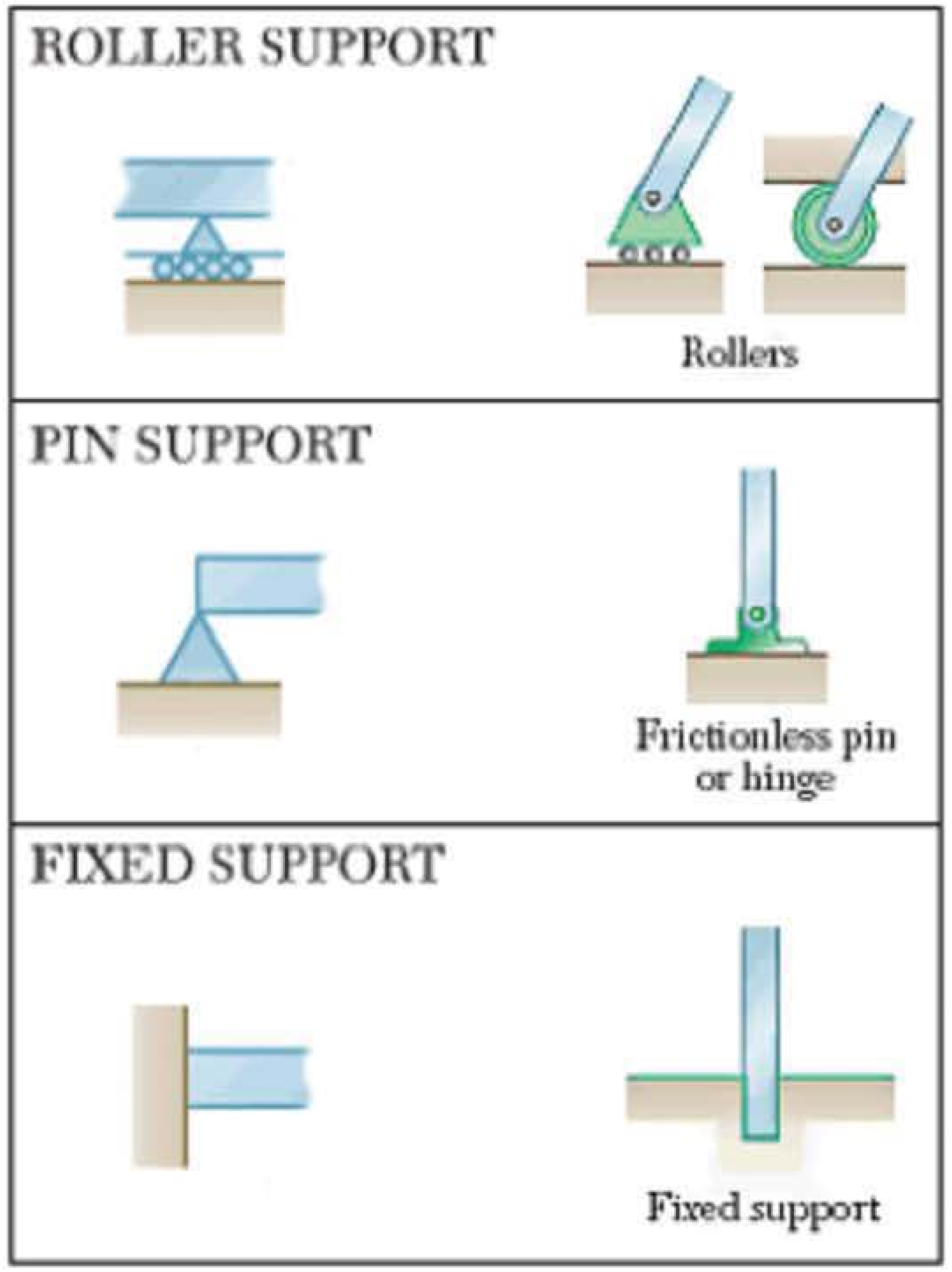
GIVE FEEDBACK

CONTINUE >

Beam analysis is used for components that are mostly under bending. This analysis starts with the forces applied at beam supports. In this chapter we consider the three types of support:



< BACK GIVE FEEDBACK OK

What are beams?

The engineering definition of a beam is pretty much the same as the common understanding. It's a long thing that carries loads, like a bridge.

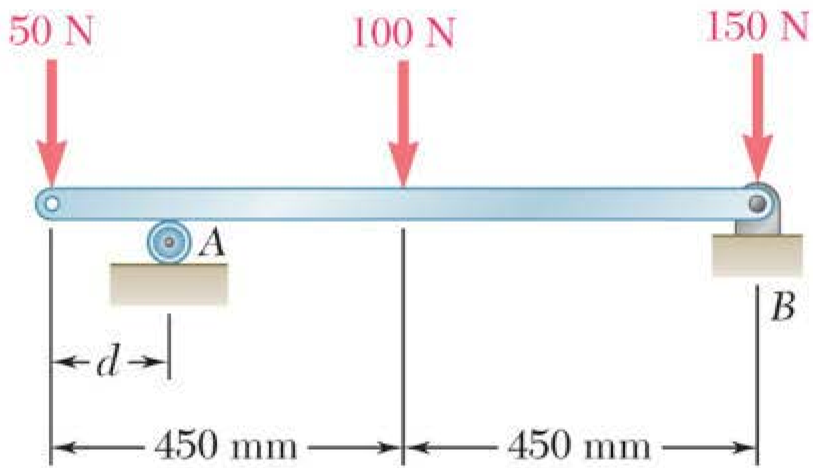
However, we can also extend the definition to include anything where loads can cause bending. So a beam can be any size, from a huge bridge girder right down to a tiny gear tooth inside a watch.

Beams are usually long, straight, solid bars with a uniform cross-sectional shape and a constant cross-sectional area. They could be made from rolled structural steel, reinforced concrete, timber or similar materials.

In most cases beams are placed in a horizontal position and carry vertical loads, i.e. forces which are perpendicular to the axis of the beam. The majority of loads supported by horizontal beams are usually weights, so these forces are applied vertically downwards. Other loads might be upwards or inclined at an angle to the axis of a horizontal beam.

GIVE FEEDBACK

CONTINUE >



Not all beams are bridge girders. This small lever could be a component in a machine. For convenience we may choose to lay a component horizontal to make analysis more familiar. To behave as a beam, the component must be relatively slender, with a length considerably longer than the width and the thickness.

< BACK

GIVE FEEDBACK

OK

For a horizontal beam carrying weights, each weight applies forces in which direction?

Click the correct answer.

- Vertically down
- Vertically up
- To the right
- To the left

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of the following statements is true regarding beam analysis?

Click the correct answer.

- All forces applied to a beam are vertical (i.e. perpendicular to the beam axis)
- A vertical beam can be turned horizontal to make analysis easier
- A beam is larger than a typical machine component
- A beam can be any shape, whether long and narrow or short and thick

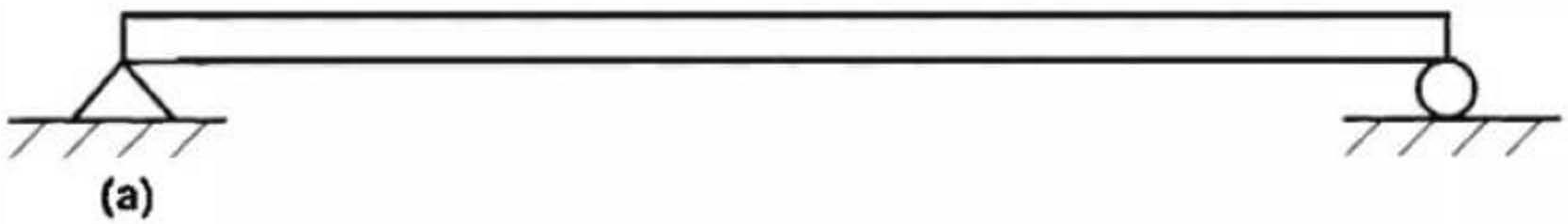
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Simply supported beams

Beams are generally classified in accordance with the way in which they are supported.

A **simply supported beam** is pinned at one end and roller-supported at the other.



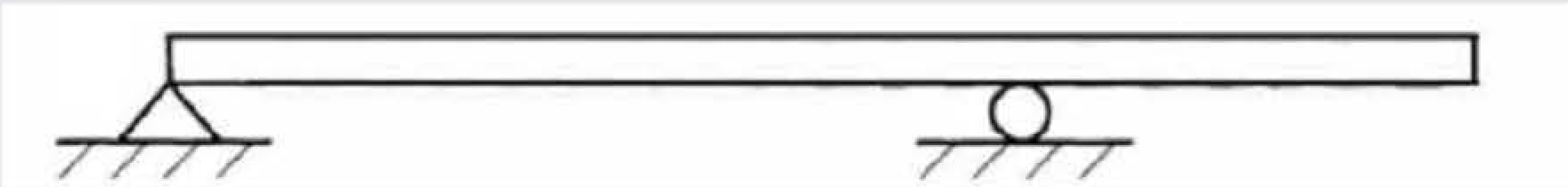
GIVE FEEDBACK

OK

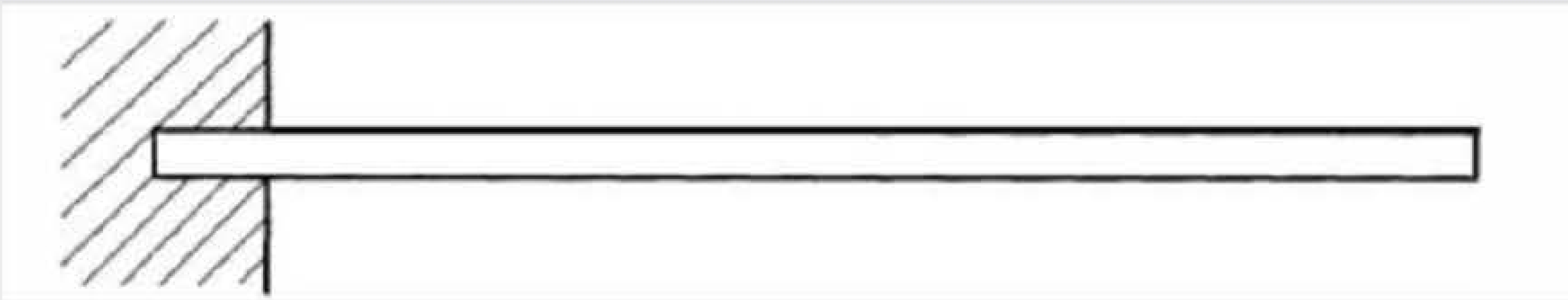
Match the name to each type of beam shown.

Drag statements on the right to match the left.

Simply supported beam



Cantilevered beam



Overhanging simply supported beam

Do you know the answer?

I KNOW IT

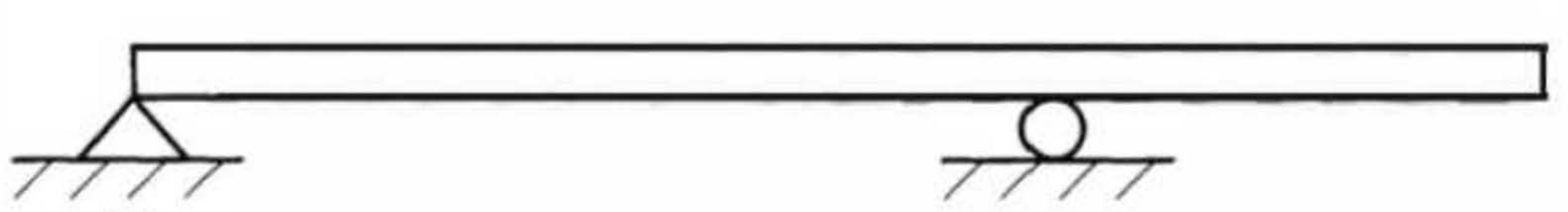
THINK SO

UNSURE

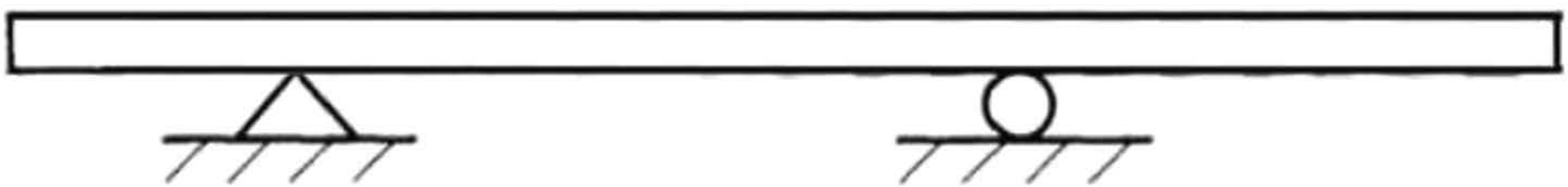
NO IDEA

Overhanging beams

An **overhanging beam** also rests on two similar supports but extends beyond one or both of its supports.



Overhang on one end

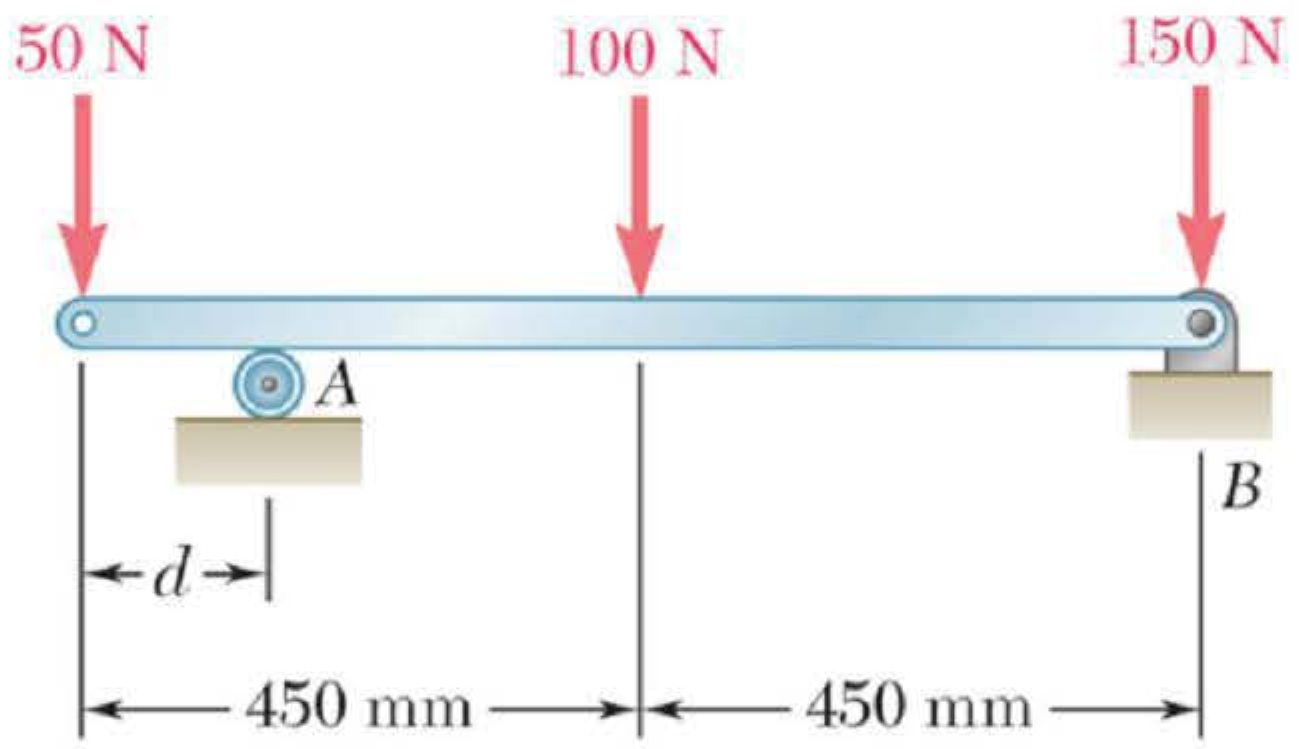


Overhang on both ends

GIVE FEEDBACK

OK

Select all correct statements about this beam arrangment:



Check all that apply.

- [ ] There are two roller joints
- [ ] It is too small to be called a beam
- [ ] This is an overhanging beam
- [ ] There is a roller and pin joint

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Cantilever beams

A **cantilever beam** is rigidly fixed at one end and free at the other.

GIVE FEEDBACK

OK

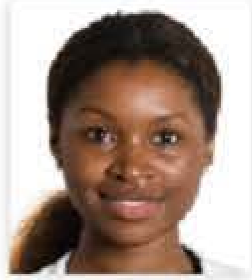
What type of support is used in a cantilever beam?

Click the correct answer.

- A fixed support
- A roller support
- A pinned support
- A hinge support

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



The loads or forces on a beam are classified according to how they contact the beam. If the force is applied over a small area it can be treated as a point force or concentrated load. Examples include a bolted joint, a cable or a narrow load. In other cases, the load is spread over a region of the beam. This is called a distributed load.

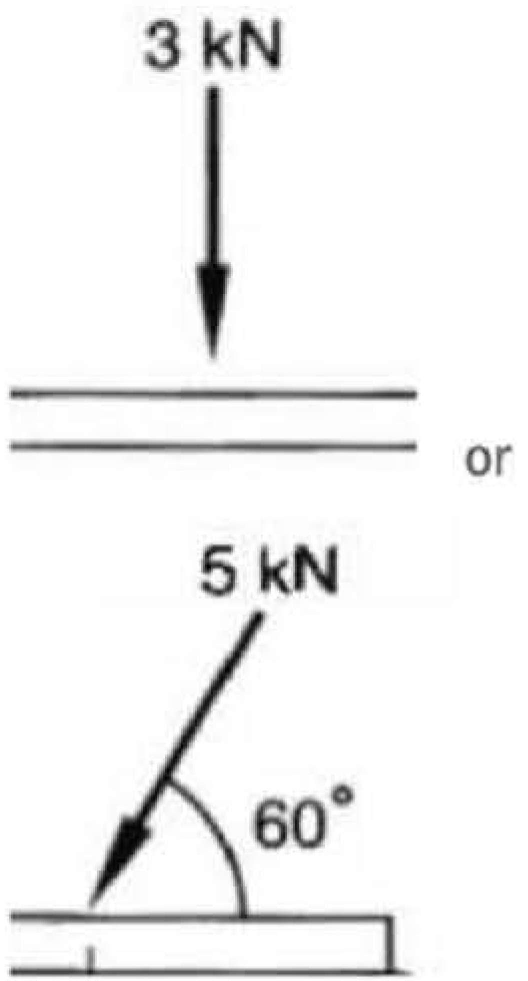
A beam may be subjected to **concentrated loads** or **distributed loads**, or both.

Concentrated loads

A concentrated load (or point load) is applied at a specific location on the beam.

It is shown as an arrow, as in the figures opposite.

It is usually vertical but can also be at an angle.

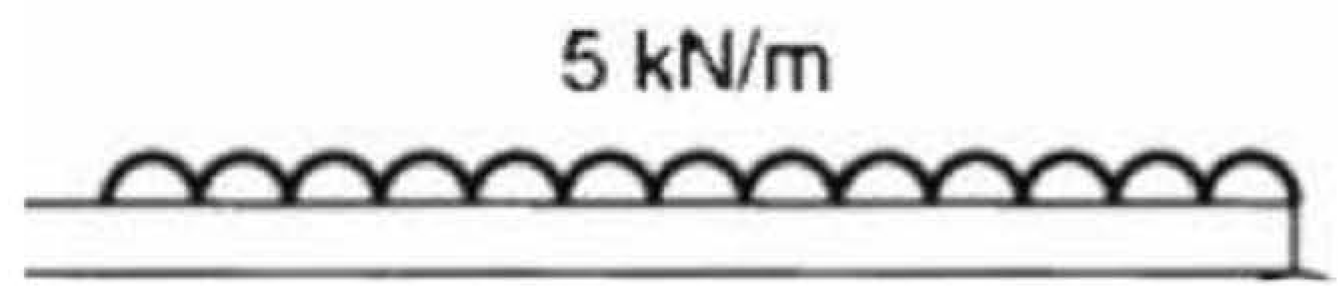


GIVE FEEDBACK

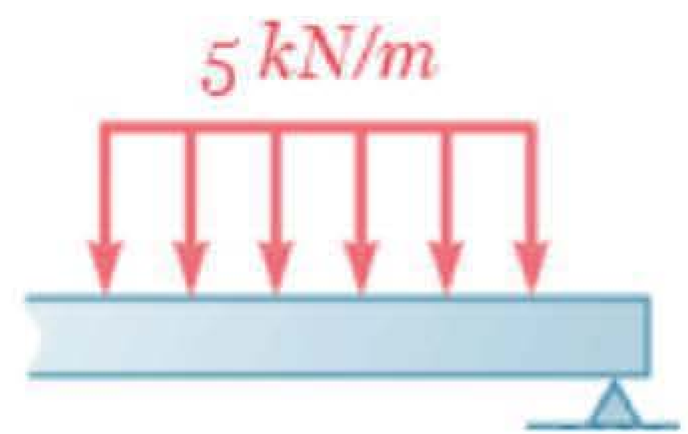
CONTINUE >

Uniformly distributed loads

A uniformly distributed load is evenly spread over a region of the beam, anywhere from a small portion to the entire beam.



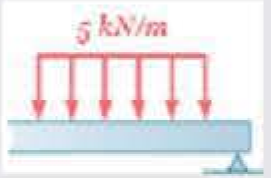
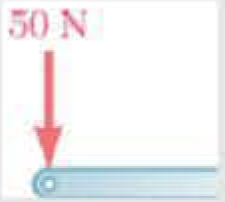
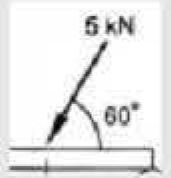
It can be shown like sandbags or arrows (usually linked together).



The number of arrows is arbitrary with nothing to do with calculation. Only the load value (N/m) and the overall length (m) of the distributed load region have any effect.

Match the following types of load on a beam:

Drag statements on the right to match the left.

Distributed load (arrows)	
Concentrated load (vertical)	
Distributed load (sandbags)	
Concentrated load (angle)	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Examples of concentrated loads include:

Check all that apply.

- [ ] A single vertical force
- [ ] A force at an angle
- [ ] The weight of a beam
- [ ] A load spread over part of the beam

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Examples of distributed loads include:

Check all that apply.

- [ ] A single vertical force
- [ ] A force at an angle
- [ ] The weight of a beam
- [ ] A load spread over part of the beam

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Sometimes we include the weight of the beam in our calculations, depending on whether the weight is significant. It also depends on how fussy you want to be. But usually if it is less than two to three per cent, it is not worth bothering with.

When to include the weight of the beam

When the weight of the beam is relatively insignificant compared to all other forces, the weight of the beam can be neglected (i.e. left out of the analysis). This is more common for smaller beams such as machine components that have high loads.

Other times, when the weight of the beam is closer to the magnitude of the applied forces, it must be included in the analysis. This is more likely for large beams such as a bridge girder.

GIVE FEEDBACK

OK

The weight of a beam should be included in analysis when the beam is ______ or ______.

Check all that apply.

- [ ] small
- [ ] large
- [ ] heavy
- [ ] vertical

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

As things get bigger, weight becomes more of a problem. Ultimately there is a size limit for every type of machine or structure.

Why weight is more significant for larger beams

The square-cube law shows that as an object is scaled up, the weight becomes more significant.

This is because weight (which is based on volume) is proportional to scale but strength (which is based on area) is also proportional to scale. Doubling the scale gives eight times the weight but only four times the area, resulting in a doubling of stress (stress = weight / area = 8 / 4 = 2).

Weight-related stresses increase in proportion to scale. This is why an ant can pick up 100 times its own weight while an elephant can lift five per cent of its body weight.

So larger beams are more likely to have their own weight included in the analysis.

GIVE FEEDBACK

OK

According to the square-cube law, what does doubling the scale of an object give (where doubling the scale means every dimension is doubled, i.e. scaled to 200% of its original size)?

Click the correct answer.

- Eight times the stress
- Four times the stress
- Double the stress
- Half the stress

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Supports are the things that hold a beam in place. They are very important in determining how the forces behave and how we go about calculating those forces.

Beam supports

When a beam carries loads and rests on its supports, the supports react to provide the necessary static balance which keeps the beam at rest, i.e. in a state of static equilibrium.

The kinds of reactions that may exist at any given support depend on the nature of the support.

GIVE FEEDBACK

OK

A beam carries loads and rests on its supports. These supports react to provide the necessary forces, preventing the beam from having any (please select) ▼ that could make the beam unstable.

These supports apply different types of reactive forces to the beam according to the (please select) ▼.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A beam is in static equilibrium when:

Check all that apply.

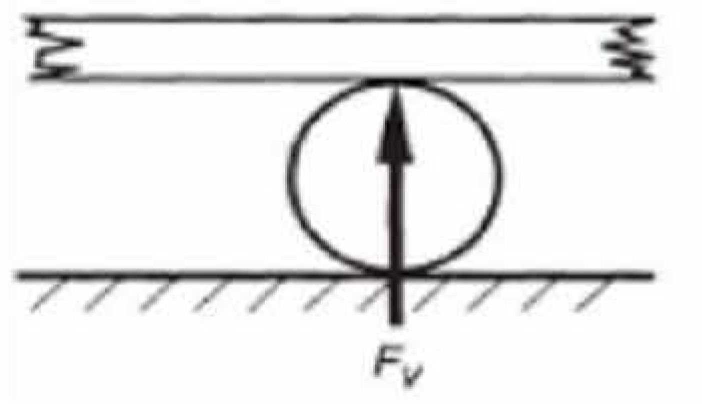
- [ ] There are no forces acting on the beam
- [ ] The weight of the beam is not included
- [ ] The forces add up to zero
- [ ] It is not accelerating
- [ ] It is not moving

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Roller supports

A roller support is the name given to any joint that slips sideways. These supports offer negligible resistance to small displacements along the axis of the beam. This movement may be necessary to account for thermal expansion and flexibility in the structure.



GIVE FEEDBACK

OK

For a horizontal beam:

Any support that allows sideways movement is known as a (please select) ▼ support.

These supports offer negligible resistance to small displacements (please select) ▼ to the beam.

Submit

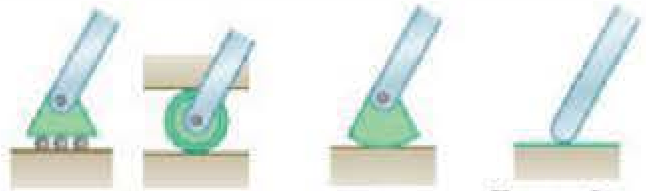
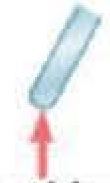
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Reactions that may exist at roller support

Roller support is a generic term for any type of support joint that supports weight but allows sideways slippage. There are many ways to achieve this, usually by rollers or slippery surfaces.

The roller support can have many forms including a slippery material (i.e. frictionless). These all have the same property: that the only force that can be transmitted through the joint is the component perpendicular to the support surface.

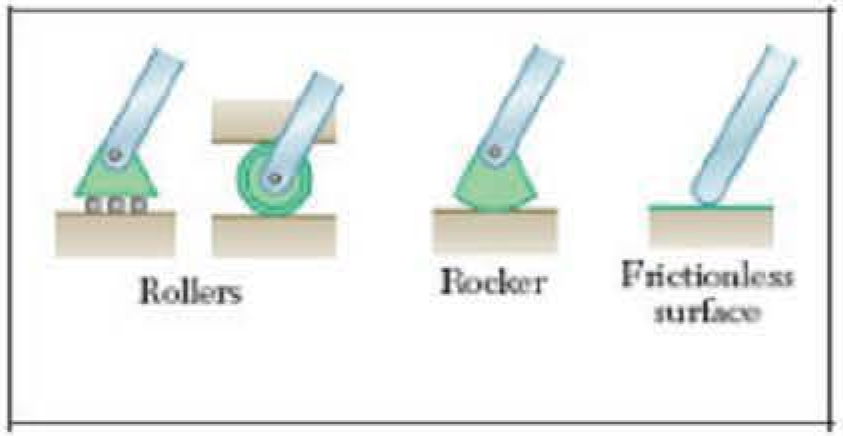
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action perpendicular to surface	1

There can be only one unknown reaction force at a roller support and it is perpendicular to the surface.

GIVE FEEDBACK

OK

These supports have some things in common. What are they?

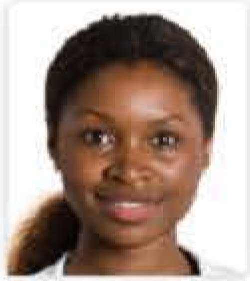


Check all that apply.

- [ ] They are all roller joints
- [ ] They only transmit forces that are perpendicular to the mounting surface
- [ ] They are all regarded as frictionless joints in calculations
- [ ] They are all pin joints

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

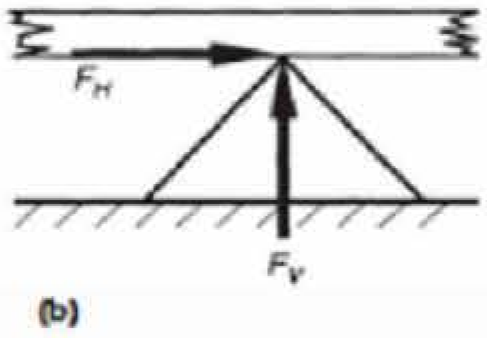


The other important type of beam support, besides the roller, is the pin support. Like the roller, the pin support can resist vertical forces. But the pin can also transmit sideways (horizontal) forces through to the foundation. In other words, they will fight to prevent sideways movement.

Pinned supports

A pinned support (or pin joint) connects the two objects through a pin or hinge arrangment. It could even be simply a rough suface (high friction), such as the foot of a ladder which is designed not to slip.

The possible direction of the reaction at a pinned support is not immediately obvious. It is often drawn as a triangle, meaning that it cannot roll sideways. These joints are needed to locate the beam and prevent it from moving horizontally.

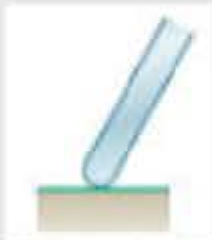
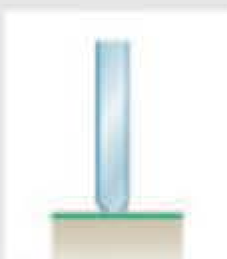


GIVE FEEDBACK

OK

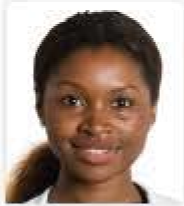
Which of the following joints act like a pinned support?

Check all that apply.

- [ ]  Rough surface joint
- [ ]  Hinge joint
- [ ]  Rocker joint
- [ ]  Frictionless joint

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

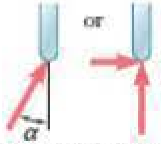


Unlike a roller joint, where the force is always perpendicular to the mounting surface, the reaction force in a pin joint could go in any direction. So it is more complicated to solve than a roller.

Reactions that may exist at pinned support

The possible direction of the reaction at a pinned support is not immediately obvious. The force transmitted through this type of joint can be in any direction. This is two unknowns: unknown force magnitude and unknown direction.

Another way to analyse a pin joint is in terms of its vertical and horizontal components, which is often simpler for mathematical solutions. In this case a pinned support still has two unknowns, a vertical force and a horizontal force.

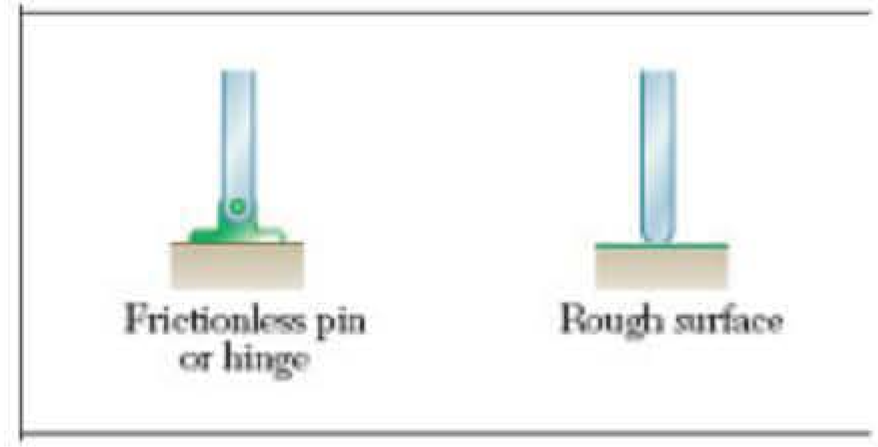
Support or connection	Reaction	Number of unknowns
 Frictionless pin or hinge; Rough surface	 Force of unknown direction	2

Pin joints are very common in structures and machines, e.g. a hinge or bearing is a pin joint.

GIVE FEEDBACK

OK

What do these types of beam support have in common?



Check all that apply.

- [ ] They do not transmit vertical forces
- [ ] They act like a hinge joint
- [ ] They only transmit vertical forces
- [ ] They are able to transmit both vertical and horizontal forces

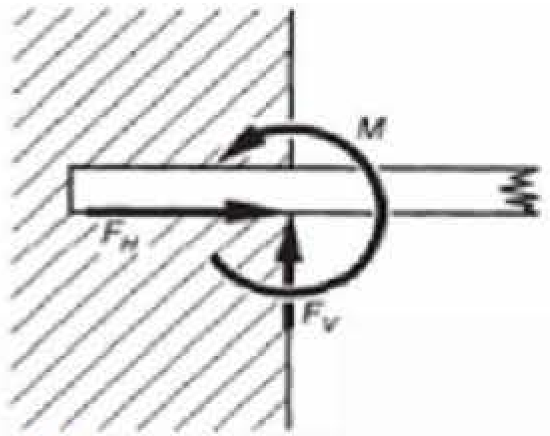
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The third type of beam support we will look at is the fixed support. Like the pin support, it resists both vertical and horizontal forces. In addition, the fixed support also prevents rotation.

Cantilever supports

The cantilever beam is a special case. Its only supported end is embedded rigidly into an unyielding solid abutment, which has to provide the total reaction against any movement or rotation.

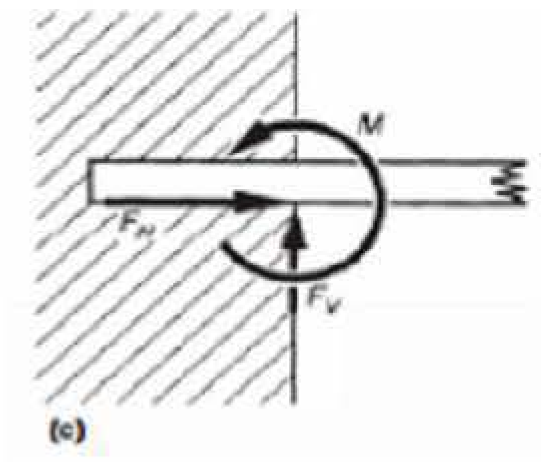


(c)

GIVE FEEDBACK

OK

Which statement is true for this type of beam support?



Click the correct answer.

The beam cannot move but it can rotate

The beam can move and rotate

The beam cannot move or rotate

The beam can move but not rotate

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The fixed support will always have three unknowns, whichever way you look at it. Two are forces and one is a moment (another word for torque).

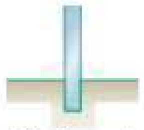
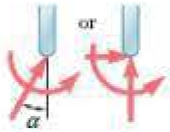
Reactions that may exist at cantilever support

The fixed support can transmit up to three types of reaction. This means there are three things that must be specified for a fixed support:

- Horizontal reaction force
- Vertical reaction force
- Reaction moment that prevents rotation of the cantilever beam, or

- Reaction force
- Angle of the reaction force
- Reaction moment that prevents rotation of the cantilever beam

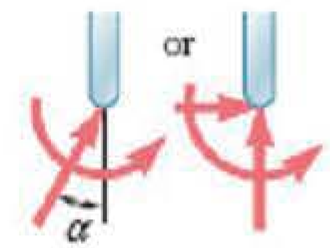
This is true for a horizontal cantilevered beam or a vertical fixed support.

Support or connection	Reaction	Number of unknowns
 Fixed support	 Force and couple	3

GIVE FEEDBACK

OK

Which of the following statements are correct regarding this image:

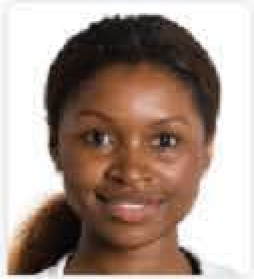


Check all that apply.

- [ ] They are both the same type of support
- [ ] They are both free to rotate
- [ ] They are both fixed supports
- [ ] They are both specified by three unknowns

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



Beams are non-concurrent bodies. All such bodies must comply with three equations of equilibrium (when working in two dimensions, which is planar).

Calculation of reactions for simply supported beams

Here we are calculating reaction forces for a simply supported beam with concentrated vertical loads only. As with any (planar) non-concurrent body, there are three equations of equilibrium:

Step 1. $\circlearrowleft^{+} \Sigma M_L = o$

Take moments about the pin joint, which gives an equation with the reaction force at the opposite roller support as the only unknown. Solve for this unknown.

Step 2. $+\uparrow \Sigma F_Y = o$

All vertical forces, including reactions, must add up to zero (vertical equilibrium). Solve this equation for the second unknown reaction force at the pin support.

Step 3. $\overset{+}{\rightarrow} \Sigma F_X = o$

There are no horizontal forces so we can skip this bit.

GIVE FEEDBACK

OK

What does this statement mean?

$$+\uparrow \Sigma F_{Y=0}$$

Click the correct answer.

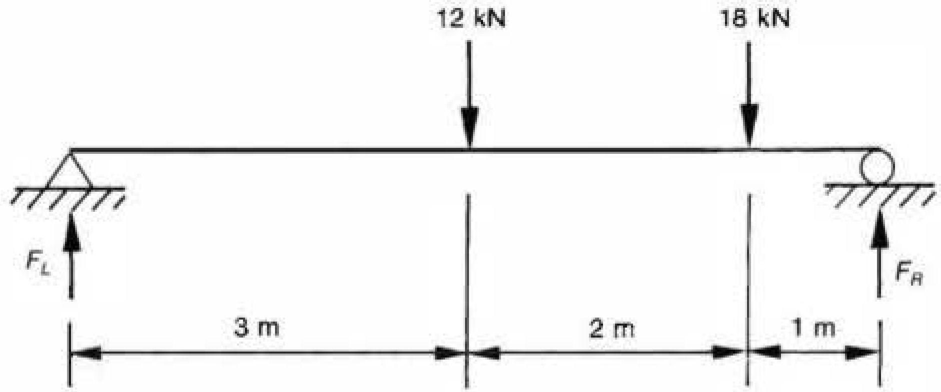
- All the forces in the *Y* direction are zero
- The sum of all the forces in the *Y* direction is zero
- The angle of force *Y* is zero
- The magnitude of force *Y* is zero

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the reactions for the simply supported beam with concentrated loads

Calculate the reactions for the simply supported beam shown.



The equations of equilibrium are:

$$\circlearrowleft + \Sigma M_L = 0$$

$$+\uparrow \Sigma F_Y = 0$$

$$\xrightarrow{+} \Sigma F_X = 0$$

We have three unknown forces:

- F_{LX}
- F_{LY}
- F_{RY}

A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

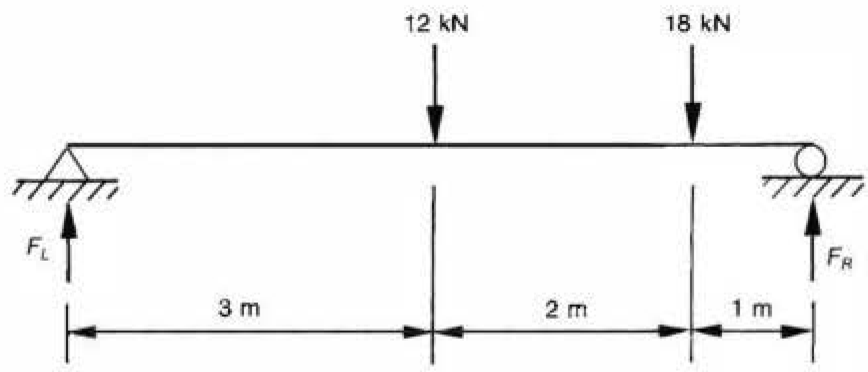
GIVE FEEDBACK

OK

Calculate the reactions for the simply supported beam with concentrated loads

As with any non-concurrent force problem, we cannot work with three variables at once. We need to start with one of these equations and force it to have only one unknown.

We achieve this using moments.



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

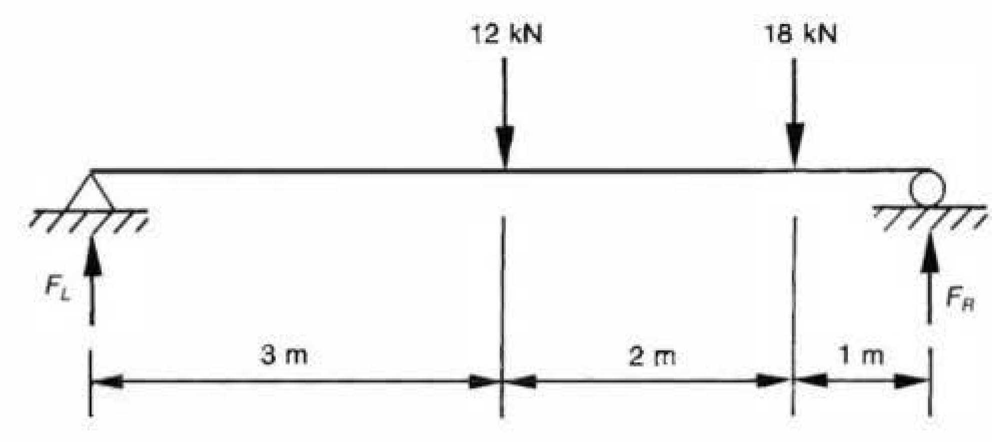
OK

Calculate the reactions for the simply supported beam with concentrated loads

We deliberately take moments right on top of the pin joint to eliminate F_L from the moment equation:

$$\circlearrowright + \Sigma M_A = 0$$

$$M_L = +(12 \times 3) + (18 \times 5) - (F_R \times 6) = 0$$



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

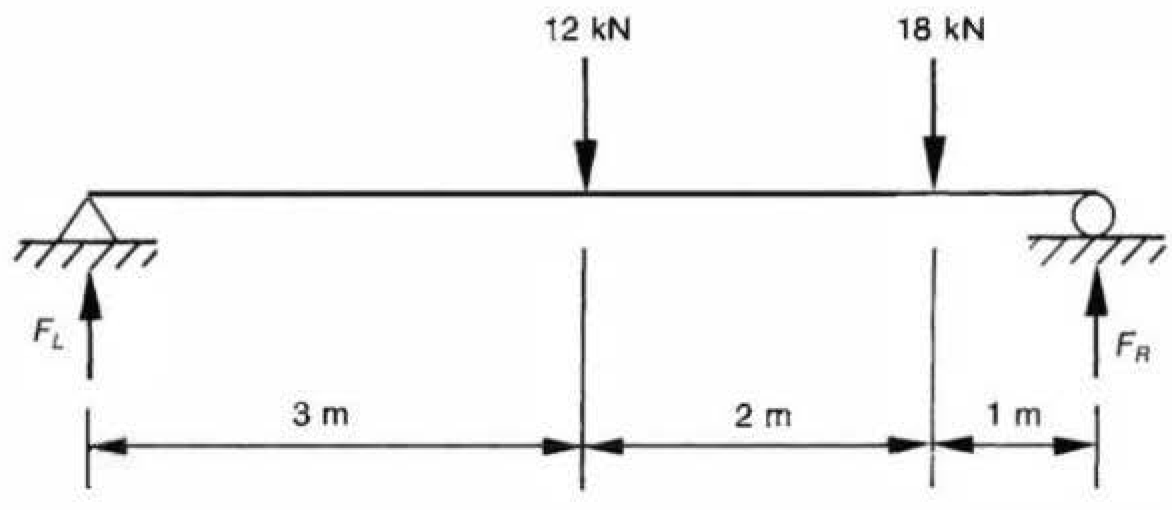
OK

Calculate the reactions for the simply supported beam with concentrated loads

$$\Sigma M_L = +(12 \times 3) + (18 \times 5) - (F_R \times 6)$$
$$= 36 + 90 - 6F_R$$
$$= 126 - 6F_R$$

So since it is in equilibrium:

$$0 = 126 - 6F_R$$
$$6F_R = 126$$
$$F_R = \frac{126}{6}$$
$$= 21 \text{ kN}$$



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

OK

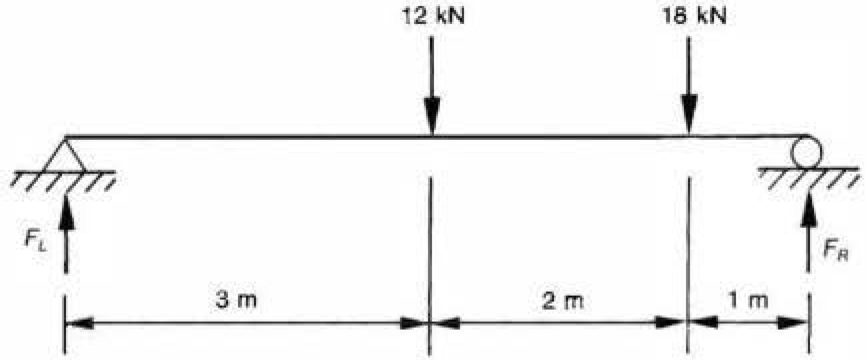
Calculate the reactions for the simply supported beam with concentrated loads

Follow the sign convention—upward forces are positive and downward forces are negative:

$$\begin{aligned}\Sigma F &= F_L - 12\text{ kN} - 18\text{ kN} + F_R \\ &= F_L - 12 - 18 + 21 \\ &= F_L - 9\end{aligned}$$

So the left reaction force is:

$F_L = 9\text{ kN}$



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

OK

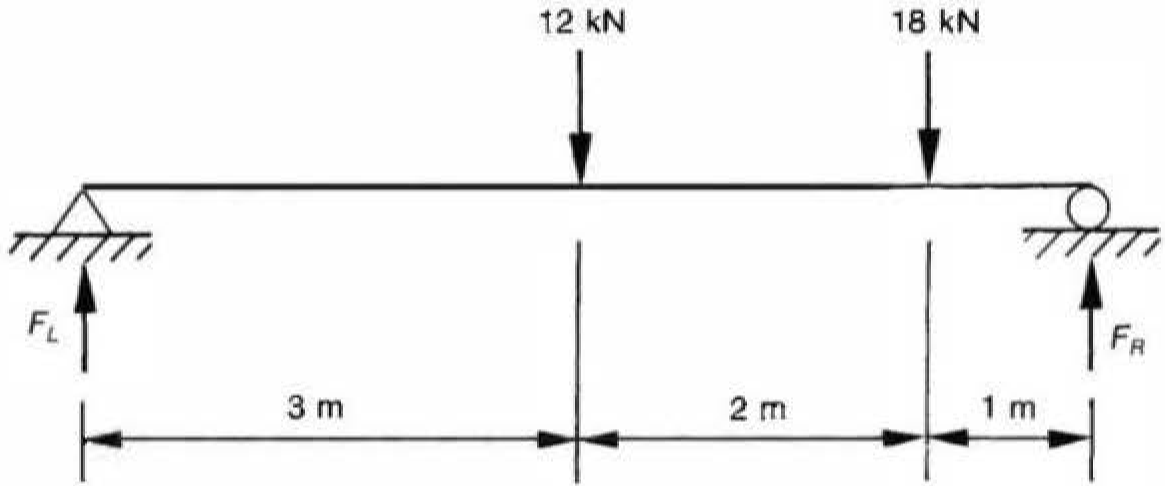
Calculate the reactions for the simply supported beam with concentrated loads

So the reaction forces are;

$F_{LX} = 0\,\mathrm{kN}$

$F_{LY} = 9\,\mathrm{kN}$

$F_R = 21\,\mathrm{kN}$



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

OK

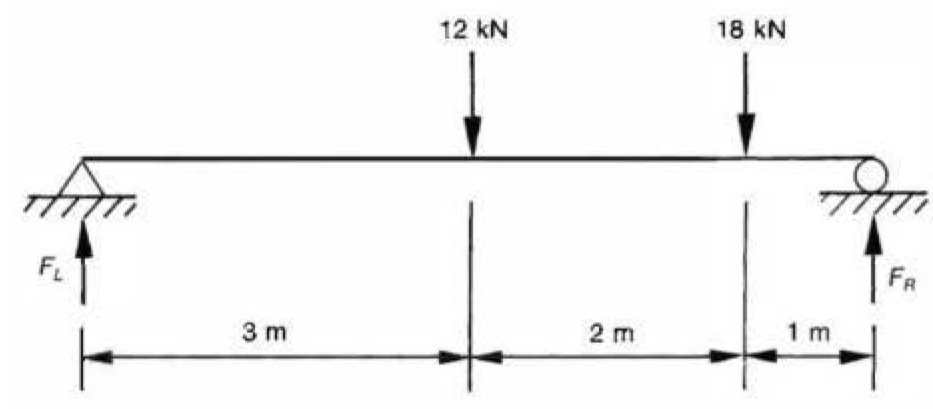
Calculate the reactions for the simply supported beam with concentrated loads

Check using moments at F_R

Take moment on the right end to find the left reaction (this is not necessary, we are just checking the answer another way):

$$\begin{aligned}\Sigma M_R &= -(12\ \text{kN} \times 3\ \text{m}) - (18\ \text{kN} \times 1\ \text{m}) + (F_L \times 6\ \text{m}) \\ &= -36 - 18 + 6F_L \\ &= -54 + 6F_L \\ F_L &= \frac{54}{6} \\ &= 9\ \text{kN}\end{aligned}$$

We get the same answer as before.



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find F_R	Vertical equilibrium	Solution	Check (alternative moment)

GIVE FEEDBACK

OK

Match the meaning of these three equations of equilibrium:

$$\circlearrowright + \Sigma M_{L}=0$$
$$+\uparrow \Sigma F_{Y}=0$$
$$\xrightarrow{+} \Sigma F_{X}=0$$

Drag statements on the right to match the left.

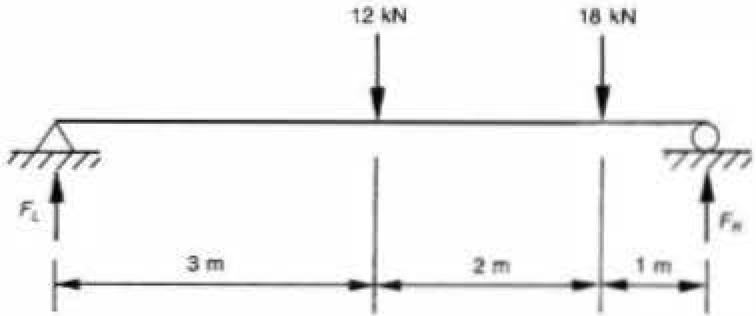
Taking clockwise as positive, the sum of all moments around point A is zero	$+\uparrow \Sigma F_{Y}=0$
Taking upwards as positive, the sum of all forces in the *Y* direction is zero	$\circlearrowright + \Sigma M_{L}=0$
Taking positive as to the right, the sum of all forces in the *X* direction is zero	$\xrightarrow{+} \Sigma F_{X}=0$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

For a simply supported beam we can have up to three unknown forces:

- F_{LX}
- F_{LY}
- F_{RY}

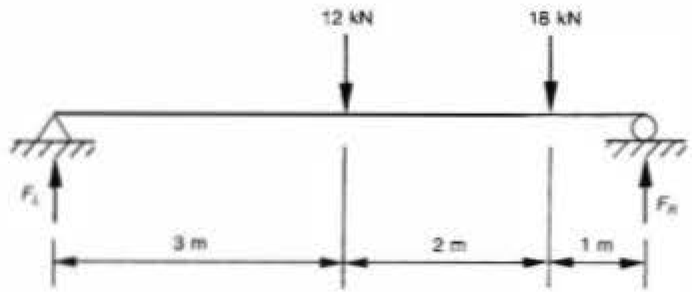


What would these symbols stand for?

Drag statements on the right to match the left.

The horizontal reaction at the pin support	F_{LX}
The vertical reaction at the pin support	F_{LY}
The vertical reaction at the roller support	F_{RY}

Calculating reactions for a simply supported beam, sort each step into order (choose to solve the horizontal reaction last):

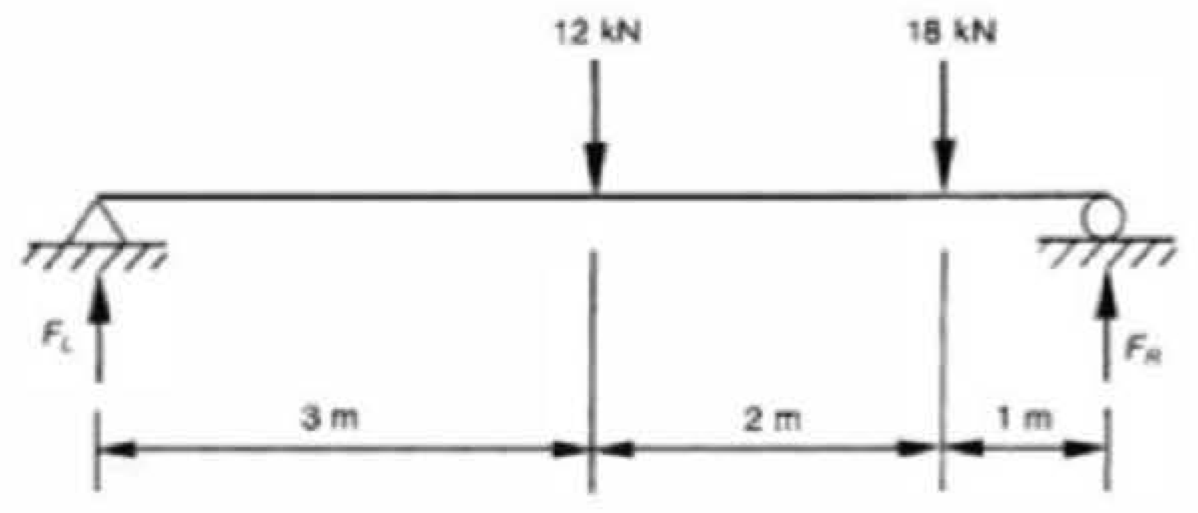


↑↓ **Place these in the proper order.**

- Take moment equilibrium at the pin support
- Solve the moment equation to find the reaction at the roller support
- Solve vertical equilibrium (using the roller reaction) to find the vertical pin reaction
- Solve horizontal equilibrium to find the horizontal pin reaction

Do you know the answer?

Calculating reactions for a simply supported beam, sort each step into order (represented by equilibrium equations; choose to solve the horizontal reaction last):



↑↓ **Place these in the proper order.**

$+\uparrow \Sigma F_Y = 0$

$\circlearrowright^{+} \Sigma M_L = 0$

$\xrightarrow{+} \Sigma F_X = 0$

Do you know the answer?

I KNOW IT

THINK SO

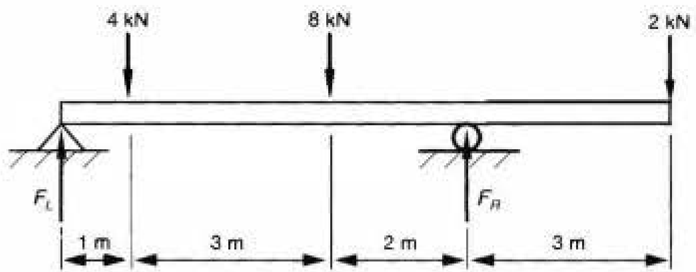
UNSURE

NO IDEA

Calculation of reactions for an overhanging beam with concentrated loads

If a beam has an overhanging portion extending beyond the supports on one or both sides, the procedure to be followed is exactly the same as a simply supported beam.

Take extra care in distinguishing between positive (clockwise) moments and negative (anticlockwise) moments relative to the selected reference point, which is usually the pin support.



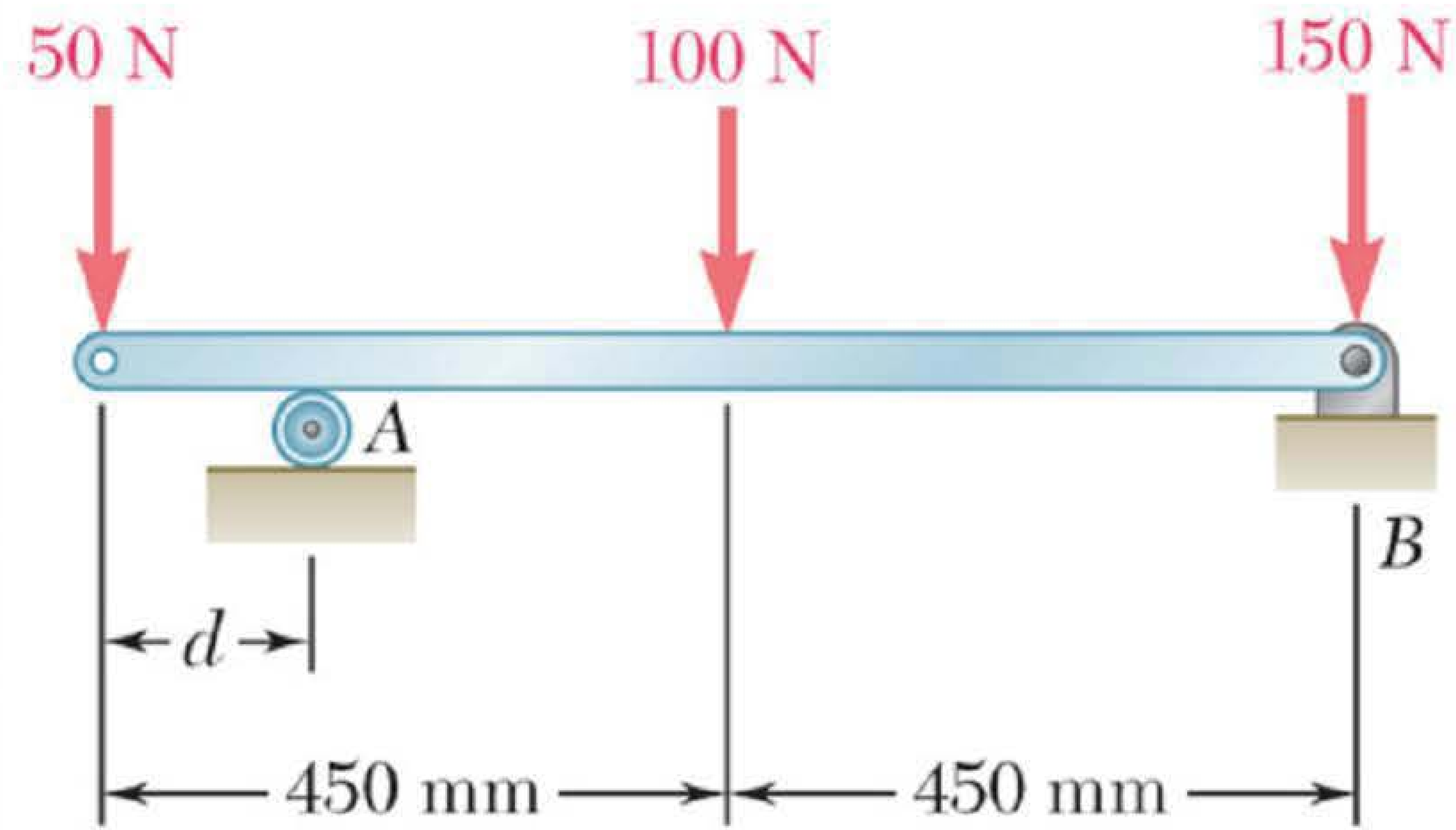
GIVE FEEDBACK

OK

Practice in taking moments

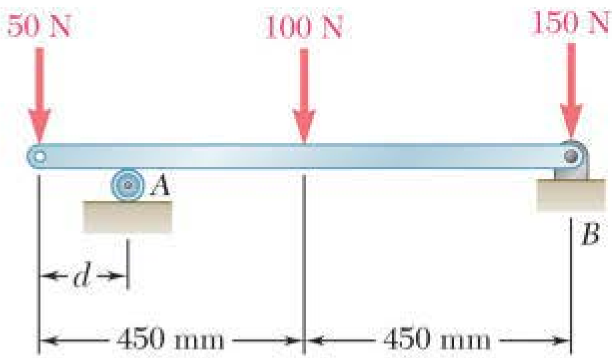
Match each moment equation.

Notes:
1. Each moment is written as (*N* x *m*);
2. Assume dimension d = 100 mm.



Drag statements on the right to match the left.

Moments at pin joint	$M = -(100 \times 0.45) - (50 \times 0.9) + (F_A \times 0.8) = 0$
Moments at roller joint	$M = -(50 \times 0.45) + (F_A \times 0.35) + (150 \times 0.45) - (F_B \times 0.45) = 0$
Moments at 50 kN force	$M = -(50 \times 0.1) + (100 \times 0.35) + (150 \times 0.8) - (F_B \times 0.8) = 0$
Moments at 100 kN force	$M = -(F_A \times 0.1) + (100 \times 0.45) + (150 \times 0.9) - (F_B \times 0.9) = 0$



If a beam has an overhanging portion extending beyond the supports on one or both sides, the procedure for finding the support reactions is

(please select)

Submit

Do you know the answer?

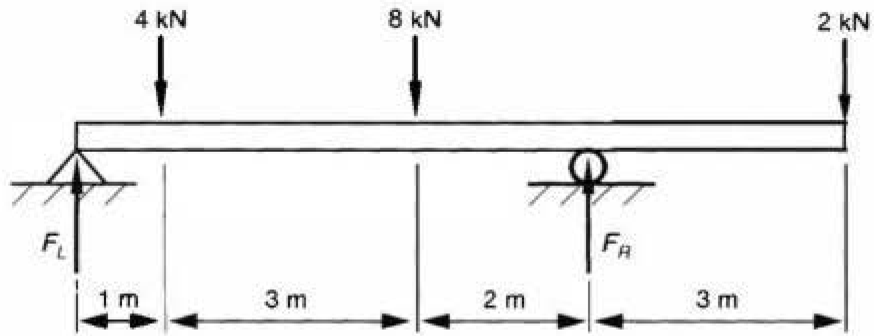
I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the reactions for an overhanging beam with concentrated loads

Calculate the reactions for the simply supported beam shown.

The equations of equilibrium are:

$$\circlearrowright + \Sigma M_L = O$$
$$+\uparrow \Sigma F_Y = O$$
$$\xrightarrow{+} \Sigma F_X = O$$



With pin and roller supports, there can be three unknown forces: F_{LX}, F_{LY} and F_R. However, there are no horizontal components of any force, so the unknown forces can be simplified to: F_L and F_R.

A simply supported beam	One equation, one unknown	Take moments at the pin joint *L*	Solve moment equation at L to find F_R.	Use vertical equilibrium to solve F_L

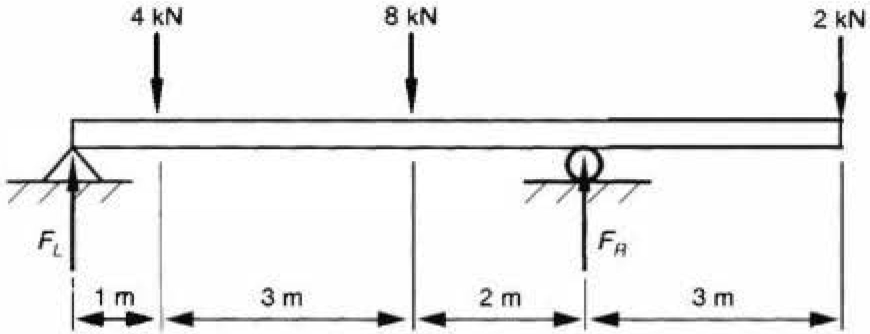
GIVE FEEDBACK

OK

Calculate the reactions for an overhanging beam with concentrated loads

As with any non-concurrent force problem, we cannot work with three variables at once. We need to start with one of these equations and force it to have only one unknown.

We achieve this using moments.



A simply supported beam	One equation, one unknown	Take moments at the pin joint *L*	Solve moment equation at L to find F_R.	Use vertical equilibrium to solve F_L

GIVE FEEDBACK

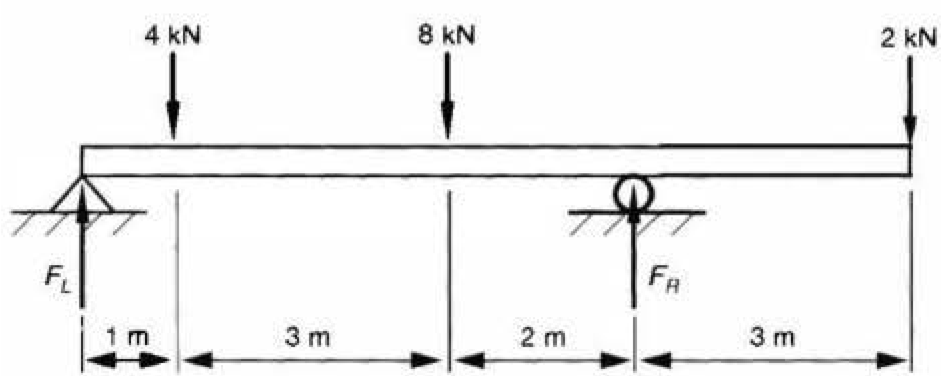
OK

Calculate the reactions for an overhanging beam with concentrated loads

We deliberately take moments right on top of the pin joint to eliminate F_L from the moment equation:

$$\circlearrowright + \Sigma M_A = 0$$

This gives an equation with one unknown, F_R, which is solvable.



$$\Sigma M_L = +(4\,\mathrm{kN} \times 1\,\mathrm{m}) + (8\,\mathrm{kN} \times 4\,\mathrm{m}) - (F_R \times 6\,\mathrm{m}) + (2\,\mathrm{kN} \times 9\,\mathrm{m}) = 0$$

A simply supported beam	One equation, one unknown	Take moments at the pin joint *L*	Solve moment equation at L to find F_R.	Use vertical equilibrium to solve F_L

GIVE FEEDBACK

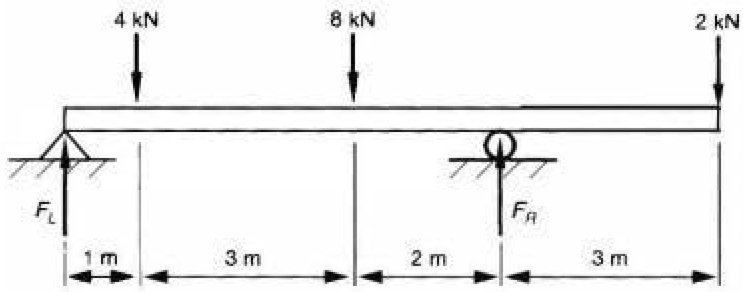
OK

Calculate the reactions for an overhanging beam with concentrated loads

Now solve moment equation taken at pin joint L to find reaction at roller R.

$\Sigma M_L = 0$

Then re-arrange to find F_R;



$$\Sigma M_L = +(4\,\text{kN} \times 1\,\text{m}) + (8\,\text{kN} \times 4\,\text{m}) - (F_R \times 6\,\text{m}) + (2\,\text{kN} \times 9\,\text{m})$$
$$= +4 + 32 - 6F_R + 18$$
$$= -54 - 6F_R$$
$$\therefore F_R = 9\,\text{kN}$$

A simply supported beam	One equation, one unknown	Take moments at the pin joint *L*	Solve moment equation at L to find F_R.	Use vertical equilibrium to solve F_L

GIVE FEEDBACK

OK

Calculate the reactions for an overhanging beam with concentrated loads

Follow the sign convention—upward forces are positive and downward forces are negative:

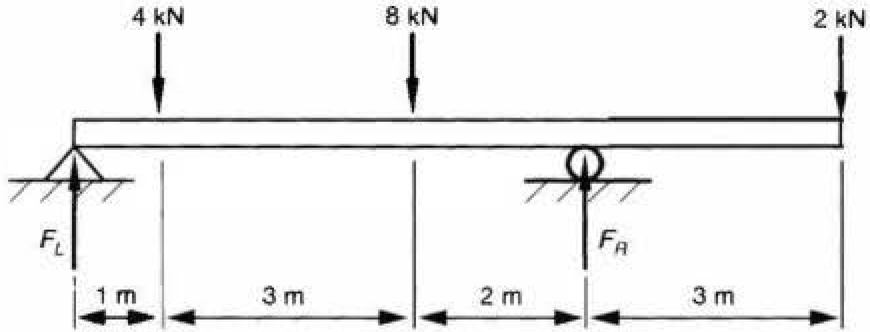
$\circlearrowright + \Sigma M_L = O$

$+\uparrow \Sigma F_Y = O$

$\Sigma F_Y = F_L - 4\,\text{kN} - 8\,\text{kN} + 9\,\text{kN} - 2\,\text{kN} = 0$

$\xrightarrow{+} \Sigma F_X = O$

$\therefore F_L = 5\,\text{kN}$



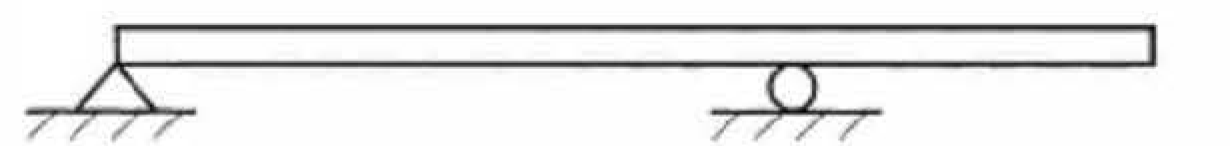
So reactions are: $F_L = 5\,\text{kN}, F_R = 9\,\text{kN}$

A simply supported beam	One equation, one unknown	Take moments at the pin joint *L*	Solve moment equation at L to find F_R.	Use vertical equilibrium to solve F_L

GIVE FEEDBACK

OK

For an overhanging beam we can have up to three unknown forces:



- F_{LX}
- F_{LY}
- F_{RY}

What would these symbols stand for?

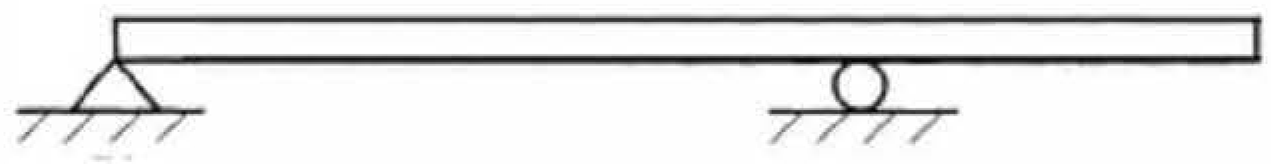
Drag statements on the right to match the left.

The horizontal reaction at the pin support	F_{LY}
The vertical reaction at the pin support	F_{LX}
The vertical reaction at the roller support	F_{RY}

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculating reactions for an overhanging beam, sort each step into order (choose to solve the horizontal reaction last):



↑↓ **Place these in the proper order.**

- Solve horizontal equilibrium to find the horizontal pin reaction
- Solve the moment equation to find the reaction at the roller support
- Solve vertical equilibrium (using the roller reaction) to find the vertical pin reaction
- Take moment equilibrium at the pin support

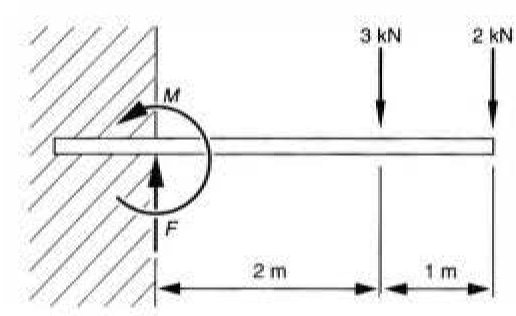
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculation of reactions for a cantilever beam with concentrated loads

To solve a cantilever beam with concentrated vertical loads, we apply the three equations of equilibrium as we would for a simply supported beam (or any non-concurrent problem).

$$\circlearrowleft^{+} \Sigma M_L = 0$$
$$+\uparrow \Sigma F_Y = 0$$
$$\xrightarrow{+} \Sigma F_X = 0$$

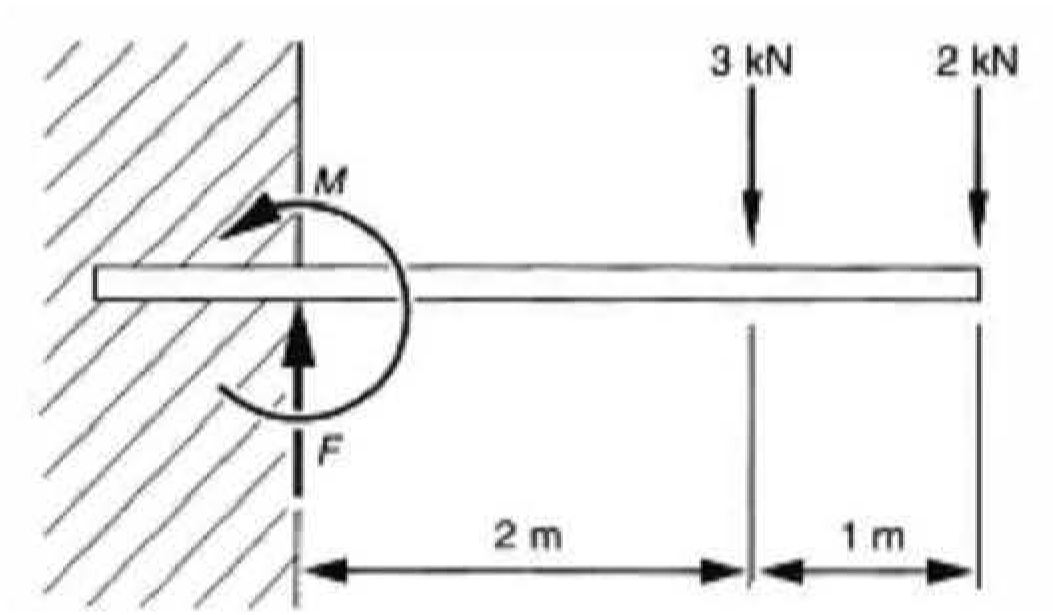


However, in this case each equation has only one unknown, so they can be solved directly, in any order. We do not need to do the moment equation first.

GIVE FEEDBACK

OK

To solve a cantilever beam with concentrated vertical loads, which equation of equilibrium must be solved first?



Click the correct answer.

$+\uparrow \Sigma F_Y = 0$

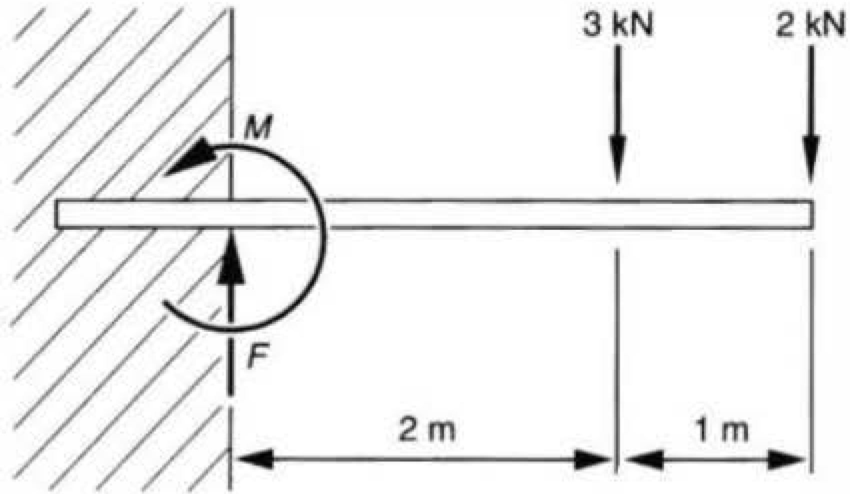
$\xrightarrow{+} \Sigma F_X = 0$

They can be solved in any order.

$\circlearrowright + \Sigma M_L = 0$

Calculate the reactions for a cantilever beam with concentrated loads

Calculate the reactions for the simply supported beam shown.



The equations of equilibrium are:

$$\circlearrowright + \Sigma M_L = 0$$
$$+\uparrow \Sigma F_Y = 0$$
$$\overset{+}{\rightarrow} \Sigma F_X = 0$$

We have three unknowns: F_X, F_Y and M. However, since there are no x components, then $F_x = 0$.

So unknowns are F_y and M.

A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

GIVE FEEDBACK

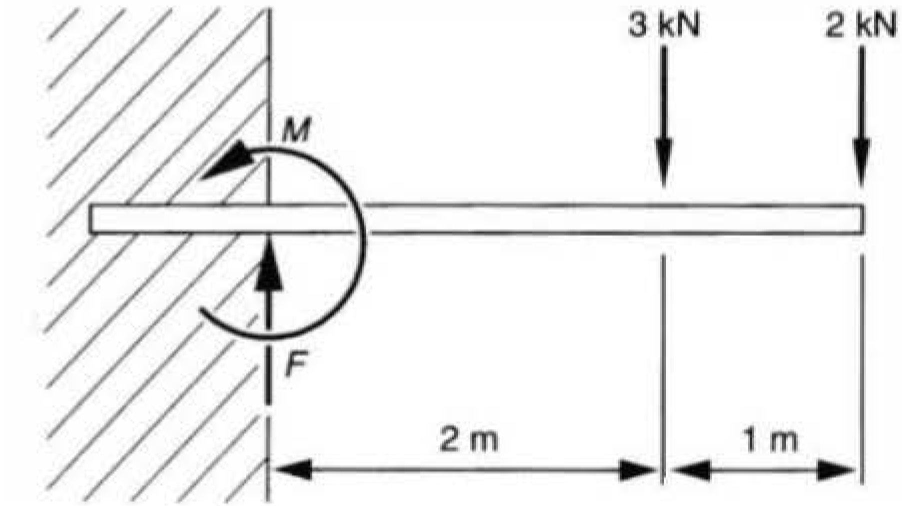
OK

Calculate the reactions for a cantilever beam with concentrated loads

Unlike the pin-and-roller supports of a simply supported beam, the cantilever has only one support.

This means that each of the three equations can be solved independently, in any order.

We can start with any one of these equations. But we will start with moment equilibrium to be consistent.



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

GIVE FEEDBACK

OK

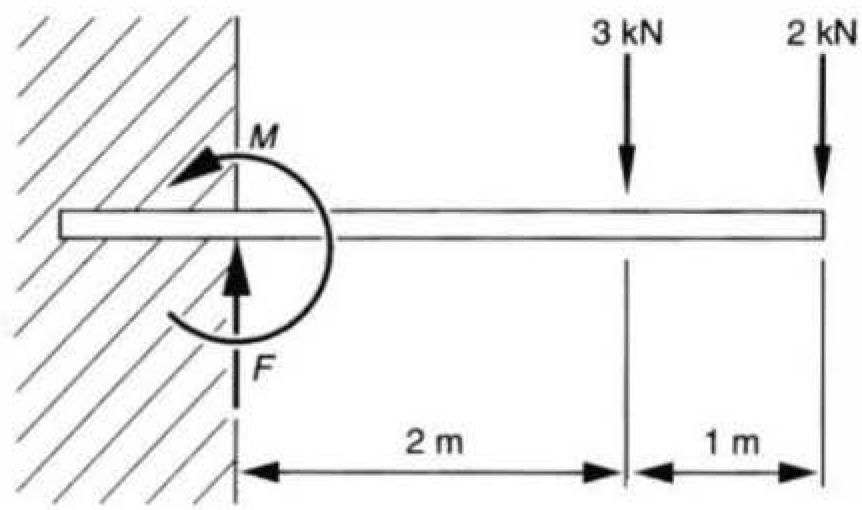
Calculate the reactions for a cantilever beam with concentrated loads

We deliberately take moments right on top of the pin joint to eliminate F_L from the moment equation:

$$\circlearrowleft + \Sigma M_A = 0$$

$$\Sigma M = +(3\,\text{kN} \times 2\,\text{m}) + (2\,\text{kN} \times 3\,\text{m}) - M = 0$$

Note: When setting up this equation, the unknown reaction moment *M* must be included in the total sum of all moments acting on the beam at the support.



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

GIVE FEEDBACK

OK

Calculate the reactions for a cantilever beam with concentrated loads

Forces are multiplied by their respective distances from the support but the reaction moment is entered as a single entity represented by the symbol *M*.

(It is negative to match the picture, which is our guess for moment reaction.)

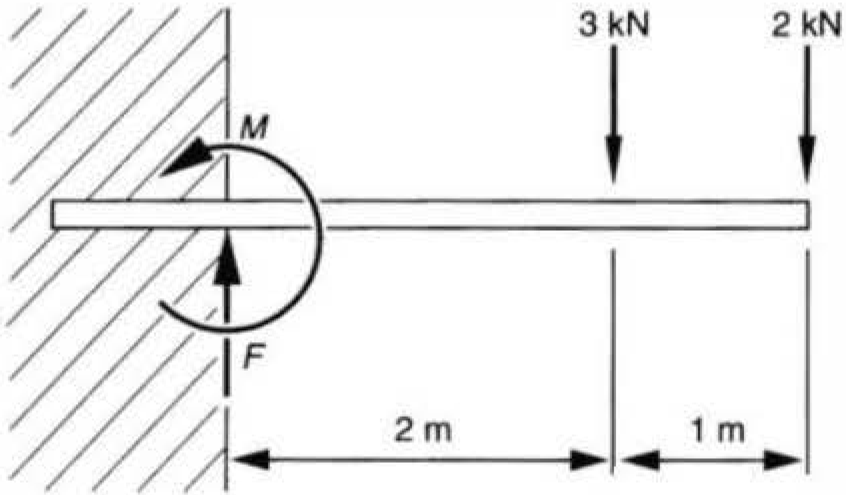
Solving for *M*:

$\Sigma M = 0$

$\Sigma M = 3\,\mathrm{kN} \times 2\,\mathrm{m} + 2\,\mathrm{kN} \times 3\,\mathrm{m} - M$

$0 = 6 + 6 - M$

$\therefore M = 12\,\mathrm{kN\,m}$



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

GIVE FEEDBACK

OK

Calculate the reactions for a cantilever beam with concentrated loads

Follow the sign convention—upward forces are positive and downward forces are negative:

$$\circlearrowright + \sum M_L = 0$$

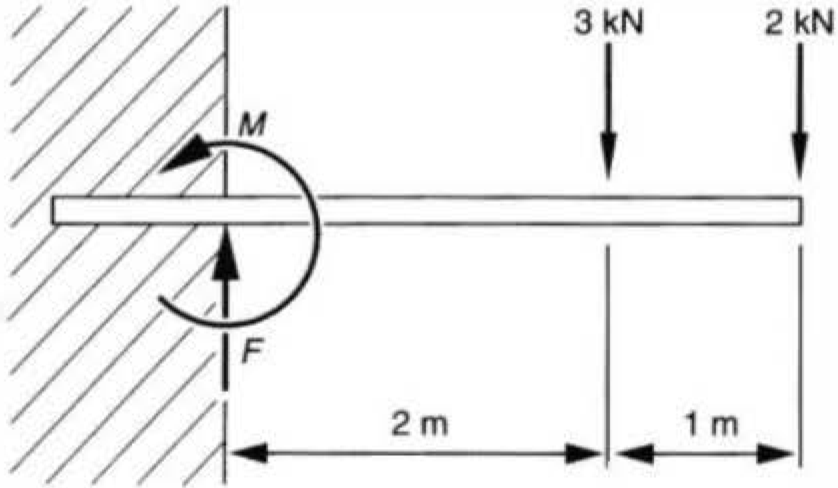
$$\sum F = F - 3\ \mathrm{kN} - 2\ \mathrm{kN}$$

$$+\uparrow \sum F_Y = 0$$

$$\therefore F = 5\ \mathrm{kN}$$

$$\xrightarrow{+} \sum F_X = 0$$

So the left reaction force is $5\ \mathrm{kN}$ upwards.



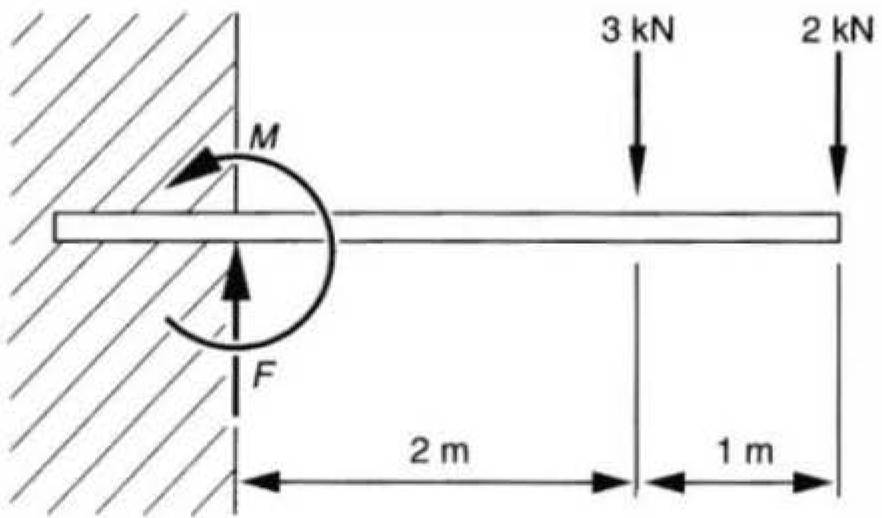
A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

GIVE FEEDBACK

OK

Calculate the reactions for a cantilever beam with concentrated loads

$F_X = 0\,\text{kN}$
$F_Y = 5\,\text{kN}$ (up)
$M = -12\,\text{kN}$ (anticlockwise)



A simply supported beam	One equation, one unknown	Take moment at pin joint	Solve moment equation to find *M*	Vertical equilibrium	Solution

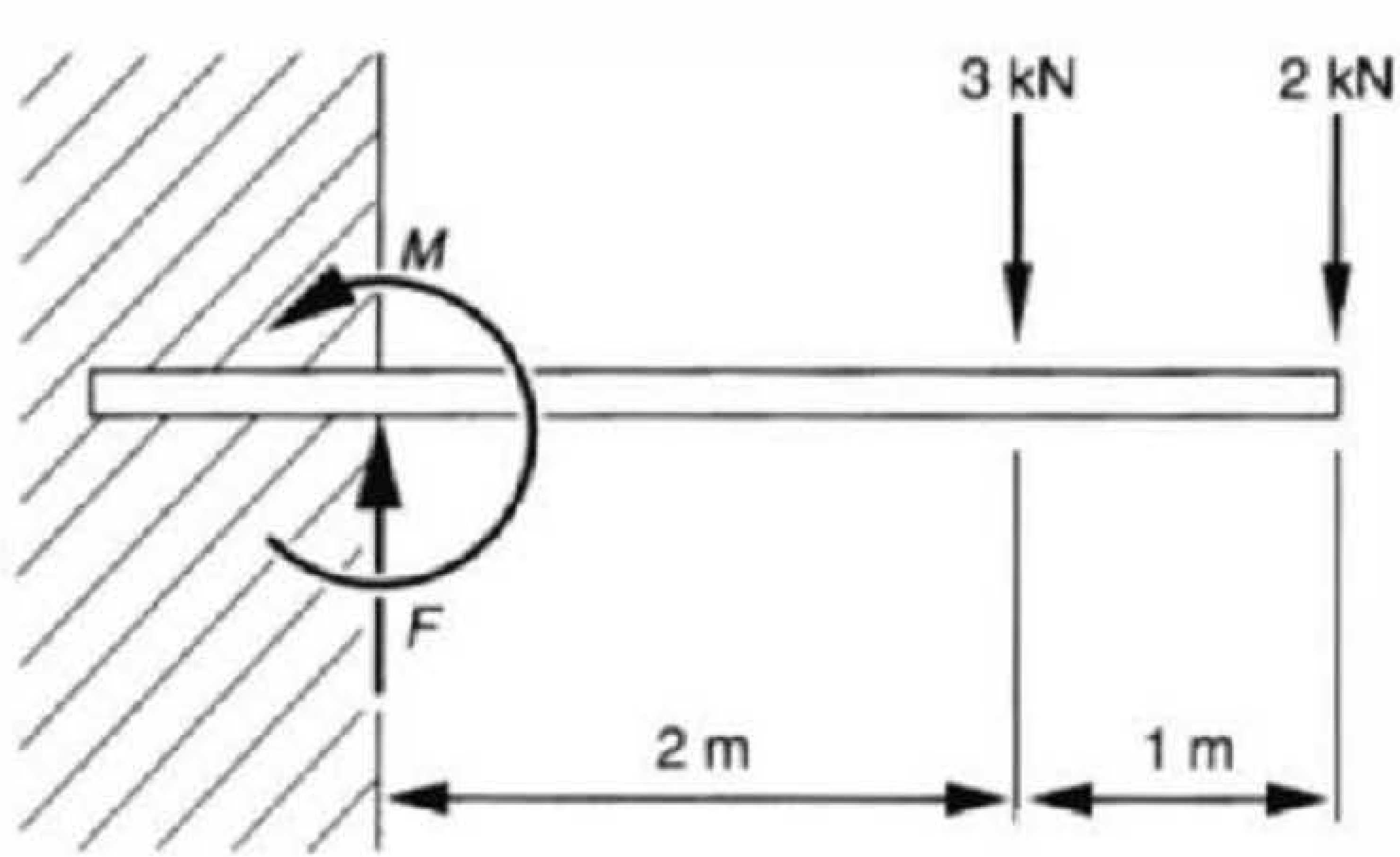
GIVE FEEDBACK

OK

For a simply supported beam, we can have up to three unknown forces:

- F_X
- F_Y
- M

What would these symbols stand for?



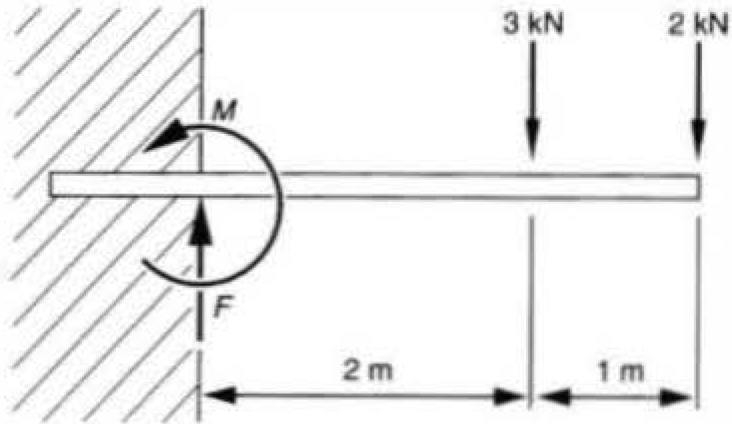
Drag statements on the right to match the left.

The horizontal reaction at the fixed support	M
The vertical reaction at the fixed support	F_Y
The moment reaction at the fixed support	F_X

For a simply supported beam, we can have up to three unknown forces:

- F_X
- F_Y
- M

What would these symbols stand for?

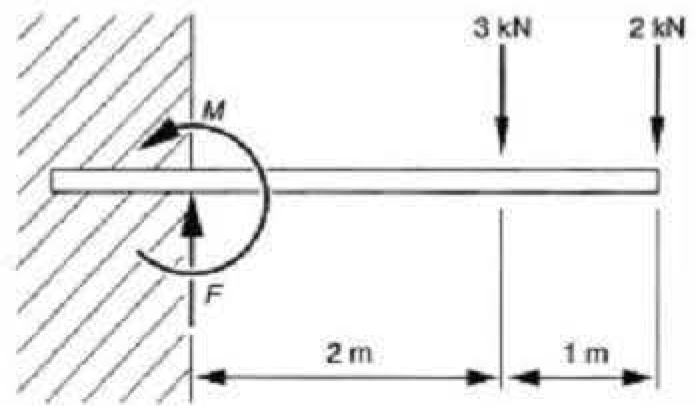


Drag statements on the right to match the left.

The horizontal reaction at the fixed support	F_X
The vertical reaction at the fixed support	F_Y
The moment reaction at the fixed support	M

Match the meaning of these three equations of equilibrium for a cantilevered beam:

$$\circlearrowright^{+} \Sigma M_L = 0$$
$$+\uparrow \Sigma F_Y = 0$$
$$\xrightarrow{+} \Sigma F_X = 0$$



Drag statements on the right to match the left.

Taking clockwise as positive, the sum of all moments around point A is zero	$\circlearrowright^{+} \Sigma M_L = 0$
Taking upwards as positive, the sum of all forces in the *Y* direction is zero	$\xrightarrow{+} \Sigma F_X = 0$
Taking positive as to the right, the sum of all forces in the *X* direction is zero	$+\uparrow \Sigma F_Y = 0$

Distributed loads are spread over a certain length of the beam.

Uniformly distributed loads

So far we have only looked at point forces, where force is applied at a single point. However, forces can also be applied over a region of the beam (e.g. weight of goods distributed over a mezzanine floor). This is known as a **distributed load.**

When a distributed load has a constant value per unit length of a beam, the load is called a **uniformly distributed load**. They can vary from quite short to taking the entire length of the beam (e.g. beam mass is a uniformly distributed load).

Such loads are expressed in units of force per unit length of beam, i.e. in newtons per metre (N/m) or kilonewtons per metre (kN/m). We only consider uniformly distributed loads in this resource.

GIVE FEEDBACK

OK

Which of the following is not a concentrated load?

Click the correct answer.

- Roller support reaction
- Point force at an inclined angle
- Distributed load
- Vertical reaction at pin support

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The first thing we do with distributed loads is convert them back to concentrated loads.

Replace distributed loads with concentrated loads

To deal with distributed loads when solving beam reactions, we convert the distributed load into a point load and then carry on with the solution as we did before.

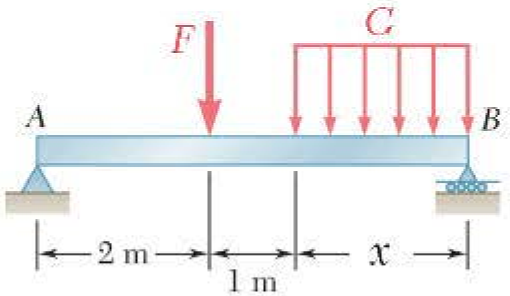
To do this conversion, calculate the total load and then replace it with a point force in the middle of the distributed region.

Note: When we multiply distributed load in N/m times the region length in m we get:

N/m . m = N (which is Newtons, a point force).

GIVE FEEDBACK

OK



Convert the distributed load into a concentrated load, where G = 29 kN/m and X = 2.5 m.

(Minimum one decimal place, type units as kN.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

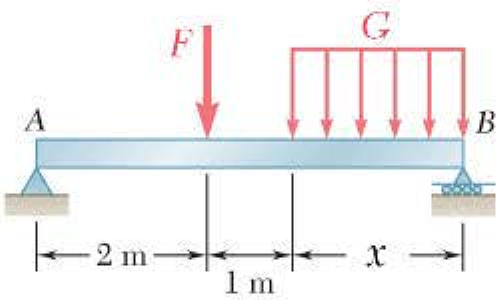
SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question



After converting the distributed load G = 29 kN/m and X = 2.5 m into a concentrated force, the value is 72.5 kN. Determine the correct location of this concentrated force (as measured from the left end of the beam, or point A).

(Minimum one decimal place, type units as m.)

+ − · ÷ π m ↵

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

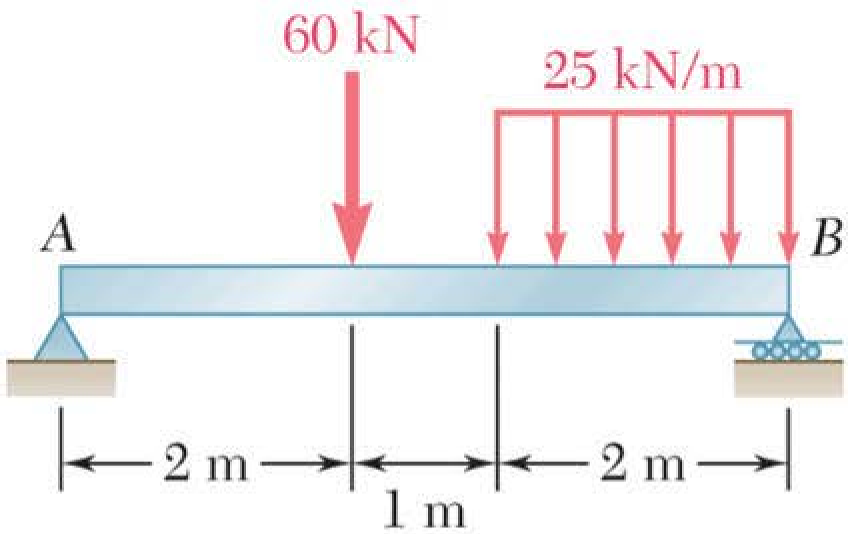
Calculate reactions for a simply supported beam with a uniformly distributed load

Determine the reactions for the overhanging beam shown.

Convert the uniformly distributed load to a point force:

$$25 \text{ kN/m} \times 2 \text{ m} = 50 \text{ kN}$$

The midpoint of this distributed load is located 1 m from the right-hand support (the roller).



Convert uniformly distributed load	Point loads	Use moment equilibrium to get F_B	Use vertical equilibrium to get F_A	Check Solution

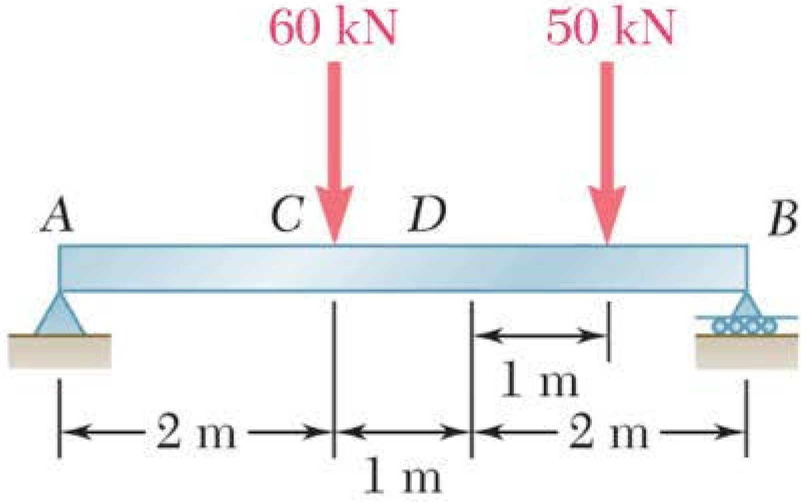
GIVE FEEDBACK

OK

Calculate reactions for a simply supported beam with a uniformly distributed load

Therefore the problem can be restated in point loads—the existing 60 kN point load plus the equivalent 50 kN concentrated load, as shown.

The usual method of solution can now be followed.



Convert uniformly distributed load	Point loads	Use moment equilibrium to get F_B	Use vertical equilibrium to get F_A	Check Solution

GIVE FEEDBACK

OK

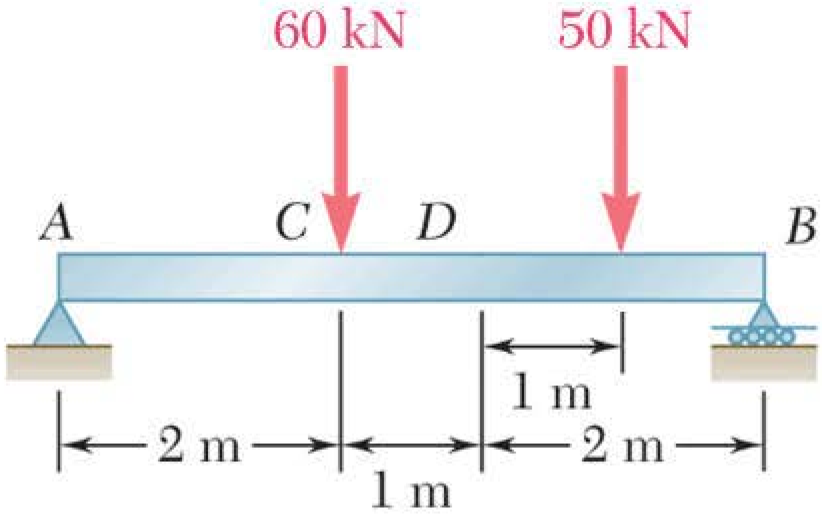
Calculate reactions for a simply supported beam with a uniformly distributed load

Solve equilibrium of moments at pin support A to get reaction force at roller support B:

$$\circlearrowright + \Sigma M_A = 0$$

$\Sigma M_A = +(60\ \text{kN} \times 2\ \text{m}) + (50\ \text{kN} \times 4\ \text{m}) - (F_B \cdot 5) = 0$

$\therefore F_B = 64\ \text{kN}$



Convert uniformly distributed load	Point loads	Use moment equilibrium to get F_B	Use vertical equilibrium to get F_A	Check Solution

GIVE FEEDBACK

OK

Calculate reactions for a simply supported beam with a uniformly distributed load

Solve vertical equilibrium to get reaction force at pin support A:

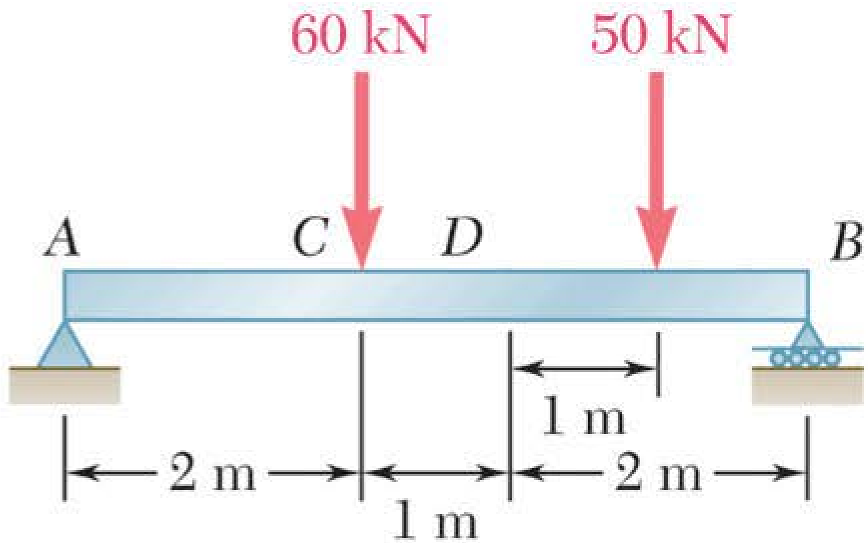
$$\circlearrowright^{+} \Sigma M_L = O$$

$$\Sigma F_y = F_A - 60\ \text{kN} - 50\ \text{kN} + 64\ \text{kN} = 0$$

$$+\uparrow \Sigma F_Y = O$$

$$\therefore F_A = 46\ \text{kN}$$

$$\xrightarrow{+} \Sigma F_X = O$$



Convert uniformly distributed load	Point loads	Use moment equilibrium to get F_B	Use vertical equilibrium to get F_A	Check Solution

GIVE FEEDBACK

OK

Calculate reactions for a simply supported beam with a uniformly distributed load

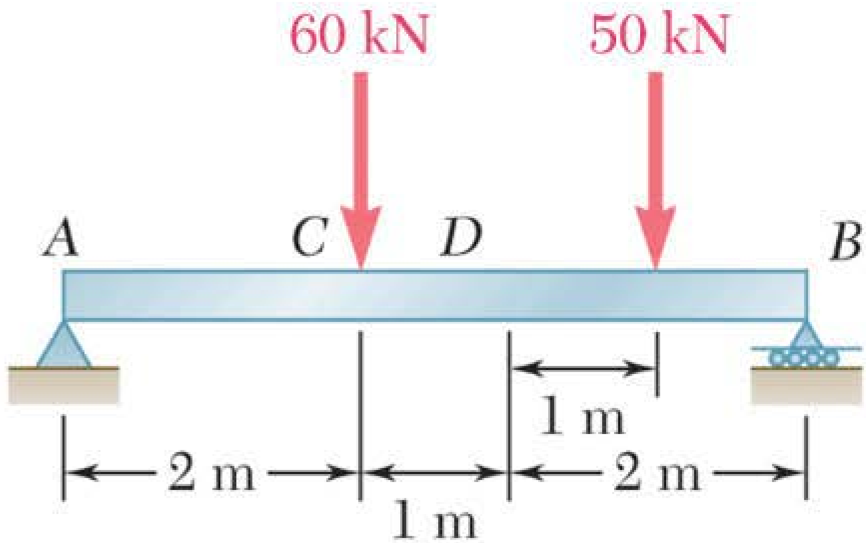
Test moment about B to re-check F_A

$\Sigma M_A = -(50\ \text{kN} \times 1\ \text{m}) + (60\ \text{kN} \times 3\ \text{m}) - (F_A \cdot 5) = 0$

$\therefore F_A = 46\ \text{kN}$

This agrees with the previous solution using vertical equilibrium.

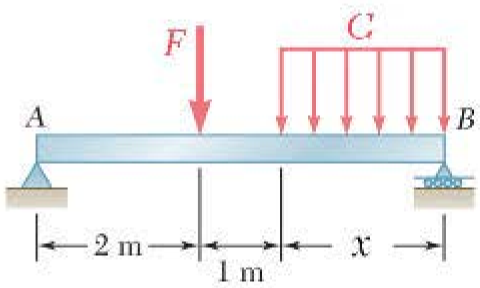
Summary: $R_A = 46\ \text{kN}, R_B = 64\ \text{kN}$



Convert uniformly distributed load	Point loads	Use moment equilibrium to get F_B	Use vertical equilibrium to get F_A	Check Solution

GIVE FEEDBACK

OK



Force F is 55 kN and a distributed load of 29 kN/m is applied over a length of 2.5 m. After converting to a concentrated force of 72.5 kN, determine the reaction force at the roller support.

(Minimum one decimal place. Type units as kN.)

+ − · ÷ kN

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

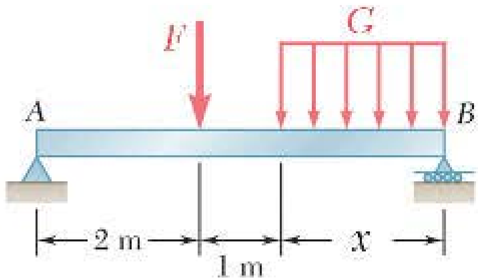
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Force F is 55 kN and a distributed load of 29 kN/m is applied over a length of 2.5 m. After converting to a concentrated force of 72.5 kN, determine the reaction force at the pin support.

(Minimum one decimal place. Type units as kN.)



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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

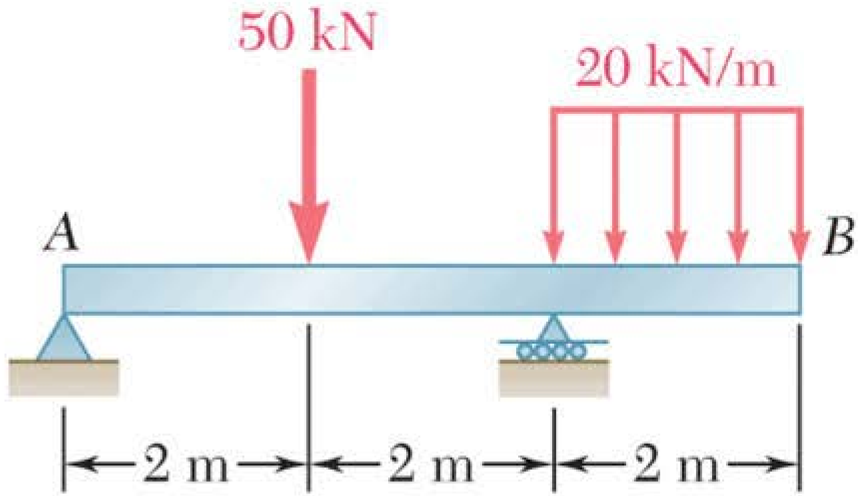
Calculate reactions for an overhanging beam with a uniformly distributed load

Determine the reactions for the overhanging beam shown.

Convert the uniformly distributed load to a point force:

$$20 \text{ kN/m} \times 2 \text{ m} = 40 \text{ kN}$$

The midpoint of this distributed load is located 1 m past the right-hand support (the roller).



Convert uniformly distributed load	Point loads	Solve moments to get F_B	Solve vertical equilibrium

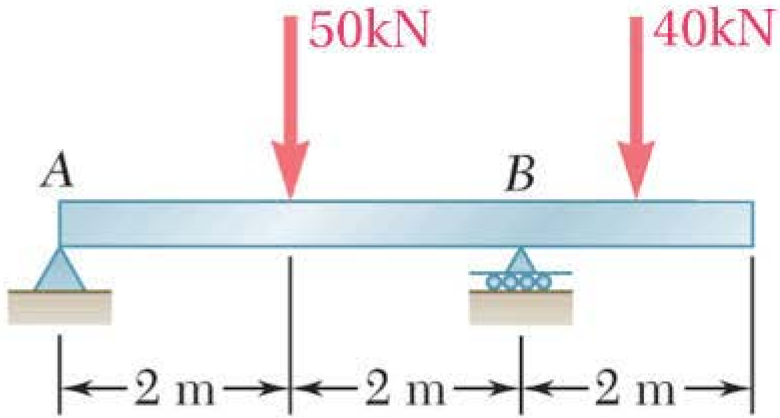
GIVE FEEDBACK

OK

Calculate reactions for an overhanging beam with a uniformly distributed load

Therefore the problem can be restated in point loads—the existing 50 kN point load plus the equivalent 40 kN concentrated load, as shown.

The usual method of solution can now be followed.



Convert uniformly distributed load	Point loads	Solve moments to get F_B	Solve vertical equilibrium

GIVE FEEDBACK

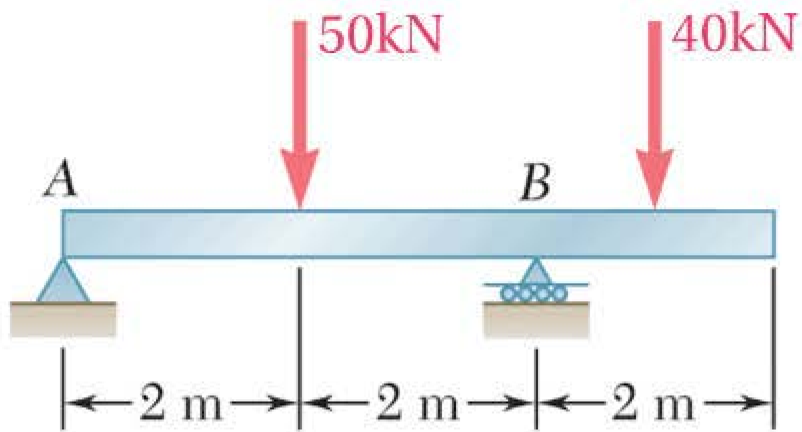
OK

Calculate reactions for an overhanging beam with a uniformly distributed load

Solve equilibrium of moments at pin support *A* to get reaction force at roller support *B*:

$$\Sigma M_A = +(50\text{ kN} \times 2\text{ m}) + (40\text{ kN} \times 5\text{ m}) - (F_B \cdot 4) = 0$$

$$\therefore F_B = 75\text{ kN}$$



Convert uniformly distributed load	Point loads	Solve moments to get F_B	Solve vertical equilibrium

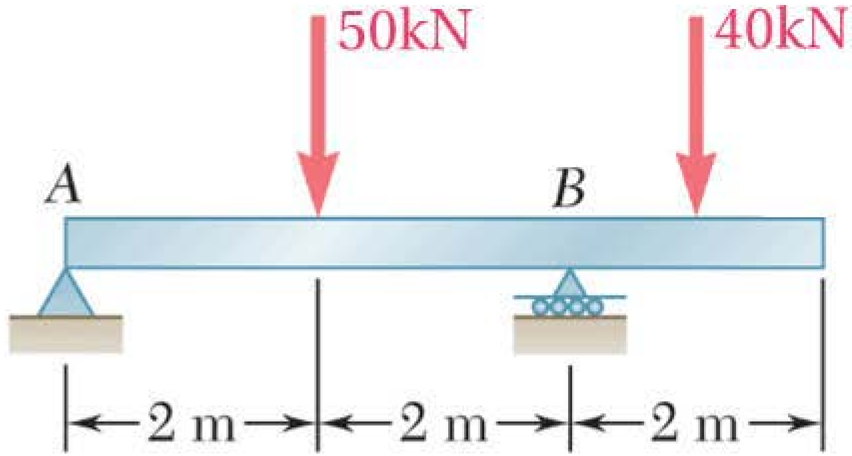
GIVE FEEDBACK

OK

Calculate reactions for an overhanging beam with a uniformly distributed load

Solve vertical equilibrium to get reaction force at pin support A:

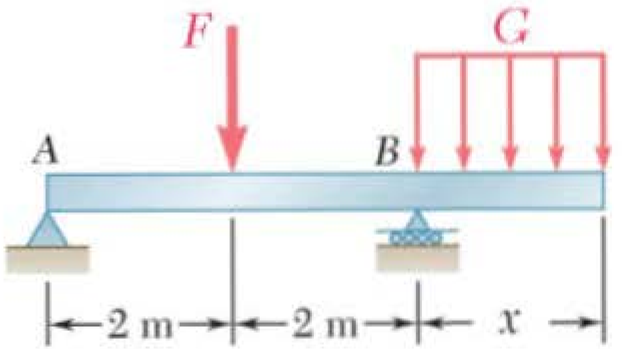
$$\Sigma F_Y = +F_A - 50\,\text{kN} + 75\,\text{kN} - 40\,\text{kN} = 0$$
$$\therefore F_A = 15\,\text{kN}$$



Convert uniformly distributed load	Point loads	Solve moments to get F_B	Solve vertical equilibrium

GIVE FEEDBACK

OK



Force F is 55 kN and a distributed load of 29 kN/m is applied over a length of 2.5 m. Determine the reaction force at the pin support.

(Minimum one decimal place, type units as kN.)

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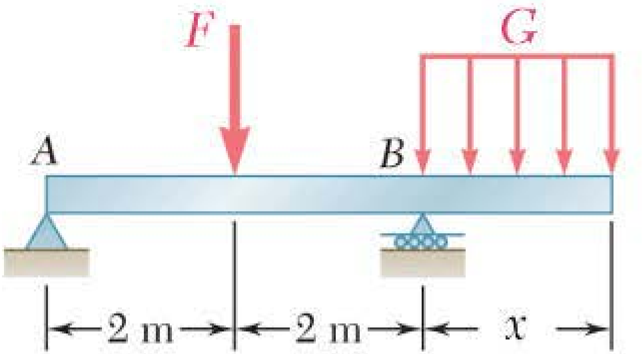
Click and type your answer here

CHALLENGE
SUBMIT
SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint
Each hint will reduce the credit received for this question



Force F is 55 kN and a distributed load of 29 kN/m is applied over a length of 2.5 m. After converting to a concentrated force of 72.5 kN, determine the reaction force at the roller support.

(Minimum one decimal place, type units as kN.)

Clear

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

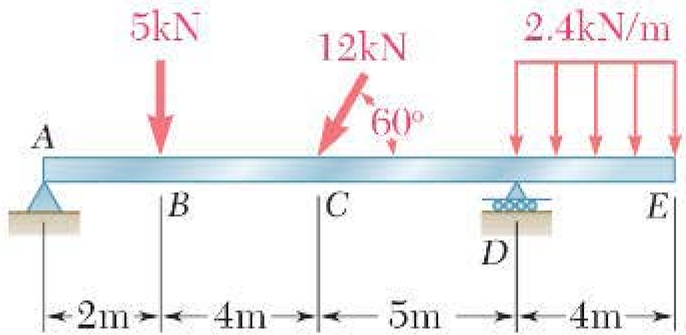
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Calculate reactions for an overhanging beam with various types of loading

Calculate the support reactions of this simply supported beam with two concentrated loads of 5 kN and 12 kN at 60°, and a uniformly distributed load of 2.4 kN/m, as shown.



Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	*Y* reaction at pin	*X* reaction at pin	Check moments

GIVE FEEDBACK

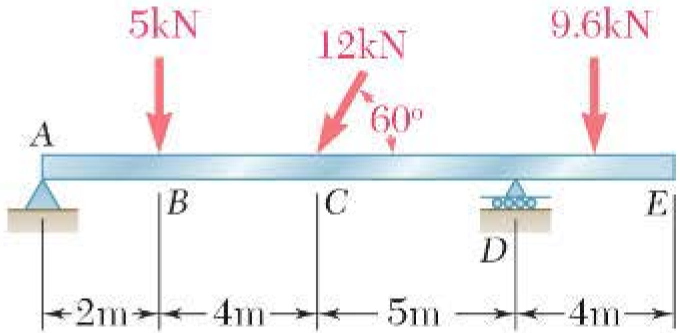
OK

Calculate reactions for an overhanging beam with various types of loading

Convert any uniformly distributed load to a point load.

Here the uniformly distributed load is 2.4 kN per metre for 4 m.

Therefore the total load is: $F = 2.4 \times 4 = 9.6$ kN



Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	*Y* reaction at pin	*X* reaction at pin	Check moments

GIVE FEEDBACK

OK

Calculate reactions for an overhanging beam with various types of loading

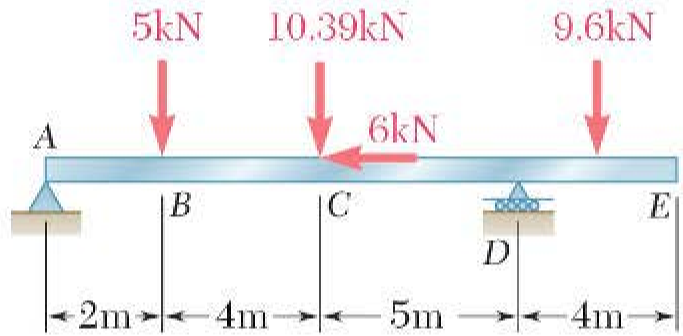
Convert any inclined forces to components.

Angle of 12 kN force is: $180° + 60° = 240°$

Calculate components:

$F_x = 12\cos 240 = -6\text{ kN}$

$F_y = 12\sin 240 = -10.39\text{ kN}$



Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	Y reaction at pin	X reaction at pin	Check moments

GIVE FEEDBACK

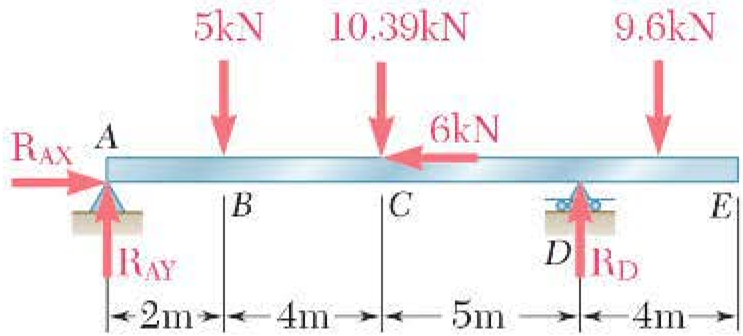
OK

Calculate reactions for an overhanging beam with various types of loading

Now solve reaction forces.

Start with moment equation at the pin joint *A* in order to determine reaction force at roller support *D*.

Take sum of moments about *A* caused by all forces:



$$\begin{aligned}\Sigma M_A &= 0 \\ &= (5\text{ kN}\times 2\text{ m}) + (10.39\text{ kN}\times 6\text{ m}) - (R_D\times 11\text{ m}) + (9.6\text{ kN}\times 13\text{ m}) \\ \therefore R_D\times 11\text{ m} &= 10\text{ kN m} + 62.35\text{ kN m} + 124.8\text{ kN m} \\ \therefore R_D &= 17.92\text{ kN}\end{aligned}$$

Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	*Y* reaction at pin	*X* reaction at pin	Check moments

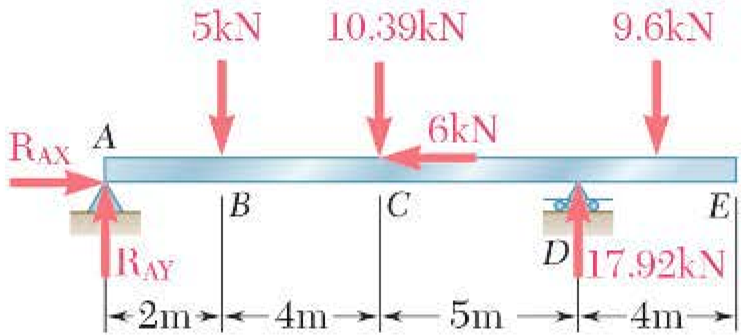
GIVE FEEDBACK

OK

Calculate reactions for an overhanging beam with various types of loading

Now that we know the right support $R_D = 17.92\ \mathrm{kN\,m}$, we can solve vertical equilibrium to get reaction force at pin support A:

$$\begin{aligned}\Sigma F_Y &= 0\\ &= R_{AY} - 5\ \mathrm{kN} - 10.39\ \mathrm{kN} + 17.92\ \mathrm{kN} - 9.6\ \mathrm{kN}\\ \therefore R_{AY} &= 7.07\ \mathrm{kN}\end{aligned}$$



Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	*Y* reaction at pin	*X* reaction at pin	Check moments

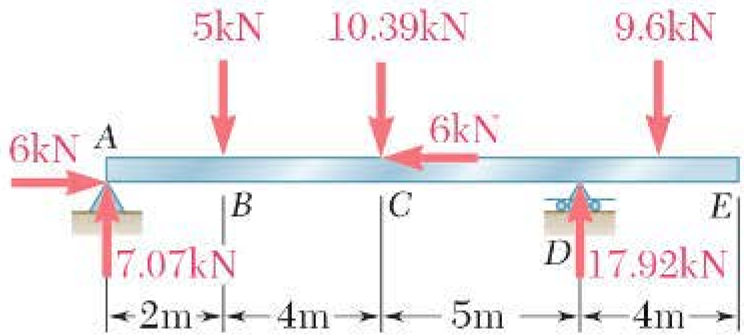
GIVE FEEDBACK

OK

Calculate reactions for an overhanging beam with various types of loading

Now solve horizontal equilibrium to get X component of reaction at A:

$$\begin{aligned}\Sigma F_X &= 0\\ &= R_{AX} - 6\text{ kN}\\ \therefore R_{AX} &= 6\text{ kN}\end{aligned}$$



Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	Y reaction at pin	X reaction at pin	Check moments

GIVE FEEDBACK

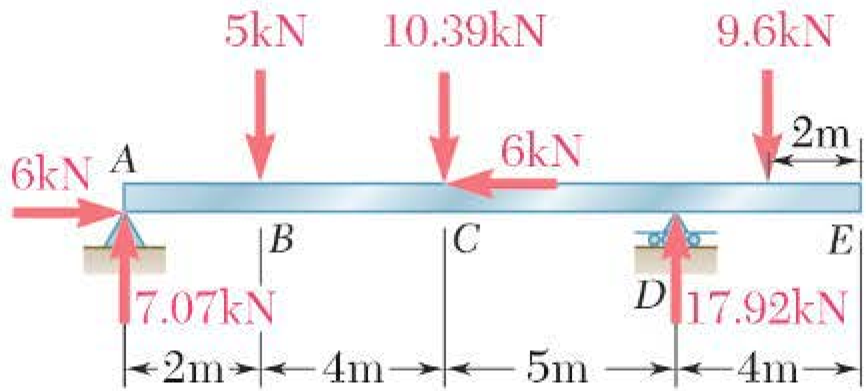
OK

Calculate reactions for an overhanging beam with various types of loading

A moment can be applied anywhere with the same effect. The reverse is also true. If a body has a moment applied, it will measure the same anywhere.

Since all seven forces applied to this beam are in moment equilibrium, we can take the moment anywhere and still get zero.

Taking moments at points A, B, C and D (using three decimal places gives an answer correct to two decimal places):



$$M_A = +(5\ \text{kN} \times 2\ \text{m}) + (10.392\ \text{kN} \times 6\ \text{m}) - (17.923\ \text{kN} \times 11\ \text{m}) + (9.6\ \text{kN} \times 13\ \text{m}) = -0.001\ \text{kN m}$$

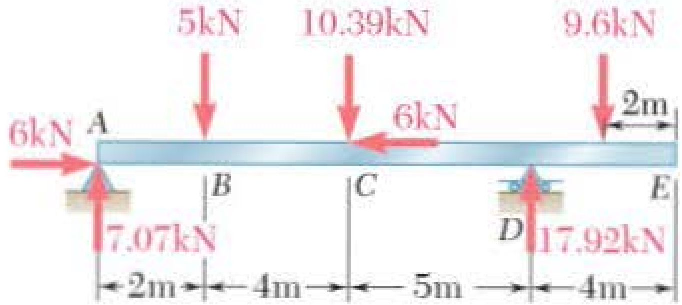
$$M_B = +(7.069\ \text{kN} \times 2\ \text{m}) + (10.392\ \text{kN} \times 4\ \text{m}) - (17.923\ \text{kN} \times 9\ \text{m}) + (9.6\ \text{kN} \times 11\ \text{m}) = -0.001\ \text{kN m}$$

$$M_C = +(7.069\ \text{kN} \times 6\ \text{m}) - (5\ \text{kN} \times 4\ \text{m}) - (17.923\ \text{kN} \times 5\ \text{m}) + (9.6\ \text{kN} \times 7\ \text{m}) = -0.001\ \text{kN m}$$

$$M_D = +(7.069\ \text{kN} \times 11\ \text{m}) - (5\ \text{kN} \times 9\ \text{m}) - (10.392\ \text{kN} \times 5\ \text{m}) + (9.6\ \text{kN} \times 2\ \text{m}) = -0.001\ \text{kN m}$$

Combined loads	Convert uniformly distributed loads	Components of inclined forces	Reaction at roller	Y reaction at pin	X reaction at pin	Check moments

OK



Match the following moment equations to the point at which they are calculated:

Drag statements on the right to match the left.

Point *A*	$+(5\text{ kN}\times 2\text{ m})+(10.39\text{ kN}\times 6\text{ m})-(17.92\text{ kN}\times 11\text{ m})+(9.6\text{ kN}\times 13\text{ m})$
Point *B*	$+(7.07\text{ kN}\times 2\text{ m})+(10.39\text{ kN}\times 4\text{ m})-(17.92\text{ kN}\times 9\text{ m})+(9.6\text{ kN}\times 11\text{ m})$
Point *C*	$+(7.07\text{ kN}\times 6\text{ m})-(5\text{ kN}\times 4\text{ m})-(17.92\text{ kN}\times 5\text{ m})+(9.6\text{ kN}\times 7\text{ m})$
Point *D*	$+(7.07\text{ kN}\times 11\text{ m})-(5\text{ kN}\times 9\text{ m})-(10.39\text{ kN}\times 5\text{ m})+(9.6\text{ kN}\times 2\text{ m})$

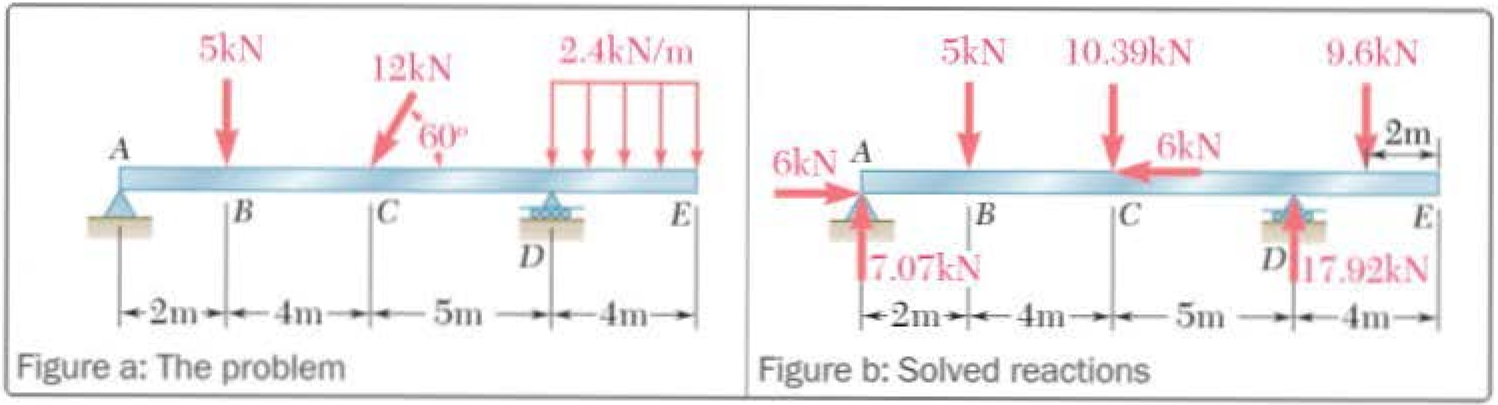


Figure a: The problem

Figure b: Solved reactions

Select the following statements about the above solution that are correct.

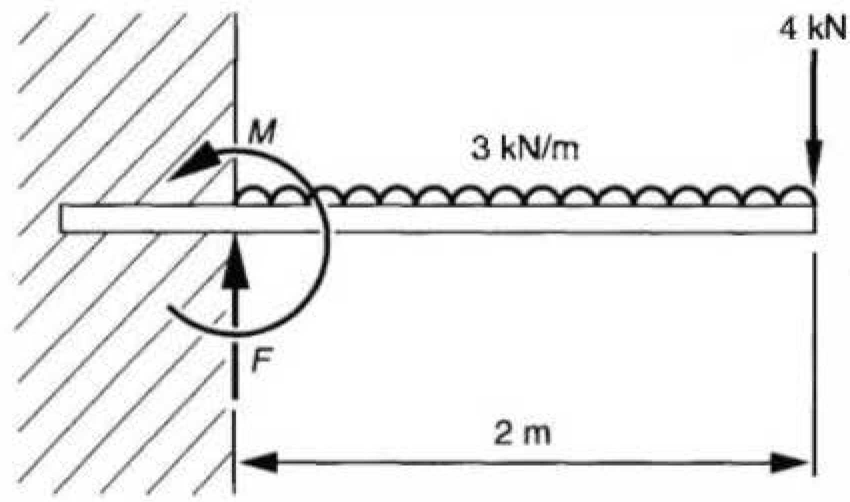
Check all that apply.

- [ ] The 2.4 kN/m distributed load applies a total of 17.92 kN to the beam
- [ ] The inclined 12 kN force applies a vertical force of 10.39 kN to the beam
- [ ] The resultant reaction at pin support *A* is $\sqrt{6^2 + 7.07^2} = 9.29 \text{ kN}$
- [ ] In Figure b, the sum of the moments of all seven forces applied to the beam is 0 kNm

Do you know the answer?

Calculate reactions for a cantilever beam with a uniformly distributed load

Determine the reactions for a cantilever beam with a uniformly distributed load of 3 kN/m as well as a single concentrated load of 4 kN located as shown.



Uniformly distributed load	Convert to point forces	Equilibrium of moments	Equilibrium of vertical forces

GIVE FEEDBACK

OK

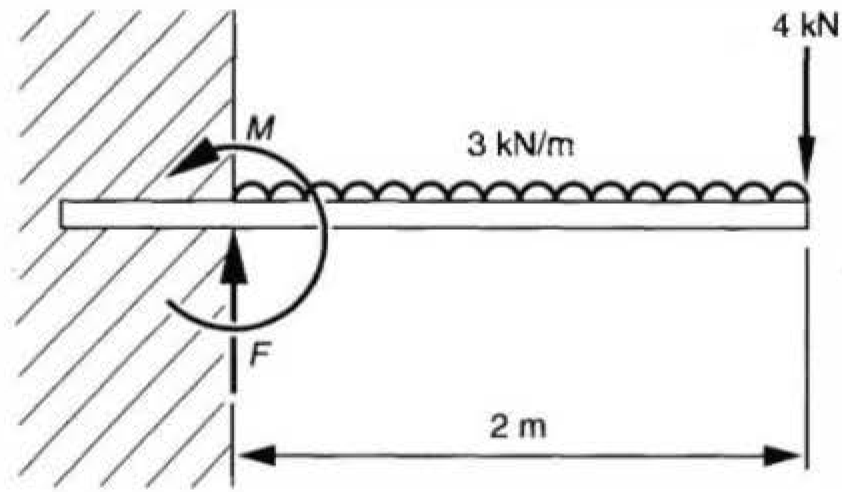
Calculate reactions for a cantilever beam with a uniformly distributed load

First job is to convert the distributed load into a point load.

The total magnitude of the distributed load is:

$$3\,\mathrm{kN/m} \times 2\,\mathrm{m} = 6\,\mathrm{kN}$$

The midpoint of this distributed load is located 1 m from the support.



Uniformly distributed load	Convert to point forces	Equilibrium of moments	Equilibrium of vertical forces

GIVE FEEDBACK

OK

Calculate reactions for a cantilever beam with a uniformly distributed load

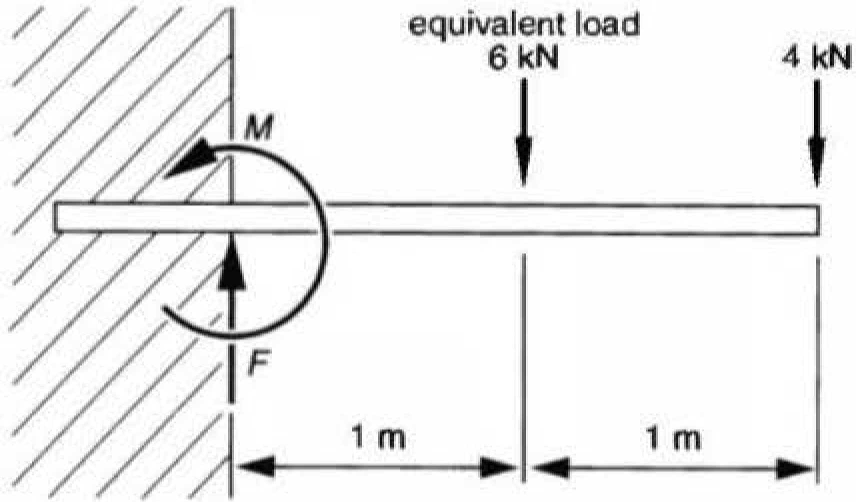
With distributed loads converted to point loads the problem becomes as shown.

Solve moment equilibrium:

$$\Sigma M = 0$$

$$0 = 6\,\mathrm{kN} \times 1\,\mathrm{m} + 4\,\mathrm{kN} \times 2\,\mathrm{m} - M$$

$$\therefore M = 14\,\mathrm{kN.m}$$



Uniformly distributed load	Convert to point forces	Equilibrium of moments	Equilibrium of vertical forces

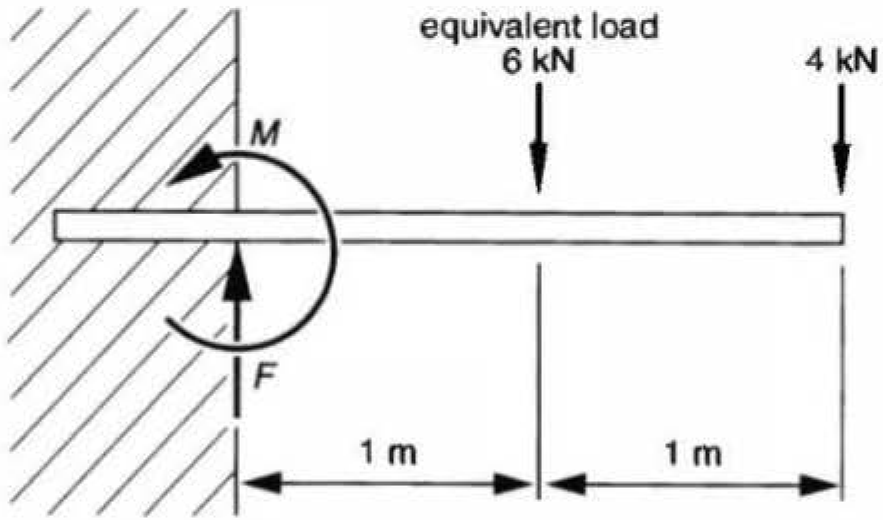
GIVE FEEDBACK

OK

Calculate reactions for a cantilever beam with a uniformly distributed load

Solve vertical equilibrium:

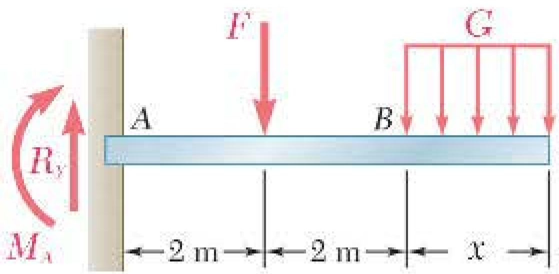
$$\Sigma F = 0$$
$$\Sigma F = F - 6\,\text{kN} - 4\,\text{kN} = 0$$
$$\therefore F = 10\,\text{kN}$$



Uniformly distributed load	Convert to point forces	Equilibrium of moments	Equilibrium of vertical forces

GIVE FEEDBACK

OK



A distributed load of 29 kN/m is applied over a length of x=2.5 m. Force *F* is 55 kN. Determine the moment reaction at the wall.

(Minimum one decimal place, type units as kN.m.)

Clear

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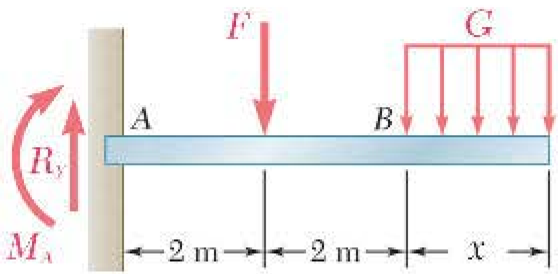
Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint Each hint will reduce the credit received for this question



A distributed load of 29 kN/m is applied over a length of 2.5 m. Force *F* is 55 kN. Determine the vertical reaction at the wall.

(Minimum one decimal place, type units as kN.)

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Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

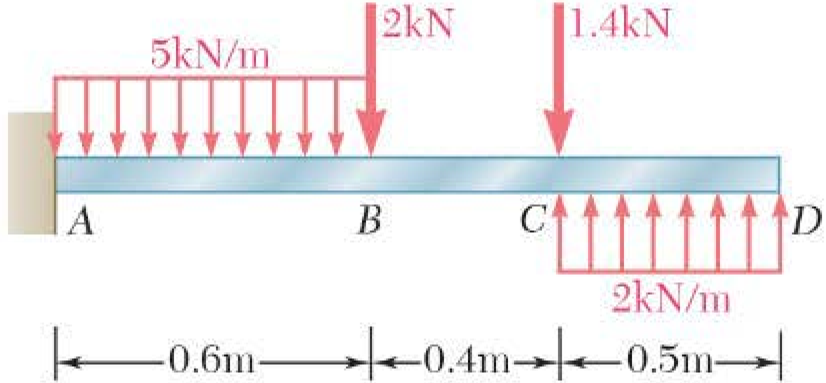
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Calculate reactions for a cantilevered beam with multiple distributed loads

Calculate the support reactions of this cantilever beam with two concentrated loads of 2 kN and 1.4 kN, and uniformly distributed loads of 5 kN/m and 2 kN/m, as shown.



Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (Y) equilibrium	Find reaction moment	Check

GIVE FEEDBACK

OK

Calculate reactions for a cantilevered beam with multiple distributed loads

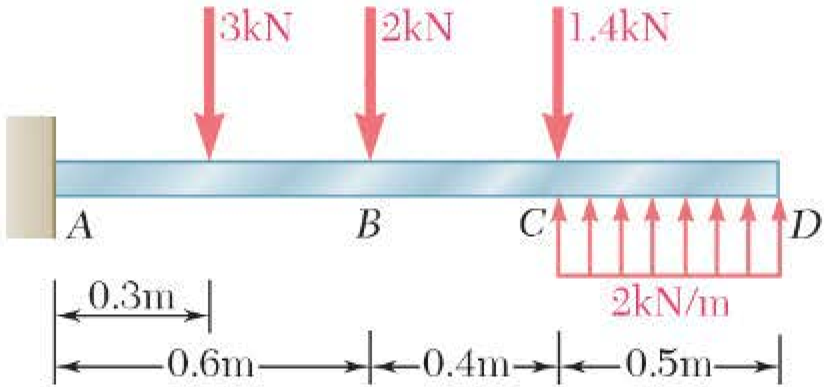
Convert the first uniformly distributed load to a point load.

The first uniformly distributed load (between *A* and *B*) is 5 kN/m for 0.6 m.

Total load:

$F = 5\,\mathrm{kN/m} \times 0.6\,\mathrm{m} = 3\,k\,N$

This force is located 0.3 m from the wall.



Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (*Y*) equilibrium	Find reaction moment	Check

GIVE FEEDBACK

OK

Calculate reactions for a cantilevered beam with multiple distributed loads

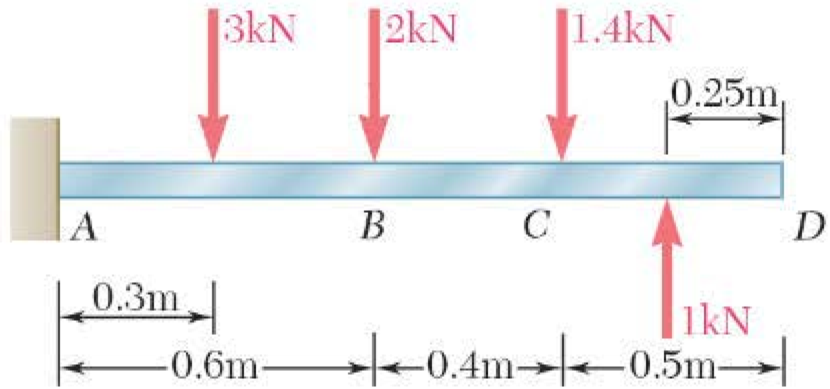
Convert the second uniformly distributed load to a point load.

The second uniformly distributed load (between *C* and *D*) is 2 kN/m for 0.5 m.

Total load:

$$F = 2\text{ kN/m} \times 0.5\text{ m} = 1\text{ kN}$$

This force is located 0.25 m from the right end, or 1.25 m from the wall.



Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (*Y*) equilibrium	Find reaction moment	Check

GIVE FEEDBACK

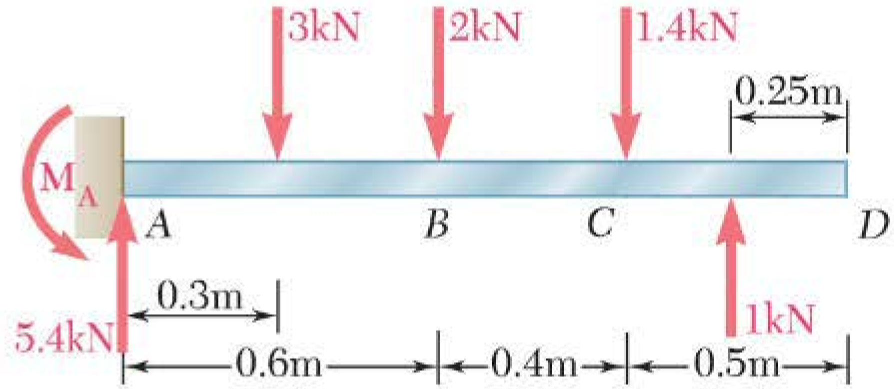
OK

Calculate reactions for a cantilevered beam with multiple distributed loads

Now solve reaction at the wall, which is both a force R_A and a moment M_A.

Start with vertical equilibrium to get **vertical reaction** at wall support:

$$\Sigma F_y = 0$$
$$= -3\,\text{kN} - 2\,\text{kN} - 1.4\,\text{kN} + 1\,\text{kN}$$
$$\therefore R_A = 5.4\,\text{kN}$$



Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (Y) equilibrium	Find reaction moment	Check

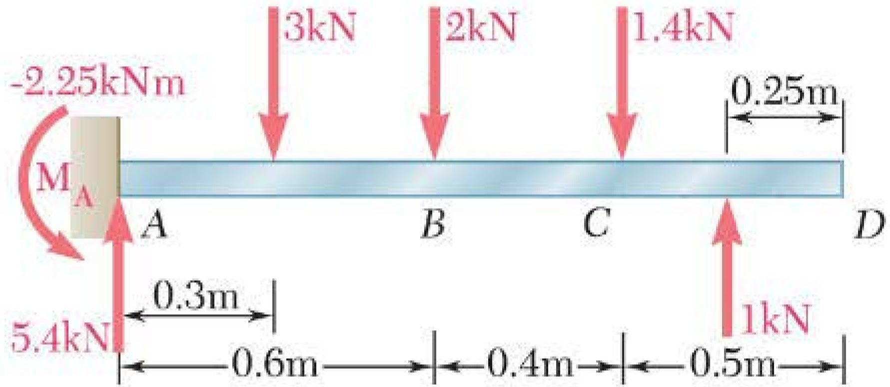
GIVE FEEDBACK

OK

Calculate reactions for a cantilevered beam with multiple distributed loads

Now find reaction moment at the wall. Solve moment equation at A to find wall reaction moment M_A.

Note that the body is the beam and the forces apply a clockwise moment to the beam. Therefore the wall must keep this beam in equilibrium by applying an anticlockwise moment:



$$
\begin{aligned}
\Sigma M_A &= 0 \\
&= M_A + (3\,\text{kN} \times 0.3\,\text{m}) + (2\,\text{kN} \times 0.6\,\text{m}) + (1.4\,\text{kN} \times 1\,\text{m}) - (1\,\text{kN} \times 1.25\,\text{m}) \\
&= M_A + 0.9\,\text{kN m} + 1.2\,\text{kN m} + 1.4\,\text{kN m} - 1.25\,\text{kN m} \\
&= M_A + 2.25\,\text{kN m} \\
\therefore M_A &= -2.25\,\text{kN m (anticlockwise)}
\end{aligned}
$$

Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (Y) equilibrium	Find reaction moment	Check

GIVE FEEDBACK

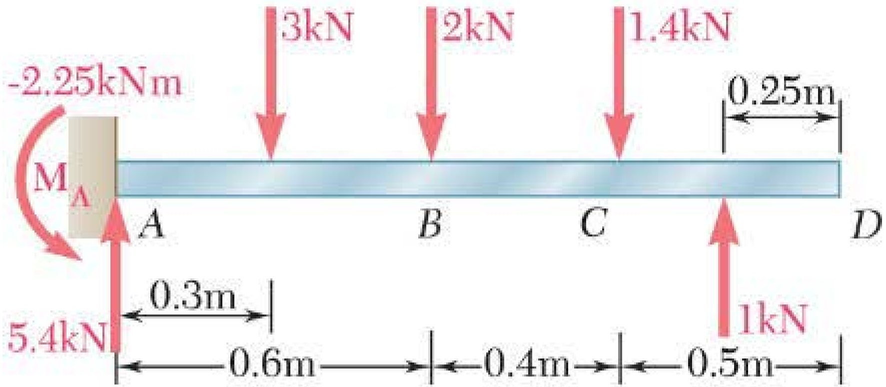
OK

Calculate reactions for a cantilevered beam with multiple distributed loads

A moment can be applied anywhere with the same effect.

The reverse is also true. If a body has a moment applied, it will measure the same anywhere.

Here the wall applies a moment of -2.25 kNm to the beam, which keeps it in equilibrium. This moment equilibrium (zero) should be the same wherever we choose to measure it.



$$M_A = +(5.4\text{ kN} \times 0\text{ m}) + (3\text{ kN} \times 0.3\text{ m}) + (2\text{ kN} \times 0.6\text{ m}) + (1.4\text{ kN} \times 1\text{ m}) - (1\text{ kN} \times 1.25\text{ m}) - 2.25\text{ kNm} = 0$$

$$M_B = +(5.4\text{ kN} \times 0.6\text{ m}) - (3\text{ kN} \times 0.3\text{ m}) + (2\text{ kN} \times 0\text{ m}) + (1.4\text{ kN} \times 0.4\text{ m}) - (1\text{ kN} \times 0.65\text{ m}) - 2.25\text{ kNm} = 0$$

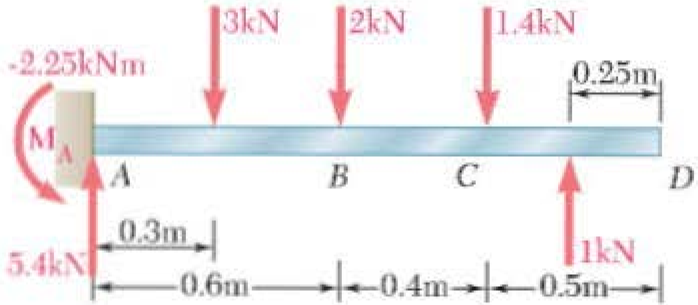
$$M_C = +(5.4\text{ kN} \times 1\text{ m}) - (3\text{ kN} \times 0.7\text{ m}) - (2\text{ kN} \times 0.4\text{ m}) + (1.4\text{ kN} \times 0\text{ m}) - (1\text{ kN} \times 0.25\text{ m}) - 2.25\text{ kNm} = 0$$

$$M_D = +(5.4\text{ kN} \times 1.5\text{ m}) - (3\text{ kN} \times 1.2\text{ m}) - (2\text{ kN} \times 0.9\text{ m}) - (1.4\text{ kN} \times 0.5\text{ m}) + (1\text{ kN} \times 0.25\text{ m}) - 2.25\text{ kNm} = 0$$

Combined loads	Convert uniformly distributed load (5 kN/m)	Convert uniformly distributed load (2 kN/m)	Vertical (Y) equilibrium	Find reaction moment	Check

GIVE FEEDBACK

OK



Select the correct statements about moments.

Check all that apply.

- [ ] The moment applied by the wall to the beam is –2.25 kNm (anticlockwise)
- [ ] The moment applied by the beam to the wall is 2.25 kNm (clockwise)
- [ ] The moment applied by the wall to the beam is 2.25 kNm (clockwise)
- [ ] The moment applied by the beam to the wall is -2.25 kNm (anticlockwise)
- [ ] The sum of all moments is highest at *A*, and decreases through points *B* and *C* until it is zero at point *D*

Do you know the answer?

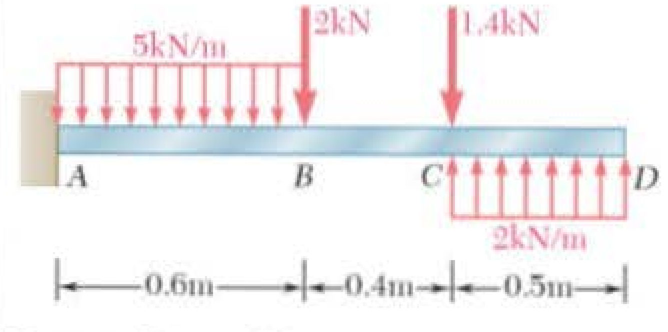


Figure a: The problem

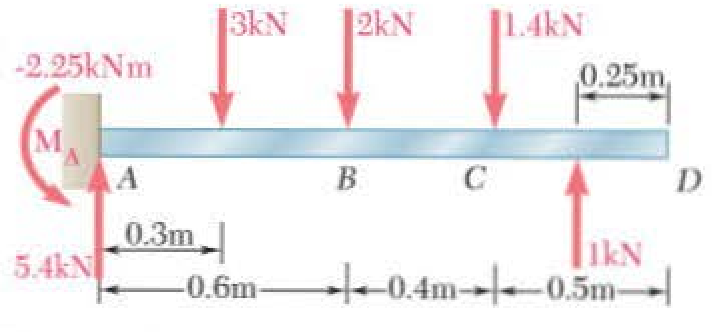


Figure b: Solved reactions

Select the following statements about the above solution that are correct.

Check all that apply.

- [ ] The 5 kN/m distributed load applies a total of 3 kN to the beam
- [ ] The beam applies a clockwise moment of 2.25 kNm to the wall
- [ ] The sum of all vertical forces applied to the beam is 5.4 kN
- [ ] The sum of the moments of every force applied to the beam is -2.25 kNm

Do you know the answer?