ	$= \omega_0 t + \frac{\alpha t^2}{2}$
$2\alpha\theta$	$= \omega^2 - \omega_0^2$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

There are two stages in this problem: motion with uniform angular velocity before the brake is applied; and uniformly decelerated motion that brings the flywheel to rest. Each stage can be considered separately and the results combined.

Example

A flywheel turns at a constant angular velocity of 150 revolutions per minute for 45 seconds, before a brake is used to bring it to rest with a retardation of 0.5 radians per second squared.

Determine the total time and the total angular displacement of the wheel.

GIVE FEEDBACK

CONTINUE >

Solution

(a) *Uniform motion*

Angular velocity (constant):

$$\begin{aligned} \omega &= 150\ \text{rpm} \\ &= 150 \times \frac{2\pi}{60} \\ &= 15.71\ \text{rad/s} \\ \text{Time: } t &= 45\ \text{s} \end{aligned}$$

Angular displacement:

$$\begin{aligned} \theta &= t \times \omega \\ &= 45\ \text{s} \times 15.71\ \text{rad/s} \\ &= 706.9\ \text{rad} \end{aligned}$$

This is equal to 112.5 revolutions.

< BACK GIVE FEEDBACK CONTINUE >

(b) *Decelerated motion*

Initial angular velocity: $\omega_0 = 15.71\ \text{rad/s}$

Final angular velocity: $\omega = 0$

Angular acceleration: $\alpha = -0.5\ \text{rad/s}^2$ (note the negative sign)

Substitute into $\omega = \omega_0 + \alpha t$ to find the time taken to bring the flywheel to rest:

$$0 = 15.71 - 0.5\,t$$
$$t = 31.4\ \text{s}$$

< BACK GIVE FEEDBACK CONTINUE >

Substitute into $\theta = t\left(\frac{\omega_0 + \omega}{2}\right)$ to find the angular displacement during the period of decelerated motion:

$$\begin{aligned}\theta &= 31.4\left(\frac{15.71 + 0}{2}\right)\\ &= 246.6 \text{ rad}\end{aligned}$$

This is also equal to 39.3 revolutions.

Combining the answers obtained in (a) and (b) yields:

$$\begin{aligned}\text{Total angular displacement} &= 706.9 + 246.6 \text{ rad}\\ &= 953.5 \text{ rad}\end{aligned}$$

$$\begin{aligned}\text{Total time taken} &= 45 \text{ s} + 31.4 \text{ s}\\ &= 76.4 \text{ s}\end{aligned}$$

$= 112.5 + 39.3$ revolutions

or $= 151.8$ revolutions

< BACK GIVE FEEDBACK OK

Determine the time taken to bring the flywheel to rest.

(Answer correct to one decimal place.)

+ − · ÷ Clear

Clear line

π Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Given that the time taken for the flywheel to come to rest is 139.6 s, calculate the angular displacement of the flywheel during braking.

(Answer in radians to the nearest whole number.)

+	−	·	÷	$\frac{\square}{\square}$	$\square^2$	$\sqrt{\square}$	Clear	
$(\square)$ ▾	≤ ▾	π	s ▾	$\square_n$	$\overline{\square}$	Clear line		
↵						?	Undo	

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

An engine flywheel turns at a constant angular velocity of 2000 revolutions per minute (209 radians per second) for 1 minute before a brake is used to bring it to rest with a retardation of 1.5 radians per second squared.

SMALL MEDIUM LARGE

Determine the time taken to bring the flywheel to rest.

(Answer correct to one decimal place.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required.
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

An engine flywheel turns at a constant angular velocity of 2000 revolutions per minute (209 radians per second) for 1 minute before a brake is used to bring it to rest with a retardation of 1.5 radians per second squared.

SMALL MEDIUM LARGE

Given that the time taken for the flywheel to come to rest is [illegible] s, calculate the angular displacement of the flywheel during braking.

(Answer in radians to the nearest whole number.)

Done

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required.
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

HELP

The analogy between rotation and linear motion is not limited to the relationships between displacement, velocity and acceleration. It can also be extended to the study of kinetics of rotation, that is, the causes of rotational motion.

Explain torque

Rotation, once started, tends to continue with constant angular velocity unless an outside influence acts to increase or decrease the velocity.

In rectilinear motion, force is the external agent which acts to change the conditions of rest, or uniform linear motion, of a body.

In rotational kinetics, **torque**, T is the analogue of force which influences change in the state of rest or rotational motion of a component capable of rotating about its axis.

GIVE FEEDBACK

OK

Type your answer in the box.

Rotation, once started, tends to continue with constant angular velocity unless a ________ acts to increase or decrease the velocity.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The differences between torque and moment

The word 'torque' derives its origin from a Latin word for twisting, or the twisted necklace as worn by the ancient Gauls.

The concept of **torque** is closely related to the concept of **moment of a force about a point**. However, the terms 'torque' and 'moment' are not completely synonymous. There are several features which distinguish them from one another.

Torque is usually used to describe a sustained turning effort applied to mechanical components, such as gears, shafts and flywheels, in situations which involve a period of continuous rotation.

Moments, on the other hand, often refer to static forces. For example, forces acting on a beam resting on its supports, or forces that produce only a small amount of discontinuous rotation through a small angle, such as the movement of a foot pedal about its hinge.

GIVE FEEDBACK

CONTINUE >

Torque is always specified in relation to the geometrical axis of a mechanical component, around which the component physically rotates or can be expected to rotate.

Moment of a force can be calculated quite meaningfully about any convenient reference point, often in circumstances which don't involve any actual rotation around the chosen reference point.

When dealing with systems in which forces and motions don't all lie in the same plane, another theoretical distinction can be made: torque relative to a given axis is a scalar quantity whereas moment of a force is a vector.

< BACK GIVE FEEDBACK CONTINUE >

Unlike moment of a force, torque doesn't have to be readily identifiable with any particular single force located at a fixed distance from the axis of rotation.

For example, within an electric motor, magnetic forces acting on the rotor are being distributed. But to an outside observer, the effort produced by the output shaft just appears as a pure turning effort.

Despite what's been said about the differences between torque and moment, there is always a close relationship between torque and the forces acting on various rotating components, such as gears, sprockets and pulleys.

Torque	Moment
A sustained turning effort applied to mechanical components, such as gears, shafts and flywheels, in situations which involve a period of continuous rotation	Refers to static forces, such as those acting on a beam resting on its supports, or to forces that produce only a small amount of discontinuous rotation through a small angle, such as the movement of a foot pedal about its hinge
Always specified in relation to the geometrical axis of a mechanical component about which the component physically rotates, or can be expected to rotate	Can be calculated quite meaningfully about any convenient reference point, often in circumstances which don't involve any actual rotation about the chosen reference point
Doesn't have to be readily identifiable with any particular single force located at a fixed distance from the axis of rotation	Must be readily identifiable with a particular single force located at a fixed distance from the axis of rotation
Torque relative to a given axis is a scalar quantity	Moment of a force is a vector

Match the following to list correct statements about torque and moments.

Drag statements on the right to match the left.

Torque	often refers to static forces.
Torque	is defined as a pure turning effort.
Torque	refers to forces that produce only a small amount of rotation, through a small angle and of discontinuous nature.
Moment	is always specified in relation to the geometrical axis of a mechanical component about which the component physically rotates, or can be expected to rotate.
A moment	of a force can be calculated quite meaningfully about any convenient reference point.
A moment	is usually used to describe a sustained turning effort.

The SI unit of torque

Torque is described as a pure turning effort.

As such, torque is similar in its effect to the turning moment of a force couple, and is measured in the same units, newton metres (N.m), or their multiples or submultiples, such as kilonewton metres (kN.m).

In fact, if all action and reaction forces acting on the component are properly identified and taken into account, torque is always a result of a force couple, or a cumulative effect of several force couples.

Examples of the application of a force couple include a wheel spanner, a tap wrench and a wingnut.

GIVE FEEDBACK

OK

What is the SI unit for torque?

Click the correct answer.

- newton metre
- newton per metre
- newton
- kilonewton
- metre

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

The newton metre is the SI unit for ______.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A gearbox consists of two spur gears in mesh. The larger gear has a 200 mm pitch circle diameter and the pinion (the smaller gear) has a 50 mm pitch circle diameter.

Note: Mating gears have their pitch circles tangent. The **pitch circle diameter** is the diameter of the circle which, by a pure rolling action, would transmit the same motion as the actual gear wheel.

The input shaft transmits a torque through the pinion of 160 N.m.

Determine the tangential force between the two gears, and the value of output torque.

Solution

Input torque T_{in} = 160 N.m. This produces a force between the two gears which makes the moment of the force about the centreline of the pinion equal to the transmitted torque:

$$\begin{aligned} \text{Force} \times \text{radius} &= \text{torque} \\ F \times 0.025\ \text{m} &= 160\ \text{N.m} \\ F &= \frac{160\ \text{N.m}}{0.025\ \text{m}} \\ &= 6{,}400\ \text{N} \\ &= 6.4\ \text{kN} \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

But the same force—or, to be precise, its equal and opposite reaction force—is also applied to the larger gear at a distance of 0.1 m from its centreline, producing a turning moment of:

$$\begin{aligned} M &= F \times d \\ &= 6{,}400\ \mathrm{N} \times 0.1\ \mathrm{m} \\ &= 640\ \mathrm{N.m} \end{aligned}$$

This is transmitted as torque through the output shaft:

$$\text{i.e. } T_{out} = 640\ \mathrm{N.m}$$

< BACK GIVE FEEDBACK OK

Type your answer in the box.

A gearbox consists of two spur gears in mesh. The larger gear has a 225 mm pitch circle diameter and the pinion, i.e. the smaller gear, has a 50 mm pitch circle diameter. The force on each pinion gear tooth is equal and opposite to the force on the corresponding larger gear tooth.

The input shaft transmits a torque through the pinion of 100 N.m. The value of output torque is

________ N.m.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A gearbox consists of two spur gears in mesh. The larger gear has a 125 mm pitch circle diameter and the pinion, i.e. the smaller gear, has a 25 mm pitch circle diameter. The force on each pinion gear tooth is equal and opposite to the force on the corresponding larger gear tooth.

The input shaft transmits a torque through the pinion of 80 N.m.

Determine the value of output torque.

+ − · ÷ Clear

? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

What is the correct definiton of pitch circle diameter?

Click the correct answer.

- The diameter of the circle which, by a pure rolling action, would transmit the same motion as the actual gear wheel
- The diameter of a circular pitch
- The diameter of the circle which, by application of torque, would transmit the same pitch as the actual gear wheel
- The diameter of the circular pitch which transmits the same motion as the input torque

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The mass moment of inertia is the property of every physical body that determines its resistance to a change in its state of rest or uniform rotational motion.

Mass moment of inertia (moment of inertia)

Rotation of a body tends to continue with constant angular velocity unless an outside, unbalanced torque acts to increase or decrease the velocity.

Therefore, it can be concluded that every physical body possesses a property of rotational inertia. This determines its resistance to a change in its state of rest or uniform rotational motion, i.e. its resistance to angular acceleration.

The correct technical term to describe this property is **mass moment of inertia**, often abbreviated to **moment of inertia,** with the symbol I.

Alternatively, J is sometimes used as a symbol for mass moment of inertia. Both are acceptable symbols according to the International Standards Organisation.

GIVE FEEDBACK

OK

What terms can be correctly applied to the property of every physical body that determines its resistance to a change in its state of rest or uniform rotational motion?

Check all that apply.

- [ ] Mass moment of inertia
- [ ] Moment of inertia
- [ ] Inertia
- [ ] Rotational inertia
- [ ] Rotational resistance
- [ ] Torque resistance

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Mathematically, mass moment of inertia of a body with respect to its axis of rotation is a function of mass distribution in the body relative to the axis.

The moment of inertia as a function of mass distribution in a body relative to its axis

A disc and a cylinder are fundamentally the same geometrical shape. A disc is a relatively thin, flat, round object of cylindrical shape, and a cylinder can be seen as a disc of relatively small diameter that's elongated in the direction of its axis.

The most common elementary shape for a rotating object is a disc (flywheel) or a cylinder (shaft), for which the mass moment of inertia about the axis is expressed as:

$$I = \frac{m r^2}{2}$$

where:

m is mass in kg
r is radius in m

It follows that the SI unit of mass moment of inertia is kg.m^2.

GIVE FEEDBACK

OK

State the equation for the moment of inertia as a function of mass distribution in a body relative to its axis.

+ − · ÷ $\frac{\square}{\square}$ $\square^2$ $\sqrt{\square}$ Clear

$(\square)$ ▼ $\leq$ ▼ π $\overline{\square}$? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

Match each of the symbols from the equation $I = \frac{1}{2} m r^2$ with its correct quantity and SI unit.

Drag statements on the right to match the left.

I	Moment of inertia in $kg.m^2$
m	Mass in kg
r	Radius in m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the mass moment of inertia for objects consisting of disc and cylindrical elements

Example

Calculate the mass moment of inertia of a flywheel in the form of a 650 mm diameter disc, 70 mm thick, with a mass of 185 kg.

GIVE FEEDBACK

CONTINUE >

Example

Calculate the mass moment of inertia of a flywheel in the form of a 650 mm diameter disc, 70 mm thick, with a mass of 185 kg.

Solution

$$\text{Moment of inertia } I = \frac{m r^2}{2}$$

$$= 185\,\text{kg} \times \frac{(0.325\,\text{m})^2}{2}$$

$$= 9.77\,\text{kg.m}^2$$

< BACK GIVE FEEDBACK OK

Calculate the mass moment of inertia of a flywheel in the form of a 300 mm diameter disc, 25 mm thick, having a mass of 50 kg. (Answer to four decimal places.)

Clear

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

Type your answer in the box.

The mass moment of inertia of an 80 kg flywheel with a diameter of 600 mm is [ ] $kg\ m^2$.

(Answer to one decimal place.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

For a rigid body rotating about a fixed axis, the laws of motion have the same form as those for rectilinear motion, with torque replacing force, mass moment of inertia replacing mass and angular acceleration replacing linear acceleration. The relationship between mass moment of inertia of a rotating body, the torque acting on it and the angular acceleration produced by the torque is identical to that between mass, force and acceleration in linear motion.

The mathematical relationship between the mass moment of inertia of a rotating body, the torque acting on it and angular acceleration produced by the torque

If rotational terms are substituted instead of linear terms, Newton's second law equation becomes:

$$T = I \times \alpha$$

where:

T is net unbalanced torque, in N.m
I is mass moment of inertia, in $kg.m^2$
α is angular acceleration, in rad/s^2

GIVE FEEDBACK

OK

Match the symbol from the equation $T = I\alpha$ with the correct quantity and SI unit.

Drag statements on the right to match the left.

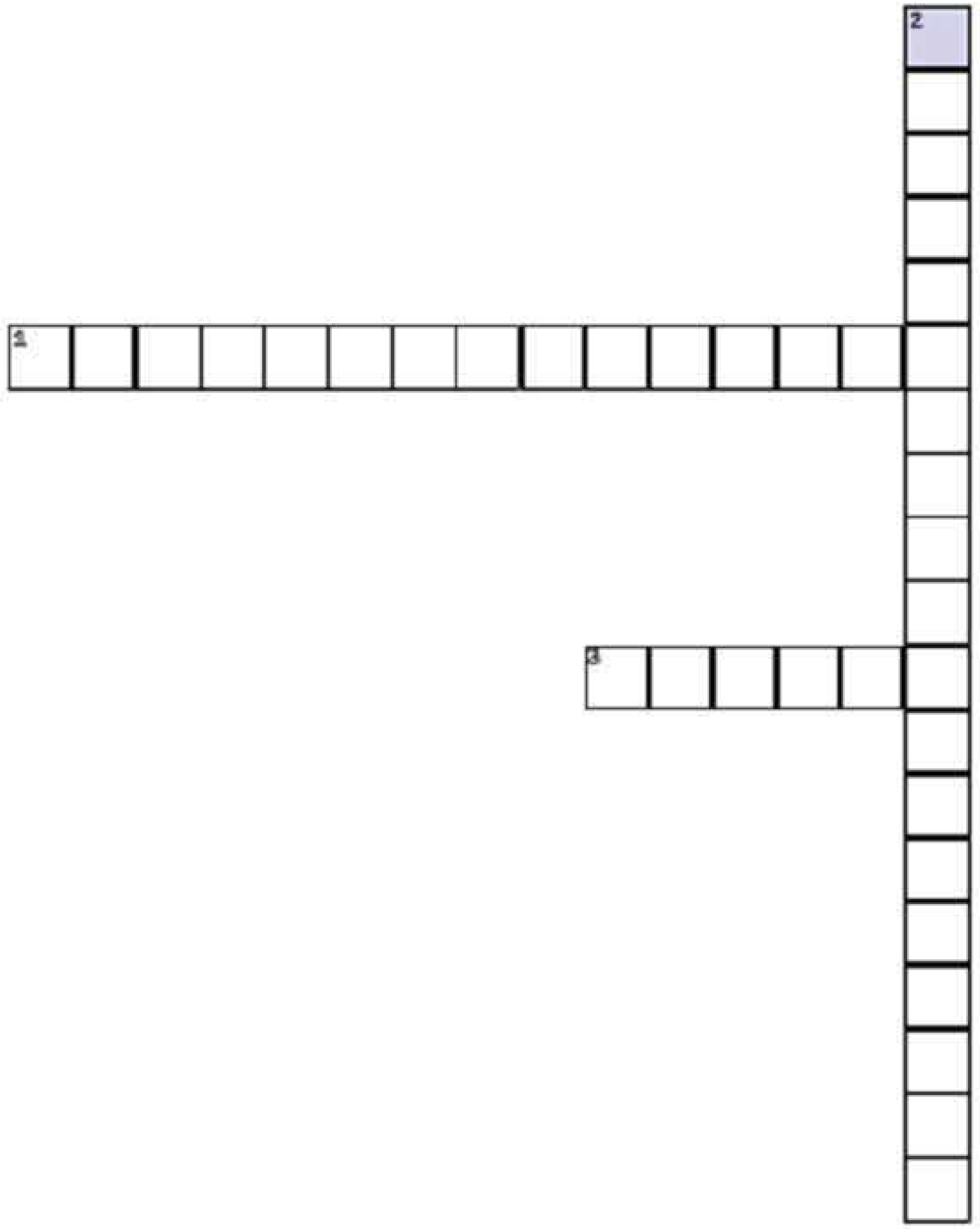
T	Torque in N.m
I	Moment of Inertia in $kg.m^2$
α	Angular acceleration in rad/s^2

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Torque equation

1



1) Denoted by the symbol I in the equation T=Iα (3 words)

2) Denoted by the symbol α in the equation T=Iα (two words)

3) Denoted by the symbol T in the equation T=Iα

Done Hint Challenge

Problems involving mass moments of inertia

Determine the net torque required to give a flywheel with a mass moment of inertia of 0.75 kg.m^2 an angular acceleration of 16 rad/s^2.

Example 1	Solution 1	Example 2	Solution 2	Example 3	Solution 3 Part A	Solution 3 Part B

$$
\begin{aligned}
T &= I \times \alpha \\
&= 0.75\,\text{kg.m}^2 \times 16\,\text{rad/s}^2 \\
&= 12\,\text{N m}
\end{aligned}
$$

Determine the torque required to accelerate a turbine rotor undergoing a dynamic balancing test, from rest to a speed of 15 000 rpm in 80 s, if the mass moment of inertia of the rotor is $11.5\,\text{kg.m}^2$.

Example 1 | Solution 1 | Example 2 | Solution 2 | Example 3 | Solution 3 Part A | Solution 3 Part B

GIVE FEEDBACK

OK

The angular acceleration required is found from $\omega = \omega_0 + \alpha t$, where $\omega = 15{,}000 \times \frac{2\pi}{60} = 1{,}571$ rad/s:

$$\omega = \omega_0 + \alpha t$$
$$1{,}571 = 0 + \alpha \times 80$$
$$\therefore \alpha = 19.63 \text{ rad/s}^2$$

Therefore, torque required is:

$$\begin{aligned} T &= I \times \alpha \\ &= 11.5 \text{ kg.m}^2 \times 19.63 \text{ rad/s} \\ &= 225.8 \text{ N.m} \end{aligned}$$

Example 1	Solution 1	Example 2	Solution 2	Example 3	Solution 3 Part A	Solution 3 Part B

GIVE FEEDBACK

OK

Problems involving mass moments of inertia

Determine the angular acceleration of a flywheel in the form of a disc 400 mm in diameter, with a mass of 60 kg, if the applied torque is 24 N.m.

Example 1	Solution 1	Example 2	Solution 2	Example 3	Solution 3 Part A	Solution 3 Part B

GIVE FEEDBACK

OK

The mass moment of inertia of the flywheel is:

$$I = \frac{m\,r^2}{2}$$
$$= \frac{60\text{ kg} \times (0.2\text{ m})^2}{2}$$
$$= 1.2\text{ kg.m}^2$$

Substituting into $T = I \times \alpha$ yields:

$$24\text{ N.m} = 1.2\text{ kg.m}^2 \times \alpha$$

Example 1 | Solution 1 | Example 2 | Solution 2 | Example 3 | Solution 3 Part A | Solution 3 Part B

GIVE FEEDBACK

OK

Therefore, the angular acceleration is:

$$\alpha = \frac{24\,\text{N.m}}{1.2\,\text{kg.m}^2}$$
$$= 20\,\text{rad/s}^2$$

Example 1 | Solution 1 | Example 2 | Solution 2 | Example 3 | Solution 3 Part A | Solution 3 Part B

GIVE FEEDBACK

OK

Type your answer in the box.

The net torque required to give a flywheel with a mass moment of inertia of 0.6 kg.m^2 an angular acceleration of 9 rad/s is ______ N.m.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the angular acceleration of a flywheel in the form of a disc 500 mm in diameter and having a mass of 80 kg, if the applied torque is 43 N.m.

(Answer in rad/s/s to one decimal place and include the units in your answer.)

+	−	·	÷	$\frac{\square}{\square}$	$\square^2$	$\sqrt{\square}$	Clear
$(\square)$ ▼	≤ ▼	π	s ▼	$\overline{\square}$	?	Undo	

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

Determine the torque required to accelerate an engine flywheel from rest to a speed of 6000 rpm in 10 s if the mass moment of inertia of the flywheel is 5 kg.metres squared. (Answer in N.m correct to one decimal place.)

$+$	$-$	$\cdot$	$\div$	$\frac{\square}{\square}$	$\square^2$	$\sqrt{\square}$	Clear
$(\square)$ ▼	$\leq$ ▼	π	m ▼	$\overline{\square}$	?	Undo	

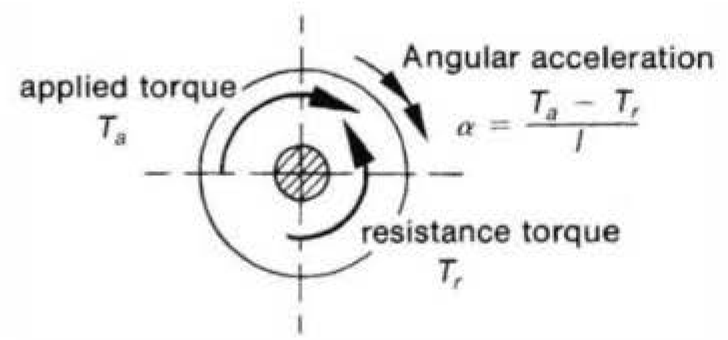
Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

In the equation $T = I \times \alpha$, the torque T is net accelerating torque, i.e. the result of all torques applied to the body.

The resultant unbalanced torque (the net accelerating torque) is the difference between the applied turning effort T_a and the resistance torque T_r.

$$T = T_a - T_r$$



The resistance is usually due to friction in the bearings, axle friction, etc.

Acceleration produced by the resultant torque is found from $T = I \times \alpha$:

$$\alpha = \frac{T}{I} = \frac{T_a - T_r}{I}$$

It follows from this equation that if the applied torque T_a is equal to the resistance torque T_r, there will be no acceleration. The rotation, if any, will continue at constant angular velocity.

Therefore, a flywheel rotating at a constant speed requires a torque equal to the friction torque to maintain uniform rotation.

Any torque in excess of friction torque will accelerate the wheel. On the other hand, an applied torque which is less than friction torque will result in retardation.

Example

A flywheel of mass moment of inertia equal to 53 $kg.m^2$ is rotating at 300 rpm. The frictional resistance is 16 N.m.

Calculate the torque that must be applied in order to accelerate the wheel to 500 rpm in 15 s.

Solution

Angular velocity can be calculated using $\omega = \omega_0 + \alpha t$, where:

$$\begin{aligned} \omega &= 500 \times \frac{2\pi}{60} \\ &= 52.36 \text{ rad/s} \\ \omega_0 &= 300 \times \frac{2\pi}{60} \\ &= 31.42 \text{ rad/s} \end{aligned}$$

Substituting:

$$\begin{aligned} 52.36 &= 31.42 + \alpha \times 15 \\ \alpha &= 1.4 \text{ rad/s}^2 \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

The net accelerating torque required to accelerate the wheel is:

$$\begin{aligned} T &= I \times \alpha \\ &= 53\,\text{kg.m}^2 \times 1.4\ \text{rad/s}^2 \\ &= 74\,\text{N.m} \end{aligned}$$

Therefore, the total torque that must be applied is the sum of the accelerating torque and resistance torque:

$$\begin{aligned} T_a &= 74\,\text{N.m} + 16\,\text{N.m} \\ &= 90\,\text{N.m} \end{aligned}$$

< BACK GIVE FEEDBACK OK

Type your answer in the box.

The torque that must be applied in order to accelerate the wheel to 800 rpm in 12 seconds is
______ N.m. (Answer to the nearest whole number.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A flywheel of mass moment of inertia equal to 90 $kg.m^2$ is rotating at 500 rpm.

The frictional resistance is 14.2 N.m.

SMALL MEDIUM LARGE

Type your answer in the box.

The angular acceleration if the flywheel accelerates to 800 rpm in 12 seconds is ______ rad/s^2. (Answer to two decimal places.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A flywheel of mass moment of inertia equal to 90 $kg.m^2$ is rotating at 500 rpm.

The frictional resistance is 14.2 N.m.

SMALL MEDIUM LARGE

Type your answer in the box.

The torque that must be applied in order to accelerate the wheel to 800 rpm in 12 seconds is ________ N.m. (Answer to the nearest whole number.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

The angular acceleration if the flywheel accelerates to 800 rpm in 12 seconds is [ ] rad/s^2. (Answer to two decimal places.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Most common types of rotating solids

Mass moment of inertia of rigid bodies

Mass moment of inertia of a rigid body about its axis of rotation is defined mathematically as the sum of the products of the mass elements of the body and the squares of their perpendicular distances from the axis.

Expressions for the mass moments of inertia of the most common rotating solids (such as a disc, a solid cylinder and a hollow cylinder), as well as for a mass concentrated at a fixed distance from the centre of rotation, have been derived by integration and are readily available.

GIVE FEEDBACK

OK

Type your answer in the box.

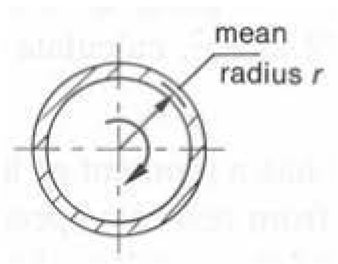
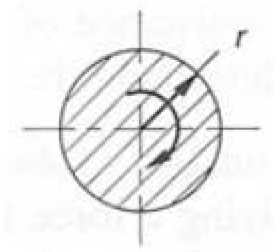
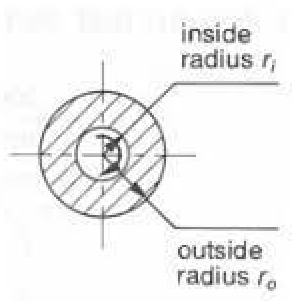
Mass moment of inertia of a rigid body about its axis of rotation is defined mathematically as the sum of the products of the ________ elements of the body, and the squares of their perpendicular ________ from the axis.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Some examples of mass moments of inertia of rigid bodies about the axis of rotation are shown here.

Expressions for the mass moments of inertia for the different types of rotating solids

Rotating body		Mass moment of inertia, I ($kg.m^2$)
Point mass at radius r		$m r^2$
Thin shell or hoop about its axis		$m r^2$
Solid cylinder about its axis		$\frac{m r^2}{2}$
Hollow cylinder about its axis		$\frac{m}{2}(r_o^2 + r_i^2)$
m = total mass		
r = radius		

GIVE FEEDBACK

OK

Match each of the rotating bodies to the correct equation for calculating its mass moment of inertia.

Drag statements on the right to match the left.

Point mass at a radius of r	$I = m r^2$
Solid cylinder or disc	$I = 0.5 m r^2$
Hollow cylinder	$I = 0.5 (r_o^2 + r_i^2)$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the mass moments of inertia if mass of the body is not given using volume and density

Density of a substance is the mass of unit volume of that substance, expressed in units such as kilograms per cubic metre.

When the mass of the body isn't given, it has to be calculated as a product of **volume** and **density** before standard formulae can be used (mass = volume x density).

Therefore, mass moment of inertia of a rotating component depends not only on its geometrical shape and size, but also on the density of the material it's made from.
The density of a material is usually given the symbol ρ (the lowercase Greek letter rho).

GIVE FEEDBACK

OK

What characteristics of a rotating component determine its mass moment of inertia?

Check all that apply.

- [ ] Geometrical shape
- [ ] Size
- [ ] Density of the material
- [ ] Applied torque
- [ ] Rotational speed
- [ ] Angular acceleration applied

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate mass moment of inertia for a rotating solid

Example

Determine the mass moment of inertia of a flywheel in the form of a disc, 50 mm wide x 300 mm diameter, if the material is steel (density of steel (ρ) is 7800 kg/m^3).

Example	Solution Volume of material	Solution Mass of flywheel	Solution Mass moment of inertia

GIVE FEEDBACK

OK

Calculate mass moment of inertia for a rotating solid

The volume of the disc is calculated by multiplying the depth and the area of the base of the disc.

Solution

Volume of material:

$$
\begin{aligned}
V &= l \times \frac{\pi D^2}{4} \\
&= 0.05 \times \frac{\pi 0.3^2}{4} \\
&= 0.00353\ \mathrm{m}^3
\end{aligned}
$$

Example	Solution Volume of material	Solution Mass of flywheel	Solution Mass moment of inertia

GIVE FEEDBACK

OK

Calculate mass moment of inertia for a rotating solid

The mass of the disc is calculated by multiplying the volume and density of the disc.

Mass of flywheel:

$$V \times \rho = 0.003\,53 \times 7800$$
$$= 27.57 \text{ kg}$$

Example	Solution Volume of material	Solution Mass of flywheel	Solution Mass moment of inertia

GIVE FEEDBACK

OK

Calculate mass moment of inertia for a rotating solid

When calculating the mass moment of inertia, the radius must be used. Remember that the radius is half of the diameter.

Moment of inertia (disc):

$$
\begin{aligned}
I &= m\frac{r^2}{2} \\
&= 27.57 \times \frac{0.15^2}{2} \\
&= 0.3101 \text{ kg.m}^2 \\
&\approx 0.310 \text{ kg.m}^2
\end{aligned}
$$

Example	Solution Volume of material	Solution Mass of flywheel	Solution Mass moment of inertia

GIVE FEEDBACK

OK

Determine the mass moment of inertia of a metal roller in the form of a disc, 400 mm wide x 300 mm diameter, if the material is steel (density of steel is 7800 kg per cubic metre).

(Answer correct to three decimal places.)

+ − · ÷ $\frac{\square}{\square}$ $\square^2$ $\sqrt{\square}$ Clear

$(\square)$ ≤ π $\square \times 10^{\square}$ m Clear line

? Undo

$\overline{\square}$ ↵

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Type your answer in the box.

The mass moment of inertia of a 2 kg metal roller with a diameter of 200 mm is ______ $kg \cdot m^2$.

(Answer correct to two decimal places.)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The transfer formula

For any element of mass in a composite body which isn't concentric with the common axis of rotation, the moment of inertia of the element about the common axis has to be calculated using the **transfer formula**:

$$I = I_c + m d^2$$

where:

I is the moment of inertia of the element about the common axis, in kg.m^2
I_c is the moment of inertia of the element about its own centroidal axis, in kg.m^2
m is the mass of the element, in kg
d is the perpendicular distance of the element from the common axis, in m

GIVE FEEDBACK

OK

Match each of the symbols from the transfer formula ($I = I_c + m\,d^2$) with the correct description.

Drag statements on the right to match the left.

I	The moment of inertia of the element about the common axis, in $kg.m^2$
I_c	The moment of inertia of the element about its own centroidal axis, in $kg.m^2$
m	The mass of the element, in kg
d	The perpendicular distance of the element from the common axis, in m

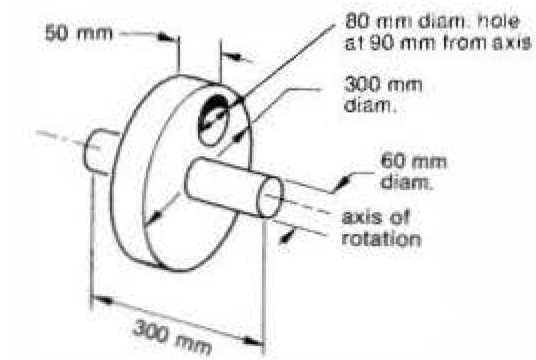
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If any mass is removed from a solid body, for example by drilling a hole, the mass removed and its corresponding moment of inertia are regarded as negative, as will be seen from part (c) of the following example.

Example

Determine the mass moment of inertia of the flywheel shown in the figure below, if the material is steel.



GIVE FEEDBACK

CONTINUE >

Solution

There are three components to be considered: the disc, shaft and hole.

(a) *Disc*

The mass moment of inertia can be calculated using volume and density to calculate the mass, and then $I = 0.5\,m\,r^2$:

$$I_{\text{disc}} = 0.3101\ \text{kg.m}^2$$

< BACK GIVE FEEDBACK CONTINUE >

(b) *Shaft*

The length of the shaft not included as part of the disc is 250 mm.

Volume:

$$\begin{aligned} V &= l\,\frac{\pi D^2}{4} \\ &= 0.25 \times \frac{\pi 0.06^2}{4} \\ &= 0.000707 \text{ m}^3 \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

Mass:

$$\begin{aligned} V \times \rho &= 0.000707 \times 7{,}800 \\ &= 5.51\ \text{kg} \end{aligned}$$

Mass moment of inertia:

$$\begin{aligned} I &= m\frac{r^2}{2} \\ &= 5.51 \times \frac{0.03^2}{2} \\ &= 0.0025\ \text{kg.m}^2 \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

(c) *Hole*

This can be regarded as a solid disc removed, therefore all values here are regarded as negative.

Volume:

$$V = l\frac{\pi D^2}{4}$$

$$= 0.05 \times \frac{\pi 0.08^2}{4}$$

$$= 0.000251 \text{ m}^3 \text{(negative)}$$

Mass:

$$V \times \rho = 0.000251 \times 7{,}800$$

$$= 1.96 \text{ kg (negative)}$$

< BACK GIVE FEEDBACK CONTINUE >

Centroidal mass moment of inertia:

$$I_c = m\frac{r^2}{2}$$
$$= 1.96 \times \frac{0.04^2}{2}$$
$$= 0.00157 \text{ (negative)}$$

Transfer term:

$$md^2 = 1.96 \times 0.09^2$$
$$= 0.0159 \text{ (negative)}$$

Mass moment of inertia about common axis of rotation:

$$I = I_c + md^2$$
$$= 0.00157 + 0.0159$$
$$= 0.0175 \text{ kg.m}^2 \text{ (negative)}$$

Therefore the total mass moment of inertia of the flywheel is:

$$\begin{aligned} I &= \Sigma(I) \\ &= 0.3101 + 0.0025 - 0.0175 \\ &= 0.2951 \text{ kg.m}^2 \end{aligned}$$

For convenience, this solution can also be tabulated as follows:

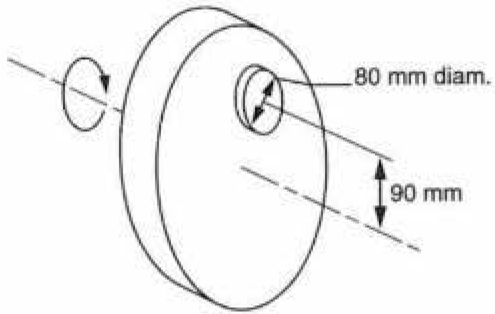
Element	m	d	I_c	$m d^2$	I
disc	27.57	—	0.3101	—	0.3101
shaft	5.51	—	0.0025	—	0.0025
hole	–1.96	0.09	–0.0016	–0.0159	–0.0175
					$\Sigma(I) = 0.2951$ kg.m^2

< BACK GIVE FEEDBACK OK

Example

A disc with a diameter of 80 mm and mass of 1.96 kg is attached to a flywheel so that it rotates at a distance of 90 mm from the axis of rotation of the wheel.

Calculate the contribution this disc makes to the total moment of inertia of the flywheel.



GIVE FEEDBACK CONTINUE >

Solution

Centroidal moment of inertia of the disc:

$$\begin{aligned} I_c &= \frac{m r^2}{2} \\ &= \frac{1.96\,\text{kg} \times (0.04\,\text{m})^2}{2} \\ &= 0.00157\,\text{kg.m}^2 \end{aligned}$$

Transfer term:

$$\begin{aligned} m d^2 &= 1.96\,\text{kg} \times (0.09\,\text{m})^2 \\ &= 0.01588\,\text{kg.m}^2 \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

Moment of inertia of the disc about the common axis of rotation:

$$
\begin{aligned}
I &= I_c + m\,d^2 \\
&= 0.00157 + 0.01588 \\
&= 0.01745\,\text{kg.m}^2
\end{aligned}
$$

Arrange in the correct sequence the steps that should be taken to determine the mass moment of inertia of a shaft mounted steel flywheel that has a hole drilled in it.

↑↓ Place these in the proper order.

- Sum the individual moments of inertia to determine the total mass moment of inertia of the flywheel assembly
- Calculate the volume of the hole using its dimensions, the mass displaced by the hole using volume and density, and the centroidal mass moment of inertia of the hole using $I_c = 0.5^2$
- Calculate the volume of the shaft using its dimensions, the mass of the shaft using volume and density, and the mass moment of inertia of the shaft using $I = 0.5\ mr^2$
- Calculate the volume of the flywheel disc using its dimensions, the mass of the flywheel disc using volume and density, and the mass moment of inertia of the flywheel disc using $I = 0.5\ mr^2$
- Calculate the transfer term for the hole using md^2
- Calculate the mass moment of inertia for the hole about the common axis of rotation using $I = I_c + md^2$