GIVE FEEDBACK

CONTINUE >

This section is about **non-concurrent force** systems. A system of forces is called non-concurrent if the forces do not meet at a common point (a point of concurrency).

When forces are non-concurrent, it means that there is always a possibility that they may have a turning effect with respect to the structure or mechanical component. The **moment of a force** is introduced; it is a measure of a force's turning effect about the reference points.

In this section the concept of **general equilibrium** is introduced, along with the concept of **moments of a couple** and the analysis of an **equivalent force-couple system**.

In engineering, non-concurrent forces and moments are essential in designing the stability of structures and machines. Moments analyses are especially important in structures and beams bearing loads.

Characteristics of a system of non-concurrent forces

A system of forces is called **non-concurrent** if it does not have a single point of concurrency, i.e. the lines of action of the forces do not all meet at a common point.

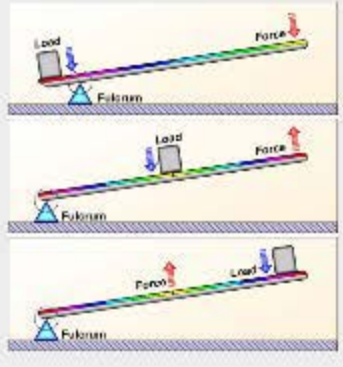
When forces are non-concurrent, there is always a possibility that they may have a turning effect with respect to the structure or mechanical component to which they are applied. The concept of **moment of a force** is introduced; it is a measure of a force's turning effect about some reference point.

Equilibrium of non-concurrent forces requires that, in addition to the force balance, the moments of all forces must also be in balance.

Every time we drive a car, tighten up a nut, ride a bicycle or open a door, we make use of the turning effect of a force or forces, applied at some distance from the axis or fulcrum about which turning takes place.

The lever is only one example of the application of the **principle of moments.**

Levers enable us to multiply the force available in our own hands to move very large loads. Today the lever, in various forms and combinations, is the most common machine element.



GIVE FEEDBACK

OK

What is the moment of a force?

Click the correct answer.

A measure of a force's turning effect about a reference point

A measure of a force's application time length

A measure of a force's importance in a system

A measure of a force's exact application point in time

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Equilibrium of non-concurrent forces requires that, in addition to the force balance, the _____ of all forces must also be in _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of these situations involves the moment of a force?

Check all that apply.

- [ ] Driving a car
- [ ] Loosening a screw
- [ ] Standing on a roof
- [ ] Tightening up a nut
- [ ] Opening a door

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

A system of forces is called [ ]-[ ] if the line of action of the forces do not meet at a common point, i.e. a point of [ ].

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Equations used to express the conditions of moment of a force

The **moment of a force** about a point is defined as the product of the force and the perpendicular distance of its line of action from the point.

$$M = F\,d$$

M = moment of a force about the point
F = force applied
d = the perpendicular distance of its line of action from the point

The perpendicular distance d is often referred to as the **moment arm** or **lever arm.**

The moment is a measure of the turning effect of the force acting on a body, relative to a specified point. As a product of force and distance, the moment of a force is measured in units derived from those of force and length.

GIVE FEEDBACK

OK

Which of the following is the correct equation describing the moment of a force about a point?

Click the correct answer.

$M = F\,d$

$F = d\,M$

$M = \frac{F}{d}$

$d = \frac{F}{M}$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

The perpendicular distance d is referred to as the [ ] [ ].

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The SI unit of the moment

The SI unit of moment of force is the product of the newton and the metre, called **newton metre,** with a compound symbol **N.m**.

The multiples and submultiples of the newton metre are formed by using decimal prefixes in front of the new unit, e.g. 1 kilonewton metre (kN.m) is equal to 1000 newton metres (N.m).

GIVE FEEDBACK

OK

Type your answer in the box.

The SI unit of the moment of force is [ ] [ ]. The symbol of the unit is [ ].

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Newton metre (Nm) is the only SI unit used in engineering.

Click the correct answer.

False

True

Do you know the answer?

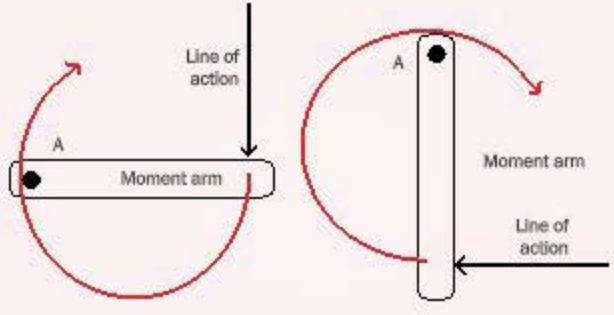
I KNOW IT | THINK SO | UNSURE | NO IDEA

The directional sense of the moment

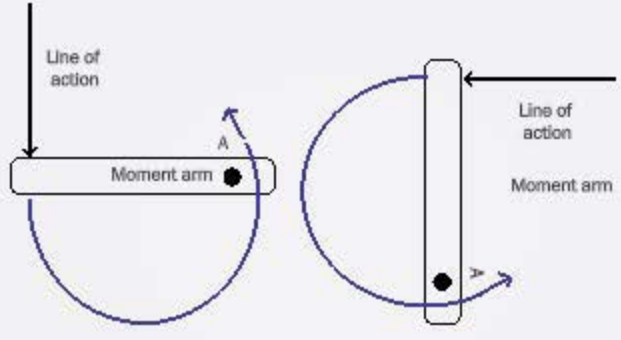
Note that the moment of a force also has a directional sense.

The directional sense is assoicated with turning motion, hence the directions are indicated as **clockwise** and **anticlockwise** (**counterclockwise**).

For mathematical calculations, a sign convention usually used is that clockwise moments are taken to be **positive** and anticlockwise moments to be **negative**.



Moments going in the clockwise direction



Moments going in the anticlockwise (counterclockwise) direction

GIVE FEEDBACK

OK

Match the moment direction with the correct mathematical sign convention.

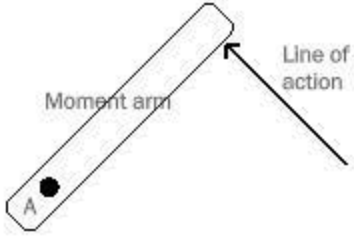
Drag statements on the right to match the left.

Clockwise direction	Positive
Anticlockwise direction	Negative

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

What is the direction of moment in the following figure?



Click the correct answer.

- Anticlockwise
- Clockwise
- North-west
- Up

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of the following are the correct direction senses for moments of a force?

Check all that apply.

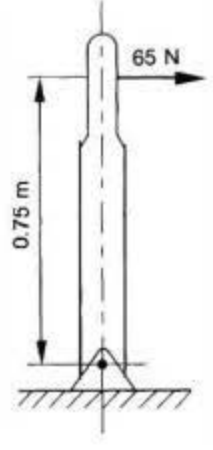
- [ ] Clockwise
- [ ] Anticlockwise
- [ ] Down
- [ ] Up
- [ ] Left
- [ ] Right

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

If a force of 65 N is applied to the lever shown in the figure below, and the length of the moment arm is 0.75 m, determine the moment of the force about the pivot point.



GIVE FEEDBACK

CONTINUE >

Solution

Moment is the product of the force and the perpendicular distance called the moment arm:

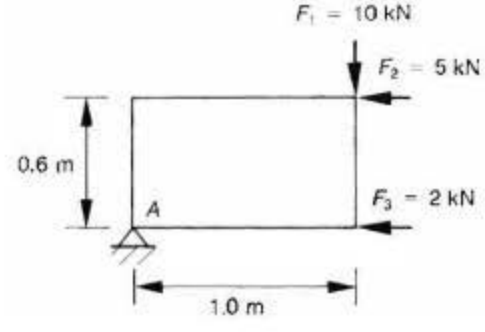
$$\begin{aligned} M &= F\,d \\ &= 65\ \text{N} \times 0.75\ \text{m} \\ &= 48.75\ \text{N}\,.\,\text{m clockwise} \end{aligned}$$

Note that the answer also indicates the directional sense of the moment, clockwise in this case. In this example, the actual length of the lever is in fact the moment arm at right angles to the force.

Example

In many problems, care must be exercised in using correct perpendicular distance with each force, as illustrated by the following example.

Determine the magnitude and sense of the moments of forces F_1, F_2 and F_3 about point A in the following figure. In this example, drawing individual diagrams for each force will be very helpful to understand which distance is in fact the moment for each of the forces in question.



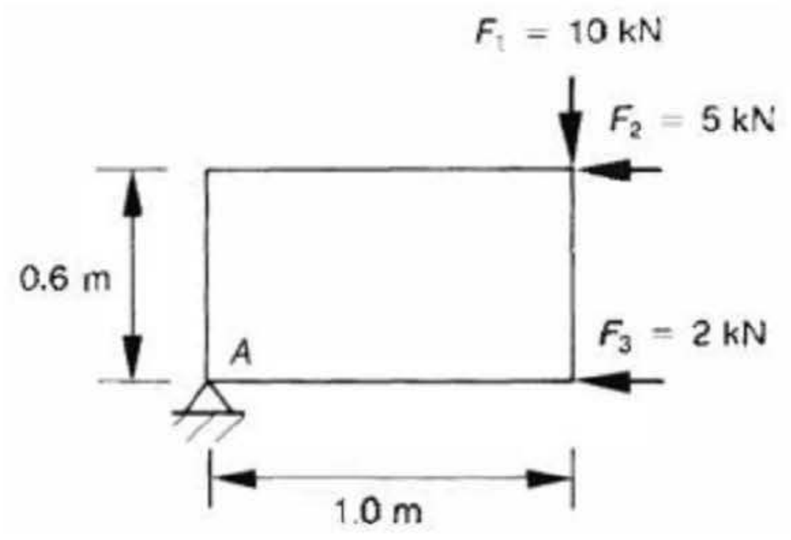
GIVE FEEDBACK

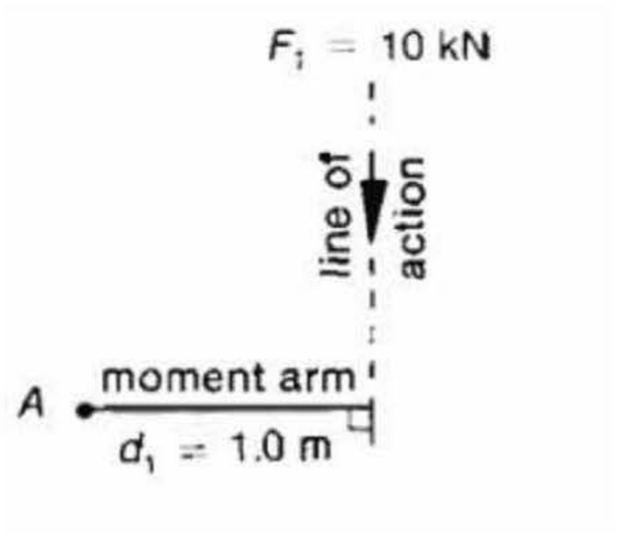
CONTINUE >

Solution

Moment of force F_1:

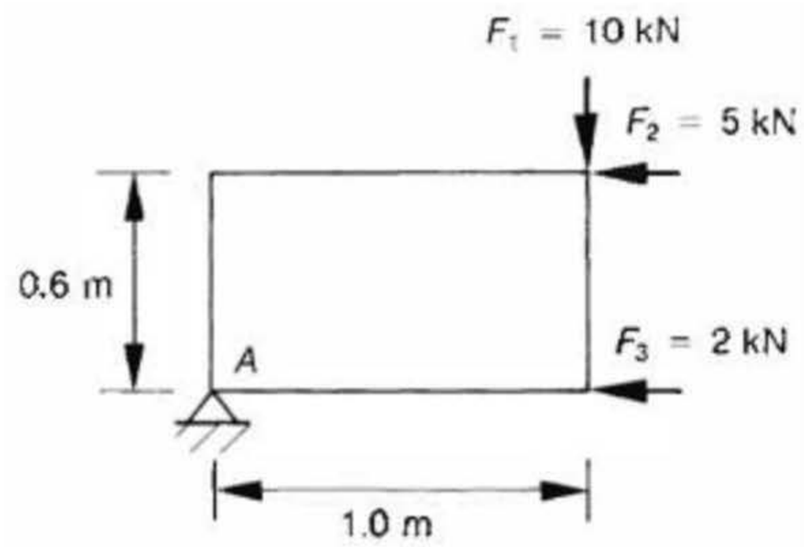
$$\begin{aligned} M_1 &= F_1 d_1 \\ &= 10\,\text{kN} \times 1\,\text{m} \\ &= 10\,\text{kN m clockwise} \end{aligned}$$

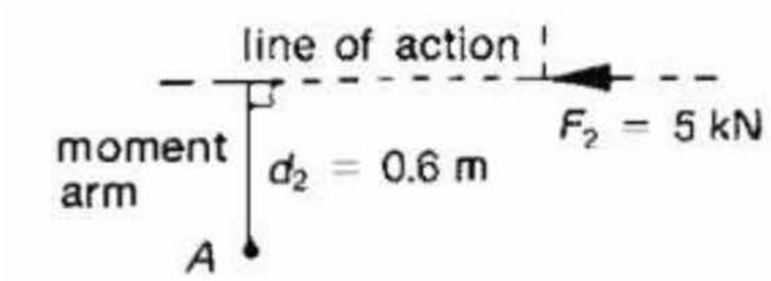




Moment of force F_2:

$$M_2 = F_2 d_2$$
$$= 5\,\text{kN} \times 0.6\,\text{m}$$
$$= 3\,\text{kN m anticockwise}$$

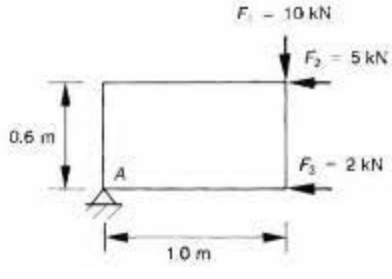




Moment of force F_3:

$$\begin{aligned} M_3 &= F_3 d_3 \\ &= 2\,\text{kN} \times 0 \\ &= 0 \end{aligned}$$

Note that force F_3 does not have a moment about point A due to the fact that its line of action passes through the point, i.e. its perpendicular distance from A is equal to zero.



$$\begin{aligned} M_2 &= F_2 d_2 \\ &= 5\,\text{kN} \times 0.6\,\text{m} \\ &= 3\,\text{kN m anticockwise} \end{aligned}$$

A force of 45 N is applied to a spanner at the distance of 300 mm from the centre of a nut. The nut has a right-hand thread.

SMALL MEDIUM LARGE

What is the maximum turning moment produced by this force?
(Show your answer in Nm and rounded to one decimal place.)

+ − · ÷ □/□ □² √□ Clear

(□) ▾ ≤ ▾ π m ▾ □ ? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

A force of 45 N is applied to a spanner at the distance of 300 mm from the centre of a nut. The nut has a right-hand thread.

SMALL | MEDIUM | LARGE

Given you are tightening a nut, what should be the direction of the force to achieve the maximum turning effect?

Click the correct answer.

- Clockwise with the line of action perpendicular to the moment arm
- Anticlockwise with the line of action perpendicular to the moment arm
- Clockwise with the line of action at any angle to the moment arm
- Anticlockwise with the line of action at any angle to the moment arm

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Given you are tightening a nut, what should be the direction of the force to achieve the maximum turning effect?

Click the correct answer.

- Clockwise with the line of action perpendicular to the moment arm
- Anticlockwise with the line of action perpendicular to the moment arm
- Clockwise with the line of action at any angle to the moment arm
- Anticlockwise with the line of action at any angle to the moment arm

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

What is the maximum turning moment produced by this force?
(Show your answer in Nm and rounded to one decimal place.)

+ − · ÷ □/□ □² √□ Clear

(□) ▼ ≤ ▼ π m ▼ □̄ ? Undo

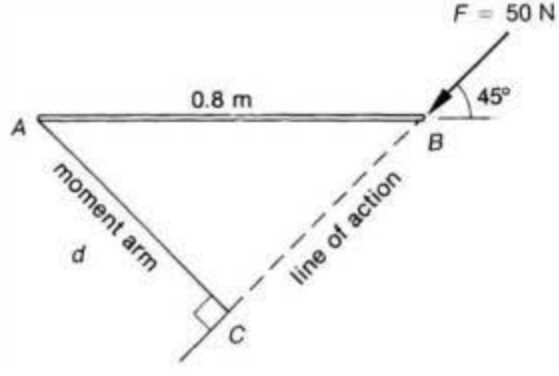
Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

Example

If a force is inclined to the convenient principal directions, such as the horizontal and vertical directions, the correct perpendicular distance can be determined graphically in the following example.

Determine the moment of force $F = 50$ N about point A if the distance AB is 800 mm.



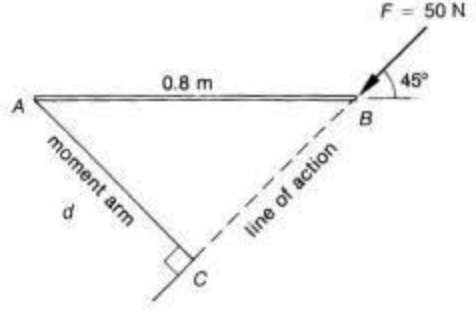
GIVE FEEDBACK

CONTINUE >

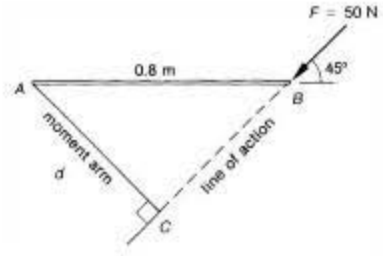
Solution

In this case the given distance AB is not the moment arm because AB is not perpendicular to the line of action of the force. To solve the problem we need distance AC. This can be found graphically or mathematically.

Graphically it can be done by drawing the diagram to scale and measuring the required distance AC. Mathematically it can be done by using trigonometry. Usually the mathematical method will generate a more reliable answer.



Determine the moment of a force by calculating the correct moment arm (perpendicular distance)—Example 3/3



From the right-angled triangle ABC:

$$\begin{aligned} \text{Moment arm } d &= AB \sin 45° \\ &= 0.8 \times 0.707 \\ &= 0.566 \text{ m} \end{aligned}$$

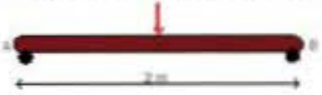
The moment of force is therefore:

$$\begin{aligned} M &= F\,d \\ &= 50 \text{ N} \times 0.566 \text{ m} \\ &= 28.3 \text{ N.m clockwise} \end{aligned}$$

< BACK GIVE FEEDBACK OK

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)

(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal

SMALL MEDIUM LARGE

Type your answer in the box.

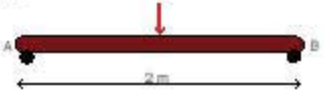
In scenario (a), the moment arm is _____ m from the left support (point A) and _____ m from the right support (point B).

Do you know the answer?

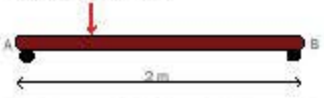
I KNOW IT | THINK SO | UNSURE | NO IDEA

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

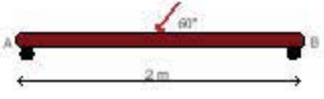
(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)



(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal



SMALL MEDIUM LARGE

Type your answer in the box.

In scenario (a), a downward force of 3 kN is applied at the midpoint of the beam.
(Show your answer in kNm and round to one decimal place if necessary.)

Given the moment arm is 1 m from the left support (point A) and 1 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

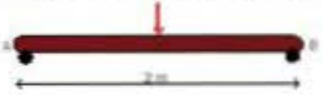
The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)

(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal

SMALL MEDIUM LARGE

Type your answer in the box.

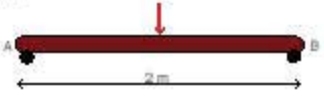
In scenario (b), the moment arm is _____ m from the left support (point A) and _____ m from the right support (point B).

Do you know the answer?

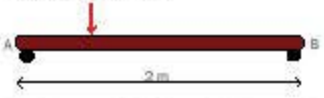
I KNOW IT | THINK SO | UNSURE | NO IDEA

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

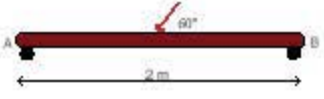
(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)



(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal



SMALL MEDIUM LARGE

Type your answer in the box.

In scenario (b), a downward force of 3 kN is applied at a point 0.5 m from the left support (point A) of the beam. (Show your answer in kNm and round to one decimal place if necessary.)

Given the moment arm is 0.5 m from the left support (point A) and 1.5 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

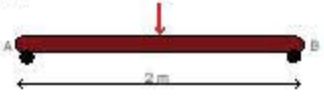
The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

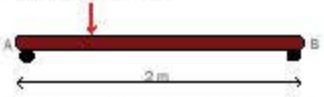
I KNOW IT | THINK SO | UNSURE | NO IDEA

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

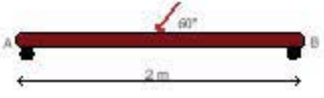
(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)



(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal



SMALL MEDIUM LARGE

Type your answer in the box.

In scenario (c), a force of 3 kN is applied at midpoint and inclined at 60° to the horizontal of the beam. (Show your answer in m and round to three decimal places.)

In this case the line of action of the force F is not perpendicular to the horizontal beam.

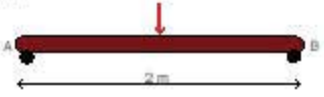
The true moment arm is _____ m from the left support (point A) and _____ m from the right support (point B).

Do you know the answer?

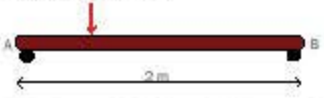
I KNOW IT | THINK SO | UNSURE | NO IDEA

A horizontal beam 2 m long is supported at its ends. In the following scenerios, determine the moments about each of the supports due to the different loads.

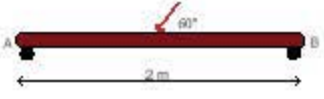
(a) Downward force of 3 kN at midpoint



(b) Downward force of 3 kN at a point 0.5 m from the left support (point A)



(c) Force of 3 kN applied at midpoint and inclined at 60° to the horizontal



SMALL MEDIUM LARGE

Type your answer in the box.

In scenario (c), a force of 3 kN is applied at midpoint and inclined at 60° to the horizontal of the beam. (Show your answer in kNm and round to one decimal place.)

In this case the beam distance is not the moment arm becuase it is not perpendicular to the line of action. Given the moment arm is 0.866 m from the left support (point A) and 0.866 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (c), a force of 3 kN is applied at midpoint and inclined at 60° to the horizontal of the beam. (Show your answer in m and round to three decimal places.)

In this case the line of action of the force F is not perpendicular to the horizontal beam.

The true moment arm is _____ m from the left support (point A) and _____ m from the right support (point B).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (a), the moment arm is [ ] m from the left support (point A) and [ ] m from the right support (point B).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (c), a force of 3 kN is applied at midpoint and inclined at $60°$ to the horizontal of the beam.
(Show your answer in kNm and round to one decimal place.)

In this case the beam distance is not the moment arm becuase it is not perpendicular to the line of action.
Given the moment arm is 0.866 m from the left support (point A) and 0.866 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (a), a downward force of 3 kN is applied at the midpoint of the beam.
(Show your answer in kNm and round to one decimal place if necessary.)

Given the moment arm is 1 m from the left support (point A) and 1 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (b), a downward force of 3 kN is applied at a point 0.5 m from the left support (point A) of the beam. (Show your answer in kNm and round to one decimal place if necessary.)

Given the moment arm is 0.5 m from the left support (point A) and 1.5 m from the right support (point B):

The moment about the left support (point A) is _____ kNm with _____ direction.

The moment about the right support (point B) is _____ kNm with _____ direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In scenario (b), the moment arm is _____ m from the left support (point A) and _____ m from the right support (point B).

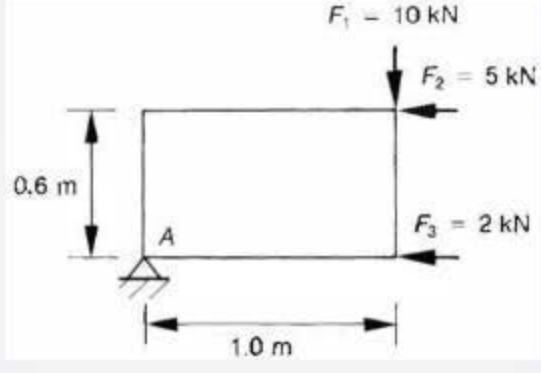
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Turning effecting or resulting moment

If more than one force is acting on a body, there is a corresponding number of moments of force, each tending to produce a turning effect about the point.

The total turning effect, or resultant moment, is the algebraic sum of the moments of all the forces acting on the body.



This figure is showing that there are three forces acting around the point A.

Hence the total turning effect is the algebraic sum of the moments of all three forces acting around point A.

GIVE FEEDBACK

OK

Type your answer in the box.

The total turning effect, or resultant moment, is the algebraic _____ of the moments of all the forces acting on the body.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

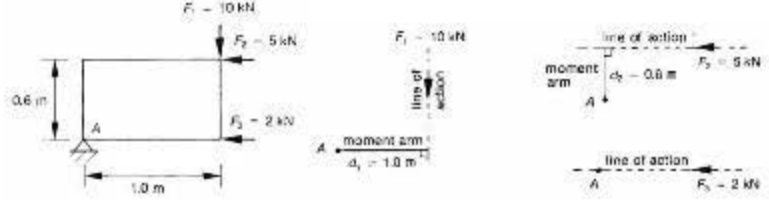
The algebraic sign convention of clockwise and anticlockwise moments

The total turning effect, or resultant moment, is the algebraic sum of the moments of all the forces acting on the body.

Algebraic sum means that the different directional sense of the various moments must be taken into account when the moments are being added.

For mathematical calculations, a sign convention usually used is that clockwise moments are taken to be positive and anticlockwise moments to be negative.

In this example, the three moments about point A can be calculated individually.



$M_1 = F_1 d_1$ $\quad$ $M_2 = F_2 d_2$ $\quad$ $M_3 = F_3 d_3$

$= 10\,\text{kN} \times 1\,\text{m}$ $\quad$ $= 5\,\text{kN} \times 0.6\,\text{m}$ $\quad$ $= 2\,\text{kN} \times 0$

$= 10\,\text{kN.m}$ clockwise $\quad$ $= 3\,\text{kN.m}$ anticlockwise $\quad$ $= 0$

Using the sign convention, the moments about point A can be shown as:

$M_1 = 10\,\text{kN.m}$ clockwise $\quad$ $M_2 = 3\,\text{kN.m}$ anticlockwise

$= +\,10\,\text{kN.m}$ $\quad$ $= -\,3\,\text{kN.m}$

$M_3 = 0$

GIVE FEEDBACK

OK

Match the moment direction with the correct mathematical sign convention.

Drag statements on the right to match the left.

Clockwise direction	Positive
Anticlockwise direction	Negative

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If a moment of a force is equal to +45 N.m, what is the direction of the moment?

Click the correct answer.

Clockwise

Anticlockwise

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If a moment of a force is equal to – 49 N.m, what is the direction of the moment?

Click the correct answer.

Anticlockwise

Clockwise

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The Varignon theorem

An extension of the principle of addition of moments is known as the **Varignon theorem,** originally proposed by the French mathematician Varignon (1654–1722).

It states that the moment of a force about any axis is equal to the sum of the moments of its components about that axis.

Type your answer in the box.

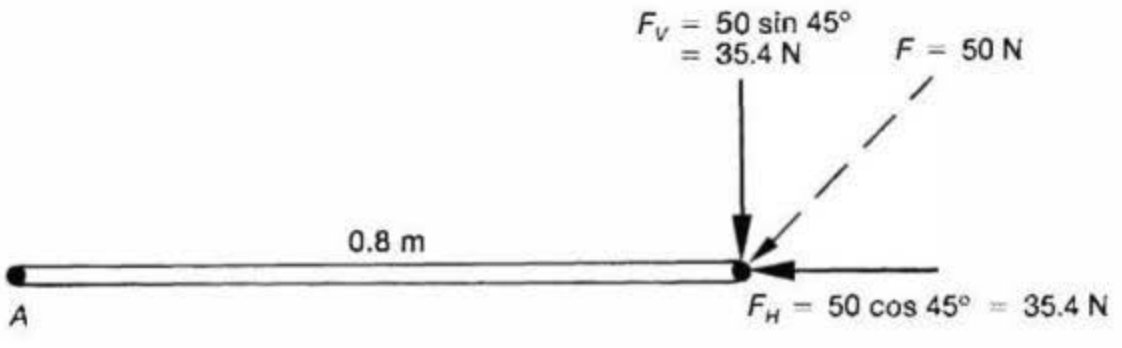
The Varignon theorem states that the _____ of a force about any axis is equal to the _____ of the moments of its components about that axis.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

By applying the Varignon theorem and the addition of moments, we can resolve a force into its horizontal and vertical components mathematically.

In the following example, we can determine the final moment about point A by resolving force F into its horizontal and vertical components, as shown in the figure.



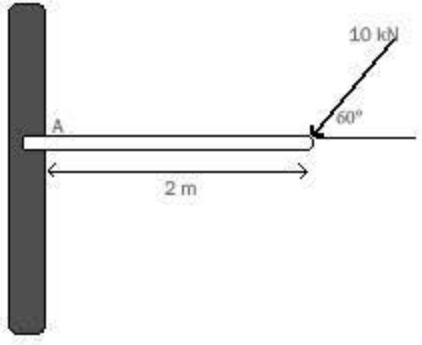
GIVE FEEDBACK

CONTINUE >

As the moment of the horizontal component is zero, due to the line of action passing through point A, the total moment about A is:

$$\begin{aligned} M &= F_V \times d + F_H \times 0 \\ &= 35.4\,\text{N} \times 0.8\,\text{m} + 0 \\ &= 28.3\,\text{N.m} \end{aligned}$$

The beam in the following figure is built into the wall and carries a load of 10 kN as shown.



SMALL MEDIUM LARGE

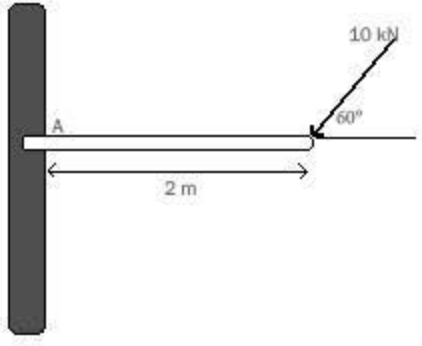
Type your answer in the box.

By resolving the 10 kN into its components, the vertical force F_V is equal to _____ kN (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The beam in the following figure is built into the wall and carries a load of 10 kN as shown.



SMALL MEDIUM LARGE

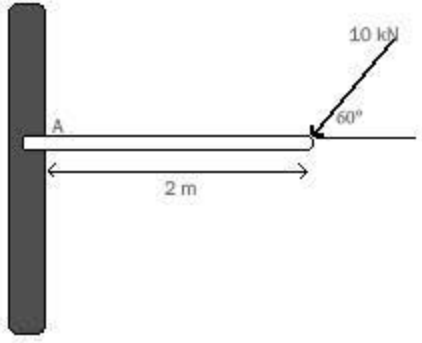
Type your answer in the box.

By resolving the 10 kN into its components, the horizontal force F_H is equal to _____ kN.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The beam in the following figure is built into the wall and carries a load of 10 kN as shown.



SMALL MEDIUM LARGE

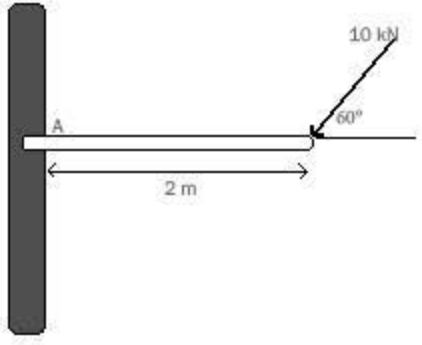
Type your answer in the box.

Given the horizontal force has a line of action passing through point A, the total moment of the horizontal component is _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The beam in the following figure is built into the wall and carries a load of 10 kN as shown.



SMALL MEDIUM LARGE

Type your answer in the box.

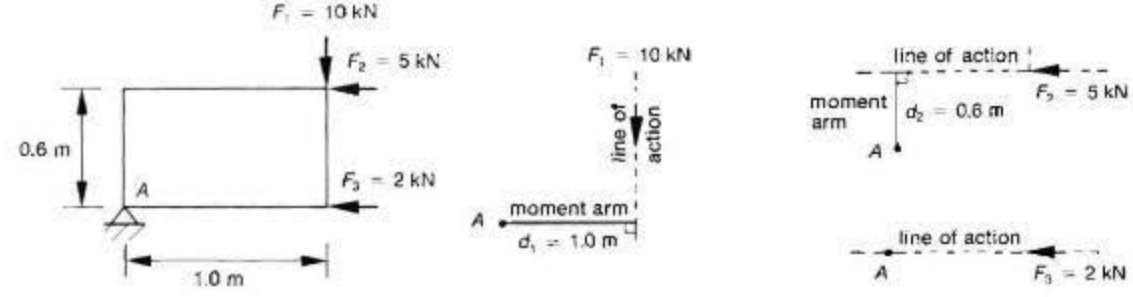
Given the vertical force component is 8.66 kN and the moment of the horizontal component is zero, the total moment M about point A is _____ kN.m in the _____ direction (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

It is important to realise that for a given system of forces there is no single answer for the moment of forces unless the reference point is specified. A moment must always be calculated with respect to a particular reference point.

In this example, the three moments about point A can be calculated individually.



$M_1 = F_1 d_1$
$= 10\,\text{kN} \times 1\,\text{m}$
$= 10\,\text{kN m}$ clockwise

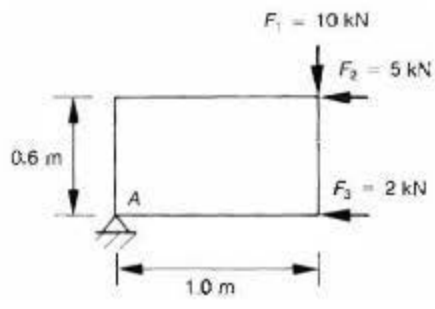
$M_2 = F_2 d_2$
$= 5\,\text{kN} \times 0.6\,\text{m}$
$= 3\,\text{kN m}$ anticockwise

$M_3 = F_3 d_3$
$= 2\,\text{kN} \times 0$
$= 0$

GIVE FEEDBACK

CONTINUE >

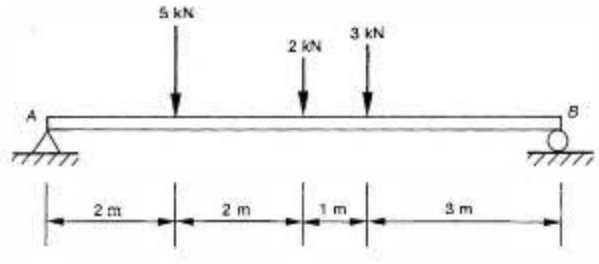
Using the sign convention, the total moment about point A of all the forces is:



$$
\begin{aligned}
M &= M_1 + M_2 + M_3 \\
&= +10\,\text{kN m} - 3\,\text{kN m} + 0 \\
&= 7\,\text{kN m clockwise} \\
&= +7\,\text{kN m}
\end{aligned}
$$

It must also be understood that in many problems there is no rotation actually taking place. The moment represents only the tendency for rotation under the influence of a force or forces and not actual rotation. The following example illustrates these points.

A horizontal beam rests on two supports, A and B, and supports three forces as shown below. Calculate the total moment due to the applied forces:
(a) about the left-hand support A
(b) about the right-hand support B



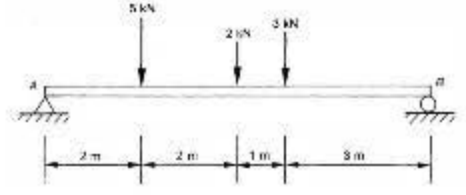
(a) The total moment about A is the sum of the moments of all forces about A.

Note: All distances are measured from point A.

$$\begin{aligned} M_A &= \Sigma(F \times d) \\ &= 5 \times 2 + 2 \times 4 + 3 \times 5 \\ &= 33 \text{ kN m} \qquad \text{clockwise} \end{aligned}$$

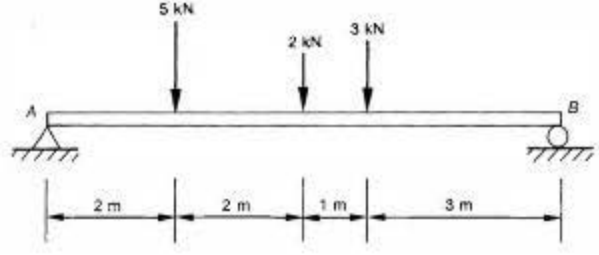
(b) The total moment about point B (distances measured from B) is:

$$\begin{aligned} M_B &= \Sigma(F \times d) \\ &= 5 \times 6 + 2 \times 4 + 3 \times 3 \\ &= 47 \text{ kN m} \qquad \text{anticlockwise} \end{aligned}$$



< BACK GIVE FEEDBACK CONTINUE >

It is obvious that the moments about A and B are different, not only in magnitude, but also in sense. There is no actual rotation taking place. However, if the right-hand support B is suddenly removed, then rotation is produced by the turning moment M_A about A in the clockwise direction. If instead the left-hand support A is removed, then the beam turns anticlockwise about B under the influence of the moment M_B.



Type your answer in the box.

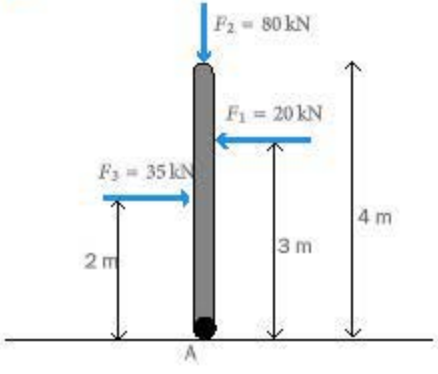
Determine the magnitude and direction sense of the moments of force F_1.

Moment	Magnitude	Direction sense
M_1	kNm	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the magnitude and sense of forces F_1, F_2 and F_3 about point A in the following figure; determine the total moment about point A of all forces.



SMALL MEDIUM LARGE

Type your answer in the box.

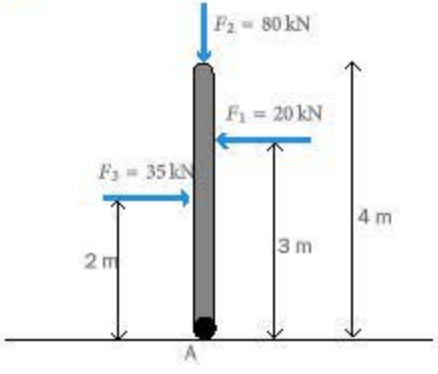
Determine the magnitude and direction sense of the moments of force F_1.

Moment	Magnitude	Direction sense
M_1	kNm	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the magnitude and sense of forces F_1, F_2 and F_3 about point A in the following figure; determine the total moment about point A of all forces.



SMALL MEDIUM LARGE

Type your answer in the box.

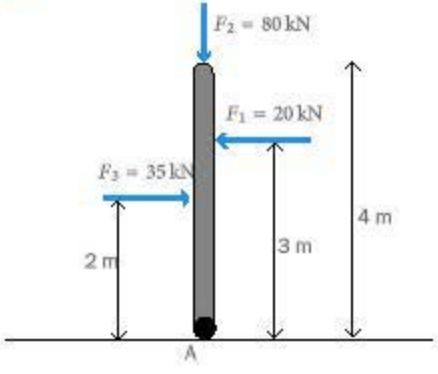
Determine the magnitude and direction sense of the moments of force F_2.

Moment	Magnitude	Direction sense
M_2	kNm	----

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the magnitude and sense of forces F_1, F_2 and F_3 about point A in the following figure; determine the total moment about point A of all forces.



SMALL MEDIUM LARGE

Type your answer in the box.

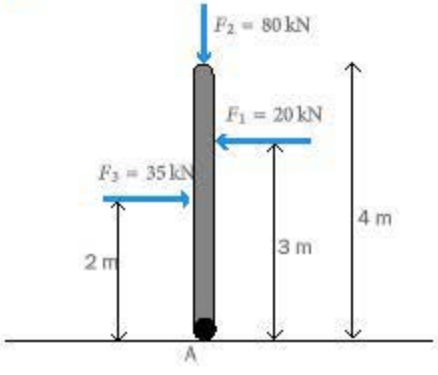
Determine the magnitude and direction sense of the moments of force F_3.

Moment	Magnitude	Direction sense
M_3	kNm	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the magnitude and sense of forces F_1, F_2 and F_3 about point A in the following figure; determine the total moment about point A of all forces.



SMALL MEDIUM LARGE

Type your answer in the box.

Determine the total moment about point A of all forces with the correct sign convention.

Moment	Magnitude	Direction sense	Moment with sign convention
M_1	60 kNm	anticlockwise	kNm
M_2	0 kNm	—	0 kNm
M_3	70 kNm	clockwise	70 kNm
Total	kNm		kNm

Do you know the answer?

I KNOW IT THINK SO UNSURE NO IDEA

Type your answer in the box.

Determine the magnitude and direction sense of the moments of force F_3.

Moment	Magnitude	Direction sense
M_3	kNm	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Determine the magnitude and direction sense of the moments of force F_2.

Moment	Magnitude	Direction sense
M_2	kNm	—

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Determine the total moment about point A of all forces with the correct sign convention.

Moment	Magnitude	Direction sense	Moment with sign convention
M_1	60 kNm	anticlockwise	kNm
M_2	0 kNm	-----	0 kNm
M_3	70 kNm	clockwise	70 kNm
Total	kNm		kNm

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The conditions of rotational equilibrium for a system of forces

When a structure or a machine component is in **equilibrium**, the moments, as well as forces, must be in a state of balance, otherwise the unbalanced resultant moment would cause rotation of the body.

Thus for a body to be in **rotational equilibrium**, the resultant moment about any point must be equal to zero, i.e. there must be no resultant turning effect.

‹

Type your answer in the box.

For a body to be in rotational equilibrium, the resultant moment about any point must be equal to [ ].

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

An unbalanced resultant moment in a body of structure would cause:

Click the correct answer.

- Rotation of the body
- The object to remain stationary
- The object to change shape
- A new equilibrium

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The equation of rotational equilibrium for a system of forces

For a body to be in rotational equilibrium, the resultant moment about any point must be equal to zero.

Mathematically this can be stated as:

$$\Sigma M = 0$$

Alternatively this principle can be expressed in terms of equivalence of the sum of the clockwise moments about any point and the sum of anticlockwise moments about the same point. That is, for equilibrium:

Sum of clockwise moments = sum of anticlockwise moments

The simplest case of this relation is the equivalence of moments of two forces in static equilibrium about a pivot point.

Which of the following equations correctly describe rotational equilibrium?

Check all that apply.

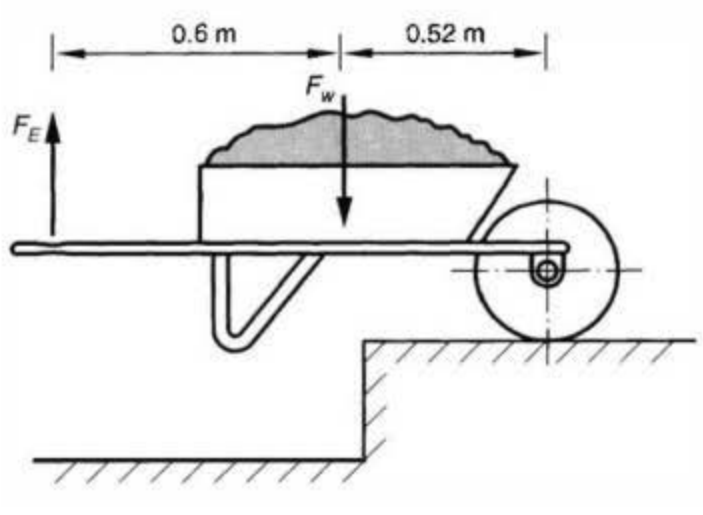
- [ ] $\Sigma M = 0$
- [ ] Sum of clockwise moments = sum of anticlockwise moments
- [ ] $\Sigma F = 0$
- [ ] $M_1 + M_2 = 0$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

The wheelbarrow shown below carries a load of 58 kg. Calculate the effort required to hold the handles in a stationary position.



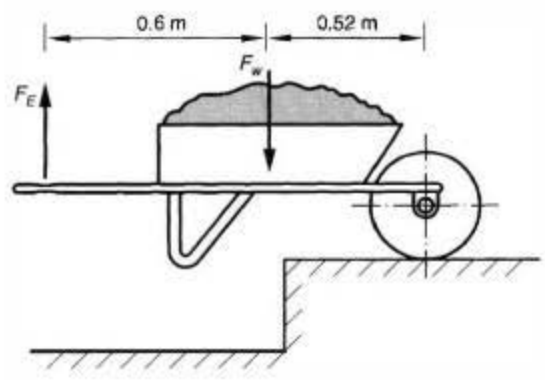
GIVE FEEDBACK

CONTINUE >

Solution

First calculate the weight of the load:

$$\begin{aligned} F_w &= m\,g \\ &= 58\ \text{kg} \times 9.81\ \text{N/kg} \\ &= 569\ \text{N} \end{aligned}$$

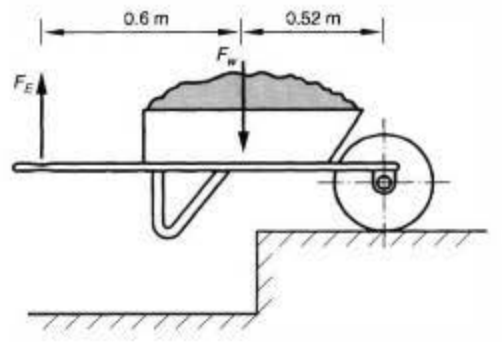


Now equate the moments of the two forces about the pivot point, which is the axis of the wheel, and solve for the unknown force:

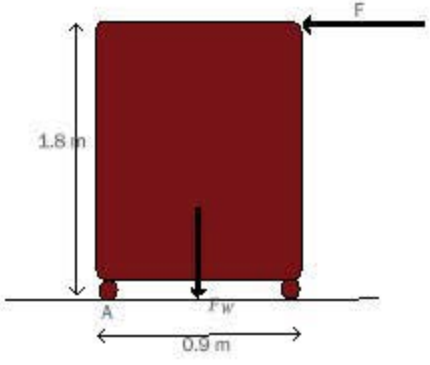
$$569 \times 0.52 = F_E \times 1.12$$

$$\therefore F_E = 264 \text{ N}$$

The total effort required is 264 N or 132 N per handle.



Determine the force F required to start to tip the cabinet shown below if the mass of the cabinet is 50 kg.



SMALL MEDIUM LARGE

Type your answer in the box.

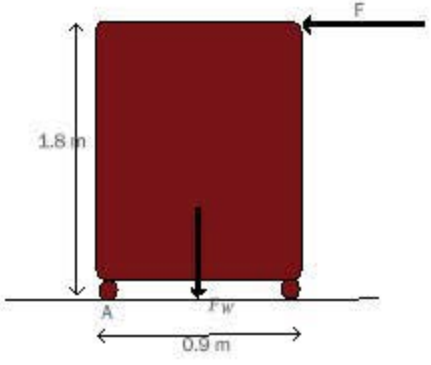
First calculate the weight of the cabinet (rounded to one decimal place).
Hint: Gravity is equal to 9.81 N/kg.

The weight is equal to _____ N.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the force F required to start to tip the cabinet shown below if the mass of the cabinet is 50 kg.



SMALL MEDIUM LARGE

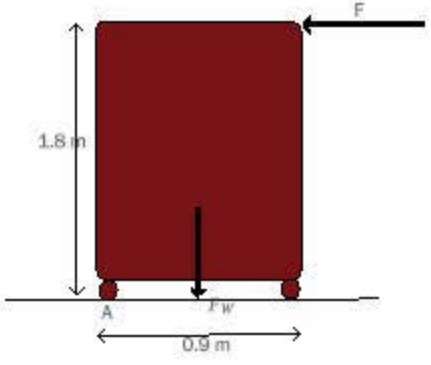
Type your answer in the box.

The moment arm distance of the weight about point A is equal to _____ m (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the force F required to start to tip the cabinet shown below if the mass of the cabinet is 50 kg.



SMALL MEDIUM LARGE

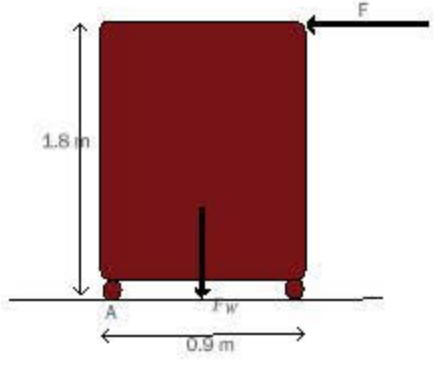
Type your answer in the box.

The moment arm distance of the pushing force F about point A is equal to _____ m (rounded to one decimal place).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the force F required to start to tip the cabinet shown below if the mass of the cabinet is 50 kg.



SMALL MEDIUM LARGE

Type your answer in the box.

Information on the forces and moment arm distances about point A appears in the table below.

Force	Magnitude	Moment arm distance
F_W	490.5 N	0.45 m
F	x	1.8 m

By equating the moments of the two forces about point A, we can solve for the unknown force F.

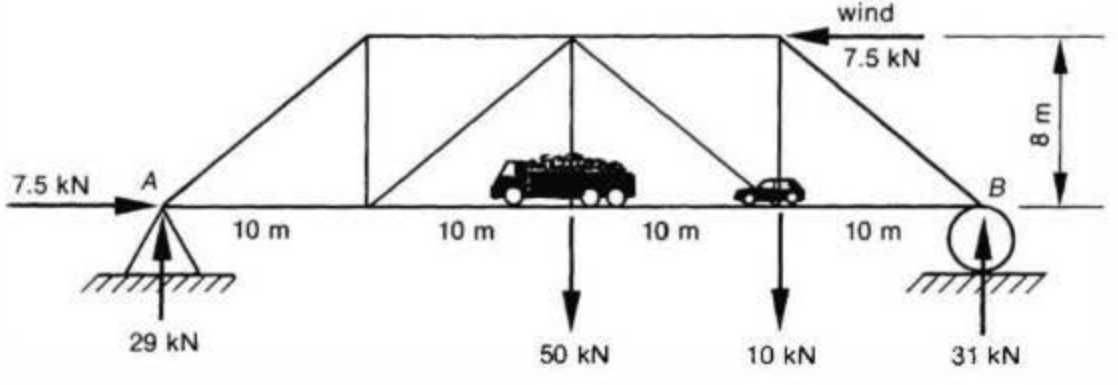
Hence the total effort required is _____ N (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

To prove that a given system of forces is in rotational equilibrium, we must demonstrate that the algebraic sum of all moments about any point is zero.

A bridge structure is subjected to forces as shown below. Prove that the structure is in rotational equilibrium.



GIVE FEEDBACK

CONTINUE >

We must demonstrate that the algebraic sum of all moments about every point is zero.
Take the algebraic sum of all moments about the left-hand support A :

$$\Sigma M_A = 50 \times 20 + 10 \times 30 - 7.5 \times 8 - 31 \times 40$$
$$\Sigma M_A = 0$$

Therefore the moments are balanced about point A .

Take the algebraic sum of all moments about the right-hand support B :

$$\Sigma M_B = 29 \times 40 - 50 \times 20 - 10 \times 10 - 7.5 \times 8$$
$$\therefore \Sigma M_B = 0$$

Therefore the moments are balanced about point B .

Type your answer in the box.

Calculate the algebraic sum of all moments about the right-hand support B.

Force magnitude	Moment arm distance	Moment about point A	Moment direction	M_A (algebraic sign)
6 kN	2 m	12 kN.m	clockwise	12 kN.m
15 kN	4 m	60 kN.m	clockwise	kN.m
8 kN	2 m	16 kN.m	anticlockwise	kN.m
7 kN	8 m	56 kN.m	anticlockwise	kN.m

ΣM_B kN.m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

This structure is in rotational equilibrium.

Click the correct answer.

True

False

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Determine all revelant information of all moments about the left-hand support A.

Force magnitude	Moment arm distance	Moment	Moment direction
6 kN	2 m	kN.m	
15 kN	4 m	60 kN.m	
8 kN	m	48 kN.m	clockwise

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Calculate the algebraic sum of all moments about the left-hand support A.

Force magnitude	Moment arm distance	Moment about point A	Moment direction	M_A (algebraic sign)
6 kN	2 m	12 kN.m	clockwise	kN.m
15 kN	4 m	60 kN.m	anticlockwise	kN.m
8 kN	6 m	48 kN.m	clockwise	kN.m

ΣM_A kN.m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

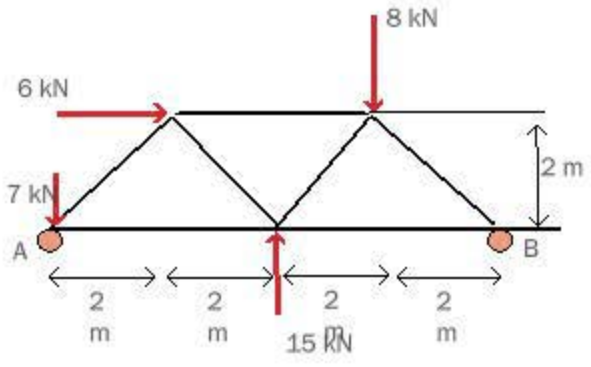
Determine all revelant information of all moments about the right-hand support B.

Force magnitude	Moment arm distance	Moment	Moment direction
6 kN	2 m	12 kN.m	clockwise
15 kN	4 m	60 kN.m	
8 kN	2 m	kN.m	anticlockwise
7kN	8 m	kN.m	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in rotational equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

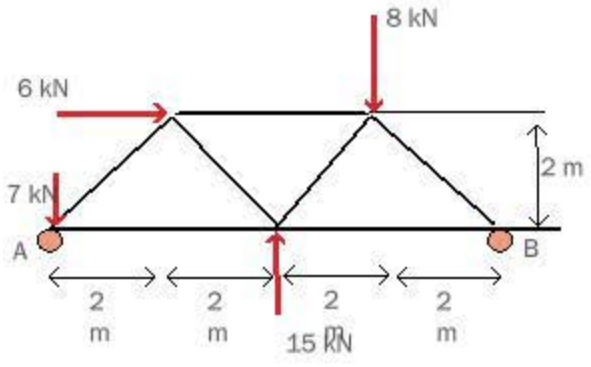
Determine all revelant information of all moments about the left-hand support A.

Force magnitude	Moment arm distance	Moment	Moment direction
6 kN	2 m	kN.m	
15 kN	4 m	60 kN.m	
8 kN	m	48 kN.m	clockwise

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in rotational equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

Calculate the algebraic sum of all moments about the left-hand support A.

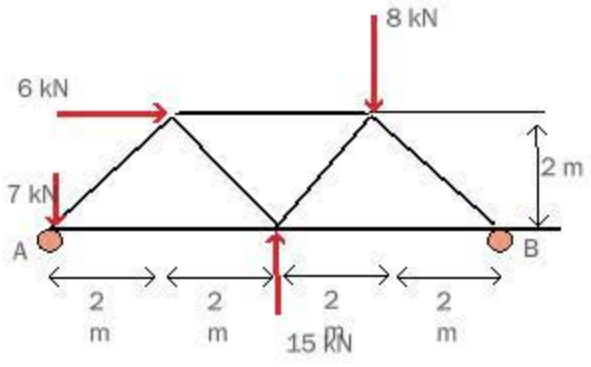
Force magnitude	Moment arm distance	Moment about point A	Moment direction	M_A (algebraic sign)
6 kN	2 m	12 kN.m	clockwise	kN.m
15 kN	4 m	60 kN.m	anticlockwise	kN.m
8 kN	6 m	48 kN.m	clockwise	kN.m

ΣM_A kN.m

Do you know the answer?

I KNOW IT THINK SO UNSURE NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in rotational equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

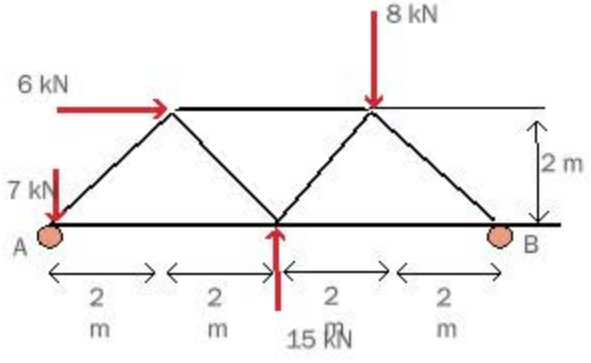
Determine all revelant information of all moments about the right-hand support B.

Force magnitude	Moment arm distance	Moment	Moment direction
6 kN	2 m	12 kN.m	clockwise
15 kN	4 m	60 kN.m	
8 kN	2 m	kN.m	anticlockwise
7kN	8 m	kN.m	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in rotational equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

Calculate the algebraic sum of all moments about the right-hand support B.

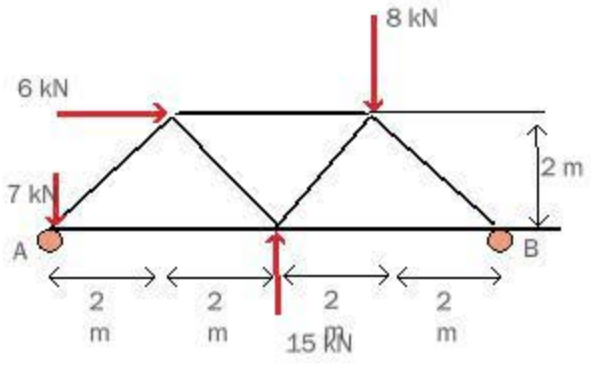
Force magnitude	Moment arm distance	Moment about point A	Moment direction	M_A (algebraic sign)
6 kN	2 m	12 kN.m	clockwise	12 kN.m
15 kN	4 m	60 kN.m	clockwise	kN.m
8 kN	2 m	16 kN.m	anticlockwise	kN.m
7 kN	8 m	56 kN.m	anticlockwise	kN.m

ΣM_B kN.m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in rotational equilibrium.



SMALL MEDIUM LARGE

This structure is in rotational equilibrium.

Click the correct answer.

True

False

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The three conditions of equilibrium for a system of non-concurrent forces

It has already been established that for a system of forces to be in equilibrium, the resultant force, i.e. the result of vectorial summation of all forces, must be equal to zero. That is, there must be no resultant push or pull.

This statement is often interpreted mathematically in terms of the perpendicular, usually horizontal and vertical, components of the resultant force and can be stated simply as:

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

Only when both of these conditions are satisfied is the force equilibrium established.

<

These two conditions were found sufficient for the study of equilibrium of concurrent force systems. However, under certain conditions rotation may occur.

An additional condition for equilibrium of a system of non-concurrent forces must, therefore, ensure the absence of a turning moment, stated as:

$$\Sigma M = 0$$

<

GIVE FEEDBACK

OK

Select the correct conditions required for equilibrium of a system of non-concurrent forces.

Check all that apply.

- [ ] The resultant sum of all horizontal component forces must equal zero
- [ ] The resultant sum of all vertical component forces must equal zero
- [ ] There must be no resultant turning moment about any points
- [ ] The body must remain stationary
- [ ] There must be no resultant turning moment about a specfic point

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The three equations of statics

Collectively the three conditions of equilibrium are known as the **three equations of statics**.

These equations may be used to prove that a particular structure is in static equilibrium under the combined action of a system of non-concurrent forces.

1. The sum of x components of all forces must equal zero:

$$\Sigma F_x = 0$$

2. The sum of y components of all forces must equal zero:

$$\Sigma F_y = 0$$

3. The sum of moments of all forces about any point must equal zero:

$$\Sigma M = 0$$

GIVE FEEDBACK

OK

Select the correct three equations of statics.

Check all that apply.

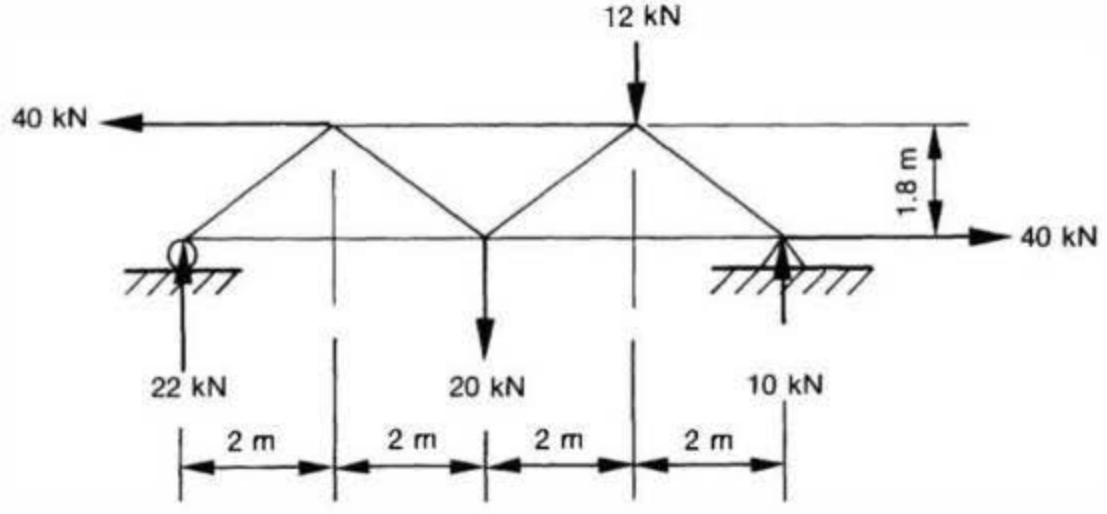
- [ ] $\Sigma F_y = 0$
- [ ] $\Sigma F_x = 0$
- [ ] $\Sigma M = 0$
- [ ] $\Sigma M_H = 0$
- [ ] $\Sigma M_V = 0$
- [ ] $\Sigma F = 0$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Prove a given structure is in static equilibrium

The structure is subjected to forces as shown below.
Using the three equations of statics, prove that the structure is in equilibrium.



Problem	Equations	Step 1	Step 2	Step 3	Conclusion

GIVE FEEDBACK

OK

Prove a given structure is in static equilibrium

The three equations of static areas follow.

The sum of x components of all forces must equal zero:

$$\Sigma F_x = 0$$

The sum of y components of all forces must equal zero:

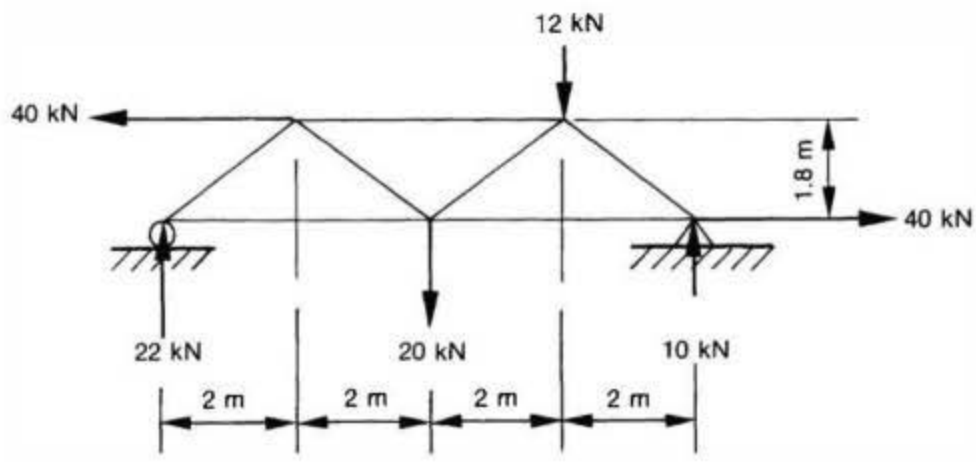
$$\Sigma F_y = 0$$

The sum of moments of all forces about any point must equal zero:

$$\Sigma M = 0$$

Prove a given structure is in static equilibrium

Note the algebraic sign convention for left direction force is negative and right direction force is positive.



From the figure we can identify two horizontal forces.

Hence the sum of horizontal forces:

$\Sigma F_x = 40\,\text{kN} - 40\,\text{kN} = 0$

Problem	Equations	Step 1	Step 2	Step 3	Conclusion

GIVE FEEDBACK

OK

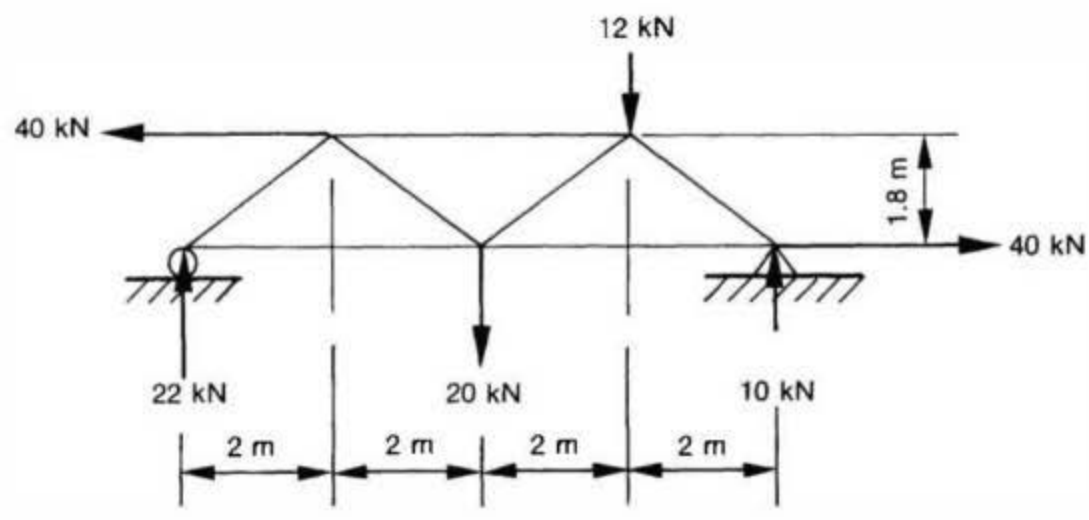
Prove a given structure is in static equilibrium

Note the algebraic sign convention for down-direction force is negative and up-direction force is positive.

From the figure we can identify four vertical forces.

Hence the sum of vertical forces:

$\Sigma F_y = 22\,\text{kN} + 10\,\text{kN} - 20\,\text{kN} - 12\,\text{kN} = 0$



Problem	Equations	Step 1	Step 2	Step 3	Conclusion

GIVE FEEDBACK

OK

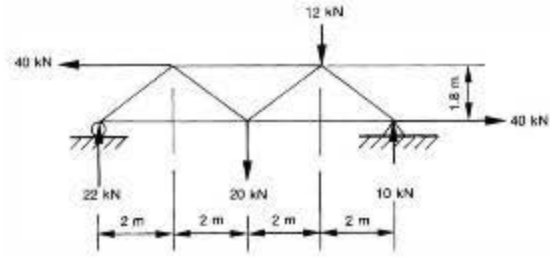
Prove a given structure is in static equilibrium

Try calculating moments about an alternative point, e.g. the right-hand support, to be satisfied that the sum of the moments is equal to zero irrespective of the position of the reference point.

$$\Sigma F_y = 22\,\text{kN} + 10\,\text{kN} - 20\,\text{kN} - 12\,\text{kN} = 0$$

Sum of moments about left-hand support:

$$\Sigma M = 20 \times 4 - 10 \times 8 + 12 \times 6 - 40 \times 1.8 = 0$$



Note the algebraic sign convention for anticlockwise direction force is negative and clockwise direction force is positive.

All three equations are satisfied. The conclusion is that the structure is in static equilibrium.

Problem	Equations	Step 1	Step 2	Step 3	Conclusion

GIVE FEEDBACK

OK

Prove a given structure is in static equilibrium

Proving equilibrium of a static structure, such as a truss or a bridge span, when all forces (i.e. all loads and all reactions) acting on it are known and taken into account does not in itself seem to be much more than a mathematical exercise of confirming by calculation the obvious fact that any static structure must always be in equilibrium.

It is quite a different matter when some of the forces, usually support reactions, are not given.

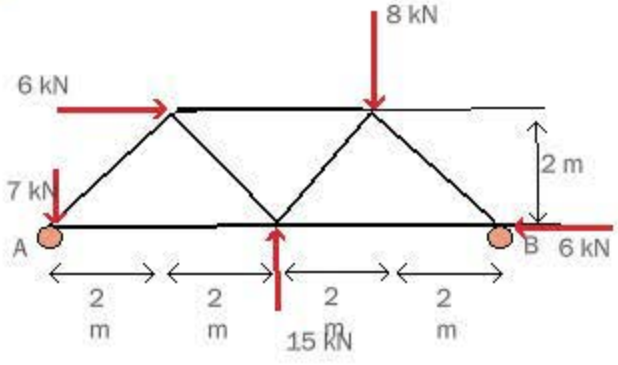
In this case the three equations of statics become a powerful tool which enables us to calculate up to three unknown reaction forces for any structure which is presumed to be in equilibrium. This will be made use of for the analysis of beams, frames and trusses.

Problem	Equations	Step 1	Step 2	Step 3	Conclusion

GIVE FEEDBACK

OK

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in static equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

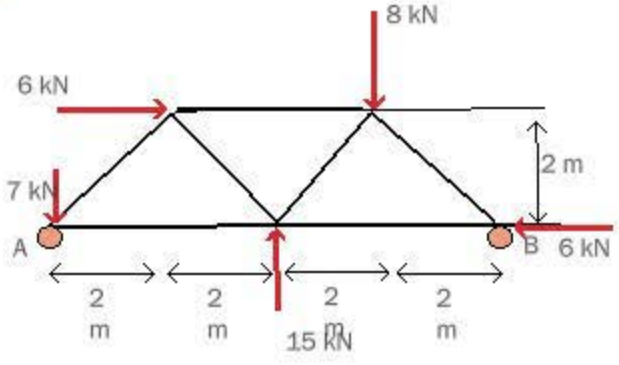
Sum of horizontal forces:

ΣF_X = [ ] kN − [ ] kN = [ ] kN

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in static equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

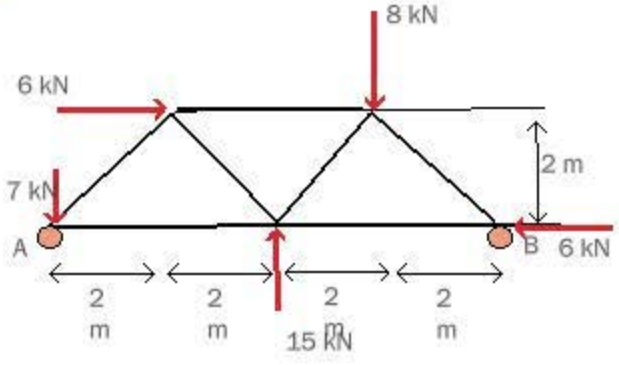
Sum of vertical forces:

$\Sigma F_y = -8$ kN + [ ] kN + [ ] kN = [ ] kN

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in static equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

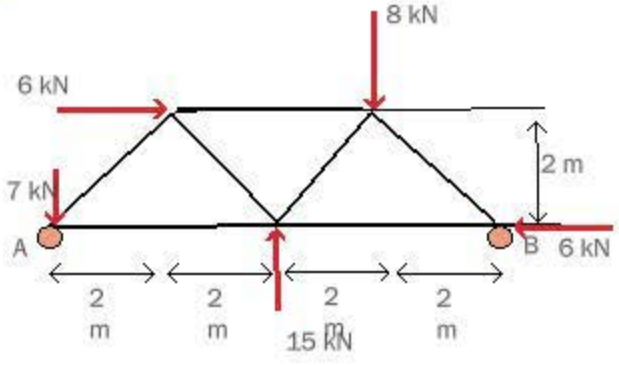
Determine all revelant information of all moments about the right-hand support B.

Force magnitude	Moment arm distance	Moment	Moment direction
6 kN	2 m	12 kN.m	clockwise
15 kN	4 m	60 kN.m	
8 kN	2 m	kN.m	anticlockwise
7kN	8 m	kN.m	

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in static equilibrium.



SMALL MEDIUM LARGE

Type your answer in the box.

Calculate the algebraic sum of all moments about the right-hand support B.

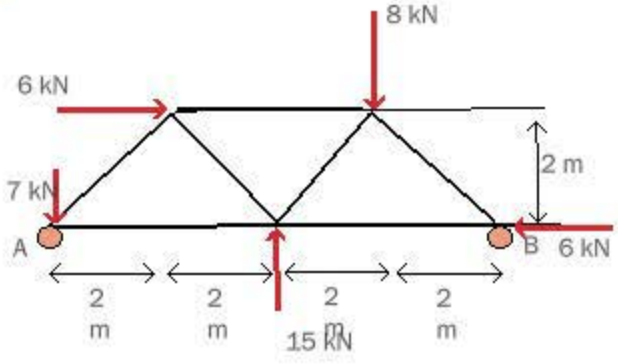
Force magnitude	Moment arm distance	Moment about point A	Moment direction	M_A (algebraic sign)
6 kN	2 m	12 kN.m	clockwise	12 kN.m
15 kN	4 m	60 kN.m	clockwise	kN.m
8 kN	2 m	16 kN.m	anticlockwise	kN.m
7 kN	8 m	56 kN.m	anticlockwise	kN.m

ΣM_B kN.m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The truss shown below is subjected to multiple loads as indicated. Prove the structure is in static equilibrium.



SMALL MEDIUM LARGE

This structure is in static equilibrium.

Click the correct answer.

True

False

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Resultant of non-concurrent force system

In studying concurrent forces and static equilibrium, we learned that the resultant force has that single force which has exactly the same effect as a given system of forces.

We also learned how to determine its magnitude and direction by the force polygon method or by mathematical addition of forces.

This is applicable to the case of non-concurrent forces acting on a rigid body, except that each force in such a system will also tend to rotate the body upon which it acts about some axis.

In order to determine the resultant of a non-concurrent force system, it is therefore necessary to consider the rotational effect of the force system in addition to the linear push–pull action of the forces.

‹

What is the main difference between determining the resultant of a concurrent force system and a non-concurrent force system?

Click the correct answer.

It is necessary to consider the rotational effect of the force system in a non-concurrent force system (in addition to the linear push–pull forces).

There is no difference.

When the resultant of a non-concurrent force system is not zero, it is can still be in equilibrium.

Mathematical addition of forces does not apply to a non-concurrent force system.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Steps required to solve for the resultant of non-concurrent forces

When solving for the resultant of a non-concurrent force system, the problem usually consists of two steps:

1. Finding the magnitude and direction of the resultant force by mathematical or graphical addition of all forces

2. Finding the location of the resultant relative to an arbitrary reference point, usually by applying the principle of moments

GIVE FEEDBACK

OK

Match the correct steps for solving the resultant of a non-concurrent force system.

Drag statements on the right to match the left.

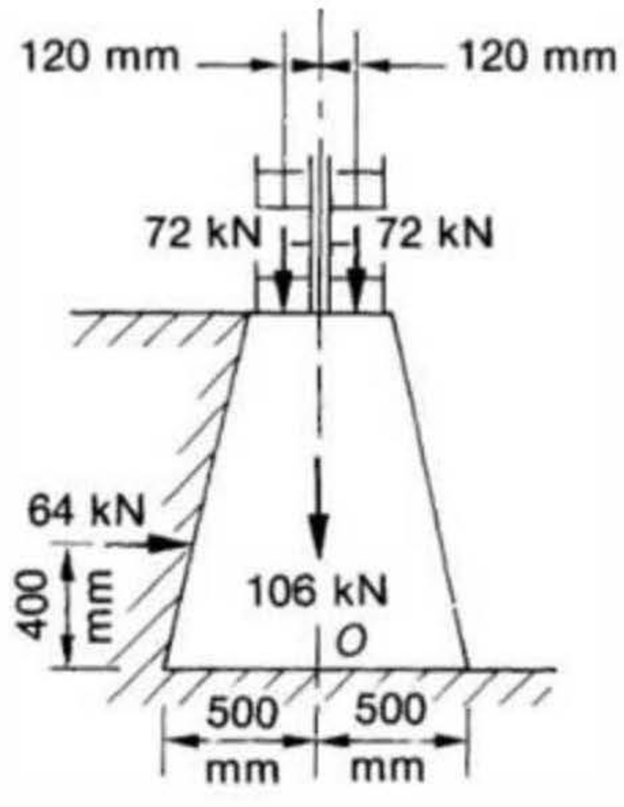
Step 1	⇔	Finding the magnitude and direction of the resultant force by mathematical or graphical addition of all forces
Step 2	⇔	Finding the location of the resultant relative to an arbitrary reference point, usually by applying the principle of moments

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

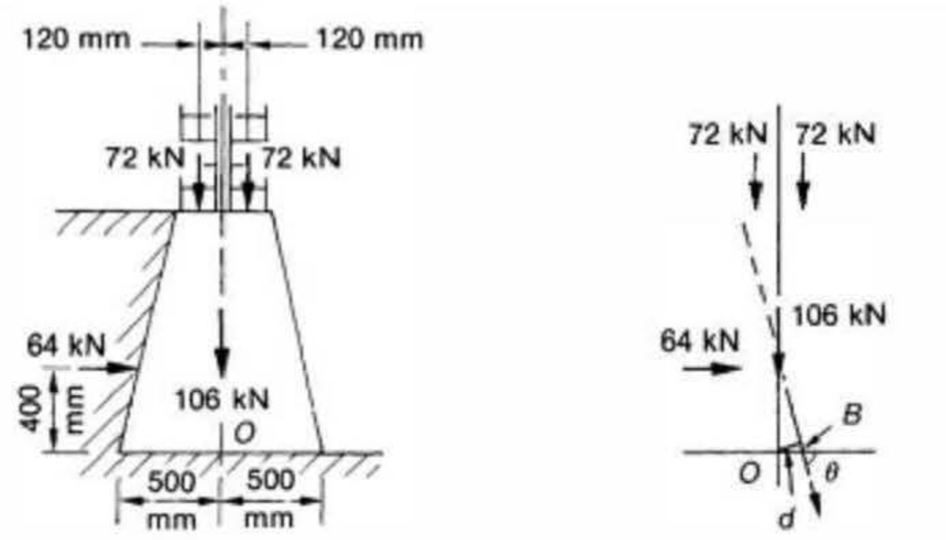
A concrete foundation for a brick wall must support two vertical loads and a horizontal load due to soil pressure on one side, as shown below. Its own weight is also shown. Determine the resultant and check if it passes through the base of the foundation, as required for stability.



GIVE FEEDBACK

CONTINUE >

Solution



Sum of vertical forces:

$$\Sigma F_V = 106 + 72 + 72$$
$$= 250\,\text{kN}\,\text{down}$$

Sum of horizontal forces:

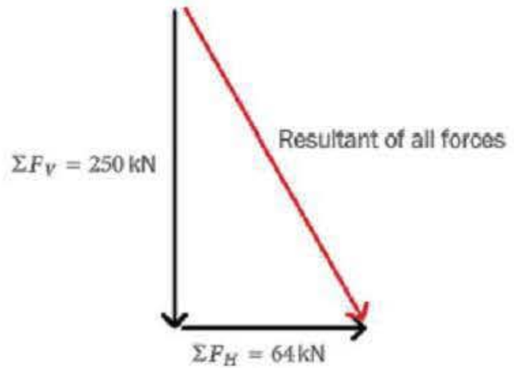
$$\Sigma F_H = 64\,\text{kN}\,\text{to the right}$$

Addition of forces states that by arranging given forces in a tip-to-tail fashion, the resultant of the forces is found by connecting the tail of the first force and the tip of the last force.

Determine the magnitude, direction and location of the resultant of the system of forces—Example

Sum of vertical forces:	Sum of horizontal forces:
$\Sigma F_V = 106 + 72 + 72$	
$= 250\,\text{kN}$ down	$\Sigma F_H = 64\,\text{kN}$ to the right

The resultant of the forces can now be solved by mathematical or graphical methods.



Using Pythagoras's theorem the resultant is:

$$F = \sqrt{250^2 + 64^2}$$
$$= 258.1\,\text{kN}$$

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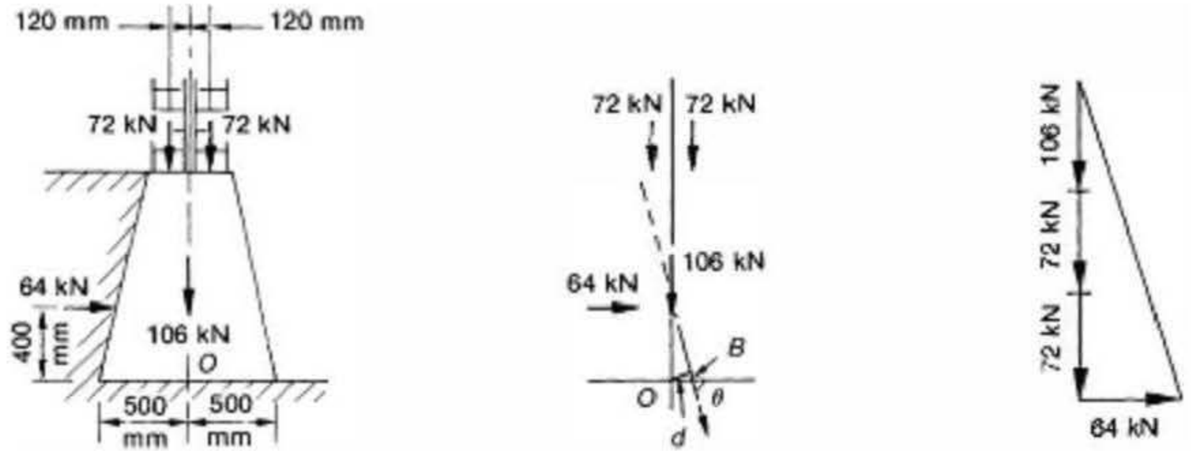


The angle to the horizontal:

$$\theta = \tan^{-1}\left(\frac{250}{64}\right)$$

$$= 75.6°$$

To locate the resultant, take moments about a convenient point, such as midpoint of the base:

$$\Sigma M = 64 \times 0.4 - 72 \times 0.12 + 72 \times 0.12$$

$$= 25.6\,\mathrm{kN.m}$$

Parts of this solution can also be done graphically. For example, the magnitude and direction of the resultant can conveniently be found by constructing the polygon of forces. However, the principle of moments, for determining the distance d is best dealt with mathematically.

The alternative graphical method, known as the funicular (from funis meaning rope) polygon, involves a special construction procedure which locates a point on the line of action of the resultant force, thus locating the force itself. It is not discussed here.

Distance of the resultant from the midpoint:

$$d = \frac{\Sigma M}{F}$$

$$= \frac{25.6\ \text{kN.m}}{258.1\ \text{kN}}$$

$$= 0.0992\ \text{m}$$

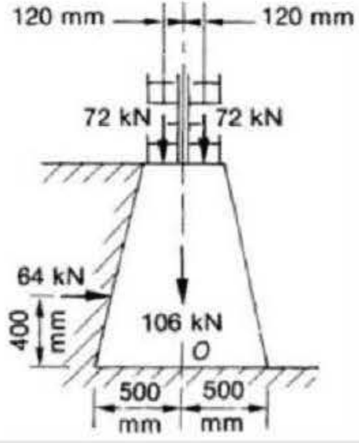
$$= 99.2\ \text{mm}$$

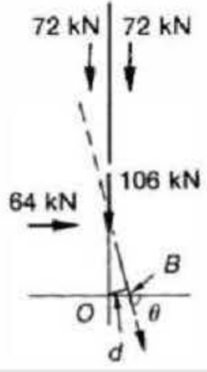
The point at which the resultant intersects with the base can be calculated from:

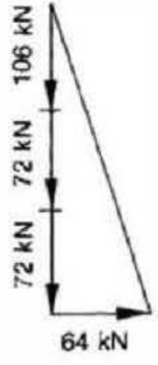
$$OB = \frac{d}{\sin 75.6°}$$

$$= \frac{992}{\sin 75.6°}$$

$$= 102.4\ \text{mm}$$





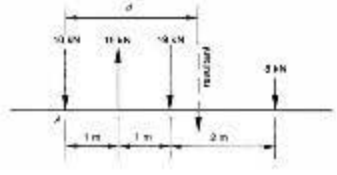


< BACK GIVE FEEDBACK OK

When solving for the resultant of a non-concurrent force system, the problem usually consists of two steps:

1. Finding the magnitude and direction of the resultant force by mathematical or graphical addition of all forces
2. Finding the location of the resultant relative to an arbitrary reference point, usually by applying the principle of moments

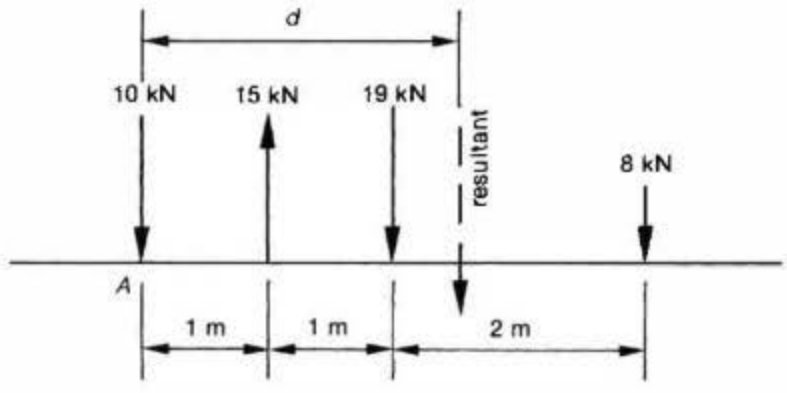
Here we can use the two-steps method to determine the magnitude, direction and location of the resultant of the system of forces.



GIVE FEEDBACK

CONTINUE >

Determine the magnitude, direction and location of the resultant of the system of forces 2/4



The magnitude of the resultant force:

$$\begin{aligned} F &= \Sigma F \\ &= -10 + 15 - 19 - 8 \\ &= -22\ \text{kN} \end{aligned}$$

That is, 22 kN down.

< BACK GIVE FEEDBACK CONTINUE >

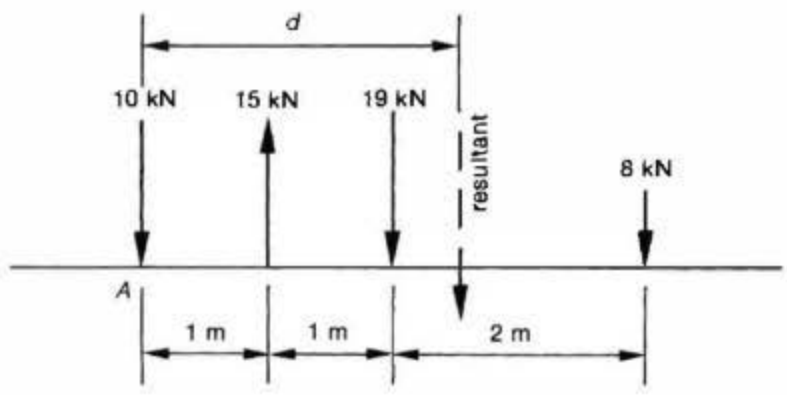
The location is found by taking moments about an arbitrary point, such as point A :

$$\begin{aligned}\Sigma M &= 10 \times 0 - 15 \times 1 + 19 \times 2 + 8 \times 4 \\ &= 55 \text{ kN.m}\end{aligned}$$

This sum must be equal to the moment of the resultant about the same point:

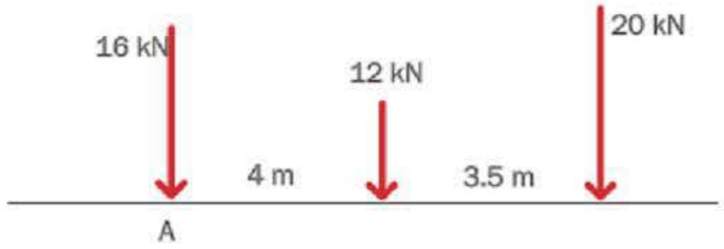
$$\begin{aligned}F \times d &= 55 \text{ kN.m} \\ 22 \text{ kN} \times d &= 55 \text{ kN.m} \\ \therefore d &= \frac{55 \text{ kN.m}}{22 \text{ kN}} \\ &= 2.5 \text{ m}\end{aligned}$$

It is therefore possible to replace the given system of forces by a single downward force of 22 kN, located at 2.5 m from point A , as shown.



< BACK GIVE FEEDBACK OK

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL MEDIUM LARGE

Type your answer in the box.

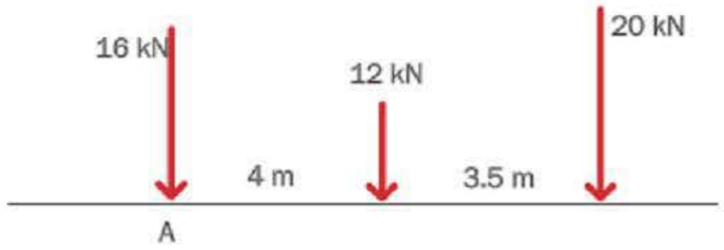
The magnitude of the resultant force is _____ kN.
Hint: $F = \Sigma F$ and show your answer with the correct sign.

The direction of the resultant force is _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL MEDIUM LARGE

Type your answer in the box.

The sum of moments about point A is equal to _____ kN.m.

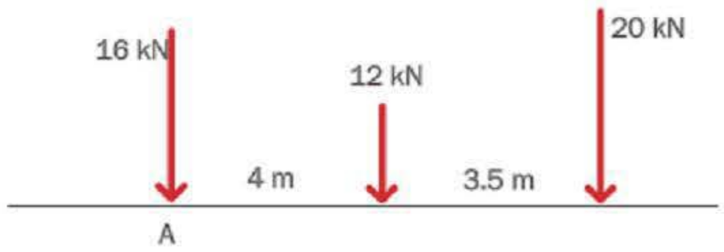
Hint: $M = \Sigma M$ and show your answer with the correct sign.

The direction of the sum of moment is _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL MEDIUM LARGE

Type your answer in the box.

The sum of moments ΣM about point A is equal to 198 kN.m.

The magnitude of the resultant force F is a single downward force of 48 kN.

With the given information, the location of the resultant force is located at _____ m from point A (rounded to two decimal places).

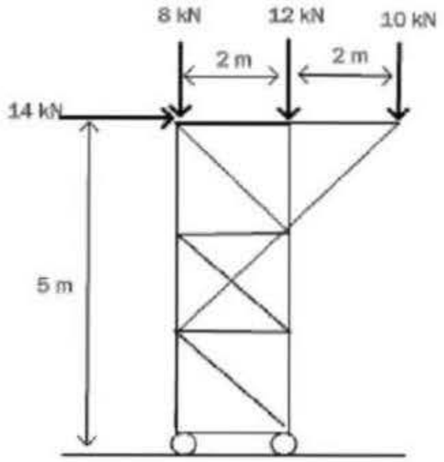
Hint: The sum of moments must equal to the moment of the resultant around point A:

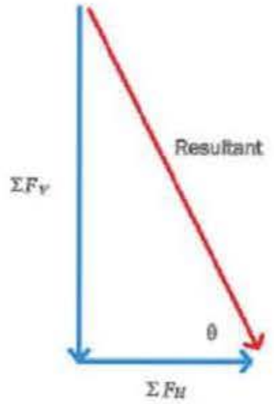
$F \cdot d = \Sigma M$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL | MEDIUM | LARGE

Type your answer in the box.

Sum of the vertical forces:

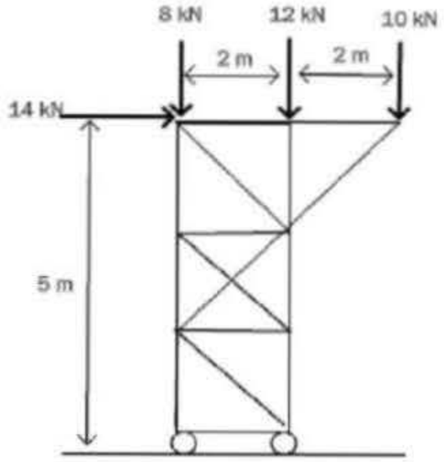
$\Sigma F_V = 8$ kN + [ ] kN + [ ] kN

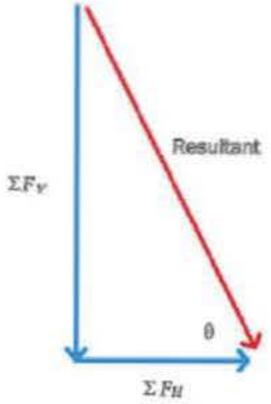
The total is equal to [ ] kN in the [ ] direction.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL | MEDIUM | LARGE

Type your answer in the box.

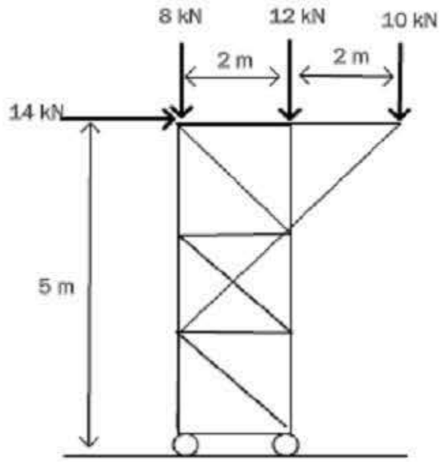
Sum of the horizontal forces:

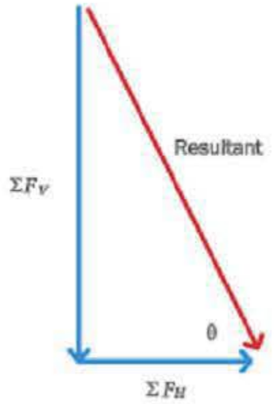
The total is equal to _____ kN to the right.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL | MEDIUM | LARGE

Type your answer in the box.

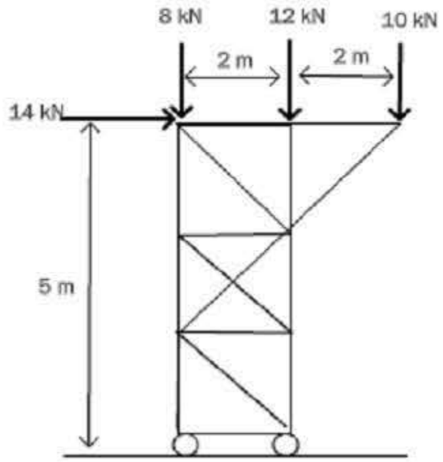
The total sum of horizontal forces is 14 kN to the right and the total sum of horizontal forces is 30 kN down.

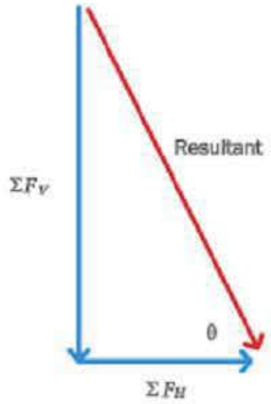
The resultant force is therefore _____ kN (rounded to one decimal place.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL | MEDIUM | LARGE

Type your answer in the box.

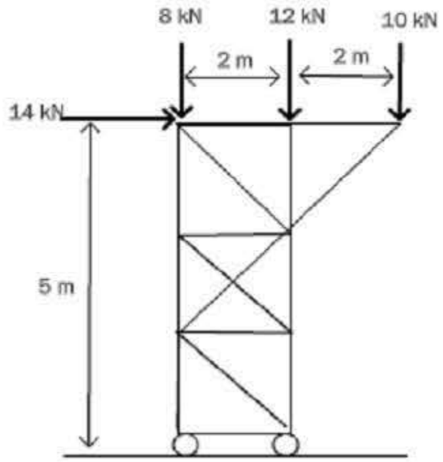
The total sum of horizontal forces is 14 kN to the right and the total sum of horizontal forces is 30 kN down.

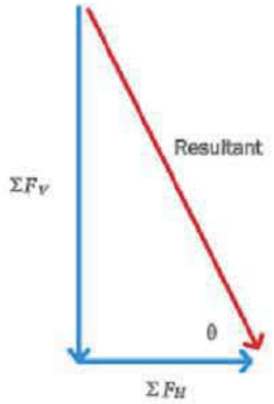
The angle between the resultant force and the horizontal θ is equal to _____ degrees (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL MEDIUM LARGE

Type your answer in the box.

To locate the resultant we must take the sum of moments about the left-hand support.

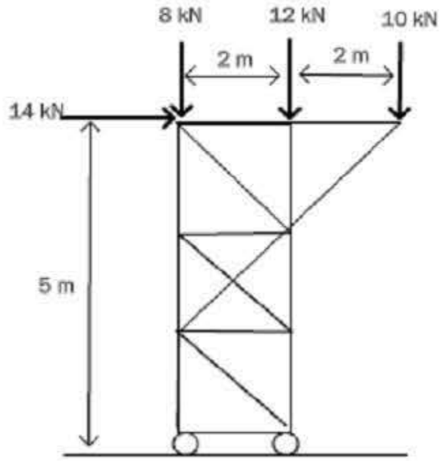
The sum of moments about the left-hand support is equal to _____ kN.m.
Hint: $M = \Sigma M$ and show your answer with the correct sign.

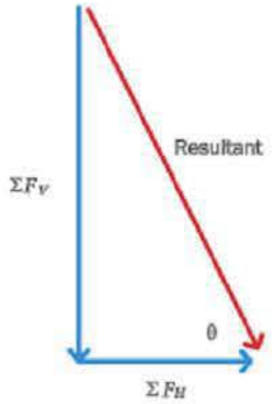
The direction of the sum of moment is _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the resultant of the loads acting on the structure shown below and locate it relative to the left-hand support.





SMALL | MEDIUM | LARGE

Type your answer in the box.

The total resultant is 33.1 kN and the total sum of moments is 134 kN.m clockwise.

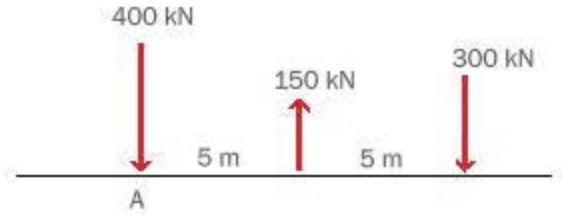
The perpendicular distance from the left hand support to the line of action of the resultant is equal to _____ m (rounded to two decimal places).

Hint: $d = \frac{(\Sigma M)}{F}$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL MEDIUM LARGE

Type your answer in the box.

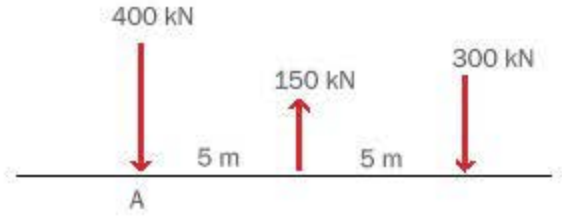
The magnitude of the resultant force is _____ kN.
Hint: $F = \Sigma F$ and show your answer with the correct sign.

The direction of the resultant force is _____.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL | MEDIUM | LARGE

Type your answer in the box.

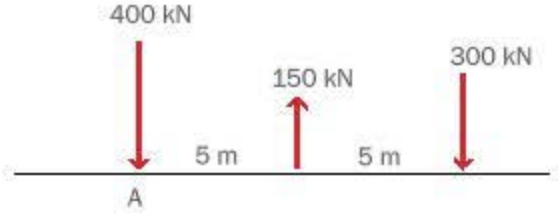
The sum of moments about point A is equal to [ ] kN.m.
Hint: $M = \Sigma M$ and show your answer with the correct sign.

The direction of the sum of moment is [ ].

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In the figure below, determine and locate the resultant force, stating the distance along the horizontal line from point A.



SMALL MEDIUM LARGE

Type your answer in the box.

The sum of moments ΣM about point A is equal to 2250 kN.m.
The magnitude of the resultant force F is a single downward force of 550 kN.

With the given information, the location of the resultant force is located at _____ m from point A (rounded to two decimal places).

Hint: The sum of moments must equal to the moment of the resultant around point A:
$F \cdot d = \Sigma M$

Do you know the answer?

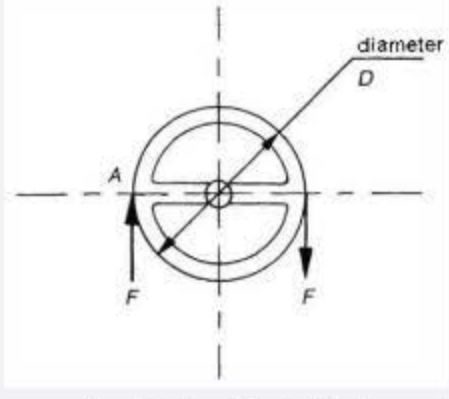
I KNOW IT | THINK SO | UNSURE | NO IDEA

Force couple

We can determine the turning effects of single forces.

A **couple** consists of two forces which have:

1. The same magnitude
2. Parallel lines of action
3. Opposite sense



Forces on a steering wheel illustrating a couple

A couple is a pair of actual forces applied to a structure or component at specified points. A couple always consists of equal and opposite forces acting along parallel lines.

When your hands are on two opposite points of the steering wheel of a car, one hand pushing up and the other pulling down with equal but opposite forces, the result is a couple.

The algebraic sum of the two forces is equal to zero, i.e. there is no net push or pull in any direction.

However, we can see from the steering wheel that there will be an overall turning effect.

GIVE FEEDBACK

OK

Which of the following are examples of a couple?

Check all that apply.

- [ ] Moving the steering wheel with two hands
- [ ] Turning a key in a lock
- [ ] Moving a steering wheel with one hand
- [ ] Pushing on the centre of steering wheel

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A couple consists of two forces which have:

Check all that apply.

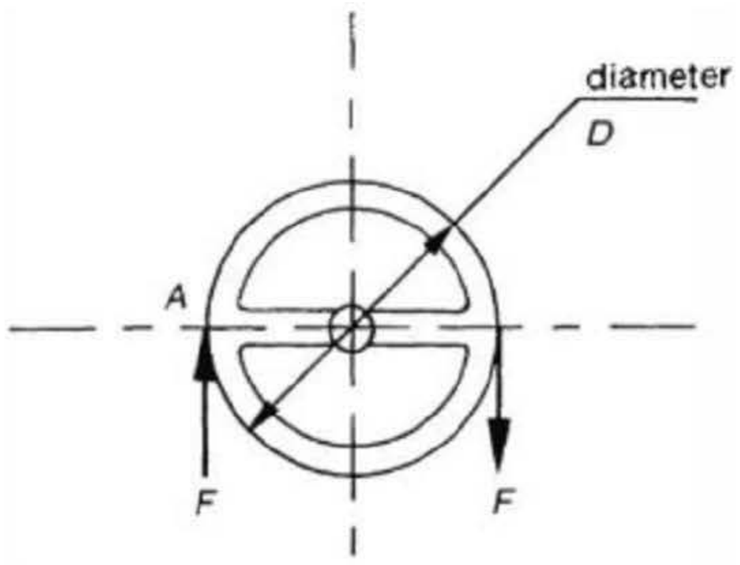
- [ ] Same magnitude
- [ ] Parallel lines of action
- [ ] Different magnitude
- [ ] Opposite direction sense
- [ ] Same direction sense

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Moment of a couple is a pure turning effect produced by the couple, expressed in newton metres. A couple has no resultant force in any direction; the pair of forces cancel each other out.

When your hands are on two opposite points of the steering wheel of a car, one hand pushing up and the other pulling down with equal but opposite forces, the result is a couple.
Even though there is is no net push or pull in any direction, there is a turning effect on the wheel.



GIVE FEEDBACK

CONTINUE >

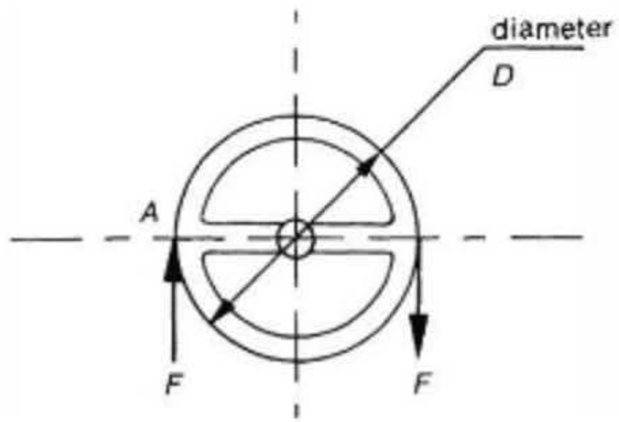
The turning effect can be calculated relative to any point. The obvious point of reference for calculating the moments is the centre point of the wheel, which is the axis of its rotation. The total moment is:

$$M = F \times \frac{D}{2} + F \times \frac{D}{2}$$
$$= F\left(\frac{D}{2} + \frac{D}{2}\right)$$
$$= F \times D$$

F is the magnitude of each of the forces in the couple.

D is the distance between them, in this case the wheel diameter.

The sense of the total moment is clockwise for the forces shown.



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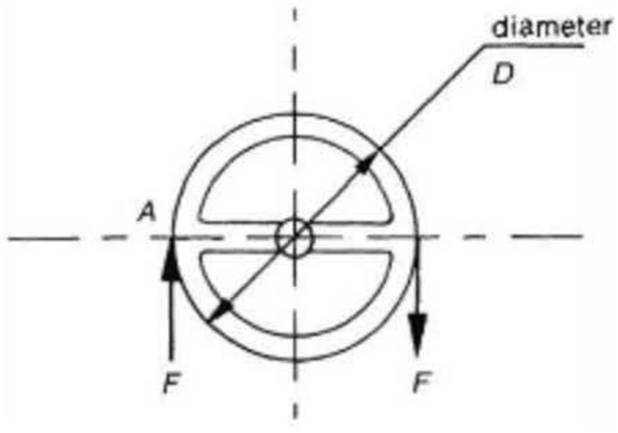
The product of one of the forces and the distance between them is called the moment of the couple. One important characteristic of a couple is that its moment, $M = F \times D$, does not depend on the choice of the reference point. For example, the moment about point A is:

$$M_A = F \times D + 0$$
$$= F \times D$$

This can be checked by repeating the same calculation about any other point on, or even outside, the wheel.



< BACK GIVE FEEDBACK OK

Select the statements that correctly describe the moment of couple.

Check all that apply.

- [ ] Moment of a couple is a pure turning effect produced by the couple
- [ ] A couple has no resultant force in any direction; the pair of forces cancel each other out
- [ ] A couple moment expressed in newtons
- [ ] The algebraic sum of forces is not zero, hence a pushing and turning effect

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of the following is the correct formula for calculating the moment of a couple?

Click the correct answer.

$M = F \cdot D$

$\Sigma M = 0$

$M = M_1 + M_2$

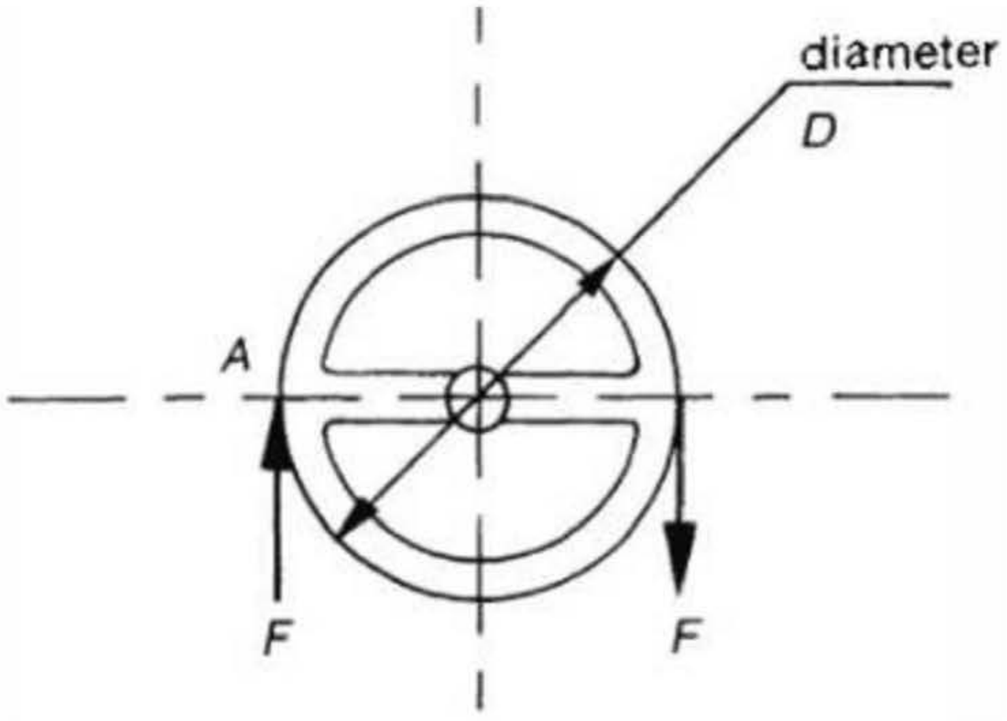
$M = \Sigma(F \cdot d)$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the moment of the couple acting on the steering wheel in the figure below if the wheel diameter is 350 mm and the forces applied as shown are each 5 N.

Also determine the forces required to produce the same moment if the hands are held on the spokes, halfway between the axis and the rim of the wheel.



GIVE FEEDBACK

CONTINUE >

Moment of the couple:

$$\begin{aligned} M &= F \times D \\ &= 5\,\text{N} \times 0.35\,\text{m} \\ &= 1.75\,\text{N.m clockwise} \end{aligned}$$

To produce the same moment if the hands are held on the spokes, halfway between the axis and the rim of the wheel, where $D = \frac{0.35\,\text{m}}{2} = 0.175\,\text{m}$, the forces required can be calculated from $M = F \times D$:

$$\begin{aligned} 1.75\,\text{N.m} &= F \times 0.175\,\text{m} \\ \therefore F &= \frac{1.75\,\text{N.m}}{0.175\,\text{m}} \\ &= 10\,\text{N each} \end{aligned}$$

A screw jack is operated by means of 40 N forces applied at each end of a double arm, at right angles to it. Determine the magnitude of the turning moment if the total length of the double arm is 700 mm.

$+$ $-$ $\cdot$ $\div$ $\frac{\square}{\square}$ $\square^2$ $\sqrt{\square}$ Clear

$(\square)$ ▼ $\leq$ ▼ π m ▼ $\overline{\square}$? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

Equivalent couple

Here are some important summary points.

1. A **couple** always consists of a pair of equal and opposite forces applied to a structure or component.
2. A **couple** has no resultant force in any direction; the pair of forces act along parallel lines and cancel each other out.
3. **Moment of a couple** is a pure turning effect produced by the couple, expressed in newton metres.

The magnitude of the moment of a couple is independent of the reference point about which it is calculated. Therefore, the simplest way to determine it is by multiplying one of the forces by the perpendicular distance between them.

The magnitude of the moment is also independent of the actual location of the couple on the structure or component to which it is applied. Therefore, a couple can be relocated to any other position without affecting the overall balance of moments.

An **equivalent couple** is any other couple with the same magnitude of turning moment and the same directional sense. A given couple can be replaced by an equivalent couple applied elsewhere on the body without changing its turning effect.

GIVE FEEDBACK

OK

Type your answer in the box.

[ ] couples are couples which involve different forces but produce with the same magnitude of turning moment and the same directional sense.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

A given couple can be replaced by an equivalent couple applied elsewhere on the body without changing its _____ effect.

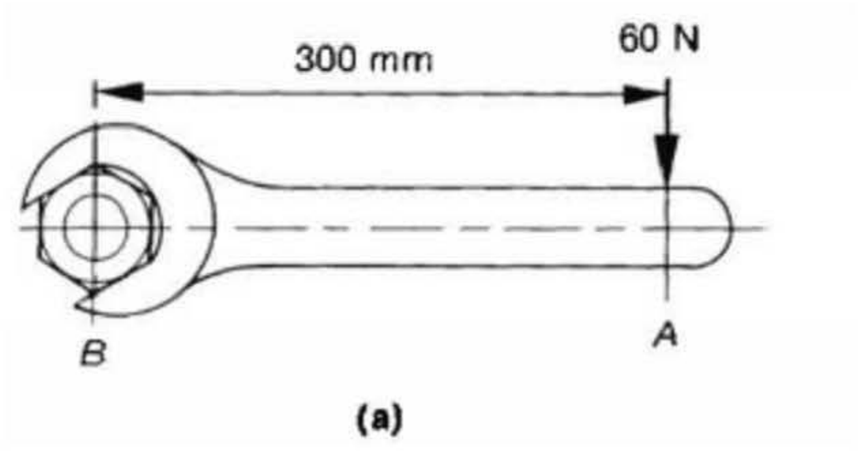
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Two forces systems are said to be **equivalent** if they result in the same resultant force (magnitude and direction) and produce the same resultant moment about any point.

When a force F is applied at a point on a rigid body, such as point A on the spanner shown in Figure (a), its effect can be viewed from some other significant point, such as the centreline of the bolt, point B.

In this case, the bolt (point B) experiences a downward push equal to the applied force F, as well as a clockwise turning moment M caused by force F about point B.

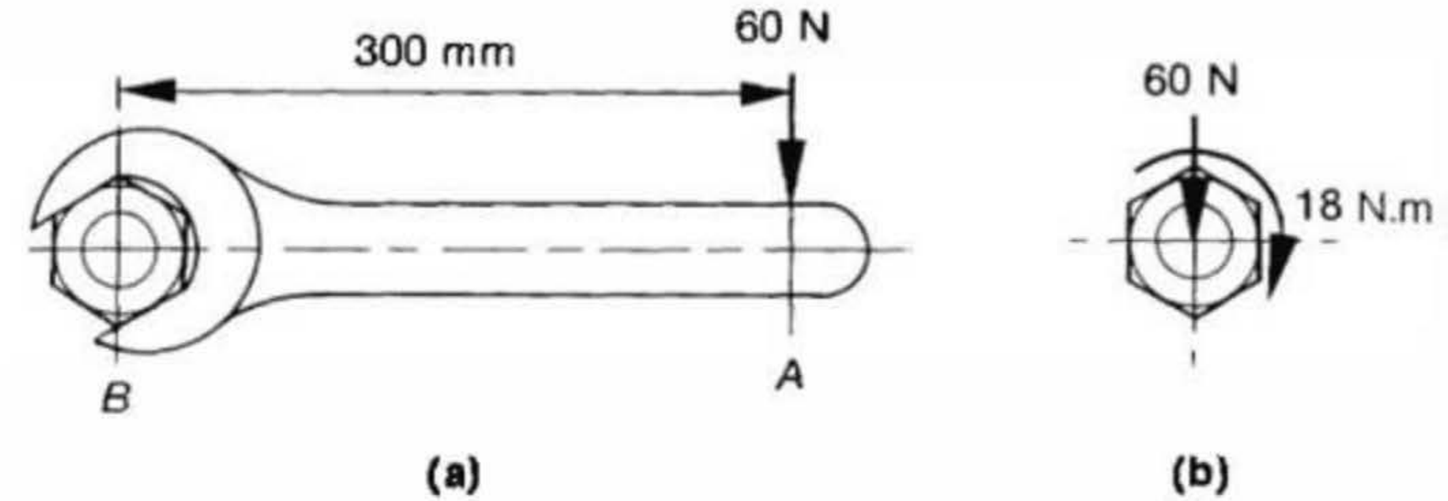


(a) Force applied to a spanner to turn a bolt

GIVE FEEDBACK

CONTINUE >

If we were to draw a separate diagram of the bolt itself, showing the effect of the applied force as viewed from B, the force F would be represented by the usual arrow symbol, and the turning moment indicated by the symbol ↷ placed around point B, as shown in Figure (b).

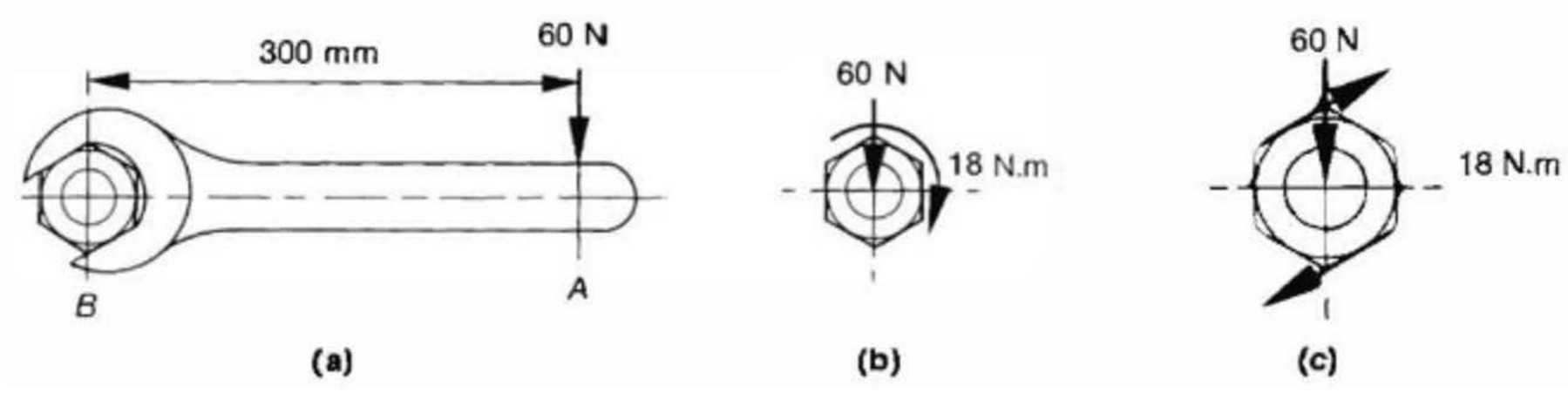


(a) Force applied to a spanner to turn a bolt (b) The head of the bolt showing the turning moment

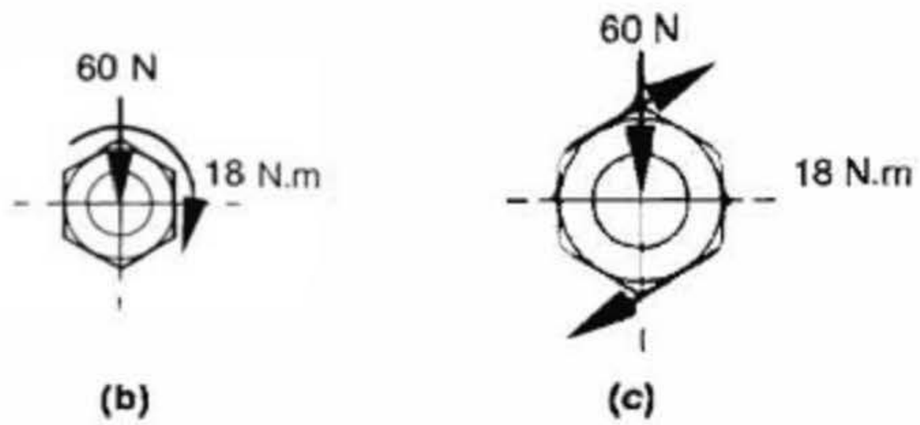
It is usually sufficient to leave the turning moment symbol as it is, without any further interpretation.

However, in reality, this pure turning moment is a result of a force couple acting on the head of the bolt, as shown in Figure (c).

In fact, if the distance between the flats of the hexagonal bolt head is known, it is possible to work out the magnitudes of the forces that constitute this couple.



(a) Force applied to a spanner to turn a bolt (b) The head of the bolt showing the turning moment (c) The force and couple acting on the head of the bolt



Diagrams (b) and (c) represent the force-and-moment system and force-and-couple system acting at point B, which is equivalent to the action of force F applied at point A.

Note that on both diagrams only the magnitude and clockwise directional sense of the turning moment are stated, without spelling out the individual force magnitudes in the pair.

There are situations, such as that illustrated by the following example, where it is necessary to know the individual forces in the couple and the perpendicular distance between them is well defined.

An equivalent force-couple system is a single force and a moment couple acting at a single point that is equivalent to the orginial set of forces and moments.

Click the correct answer.

True

False

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

Two forces systems are said to be _____ if they result in the same resultant force (magnitude and direction) and produce the same resultant _____ about any point.

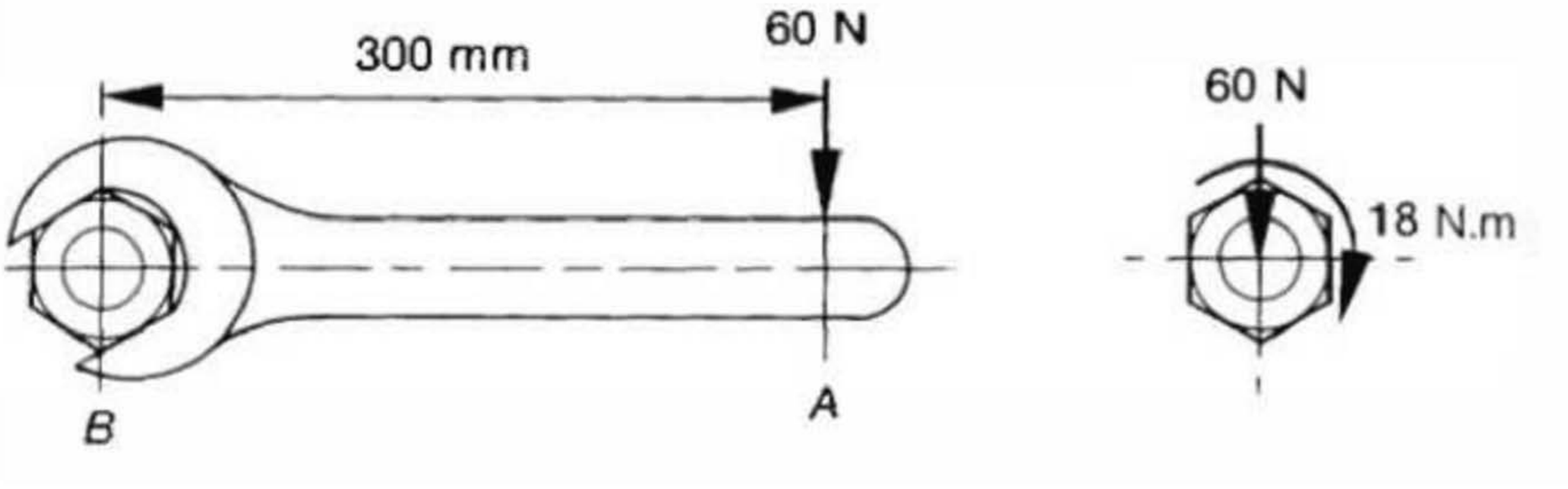
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

A bolt is being tightened by a force of 60 N applied to a spanner at a distance of 300 mm, as shown below.

What is the force and turning moment acting on the bolt at point B?



GIVE FEEDBACK

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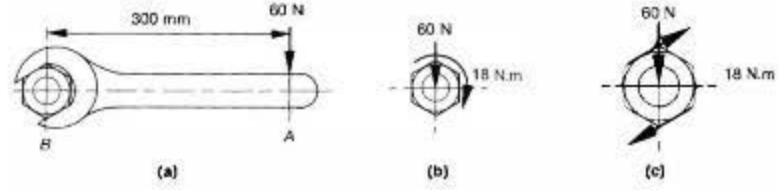
Solution

If the distance between the flats of the hexagonal bolt head is known, it is possible to work out the magnitudes of the forces that constitute this couple.

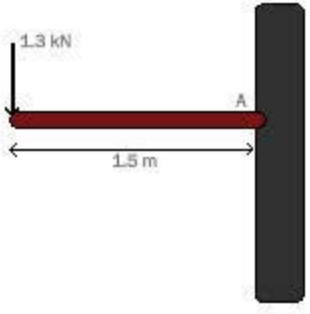


(a) Force applied to a spanner to turn a bolt (b) The head of the bolt showing the turning moment (c) The force and couple acting on the head of the bolt

The force-moment system at B comprises a force of 60 N and a turning moment equal to:

$$M = 60\ \text{N} \times 0.3\ \text{m}$$
$$= 18\ \text{N.m clockwise}$$

Determine the equivalent force-moment system at point A (the built-in end of the cantilever beam) shown below.



SMALL MEDIUM LARGE

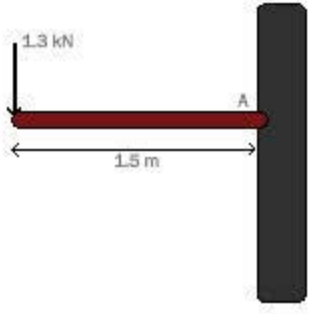
Type your answer in the box.

The force-moment system comprises a [ ] force of [ ] kN at 1.5 m from point A.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Determine the equivalent force-moment system at point A (the built-in end of the cantilever beam) shown below.



SMALL MEDIUM LARGE

Type your answer in the box.

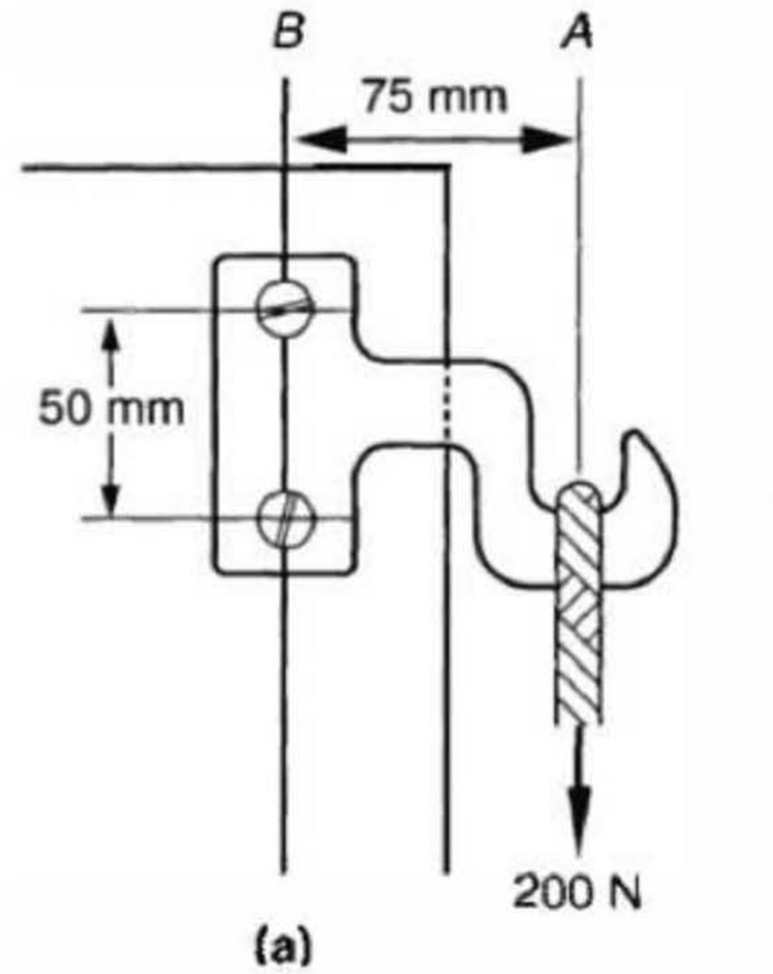
The force-moment system at A comprises a force of 1.3 kN and a turning moment of _____ kN.m in the _____ direction (rounded to two decimal places).

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

For the hook shown in Figure (a), replace the force of 200 N acting in plane A by an equivalent force-and-couple system at B.



(a)

GIVE FEEDBACK

CONTINUE >

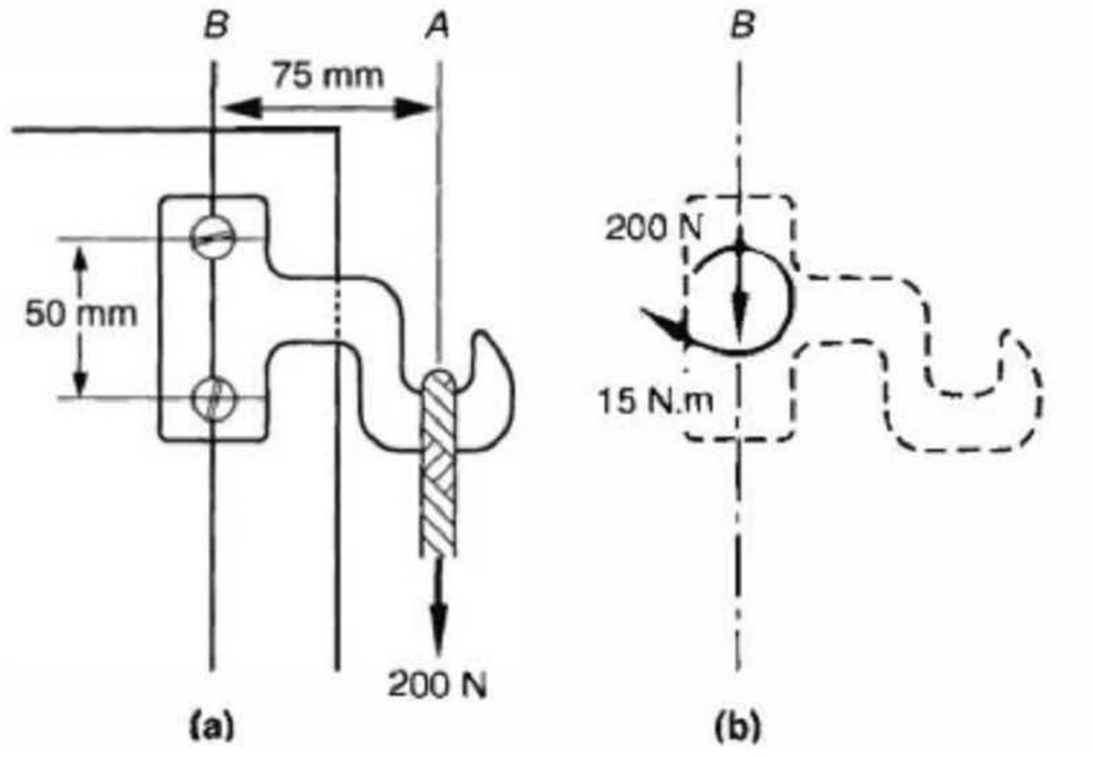
Solution

The equivalent force-moment system at B consists of a force of 200 N and a clockwise turning moment equal to:

$$M = 200\,\mathrm{N} \times 0.075\,\mathrm{m}$$

$$= 15\,\mathrm{N.m}$$

This is shown in Figure (b).



In this case, where the moment is taken by the two screws, it is meaningful to interpret the moment as an equivalent couple with a distance of 50 mm between forces:

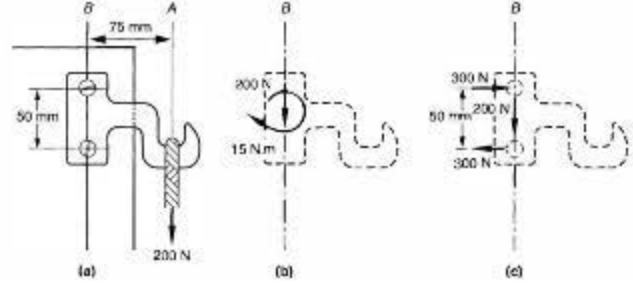
$$M = F\,d$$

$$15\ \text{N.m} = F \times 0.05\ \text{m}$$

$$\therefore F = \frac{15}{0.05}$$

$$= 300\ \text{N}$$

This is shown in Figure (c).



Type your answer in the box.

The equivalent force-moment system at the bolts consists of a force of 49.05 N (weight of the mass) and a [ ] direction turning moment equal to [ ] N.m (rounded to two decimal places).

Hint: Moment $M = F \cdot d$ where d is the moment arm distance.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

In this case, where the moment is taken by the two bolts, it is meaningful to interpret the moment as an equivalent couple with a distance of [ ] m between forces.

Hence the equivalent force couple is equal to [ ] N each going through the bolts (rounded to two decimal places).

Hint: $M = F \cdot D$

M = moment of a couple

F = force of a couple

D = distance between the force

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Type your answer in the box.

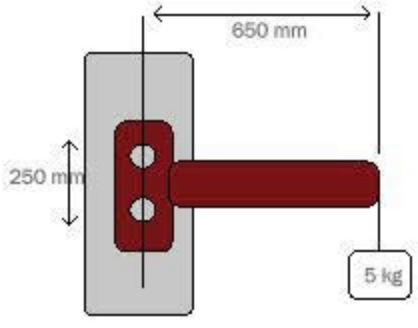
The weight of the mass is [ ] N (rounded to two decimal places).

Hint: Weight $F_W = m\,g$ where g = 9.81 N/kg.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If the mass supported by the bracket in the figure below is 5 kg, interpret the load as a force and a couple acting through the bolts.



SMALL MEDIUM LARGE

Type your answer in the box.

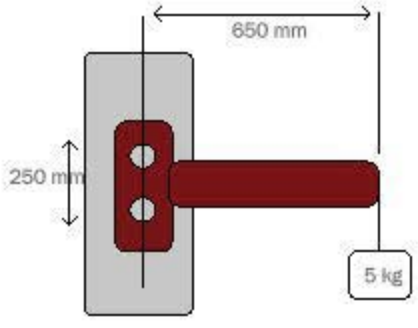
The weight of the mass is _____ N (rounded to two decimal places).

Hint: Weight $F_W = m\,g$ where g = 9.81 N/kg.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If the mass supported by the bracket in the figure below is 5 kg, interpret the load as a force and a couple acting through the bolts.



SMALL MEDIUM LARGE

Type your answer in the box.

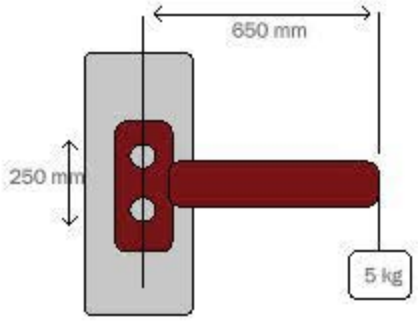
The equivalent force-moment system at the bolts consists of a force of 49.05 N (weight of the mass) and a [ ] direction turning moment equal to [ ] N.m (rounded to two decimal places).

Hint: Moment $M = F \cdot d$ where d is the moment arm distance.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

If the mass supported by the bracket in the figure below is 5 kg, interpret the load as a force and a couple acting through the bolts.



SMALL MEDIUM LARGE

Type your answer in the box.

In this case, where the moment is taken by the two bolts, it is meaningful to interpret the moment as an equivalent couple with a distance of _____ m between forces.

Hence the equivalent force couple is equal to _____ N each going through the bolts (rounded to two decimal places).

Hint: $M = F \cdot D$
M = moment of a couple
F = force of a couple
D = distance between the force

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA