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Engineering
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Friction on an inclined plane is the next step from friction on a horizontal surface.

Now we will have to take gravity into account, which is trying to pull the object down the slope. The steepness of the slope and the amount of friction will determine whether the object slides down the plane or holds still.

In this chapter we look at friction on an inclined plane.

When the ground is on a slope, the object requires more force to pull it uphill than downhill.

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This type of problem can be extended to include other engineering designs such as screw threads, cams and wedges (which we study in later chapters).

We solve inclined friction two ways:

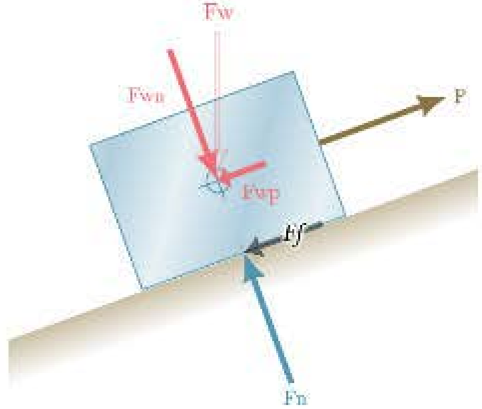
- By converting all forces to components parallel and perpendicular to the plane
- By solving the forces graphically using a force polygon

Here we are again with the familiar block-sliding-down-a-plane problem. As it turns out, the science behind it can be applied to many engineering uses, from screw jacks and power screws to wedges and cams.

A common problem involving friction is that of starting or maintaining motion of a body on a sloping surface connecting two different levels.

This may seem like a very specific case.

However, by studying an object moving on an inclined plane, we can use the same methods to analyse screw jacks, power screws, cams and wedges. They all operate on the principle of friction on an inclined plane.



GIVE FEEDBACK

CONTINUE >

Screw threads are an application of friction on an inclined plane.

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Screw jacks, power screws, cams and wedges all operate on the principles of:
(Select all correct answers)

Check all that apply.

- [ ] Friction on an inclined plane
- [ ] Zero friction
- [ ] Negative friction
- [ ] Equilibrium of forces

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Impending motion

We need to define impending motion. An object with sufficient friction has an impending motion due to applied force P, but motion is prevented by the friction force. Any increase in the applied force will cause the object to travel in the direction of impending motion.

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In the earlier study of friction on a level surface, the applied force attempted to move the block. If this force does not overcome the friction there is no motion at all, i.e. nothing happens.

However, the friction force was still resisting the attempted motion.

When the body is on the verge of sliding motion, the condition is referred to as **impending motion** and the coefficient of static friction applies.

GIVE FEEDBACK

OK

Impending motion means that the body is:

Check all that apply.

- [ ] On the verge of sliding
- [ ] Accelerating
- [ ] In equilibrium
- [ ] On a level surface

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

There are four possible modes for an object on an incline: it sits there motionless; it sits there with the maximum force it can handle without moving; it moves at constant velocity under an applied force; or it accelerates up or down.

The four possible situations of a block on an inclined plane

For a block on a horizontal plane the impending motion is in the direction of the force. On an inclined plane it is not so simple and there are four possible situations:

1. No applied force and friction is greater than gravity down the plane (equilibrium)
2. A force is applied and friction is equal to all other forces parallel to the plane (equilibrium)
3. Same as case 2 above, but with constant speed (equilibrium)
4. The force is such that the body accelerates (not in equilibrium)

We will study these four situations in more detail.

GIVE FEEDBACK

OK

For a body with friction on an inclined plane there are four possible situations.

Which of the following situations are in static equilibrium?

Check all that apply.

- [ ] No applied force and friction is greater than gravity force down the plane
- [ ] The body is stationary; a force is applied and friction is equal to all other forces parallel to the plane
- [ ] The body is moving; a force is applied and friction is equal to all other forces parallel to the plane
- [ ] The force is such that the body accelerates

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

For a body with friction on an inclined plane there are four possible situations.

Which of the following situations are in equilibrium?

Check all that apply.

- [ ] No applied force and friction is greater than gravity force down the plane
- [ ] The body is stationary; a force is applied and friction is equal to all other forces parallel to the plane
- [ ] The body is moving; a force is applied and friction is equal to all other forces parallel to the plane
- [ ] The force is such that the body accelerates (either up or down the inclined plane)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

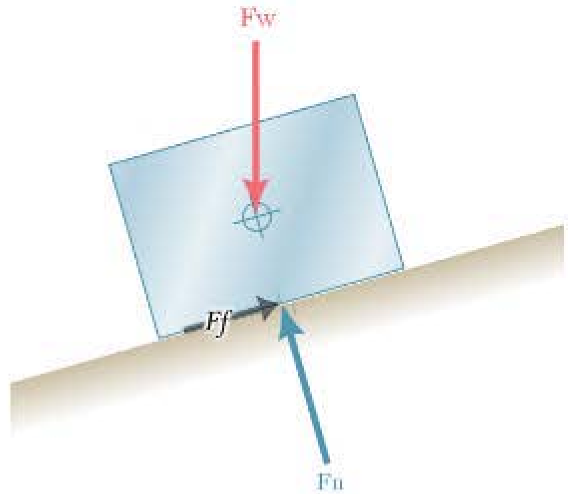
Describe hints for sketching a typical free body diagram of a body on an inclined plane

Recall that a free body diagram requires a body to be chosen. In this case our body is the block.

It is also important to consider all the forces as they are applied to the body.

In this simple diagram (without applied force P) the forces are:

F_w—the force of gravity to the block
F_n—the force of the plane to the block
F_f —the force of friction to the block



Forces on a block on an inclined plane

Revise free body diagrams	Components replace the forces	Balance the forces

GIVE FEEDBACK

OK

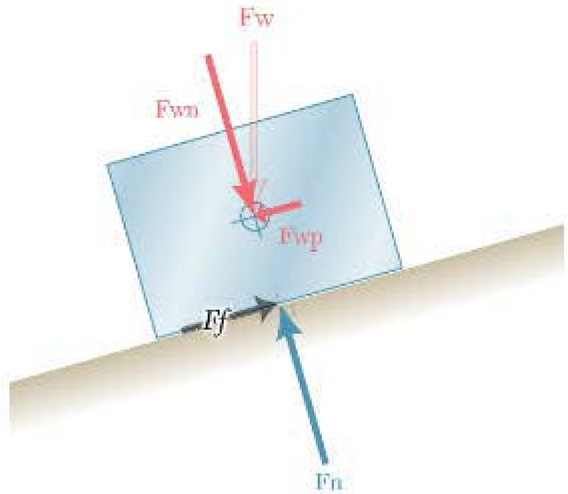
Describe hints for sketching a typical free body diagram of a body on an inclined plane

Take care that when components are used, they completely replace the original force.

This is why the original force F_w has been removed.

The two components are inserted in its place, where:

$$F_{wn} = F_w \cdot \cos\theta$$
$$F_{wp} = F_w \cdot \sin\theta$$



Components replace the original force

Revise free body diagrams	Components replace the forces	Balance the forces

GIVE FEEDBACK

OK

Describe hints for sketching a typical free body diagram of a body on an inclined plane

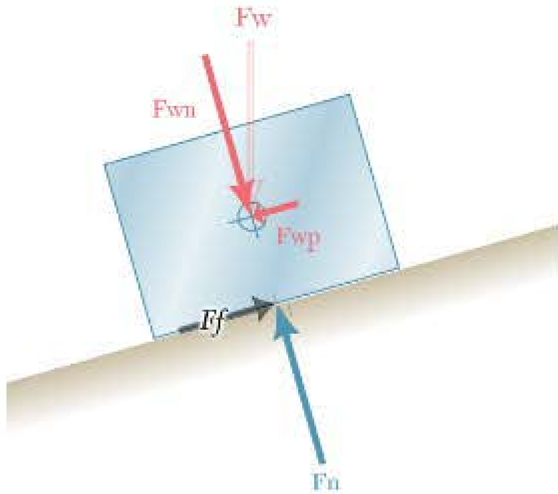
With components, all the forces in the free body diagram are:

F_n— the force of the plane to the block
F_f — the force of friction to the block
F_{wn}—the normal of gravity to the block
F_{wp}—the parallel of gravity to the block

Due to equilibrium, normal forces and parallel forces will balance:

$F_n - F_{wn} = 0$

$F_f - F_{wp} = 0$



Components applied to a free body diagram

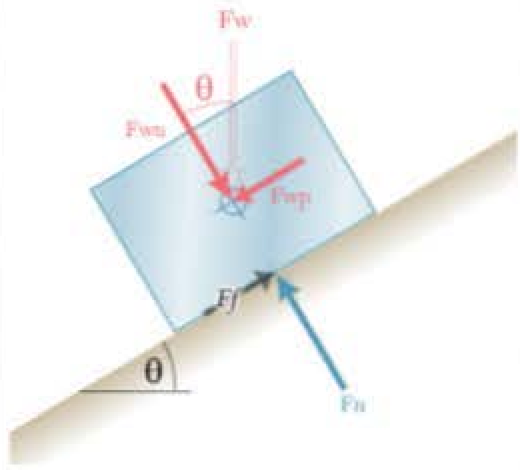
Revise free body diagrams	Components replace the forces	Balance the forces

GIVE FEEDBACK

OK

This FBD shows the original weight of the block converted to parallel and normal components.

Match the equations below to calculate the correct values for these components.



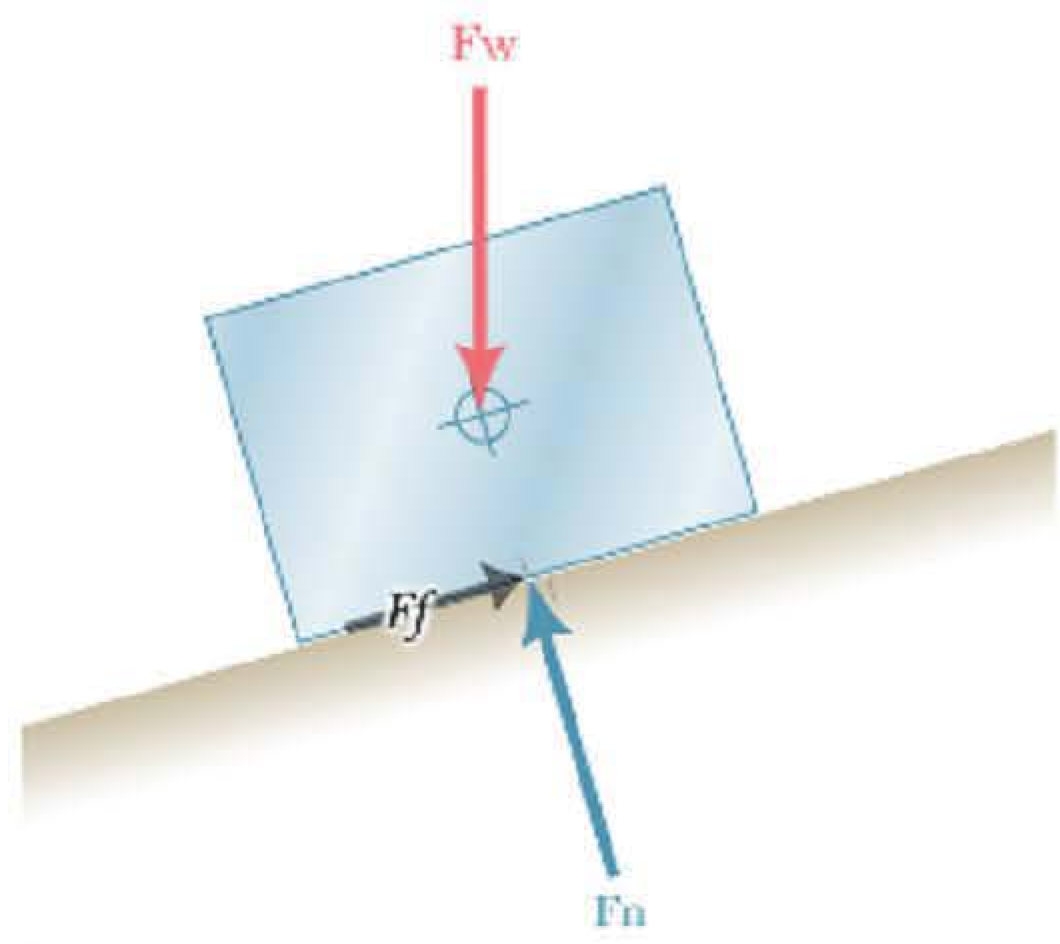
Drag statements on the right to match the left.

F_{wp}	$F_w \sin\theta$
F_{wn}	$F_w \cos\theta$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

According to the definition of a free body diagram of the block, which of the following forces are correctly described?



Check all that apply.

- [ ] F_f—the force of friction to the block
- [ ] F_n—the force of the plane to the block
- [ ] F_n—the force of the block to the plane
- [ ] F_f—the force of friction to the plane
- [ ] F_w—the force of gravity to the block

Understand how friction always resists impending motion

Impending motion up the incline can only occur if the force is high enough to (almost) overcome both weight and friction components. Impending means it doesn't quite get to move.

On the downhill, things are a little different and it depends on whether the block will slide on its own. If it can't, the applied force will have to be reversed to push the object downhill.

Friction always resists impending motion, not necessarily the direction of applied force.
In this example, the friction force is unable to hold the block on the plane.

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As usual, P must overcome both friction and weight force to pull uphill, but downhill motion also occurs while force P is uphill (but smaller).

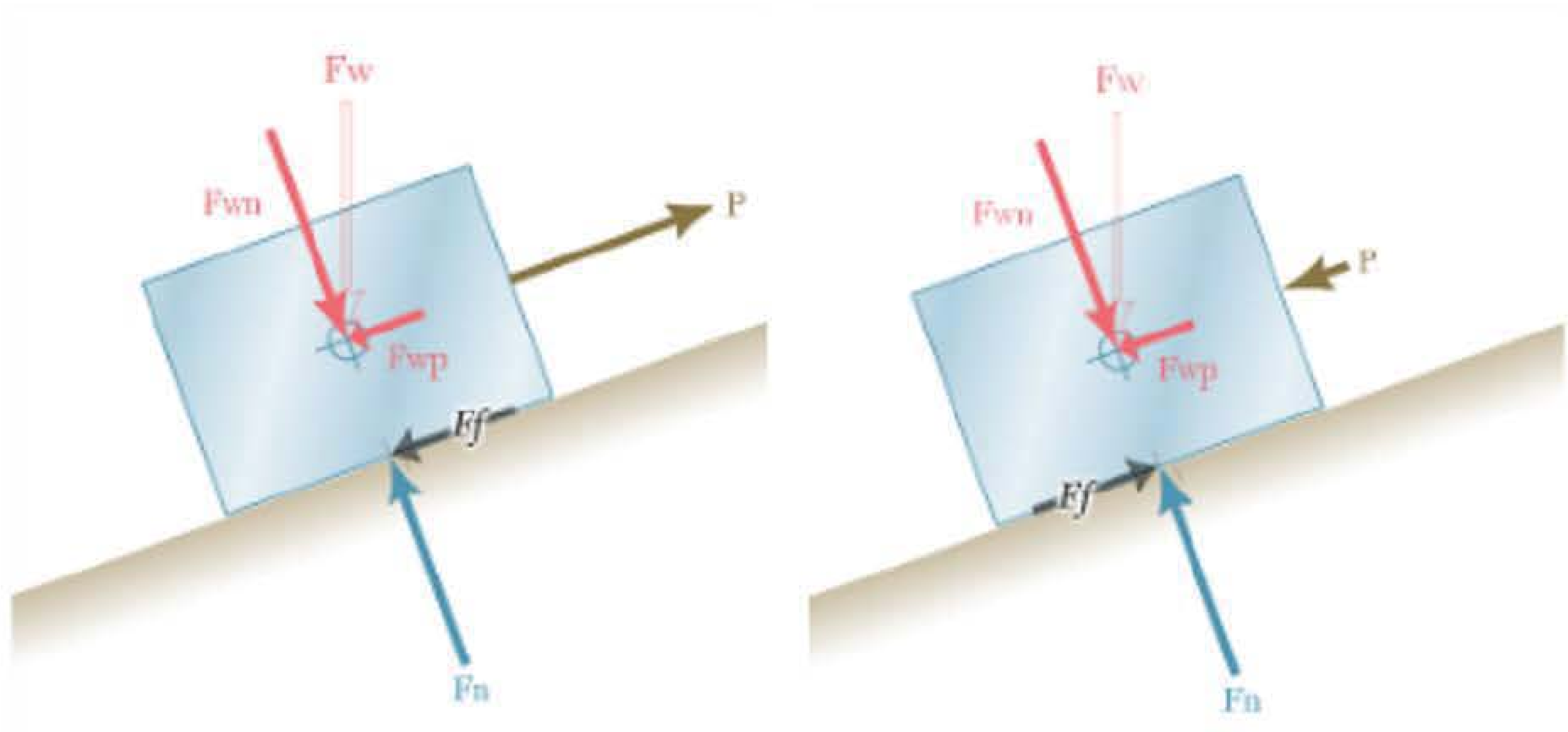
The three possibilities for the movement of the block on the incline are:

- $P > F_{wp} + F_f$ —moves uphill
- $P < F_{wp} - F_f$ —moves downhill
- $(F_{wp} - F_f) < P < (F_{wp} + F_f)$—stationary

GIVE FEEDBACK

OK

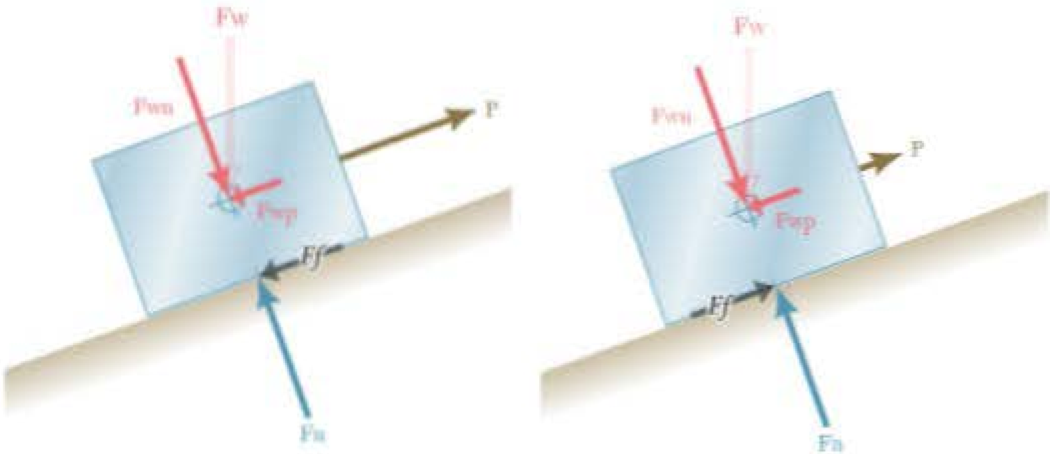
Without force P this block will remain stationary. Match the four possibilities for the movement of this block:



Drag statements on the right to match the left.

$P_{\text{uphill}} > F_{\text{wp}} + F_f$	The block remains stationary at the threshold of motion
$P_{\text{downhill}} > F_f - F_{\text{wp}}$	The block moves uphill
$P_{\text{downhill}} = F_{\text{wp}} + F_f$	The block remains stationary
$P_{\text{downhill}} = 0$	The block moves downhill

Without force P this block will slide downhill. Match the three possibilities for the movement of this block:



Drag statements on the right to match the left.

$P > F_{\text{wp}} + F_f$	The block moves uphill
$P < F_{\text{wp}} - F_f$	The block moves downhill
$(F_{\text{wp}} - F_f) < P < (F_{\text{wp}} + F_f)$	The block remains stationary

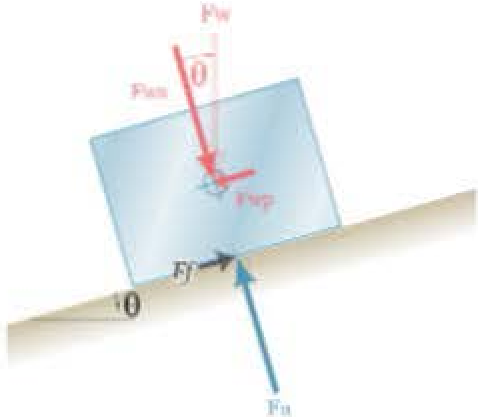
In the first of four possible situations, the block sits still on the incline without any applied force.

Describe a body on an inclined plane without an applied force

This is the first of four possibilities for the mechanics of friction on an inclined plane.

There is no external push or pull applied to the body, apart from the force of gravity, and there is sufficient friction to prevent sliding. The body remains stationary on the incline.

This situation occurs when the angle of inclination of the plane θ is less than the **angle of repose** (which is the steepest angle before motion occurs, e.g. a car safely parked on a gentle slope).



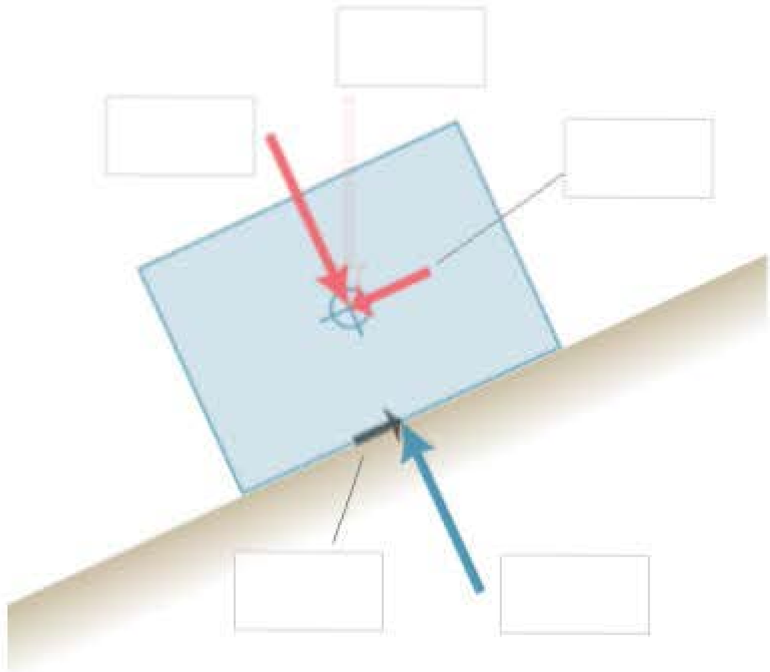
GIVE FEEDBACK

OK

Drag forces onto the free body diagram for a block accelerating down an incline.

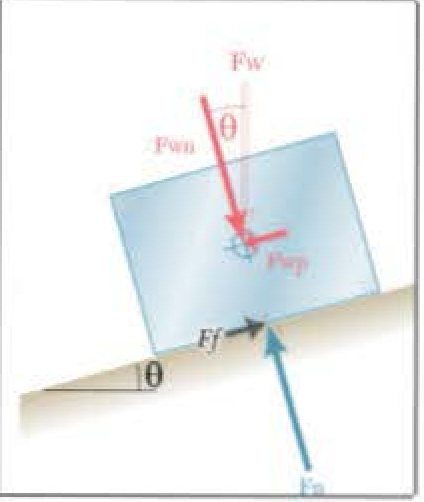
F_w | F_n | F_f

$F_w \cos \theta$ | $F_w \sin \theta$



Assume there is no external push or pull applied to the body, apart from the force of gravity.

For which of the following does the body remain stationary on the incline?

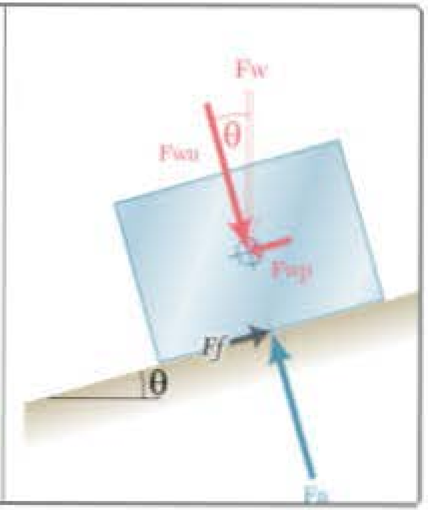


Check all that apply.

- [ ] The angle of inclination is less than the angle of friction
- [ ] The angle of inclination is less than the angle of repose
- [ ] The angle of inclination is greater than the angle of friction and $F_f < F_{wp}$
- [ ] The angle of inclination is greater than the angle of friction and $F_f = F_{wp}$

Assume there is no external push or pull applied to the body, apart from the force of gravity.

Select all true statements about the movement of the block.



Check all that apply.

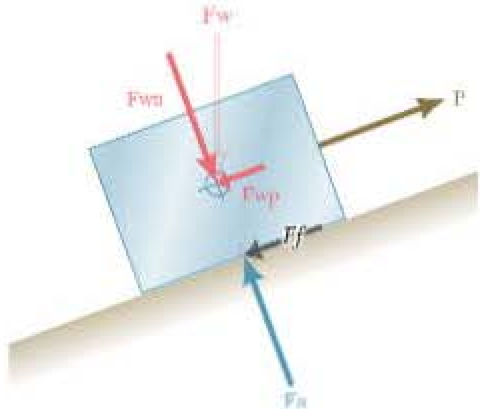
- [ ] The block slides whenever $F_{wp} > F_f$
- [ ] If F_w keeps increasing, there is a point where the block will start to slide
- [ ] Because there is no external force, the block will not slide since it is unable to overcome the friction force F_f
- [ ] As θ increases, so does F_{wp} but F_f will reach a limit and the block will slide

In the second of four possible situations, the maximum force is applied without the block moving.

Describe a block at static threshold under an applied force on an inclined plane

This is the second of four possibilities for friction on an inclined plane.

An external force P is applied to the body, to be just on the verge of overcoming static friction. The body is almost set in motion (i.e. **impending motion**).



All forces acting on the body are in static equilibrium and the law of friction applies, which gives the following three equations:

$\Sigma F_x = 0, \Sigma F_y = 0$ and $F_f = \mu F_n$

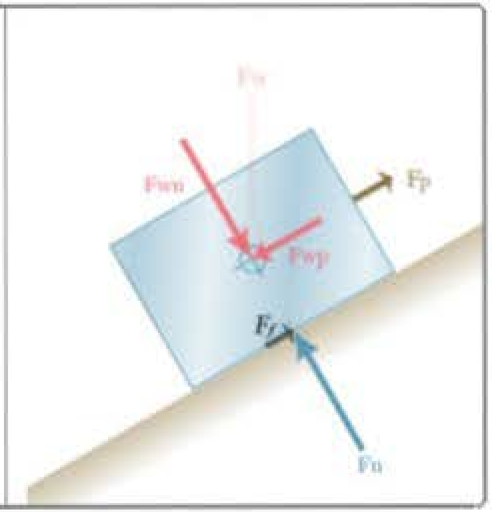
Note: Since it is not moving we use the static coefficient of friction.

GIVE FEEDBACK

OK

The applied force F_p has this body in impending downhill motion.

Which of the following statements are true?

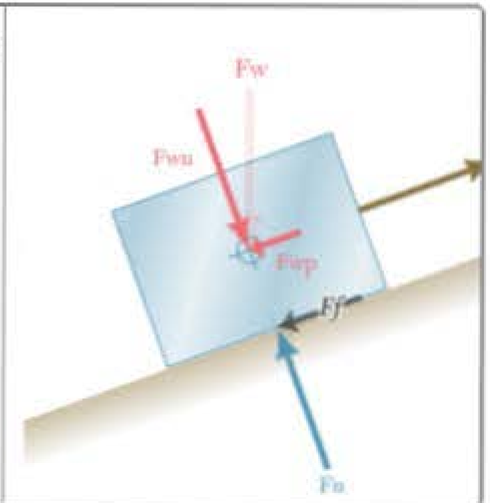


Check all that apply.

- [ ] All forces acting on the body are in static equilibrium
- [ ] The block is moving at constant velocity uphill
- [ ] A small increase in P will move the block uphill
- [ ] A small decrease in P will move the block downhill

Do you know the answer?

The applied force P has this body in impending uphill motion. Which of the following are true?



Check all that apply.

- [ ] All forces acting on the body are in static equilibrium
- [ ] The block is moving at constant velocity uphill
- [ ] A small increase in P will move the block uphill
- [ ] A small decrease in P will move the block downhill

Do you know the answer?

In the third of four possible situations, the block is moving at a constant speed.

Describe a block at constant velocity under an applied force on an inclined plane

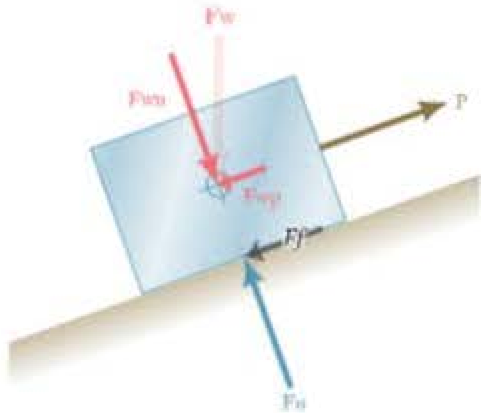
This is the third of four possibilities for friction on an inclined plane.

The body is moving along the plane with **uniform speed**, which may be up or down the plane.

This takes a slightly different force than was required to get it moving in the first place, since it depends on the value of the coefficient of **kinetic friction**.

In every other way this body is in dynamic equilibrium, giving the usual equations for static equilibrium.

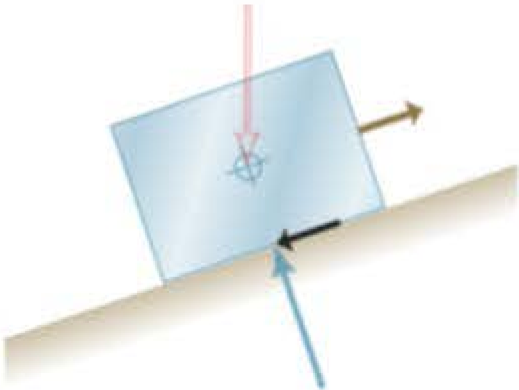
Note: In most calculations, we usually ignore any difference between static and kinetic coefficients of friction.



GIVE FEEDBACK

OK

If this block moving, is it going uphill or downhill?



Click the correct answer.

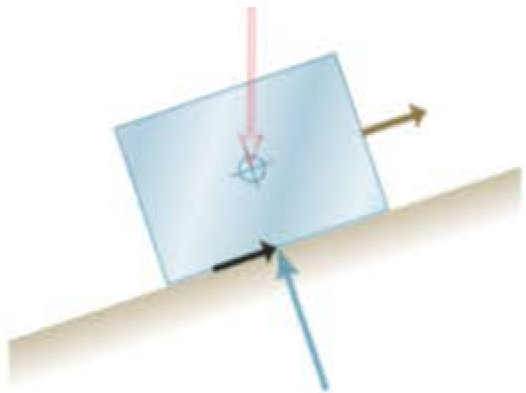
Downhill

Uphill

Either direction

Do you know the answer?

If this block is moving, is it going uphill or downhill?



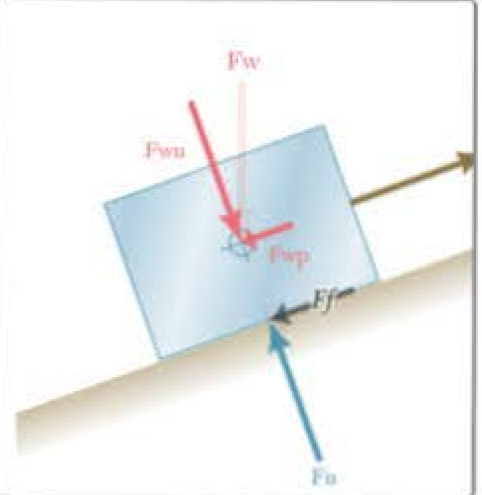
Click the correct answer.

- Downhill
- Uphill
- Either direction

Do you know the answer?

The body is moving up the plane with **uniform speed.**

If the kinetic coefficient is 80% of the static friction, which of the following are true?



Check all that apply.

- [ ] The static coefficient of friction is applied
- [ ] The kinetic coefficient of friction is applied
- [ ] Force P is lower than the force required to start the uphill motion
- [ ] Force P is higher than the force required to start the uphill motion

Do you know the answer?

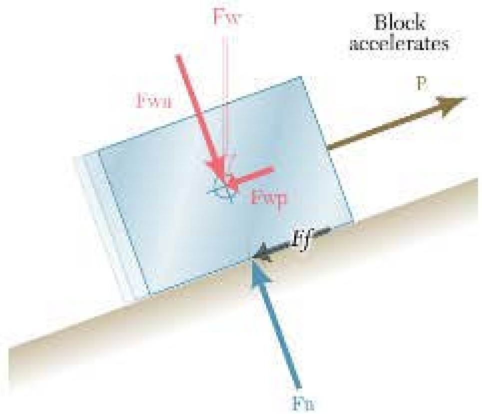
Describe a block under acceleration due to an applied force on an inclined plane

In the last of four possible situations, the block is accelerating.

This is the last of four possibilities for a body on an inclined plane.

The body is moving along the plane, assisted by an external force P which overcomes the frictional resistance, and a bit more. These forces are **not in equilibrium.**

This causes acceleration, so such problems cannot be solved by the methods of statics alone but rather using methods of dynamics.

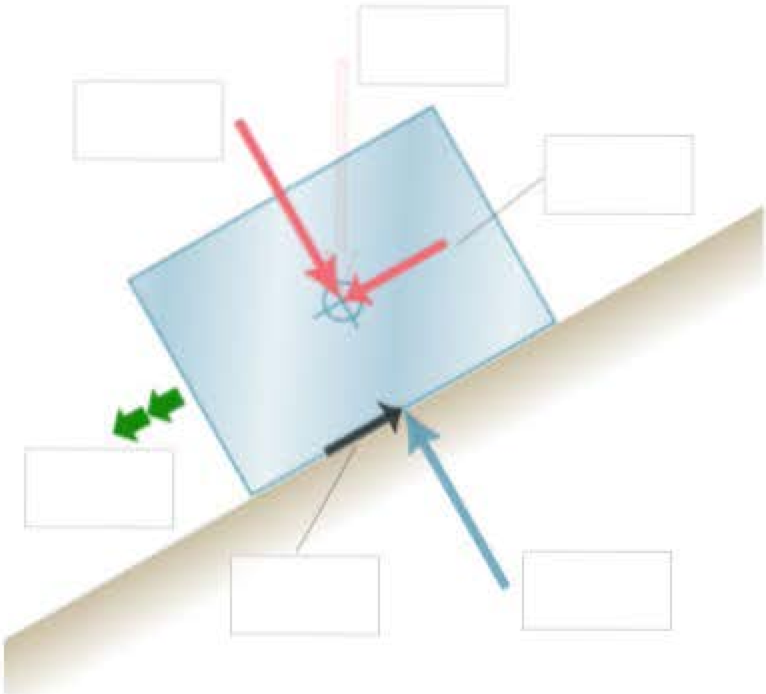


GIVE FEEDBACK

OK

Drag forces onto the free body diagram for a block accelerating down an incline.

F_w	F_n	F_f
a	$F_w \cos\theta$	$F_w \sin\theta$



Impending motion up the incline can only occur if the force is high enough to (almost) overcome both weight and friction components. Impending means it doesn't quite get to move. On the downhill, things are a little different and it depends on whether the block will slide on its own. If it can't, the applied force will have to be reversed to push the object downhill.

Difference between impending motion up or down an inclined plane

On an inclined plane there is always a parallel component of gravity pushing downhill F_{wp}.

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In response, the friction force F_f will push uphill to resist the **impending downward motion**.

However, as the applied force P increases, F_f will decrease, until it matches F_{wp}. At this point the block is perfectly balanced without any need for friction to hold it there.

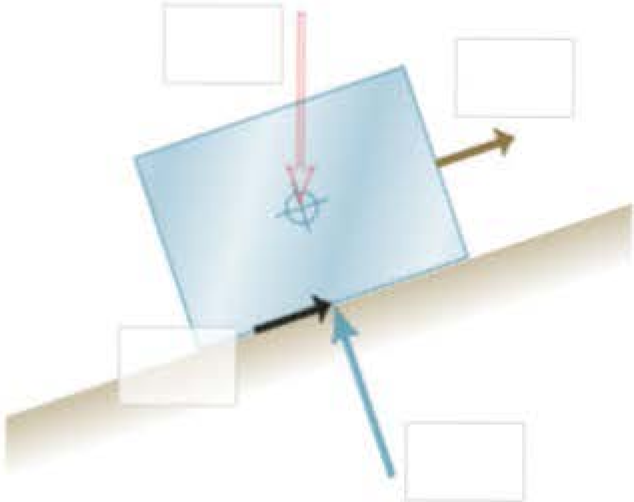
If we increase P some more, the **impending motion is up**, so F_f will push downhill.

GIVE FEEDBACK

OK

Drag the forces to their correct positions in the free body diagram.

F_w F_n F_p F_f



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Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Force P is applied to a block on an inclined plane. Which of the following statements about angles is true?	Video object isn't supported in c++ version

Click the correct answer.

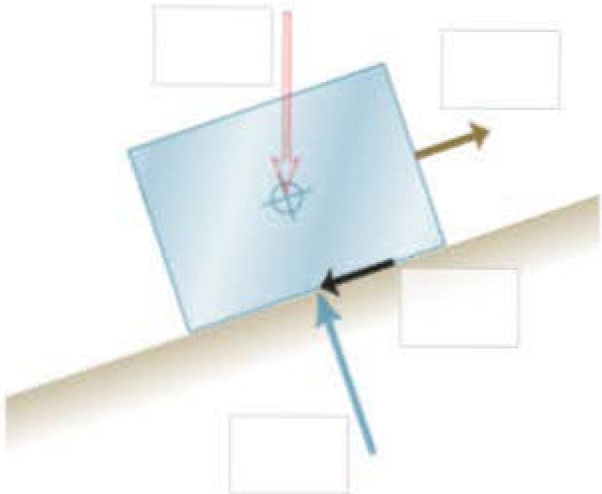
- Angle of inclination < angle of repose
- Angle of inclination > angle of repose
- Angle of inclination = angle of repose
- Angle of repose = 0

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Drag the forces to their correct positions in the free body diagram.

F_w	F_n	F_p	F_f



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Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

There are three ways to solve typical block-on-incline problems. The mathematical method, which uses force components; the graphical method that draws the forces to scale; and a semi-graphical method that also draws forces but solves them with trigonometry. There are positives and negatives for each method.

Three methods to solve problems involving friction and equilibrium of forces on an inclined plane

Problems involving friction and equilibrium of forces on an inclined plane can be solved by any one of three methods:

1. Mathematically by resolving all forces into parallel (p) and normal (n) components, and then solving the equilibrium equations in each of the two directions
2. Graphically by constructing a force triangle to scale representing the equilibrium of forces acting on the body
3. Semi-graphically by sketching the triangle of forces and then solving it using the sine rule

GIVE FEEDBACK

OK

Problems involving friction and equilibrium of forces on an inclined plane can be solved by any one of three methods.

Match the name of each method to a summary of its process:

Drag statements on the right to match the left.

Mathematically	Solve equilibrium equations in parallel (p) and normal (n) directions
Graphically	A force triangle drawn to scale
Semi-graphically	Solve using the sine rule

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Problems involving friction and equilibrium of forces on an inclined plane can be solved by any one of three methods. Match the names of each method.

Select... ▼ This involves resolving all forces into parallel (p) and normal (n) components, and then solving the equilibrium equations in each of the two directions.

Select... ▼ This involves constructing a force triangle to scale, representing the equilibrium of forces acting on the body.

Select... ▼ This involves sketching the triangle of forces and then solving it using the sine rule.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

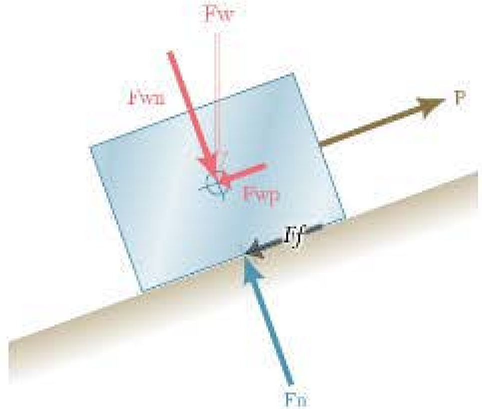
The mathematical method involves converting all forces to their parallel and perpendicular components measured from the incline. It is slow but well suited to computer automation.

Advantages of the mathematical method for solving friction

The mathematical method can solve any problem involving friction on an inclined plane.

It is the best choice for computer automation using spreadsheets or programs.

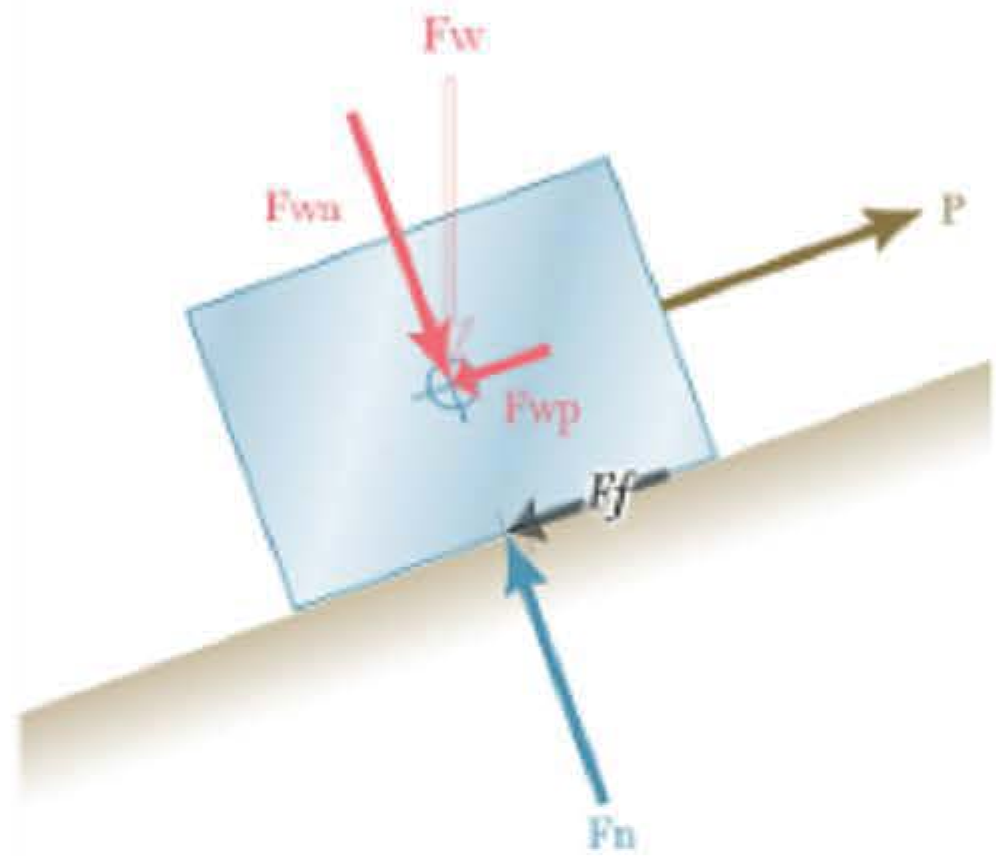
For hand calculations, the more complex problems can become mathematically challenging. For this reason we will limit this technique to situations where the applied force F_p is parallel to the support plane.



GIVE FEEDBACK

OK

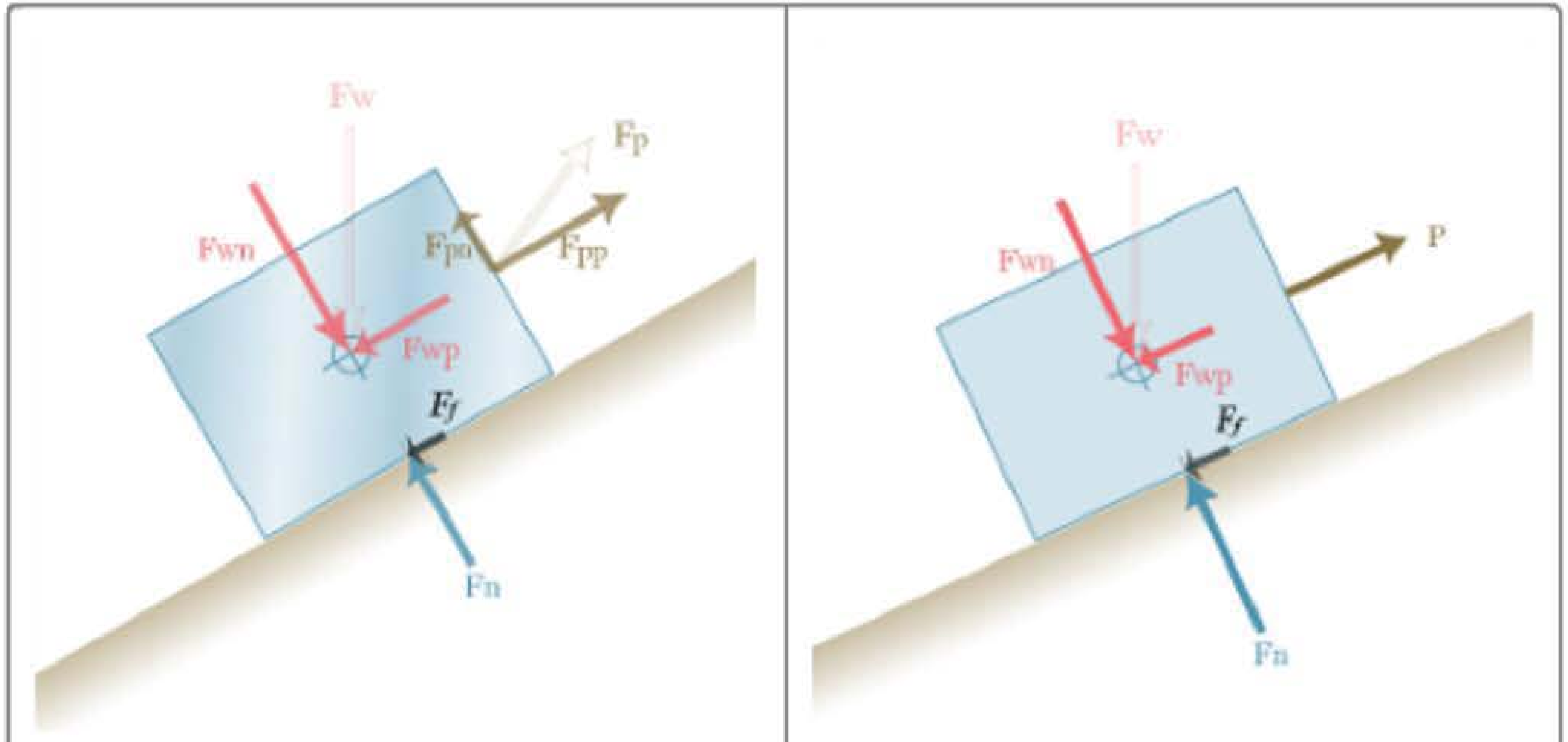
Which of the following are true for the mathematical method of solving friction?



Check all that apply.

- [ ] Can solve any problem involving friction on an inclined plane
- [ ] Is suitable for automating the calculations using computer programs
- [ ] Requires all forces to be resolved into components that are parallel and normal to the inclined plane
- [ ] Requires all forces to be resolved into horizontal and vertical (X and Y) components

In our study of friction on an inclined plane, the mathematical method will be restricted to applications where the applied force F_p is parallel to the support plane. Why?



Angled force F_p | Parallel force F_p

Click the correct answer.

- Because components cannot be used in this case
- Because the applied force cannot be any other angle
- Because this would give too many variables to solve using the equilibrium equations
- Because the mathematics gets too complicated

The forces components are solved by equilibrium equations along the parallel and perpendicular axes.

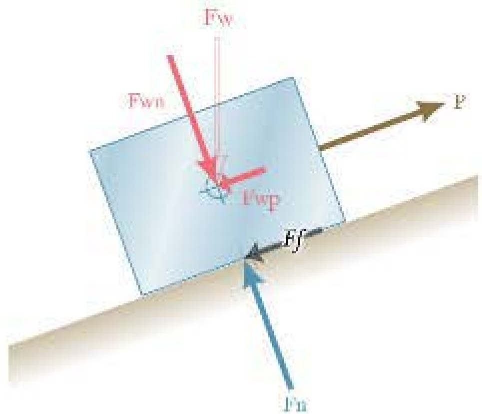
Three equations required for solving friction by the mathematical method

The mathematical method converts all forces into components parallel (p) and normal (n) to the inclined plane.

Since the body is in equilibrium, there are three equations to solve:

$$\Sigma F_p = 0$$
$$\Sigma F_n = 0$$
$$F_f = \mu F_n$$

Remember that these only apply where acceleration does not occur (all the forces balance).

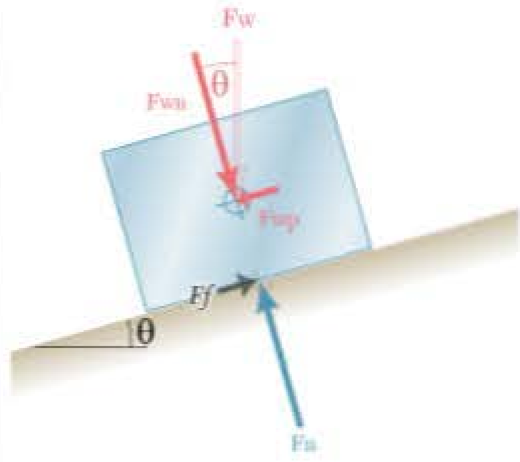


The mathematical method employs parallel and normal force components

GIVE FEEDBACK

OK

For a block in equilibrium, the friction force F_f can be determined
from the normal force F_n by:

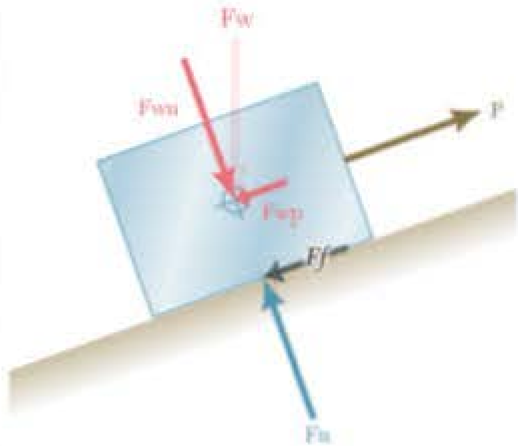


Click the correct answer.

- $F_f = \mu F_n$
- $F_f = F_n \cdot \sin\theta$
- $F_f = F_n \cdot \cos\theta$
- $F_f = \sqrt{F_n^2 + F_p^2}$

This block shown in the free body diagram is in equilibrium.

Match the equilibrium equations with each statement of the forces.



Drag statements on the right to match the left.

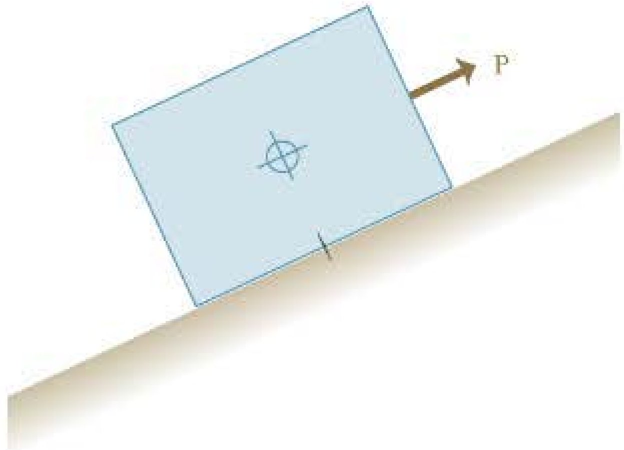
$\Sigma F_{\text{parallel}} = 0$	$P - F_{\text{wp}} - F_f = 0$
$\Sigma F_{\text{normal}} = 0$	$F_n - F_{\text{wn}} = 0$
A completely wrong equation	$F_w - F_n + F_f = 0$

Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

A 200 kg block rests on a 25° incline. The coefficient of friction between the block and the plane is 0.2.

Determine the magnitude of the force F_P acting along the inclined plane, required to:

(a) Start the block moving **up** the inclined plane
(b) Start the block slipping **down** the inclined plane



The problem	Free body diagram for impending upward motion	Weight components	Solve impending uphill	Free body diagram for impending downward motion	Solve impending downhill	Compare forces for up and down

GIVE FEEDBACK

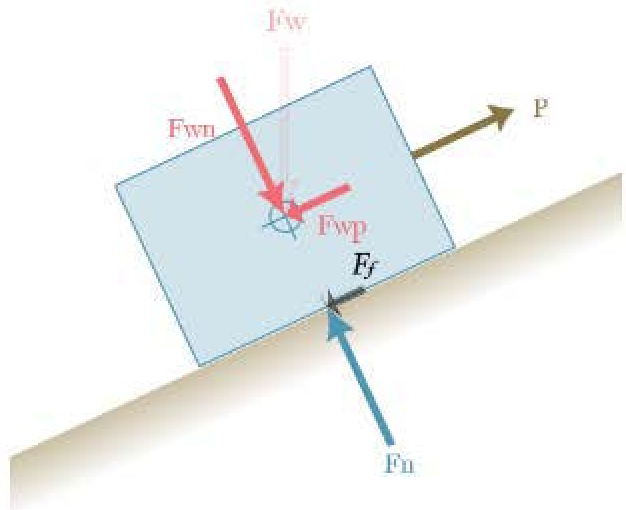
OK

Solution (a)

While motion is about to begin up the plane, the force of friction (F_f) will act **down** the plane in opposition to the impending motion.

A free body diagram is now constructed using force components parallel and normal to the inclined plane.

The block will slide as soon as the applied force P overcomes the parallel weight component F_{wp} as well as the friction force F_f .

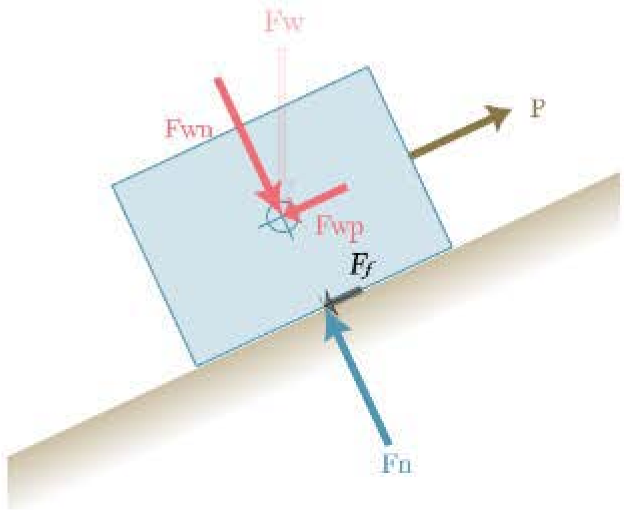


Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

The weight of the block can be resolved into components parallel and normal to the plane:

$$
\begin{aligned}
F_{wp} &= F_w \sin 25^\circ \\
&= 200 \times 9.81 \times \sin 25^\circ \\
&= 829.2\ \text{N}
\end{aligned}
$$

$$
\begin{aligned}
F_{wn} &= F_w \cos 25^\circ \\
&= 200 \times 9.81 \times \cos 25^\circ \\
&= 1{,}778\ \text{N}
\end{aligned}
$$



The problem	Free body diagram for impending upward motion	Weight components	Solve impending uphill	Free body diagram for impending downward motion	Solve impending downhill	Compare forces for up and down

GIVE FEEDBACK

OK

Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

Summation of normal forces:

$$\Sigma F_{\text{normal}} = F_{\text{wn}} + F_n = 0$$

$$F_n = -F_{\text{wn}}$$

$$\therefore F_n = 1{,}778 \text{ N}$$

Find F_f using the law of friction:

$$F_f = \mu F_n$$

$$= 0.2 \times 1{,}778$$

$$= 355.6 \text{ N}$$

Summation of parallel forces:

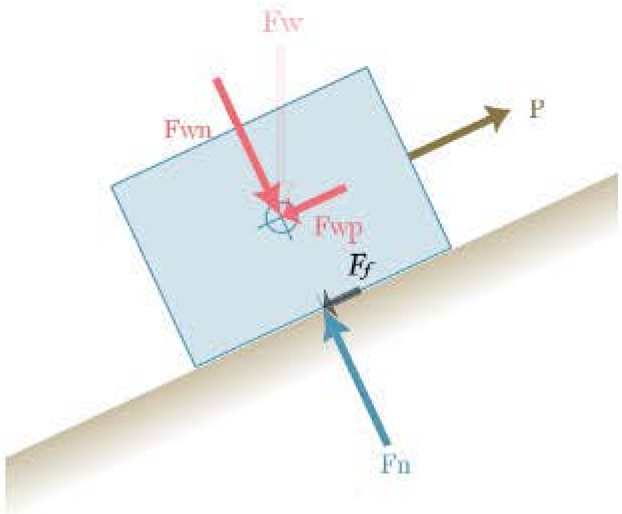
$$\Sigma F_{\text{parallel}} = -F_{\text{wp}} - F_f + P = 0$$

$$P = F_{\text{wp}} + F_f$$

$$P = 829.2 + 355.6$$

$$\therefore P = 1{,}184.8 \text{ N}$$

Above 1,184.8 N the block will move up the plane.



The problem	Free body diagram for impending upward motion	Weight components	Solve impending uphill	Free body diagram for impending downward motion	Solve impending downhill	Compare forces for up and down

GIVE FEEDBACK

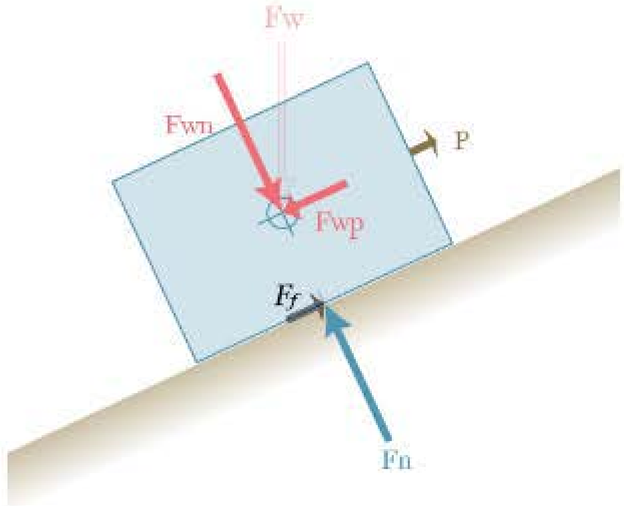
OK

Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

Solution (b)

The tendency for motion is now down the plane and the direction of friction F_f will be reversed.

The gives a free body diagram as shown.



Weight components are the same as before:

$$\begin{aligned} F_{wp} &= F_w \sin 25° \\ &= 200 \times 9.81 \times \sin 25° \\ &= 829.2 \text{ N} \end{aligned}$$

$$\begin{aligned} F_{wn} &= F_w \cos 25° \\ &= 200 \times 9.81 \times \cos 25° \\ &= 1{,}778 \text{ N} \end{aligned}$$

The problem	Free body diagram for impending upward motion	Weight components	Solve impending uphill	Free body diagram for impending downward motion	Solve impending downhill	Compare forces for up and down

GIVE FEEDBACK

OK

Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

$\Sigma F_{\text{normal}} = F_n - F_{\text{wn}} = 0$

$\therefore F_n = 1{,}778 \text{ N}$

Now use friction to find F_f :

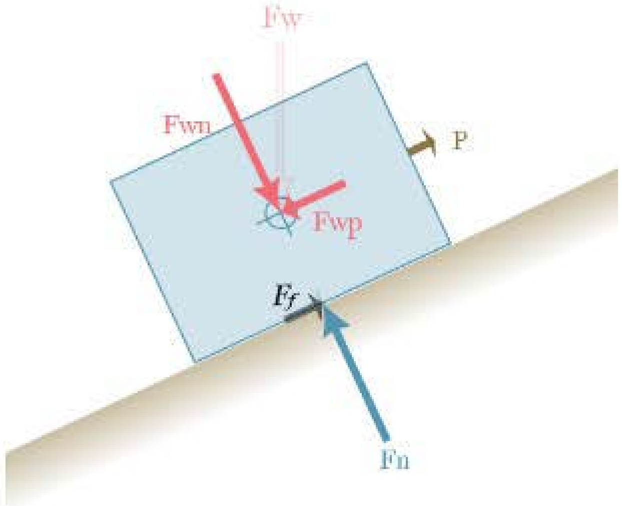
$F_f = \mu F_n$:

$\therefore F_f = 0.2 \times 1{,}778 = 355.6 \text{ N}$

Summation of parallel forces:

$\Sigma F_{\text{parallel}} = F_f - F_{\text{wn}} + P = 0$:

$$F_p = F_{\text{wp}} - F_f$$
$$= 829.2 - 355.6$$
$$\therefore F_p = 473.5 \text{ N}$$



The problem	Free body diagram for impending upward motion	Weight components	Solve impending uphill	Free body diagram for impending downward motion	Solve impending downhill	Compare forces for up and down

GIVE FEEDBACK

OK

Apply the mathematical method of solving friction to a load pulled up or down an inclined plane

By comparing the answers to parts (a) and (b), it can be seen that there is a range of magnitudes of force F_p between 473.5 N and 1184.8 N within which the block remains stationary on the plane. However, if the force is 1184.8 N or greater, the block will move up the plane. On the other hand, if the force is equal to or less than 473.5 N, the block will slip down the plane.

It is also worth noting that while the weight of the block is 1962 N, it can be pushed up the plane with a force of only 1184.8 N (depending on the coefficient of friction).

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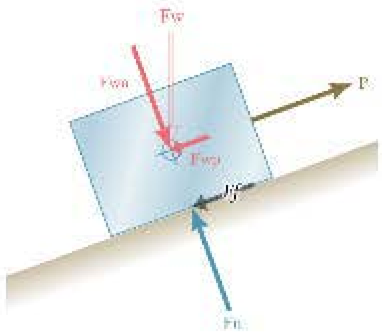


This 200 kg block is at the point of impending motion uphill. The incline is at 26° and the coefficient of friction is 0.18.

Find the force P that is keeping the block at the threshold of motion.

(Round off to nearest integer, include units.)

+ − · ÷ Clear

Clear line

? Undo

sin

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

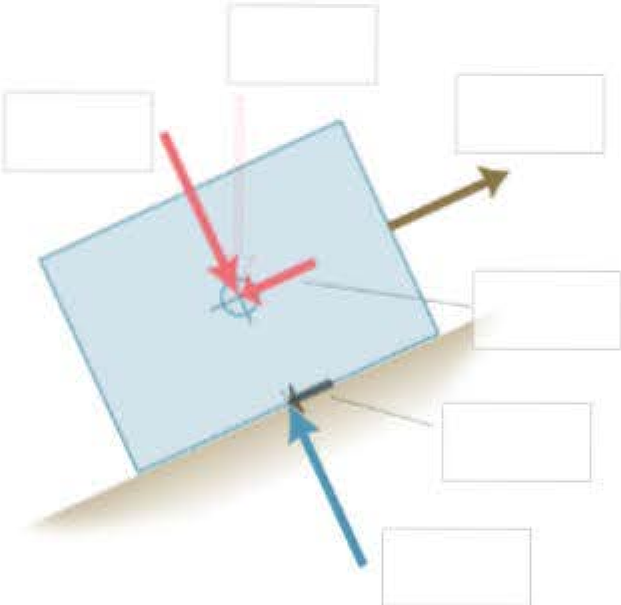
INSTRUCTIONS

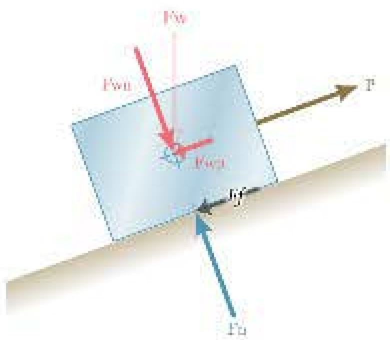
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint Each hint will reduce the credit received for this question

Label the free body diagram.

F_w	F_n	F_f
F_p	$F_w \cos \theta$	$F_w \sin \theta$





This 200 kg block is at the point of impending motion downhill. The incline is at 26° and the coefficient of friction is 0.18.

Find the force P that is keeping the block at the threshold of downhill motion.

(Round off to nearest integer, include units.)

Clear
Clear line
Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Apply the mathematical method of solving friction to a tilted force on a horizontal plane

Here is an interesting example that shows there is an optimal angle for the applied force. Even this simple example gets quite complex mathematically. Most of the time we will only use the mathematical method for problems without an angle on the applied force.

With a relatively high coefficient of friction, there is an optimal angle for dragging the block along the horizontal surface.

Here, the applied force is set just below the threshold of impending motion at angle $\beta = 0$
.

The block slides for a certain range of angle β.

Video object isn't supported in c++ version

Optimum angle of applied force

GIVE FEEDBACK

CONTINUE >

When the applied force F_p is at an angle β to the support plane, there will be a vertical component. This makes the block lighter, reducing the normal force F_n and also F_f. In practice it works as follows.

Find the normal force:

$$F_n = F_w - F_p \sin\beta$$

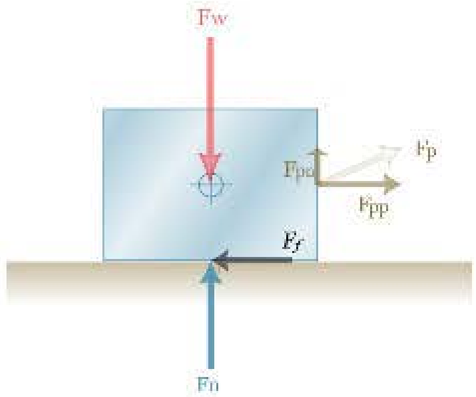
Use this to find the friction force:

$$F_f = \mu F_n = \mu(F_w - F_p \sin\beta)$$

But we know $F_f = F_{pp} = F_p \cos\beta$, so:

$$F_w = \frac{F_p \cdot \sin(0.5\pi - \Phi + \beta)}{\sin\Phi}$$

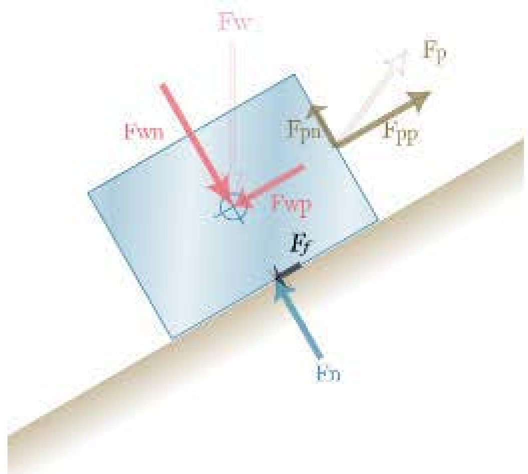
$$F_p = \frac{\sin\Phi \cdot F_w}{\sin(90 - \Phi + \beta)}$$



Applied force at an angle

< BACK GIVE FEEDBACK CONTINUE >

Notice how complex the equations become for a relatively simple problem.
Imagine what would happen if we analyse this problem on an inclined plane:



There is an easier way to solve problems on inclined plane—graphically.

< BACK GIVE FEEDBACK OK

Video object isn't supported in c++ version

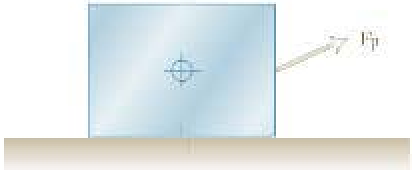
What is happening here?
(Assume angle of force F_p is β.)

Check all that apply.

- [ ] When $\beta = 0$, the weight of the block is too high for impending motion
- [ ] When β approaches $90°$, the horizontal component of force F_p is too small for impending motion
- [ ] Force F_p increases in a certain range of angle β, which exceeds the threshold of impending motion
- [ ] The coefficient of friction changes because the weight changes

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



Force Fp is 1700 N at 30°. What is the friction force Ff?

(Round off to nearest integer, include units.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint Each hint will reduce the credit received for this question

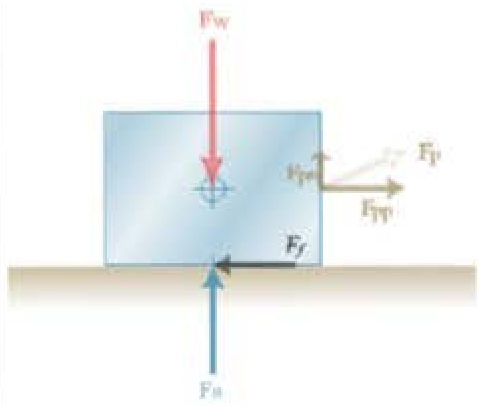
In these equations of friction, applied force F_p is applied at angle β to a block of weight F_w, and a coefficient of friction is converted to an angle of friction Φ.

$$F_w = \frac{F_p \cdot \sin(0.5\pi - \Phi + \beta)}{\sin \Phi}$$

$$F_p = \frac{\sin \Phi \cdot F_w}{\sin(90 - \Phi + \beta)}$$

$$F_f = \frac{\sin \Phi \cdot F_w \cdot \cos \beta}{\sin(90 - \Phi + \beta)}$$

Match the correct equation to each question.



Applied force at an angle β

Drag statements on the right to match the left.

For a certain pulling force, what weight can be dragged along the horizontal plane?	$F_w = \frac{F_p \cdot \sin(0.5\pi - \Phi + \beta)}{\sin \Phi}$
How much force is required to drag a block of a certain weight?	$F_p = \frac{\sin \Phi \cdot F_w}{\sin(90 - \Phi + \beta)}$
How much friction force is generated by dragging the block along the floor?	$F_f = \frac{\sin \Phi \cdot F_w \cdot \cos \beta}{\sin(90 - \Phi + \beta)}$

Do you know the answer?

The graphical method is quick and simple but not as precise as the mathematical method.

Advantage of the graphical method for solving friction

The graphical method is relatively quick and simple.

The simplicity comes from the fact that a friction problem is essentially a set of three forces in equilibrium. From the three force principle, these three forces must be concurrent and solvable by a force triangle.

The three forces are six pieces of information (three magnitudes and three directions). So we can solve this triangle with any of these possibilities:

- Two unknown force magnitudes
- One unknown force magnitude and one unknown angle
- Two unknown angles

It can be done manually, giving acceptable accuracy for most practical purposes, provided the force triangle is drawn to a sufficiently large scale.

GIVE FEEDBACK

OK

Compared to the mathematical method, the graphical method is relatively

Select... ▼

The graphical method is based on the fact that a friction problem can be analysed as a set of Select... ▼ forces in equilibrium.

It can even be done by sketching the forces on paper, which gives acceptable accuracy for most practical purposes, provided the force triangle is drawn

Select... ▼

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

An inclined plane friction problem can be solved using three forces.
There is a minimum amount of information necessary to solve these three forces.
Each force is only solved when two values are known, magnitude and direction.
Examples of the maximum number of unknowns that can be solved include:

Check all that apply.

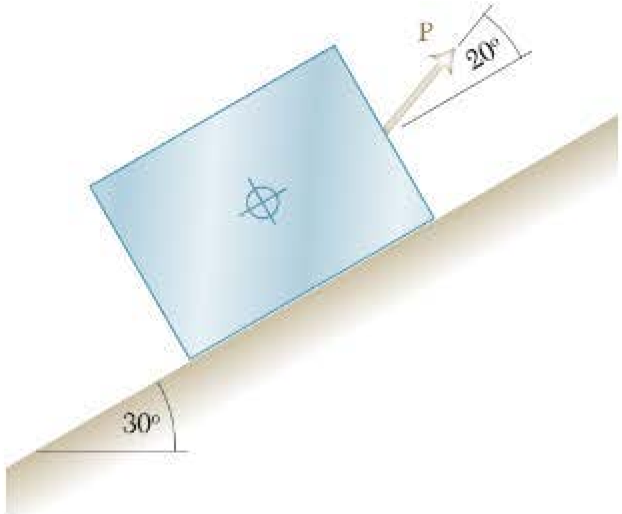
- [ ] 1 unknown magnitude
  1 unknown direction
- [ ] 2 unknown magnitudes
- [ ] 1 unknown magnitude
  2 unknown directions
- [ ] 2 unknown directions
- [ ] 2 unknown magnitudes
  1 unknown direction

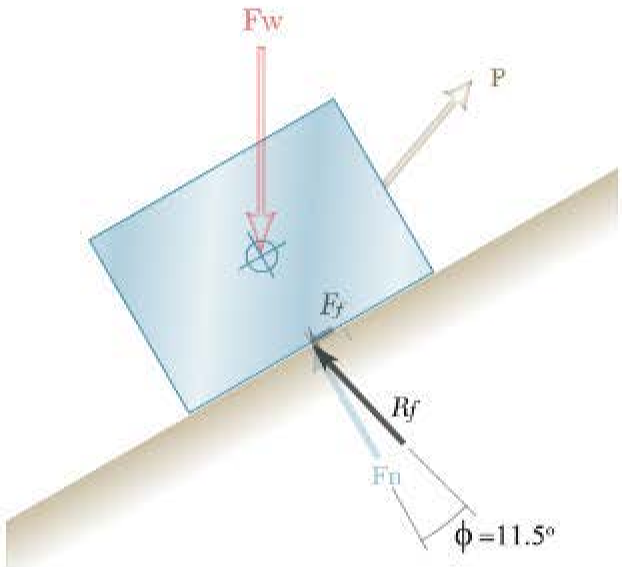
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The graphical method of solving friction on an inclined plane

Identify all the forces.
There should be three: F_w, F_r and P.





The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

The graphical method of solving friction on an inclined plane

Take care with the direction of the friction force F_f .

If the impending motion is uphill, friction will act downhill and vice versa.

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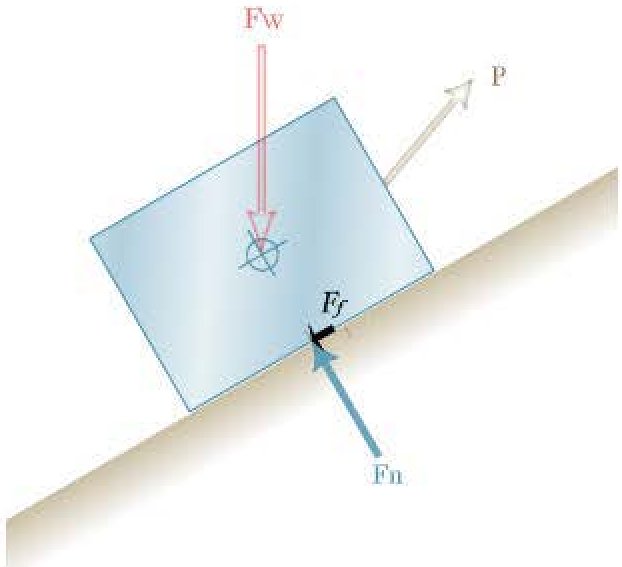


The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

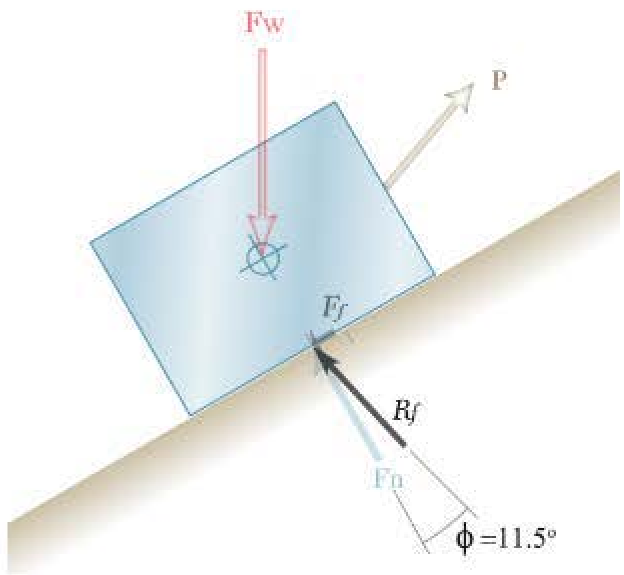
The graphical method of solving friction on an inclined plane

If the problem is solvable, there must be no more than two unknowns out of the six possible magnitudes and angles.

The six unknowns of a three-force body:

$$F_w = 1?\ \mathrm{N} @ 270^\circ$$
$$F_r = 3?\ \mathrm{N} @ 4?^\circ$$
$$F_p = 5?\ \mathrm{N} @ 6?^\circ$$

Four of these must be determined before a force triangle can be constructed.



The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

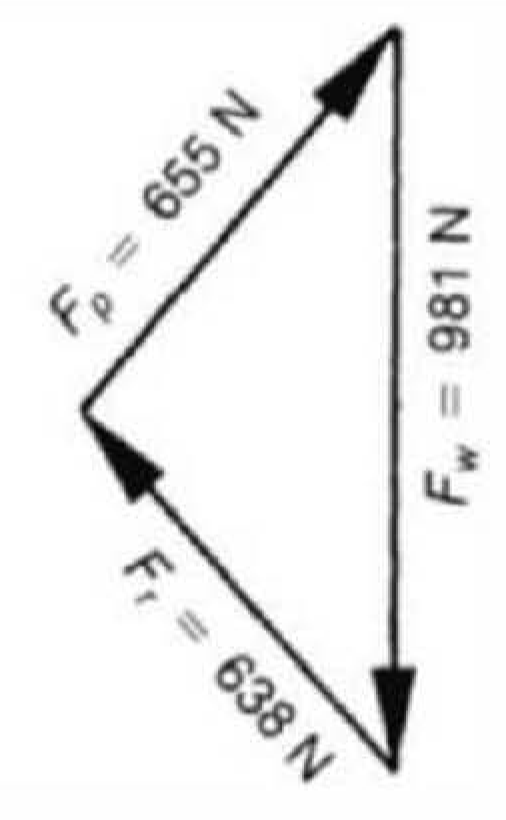
OK

The graphical method of solving friction on an inclined plane

Solving the force triangle can be done graphically on paper or drawn using a computer-aided design program.

When all three angles are known, there will be two magnitudes to be determined in the force triangle.

This is the equivalent of finding the intersection of two lines.



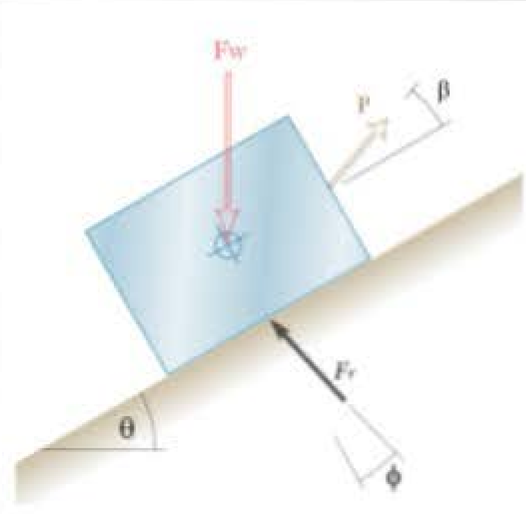
The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

Assume this block is at the point of impending motion.
To solve this question we would need to know some minimum information.

Match the following sets of known data to the unknown data the we are trying to solve.



Drag statements on the right to match the left.

We know F_w, θ, β, μ	Trying to find P or F_r
We know P, θ, β, Φ	Trying to find F_w or F_r
We know F_w, P, β, F_r	Trying to find θ or φ
We know P, θ, Φ, F_r	Trying to find F_w or β

Watch the video and select all true statements regarding it.

Video object isn't supported in c++ version

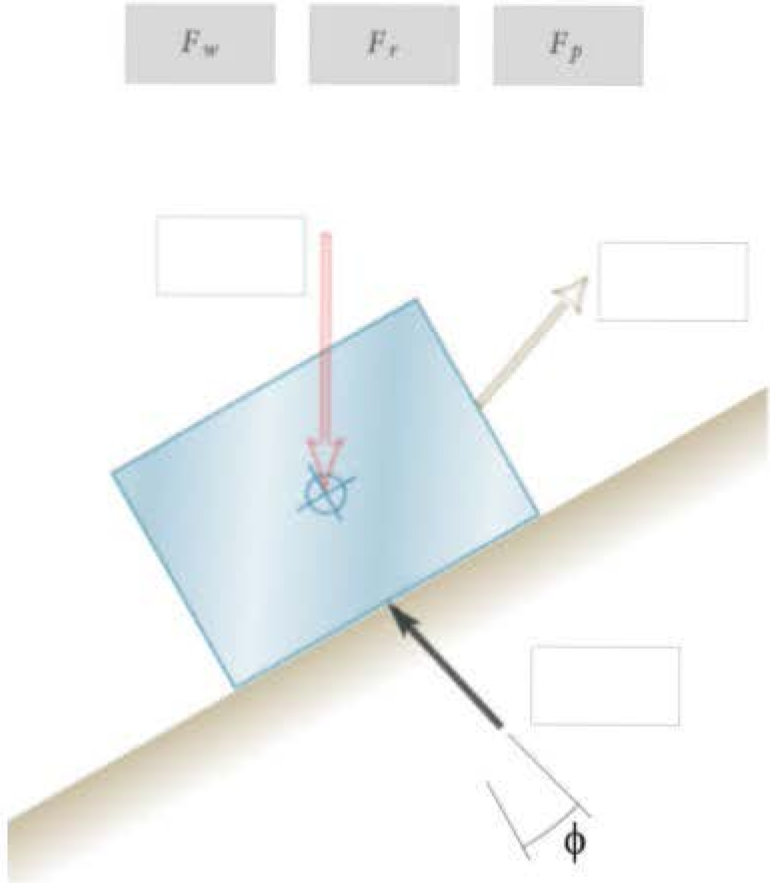
Check all that apply.

- [ ] If the impending motion is uphill, friction will act downhill
- [ ] If the impending motion is downhill, friction will act uphill
- [ ] If the impending motion is uphill, friction will act uphill
- [ ] If the impending motion is downhill, friction will act downhill

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Drag forces onto the free body diagram for the block shown, where:
Fw = Weight of block, Fr = reaction force, Fp = applied force.

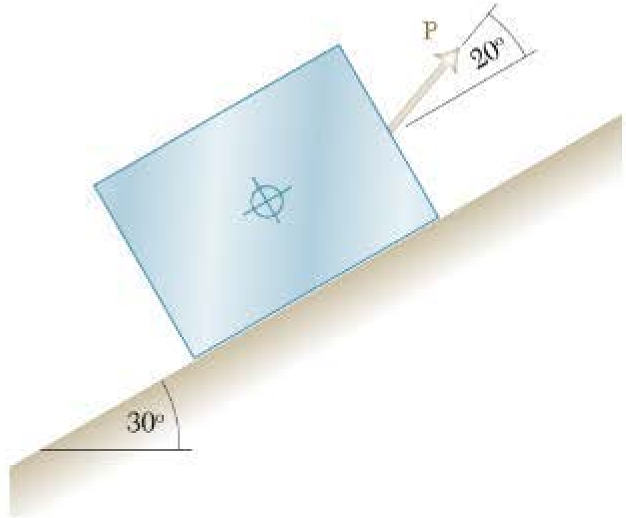


Submit

Solve friction problems on an inclined plane by the graphical method

A 100 kg block resting on a 30° incline is acted upon by a force P at 20° to the plane as shown.

Determine the magnitude of the force required to start motion up the plane if the coefficient of static friction is 0.2.



The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

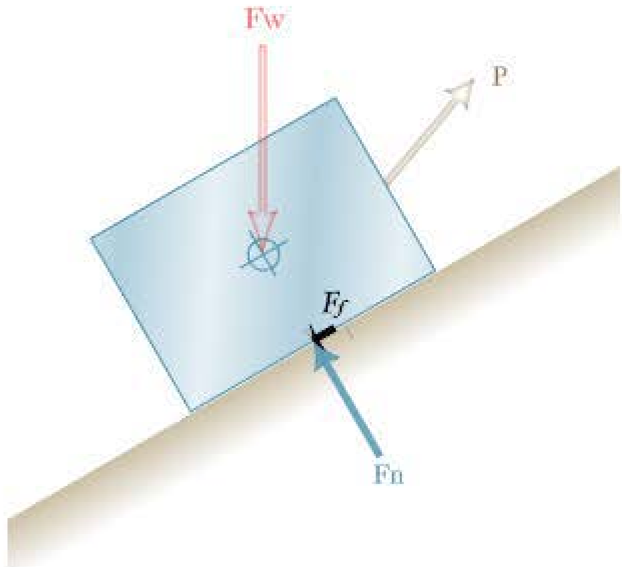
OK

Solve friction problems on an inclined plane by the graphical method

Take care with the direction of the friction force F_f .

The impending motion is uphill, so friction will act downhill.

Video object isn't supported in c++ version



The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

Solve friction problems on an inclined plane by the graphical method

The weight of the block is:

$$F_w = 100 \times 9.81 = 981\ \text{N}$$

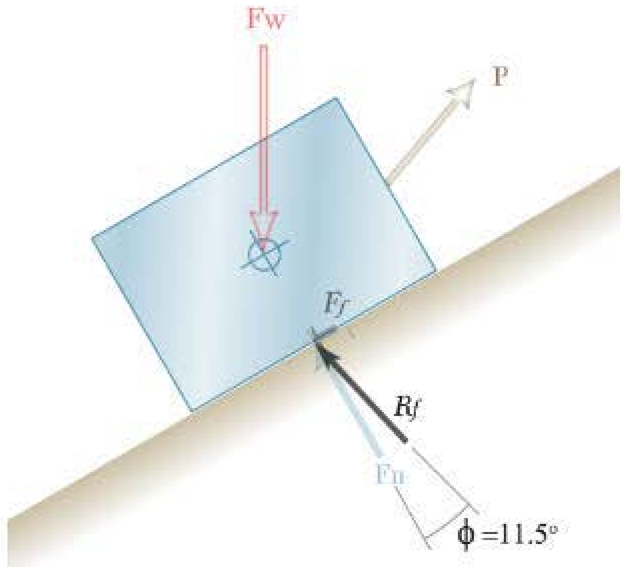
which completes the first force:

$$F_w = 981\ \text{N} @ 270°$$

The resultant support reaction at the surface is the sum of F_f and F_n. Both of these are unknown because force P is altering the weight of the block by lifting away from the surface.

But we know it is aligned at the angle friction φ, which is:

$$\Phi = \tan^{-1}\mu = \tan^{-1}0.2 = 11.3°$$



The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

Solve friction problems on an inclined plane by the graphical method

Converting the friction reaction F_r to 360° format:

Angle $= 90 + \theta - 11.3 = 90 + 30 - 11.3 = 108.7°$

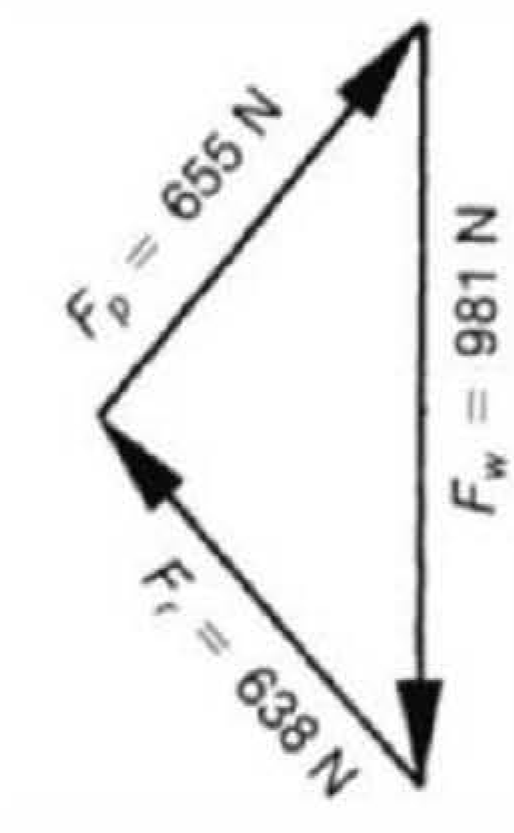
Converting the applied force P to 360° format:

Angle $= \theta + 20 = 30 + 20 = 50°$

The triangle of forces is now constructed:

- $F_w = 981\ \text{N} @ 90°$
- $F_r = ?\ \text{N} @ 108.7°$
- $F_p = ?\ \text{N} @ 50°$

Solving this triangle, the magnitude of the applied force F_p is found to be 655 N.

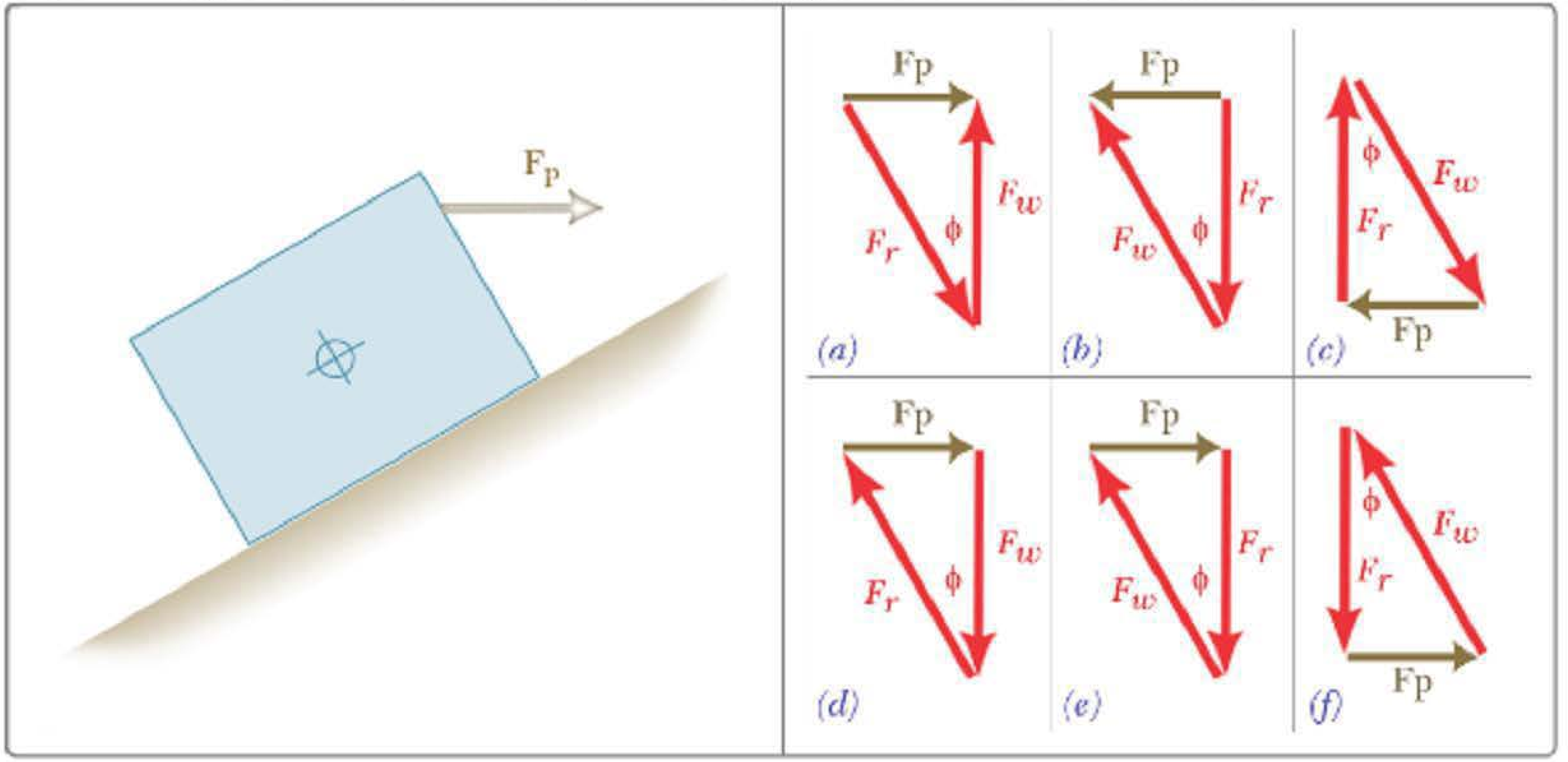


The problem	Impending motion and friction force	Known forces	Solve the force triangle

GIVE FEEDBACK

OK

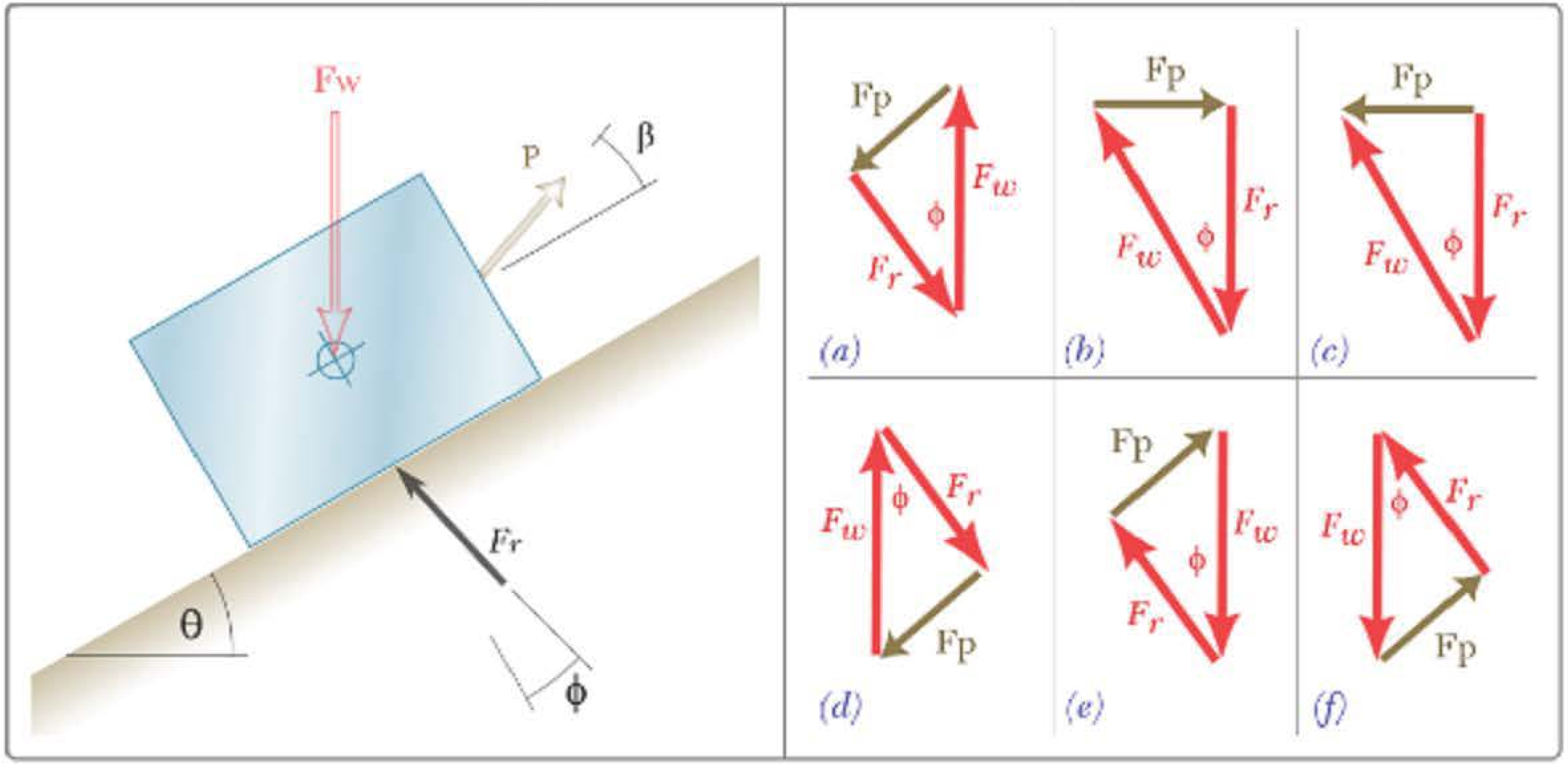
Select every correct force triangle for solving the block in equilibrium.



Check all that apply.

- [ ] (d)
- [ ] (a)
- [ ] (b)
- [ ] (c)
- [ ] (e)
- [ ] (f)

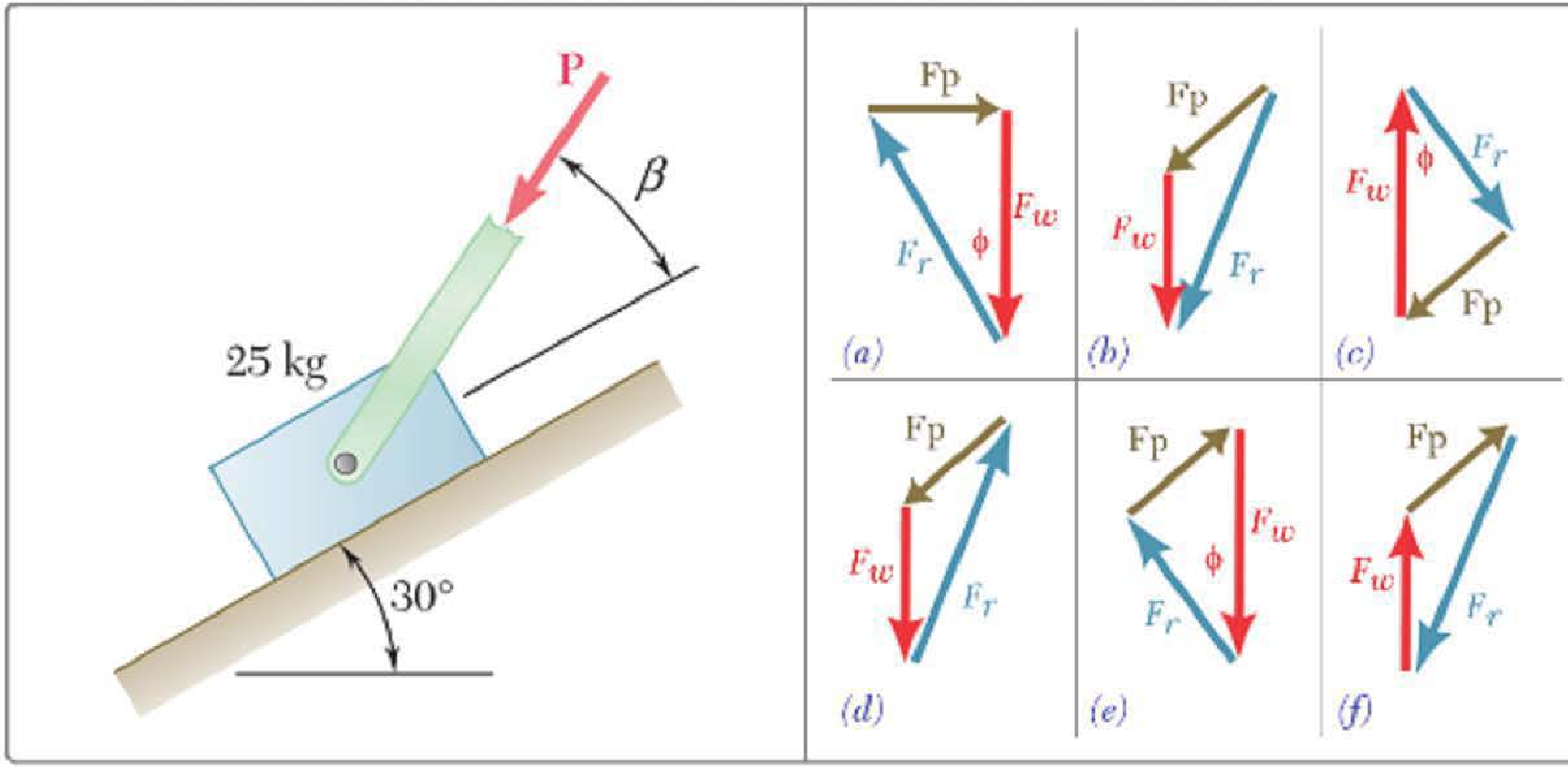
Select the correct force triangles for solving the block in equilibrium.



Check all that apply.

- [ ] (c)
- [ ] (d)
- [ ] (f)
- [ ] (b)
- [ ] (e)
- [ ] (a)

Select every correct force triangle for solving the block in equilibrium.

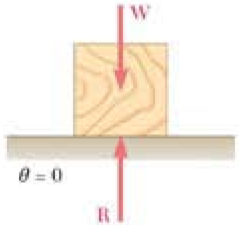


Check **all** that apply.

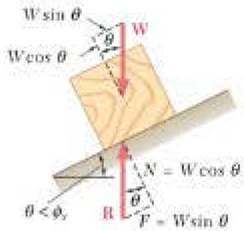
- [ ] (c)
- [ ] (b)
- [ ] (e)
- [ ] (a)
- [ ] (f)
- [ ] (d)

The states of impending motion for an object without an applied force

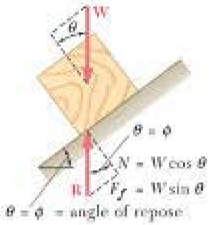
(a) No friction force



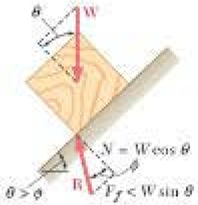
(b) No motion



(c) Impending motion



(d) Acceleration



In the cases shown, all are in equilibrium except (d), which is accelerating.

All cases have only two forces, which must be equal and opposite for equilibrium.

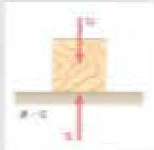
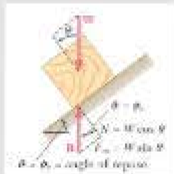
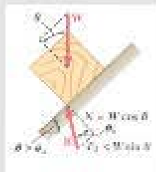
Impending motion occurs at the angle of repose, when $\theta = \Phi$. Beyond that point the friction reaction R is no longer able to balance the weight force W with an equal and opposite force. Hence equilibrium cannot be maintained and the block accelerates.

GIVE FEEDBACK

OK

Match the free body diagram to each description of the behaviour of the block.

Drag statements on the right to match the left.

No friction force and no motion	
Friction force with no motion	
Maximum friction force with no motion or with constant velocity	
Friction force with acceleration	

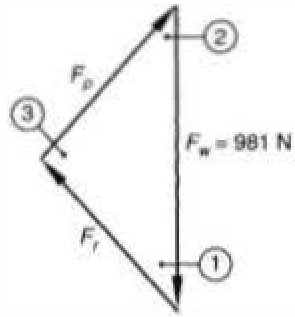
The semi-graphical method by trigonometry of solving friction

The semi-graphical method is the same as the graphical method but the final triangle of forces is solved using trigonometry for accuracy.

This semi-graphical solution involves sketching the triangle of forces and then solving it (usually) by the sine rule.

The sine rule (or cosine rule) is needed because the triangle of forces is not usually a right triangle.

A typical force triangle is shown opposite. There is no 90° angle in this triangle.

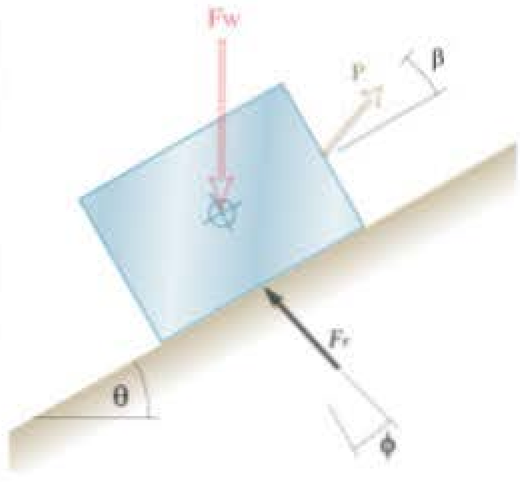


GIVE FEEDBACK

OK

The semi-graphical solution involves sketching the triangle of forces and then solving it (usually) by the sine rule.

The reason the sine rule (or cosine rule) must be used is because:



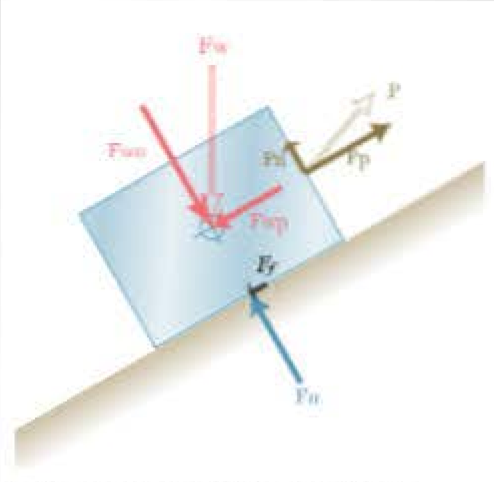
Click the correct answer.

- The triangle of forces is not usually a right triangle
- Force Fw is not necessarily vertical
- There are more than three forces
- The angle of inclination θ is not known

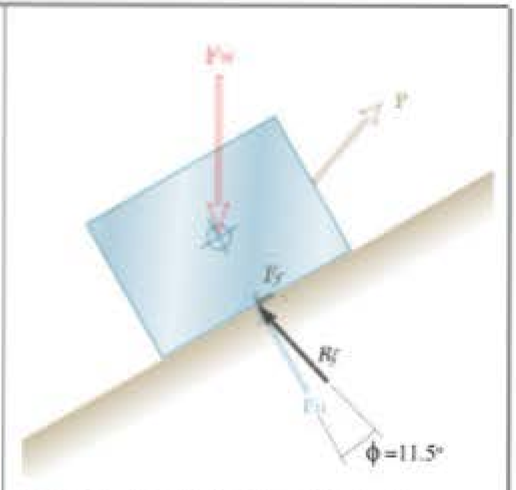
The semi-graphical method is just as accurate as the mathematical method but as easy to follow as the graphical method.

Advantages of the semi-graphical method (trigonometry) of solving friction

The semi-graphical solution (trigonometry) combines the accuracy of the mathematical method (components) with the helpful visual representation of the graphical solution.



Mathematical solution free body diagram



Graphical solution free body diagram

GIVE FEEDBACK

OK

Some advantages of the method of trigonometry for solving friction include that it is:

Check all that apply.

- [ ] Faster than the mathematical components method
- [ ] More accurate than the graphical hand-sketch method
- [ ] More accurate than the mathematical components method
- [ ] Faster than the graphical hand-sketch method

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Method of trigonometry for solving friction on an inclined plane

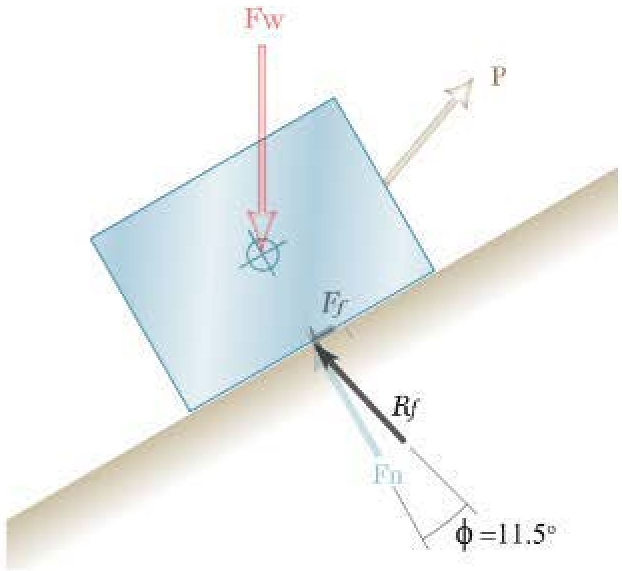
The three forces in equilibrium are identified in the free body diagram.

We can have a maximum of two unknowns out of a possible six.

For example:

- $F_w = 981\ \mathrm{N}@90°$
- $F_p = ?\ \mathrm{N}@50°$
- $F_p = ?\ \mathrm{N}@50°$

The two unknowns are to be determined using a force triangle.



The problem

Set up the triangle

GIVE FEEDBACK

OK

Method of trigonometry for solving friction on an inclined plane

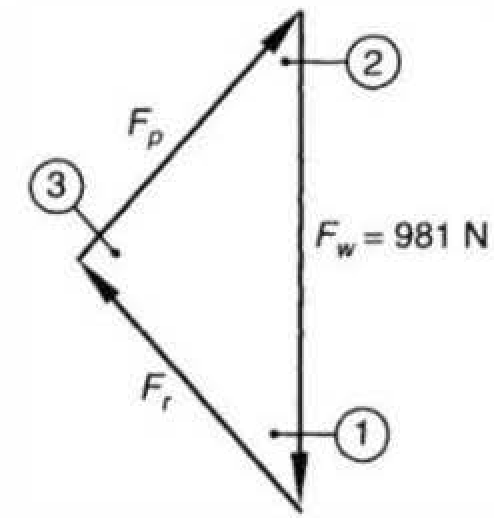
After sketching the triangle, the three angles within the triangle are evaluated by reference to the angles shown on the free body diagram.

The sine rule is used when we are given either:

- Two angles and one side
- Two sides and a non-included angle

The cosine rule is used when we are given either:

- Three sides
- Two sides and the included angle



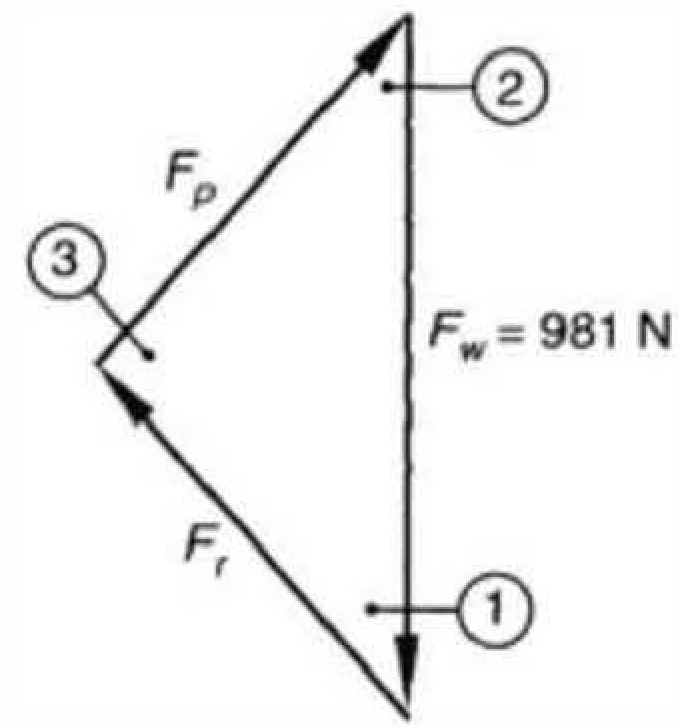
The problem | Set up the triangle

GIVE FEEDBACK

OK

After sketching the triangle, the unknown force magnitudes (lengths) or angles can be determined by using the sine or cosine rule.

Match when to use either the sine or cosine rule.



Drag statements on the right to match the left.

The sine rule is used when we are given:	Two sides and the included angle
The sine rule is used when we are given:	Two angles and one side
The cosine rule is used when we are given:	Two sides and a non-included angle
The cosine rule is used when we are given:	Three sides

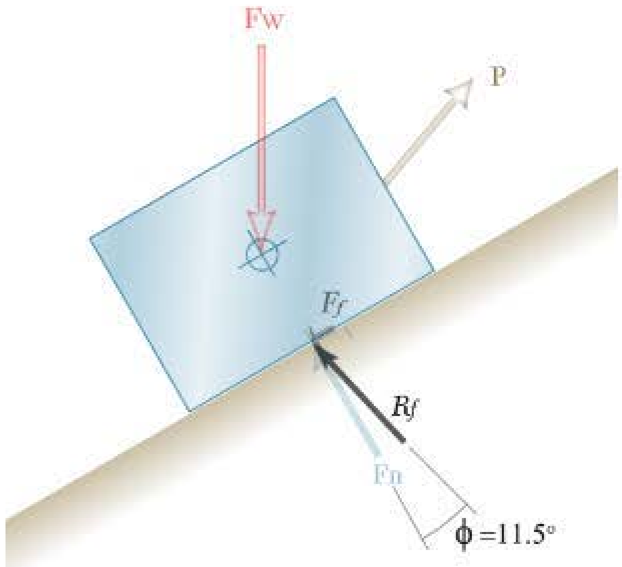
Solve friction on an inclined plane using the method of trigonometry

Determine the magnitude of force F_p by using the sine rule.*

The triangle of forces is constructed from the following forces:

- $F_w = 981$ N@90°
- $F_r = ?$ N@108.7°
- $F_p = ?$ N@50°

*The sine rule is used when we are given either two angles and one side, or two sides and a non-included angle. The cosine rule is used when we are given either three sides or two sides and the included angle.



The problem | Set up the triangle | Solve by the sine rule

GIVE FEEDBACK

OK

Solve friction on an inclined plane using the method of trigonometry

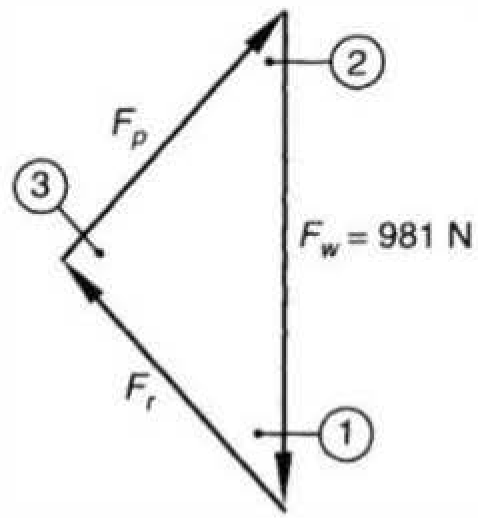
After sketching the triangle, the three angles within the triangle are evaluated by reference to the angles shown on the free body diagram.

In this case:

$$\text{Angle no. 1} = 30° + 11.3° = 41.3°$$

$$\text{Angle no. 2} = 90° - (30° + 20°) = 40°$$

$$\text{Angle no. 3} = 180° - (41.3° + 40°) = 98.7°$$



The problem	Set up the triangle	Solve by the sine rule

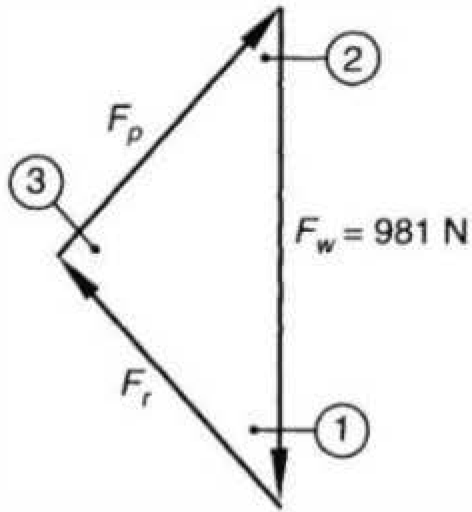
GIVE FEEDBACK

OK

Solve friction on an inclined plane using the method of trigonometry

Now apply the sine rule, where:

$$\text{Angle no. 1} = 41.3°$$
$$\text{Angle no. 2} = 40°$$
$$\text{Angle no. 3} = 98.7°$$



Force F_p can now be calculated quite simply using the sine rule:

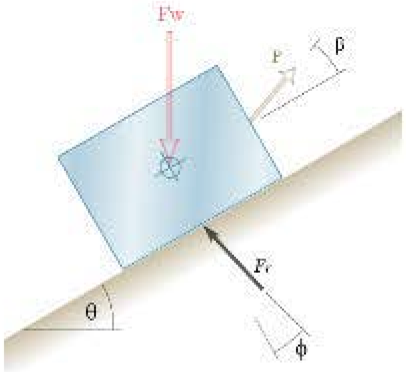
$$\frac{981 \text{ N}}{\sin 98.7°} = \frac{F_p}{\sin 41.3°}$$
$$\therefore F_p = 655 \text{ N}$$

Note: These problems will most often use the sine rule because the angles are known. If two angles are missing the cosine rule must be used.

The problem	Set up the triangle	Solve by the sine rule

GIVE FEEDBACK

OK



Angles Θ = 20°, Φ = 10°, β = 15°. What is the angle of Force P in 360° format? (Round off to nearest integer, do not include units.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

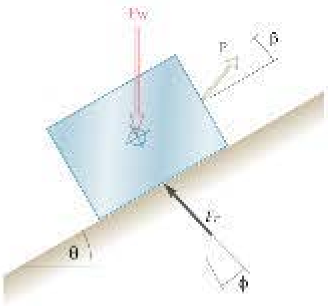
SHOW ANSWER

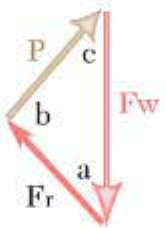
INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question





Angles Θ = 20°, Φ = 10°, β = 15°. What is angle b? (Round off to nearest integer, do not type units.)

Clear
Clear line
Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

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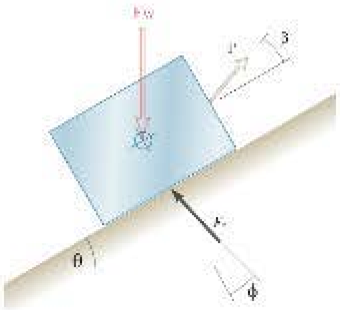

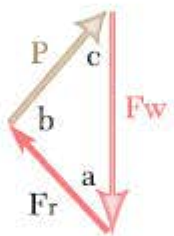

Angles ϴ = 20°, Φ = 10°, β = 15°. What is angle a? (Round off to nearest integer, do not type units.)

Clear
Clear line
Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

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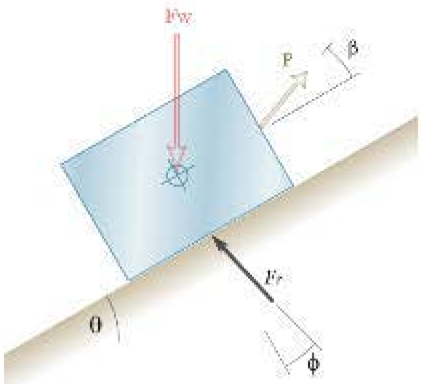

Angles Θ = 20°, Φ = 10°, β = 15°. What is the angle of the frictional resultant force Fr in 360° format?

(Round off to nearest integer, do not type units.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

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Hint

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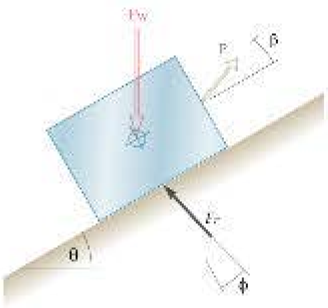

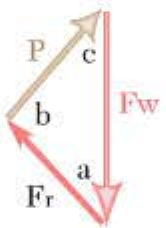

Angles Θ = 20°, Φ = 10°, β = 15°. What is angle c? (Round off to nearest integer, do not type units.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

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