

GIVE FEEDBACK

CONTINUE >

When a structure or machine component is at rest or in a state of balance, it is said to be in **static equilibrium**. Even though the object appears to be in a state of rest, there are multiple forces acting on the object that counteract each other.

In this section the concept of static equilibrium is introduced together with some important principles associated with the mathematical and graphical analysis of **concurrent forces** and the **point of concurrency**.

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CONTINUE &gt;

In engineering, concurrent forces and static equilibrium are essential in designing stability for all structures and machines. A thorough understanding of the effects of forces is required for the construction of structures and devices such as bridges, buildings and engines.



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GIVE FEEDBACK

OK



The whole structure must be completely stationary to be considered in static equilibrium.



### Define static equilibrium

When a structure or machine component is at rest, it is said to be in static equilibrium.

For example, an operating Ferris wheel is not in static equilibrium. Even though the overall structure is stationary, it has many components in motion.



GIVE FEEDBACK

OK

Which of the following scenarios are in static equilibrium?

---

Check **all** that apply.

☐ Car parked in a sloped driveway

☐ Tennis ball travelling at 130 km/h

☐ Book on a bookshelf

☐ Rotating Ferris wheel

☐ Poster pinned on a wall

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

If all components of the structure are stationary, then the structure is considered in static equilibrium.

---

**Click the correct answer.**

True

False

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

## Equilibrium of forces

In general, the state of **equilibrium** can be defined as a state of rest or balance under the action of forces which counteract each other.

In other words, all forces balance each other in such a way that there is no resultant push or pull acting on the body.



GIVE FEEDBACK



OK

Static equilibrium of forces in a structure means that \_\_\_\_\_.

---

**Click the correct answer.**

it is in a state of rest or balance where all actions of forces counteract each other

there are no forces acting on the structure

the structure is moving in the same direction at a constant speed

the structure is rigid and not falling apart

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



Forces don't have to balance each other out for the body to be in equilibrium.

---

**Click the correct answer.**

False

True

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



This diagram shows four forces acting on a joint of a pin-jointed truss.



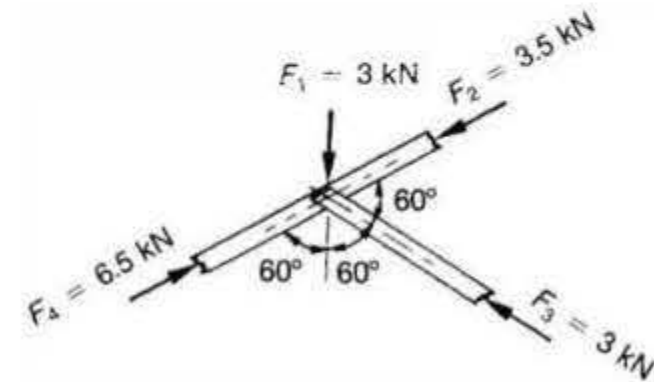
### Define concurrent forces

**Concurrent forces** are two or more forces whose lines of action intersect at the same common point.

The **line of action** is the geometric representation of how the force is applied.

A system of forces is called concurrent when the lines of action of all forces intersect at a common point, the point of concurrency.

All of the lines of actions below intersect at the joint and hence the joint is the point of concurrency.



GIVE FEEDBACK

OK

**Type your answer in the box.**

When the lines of action of all forces intersect at a common point (point of concurrency), the system is called

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

**Type your answer in the box.**

A system of forces is called concurrent when the lines of  of all forces intersect at a common point.

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

## The conditions of equilibrium for a system of concurrent forces

For a system of concurrent forces to be in equilibrium, the resultant force, i.e. the resulting summation of all forces acting through that point, must be equal to zero.

In other words, all forces balance each other out in such a way that there is no resultant push or pull acting on the body at the point of application of the forces.

To prove that a given system of forces is in fact in equilibrium, we must demonstrate, graphically or mathematically, that the forces add up to zero.

Naturally, since forces are vector quantities, the addition must be vectorial additions (graphically or mathematically).

Graphically it must show that the force polygon diagram is drawn to scale and all forces must close (head to tail) properly.



Mathematically for a given system of forces, the sum of all components in any direction must be zero. This is usually expressed in terms of mutually perpendicular directions  $x$  and  $y$ , often horizontal and vertical.



GIVE FEEDBACK



OK

What are the conditions of equilibrium for a system of concurrent forces?

---

Check **all** that apply.

- ☐ Graphically it must show that the force polygon diagram is to be drawn to scale and all forces must close (head-to-tail) properly
- ☐ Mathematically for a given system of forces, the sum of all force components in any direction must be zero. The force components are usually expressed in terms of horizontal and vertical directions, often shown in x and y axes
- ☐ As long as the body is not moving, it is in equilibrium
- ☐ Forces are all pointing towards the point of concurrency

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

When the system of concurrent forces is in equilibrium, it means:

---

Check **all** that apply.

- ☐ The result of summation of all forces acting through the point of concurrency must equal zero
- ☐ The moments must add up
- ☐ All forces balance each other out at the point of application and there is no resultant push or pull acting on the body
- ☐ The structure is at rest

Do you know the answer?

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

**Type your answer in the box.**

The conditions of equilibrium for a system of concurrent forces is that all forces must add up to

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



All forces do not have to balance each other out as long as there is no resultant push or pull acting on the body at the point of application of the forces.

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**Click the correct answer.**

False

True

**Do you know the answer?**

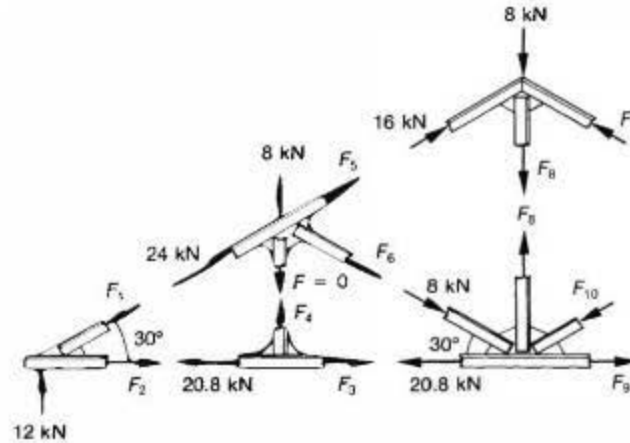
**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

A pin-jointed truss with internal and external forces is shown below.

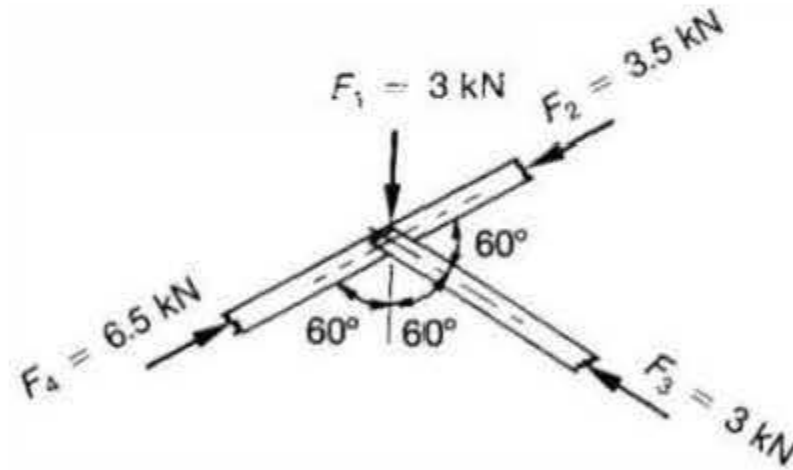


It is important to know how to prove that the joints are in equilibrium.

GIVE FEEDBACK

CONTINUE >

Our task now is to prove that the following joint is in equilibrium graphically.  
In this task, the forces are only expressed in two dimensions, horizontal and vertical directions (written as x and y directions components).



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GIVE FEEDBACK

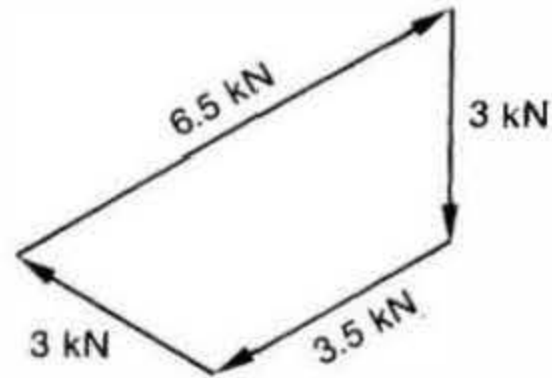
CONTINUE &gt;

In order to prove the equilibrium of this system of concurrent forces graphically, it is necessary to:

1. construct the polygon of forces using all of the applied forces, external ( $F_1$ ) as well as internal ( $F_2$ ,  $F_3$  and  $F_4$ ) forces
2. show that the starting point of the construction coincides with the end point, i.e. the polygon must close properly
3. ensure that the diagram is drawn to scale and all forces close, leaving no room for a gap representing a resultant force.

Note: The resultant must be zero.

The completed force polygon diagram is shown below.

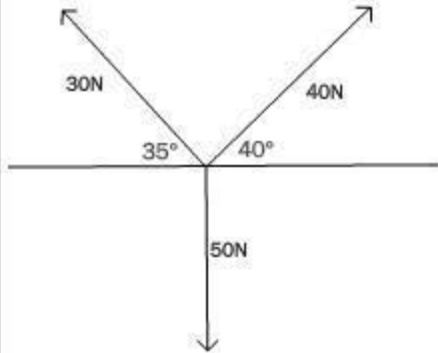


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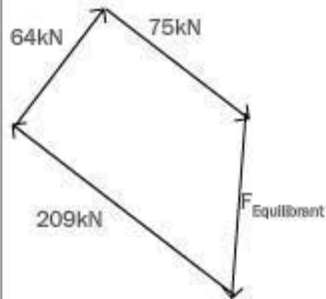
GIVE FEEDBACK

OK

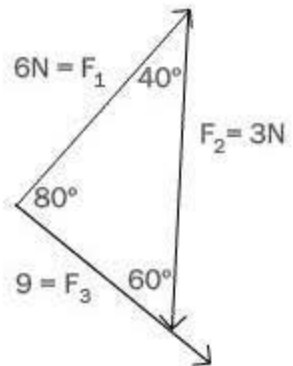
1.



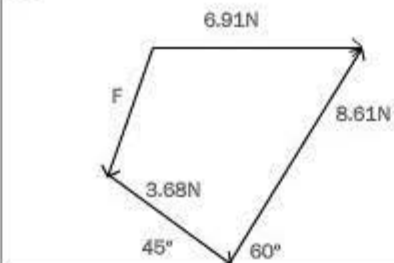
2.



3.



4.



Which force polygon proves the joint is in equilibrium?

Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

To prove the system is in equilibrium graphically, you must:

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Check **all** that apply.

- ☐ Construct a force polygon
- ☐ Show that the external and internal forces are equal
- ☐ Ensure that the force polygon closes where the starting point coincides with the end point of forces
- ☐ Show that all forces intersect at one point

Do you know the answer?

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



There are three equations you must remember.

The first equation is that the sum of all forces must equal zero.

The second equation is that the sum of all forces in the x direction must equal zero.

The third equation is that the sum of all forces in the y direction must equal zero.



### Equations used to express the conditions of equilibrium for a system of concurrent forces

Mathematically the summation of forces is signified by  $\Sigma F$ , meaning that the resultant force is the sum of all the applied forces. For a system in equilibrium, the resultant force is equal to zero:

$$\Sigma F = 0$$

This is the useful equation of forces in equilibrium. However, this equation is often interpreted in terms of the perpendicular components of the resultant force. If the resultant force is equal to zero, its components must also be equal to zero, i.e. no force, no components.

It follows that, for a given system of forces, the sum of all components in any direction must be zero. This is usually expressed in terms of mutually perpendicular horizontal and vertical directions, often notated as the  $x$  and  $y$  directions.

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

GIVE FEEDBACK

OK



State collectively all the mathematical conditions of equilibrium for a system of concurrent forces.

---

Check **all** that apply.

☐  $\Sigma F_x = 0$

☐  $\Sigma F_y = 0$

☐  $\Sigma F = 0$

☐  $\Sigma F_{xy} = 0$

☐  $\Sigma F_{\text{internal}} = 0$

☐  $\Sigma F_{\text{external}} = 0$

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

In order to prove the equilibrium of a system of concurrent forces mathematically, it is necessary to demonstrate that the sum of the horizontal and vertical components add up to zero.

---

**Click the correct answer.**

True

False

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

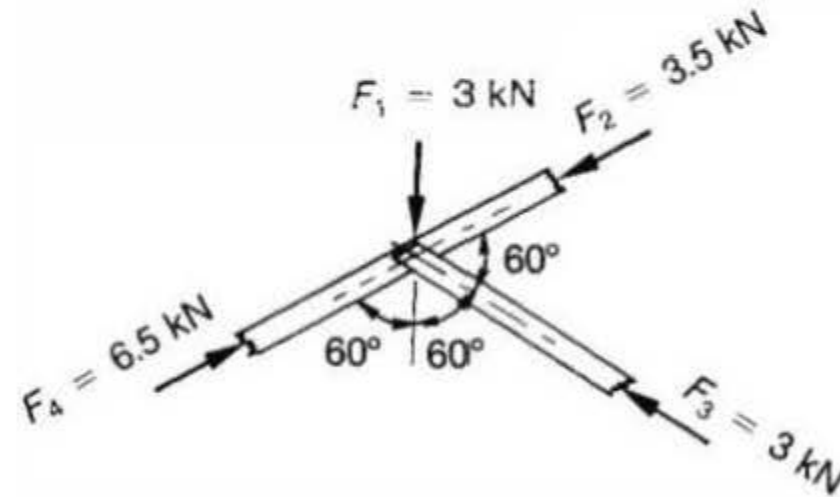
**UNSURE**

**NO IDEA**

A joint of a pin-jointed truss with internal and external forces is shown in the following figure.

Our task is to prove that the joint is in equilibrium mathematically.

In this task, the forces are only expressed in two dimensions, horizontal and vertical directions (written as  $x$  and  $y$ ).

[GIVE FEEDBACK](#)[CONTINUE >](#)

In order to prove the equilibrium of this system of concurrent forces mathematically, we should:

1. demonstrate that the sum of the horizontal, or  $x$ -direction, components add up to zero ( $\Sigma F_x = 0$  must be satisfied)
2. demonstrate that the sum of the vertical, or  $y$ -direction, components add up to zero ( $\Sigma F_y = 0$  must be satisfied)
3. use a table to record all the force components including their positive and negative signs.

Note:

The sum of vertical and horizontal components must be zero.

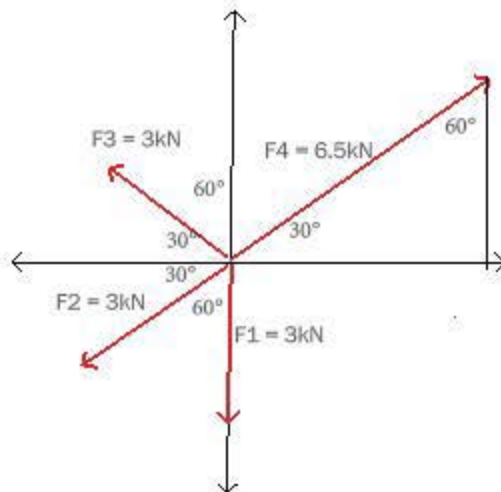
The relationships between a force  $F$  and its rectangular components in the mutually perpendicular  $x$  and  $y$  directions,  $F_x$  and  $F_y$ , are

$$F_x = F \cos \theta \text{ and } F_y = F \sin \theta$$

where  $\theta$  is the angle between the force and the  $x$ -direction

The completed force table is shown below.

Force	Magnitude	x-component	y-component
$F_1$	3.0	0	-3
$F_2$	3.5	-3.03	-1.75
$F_3$	3.0	-2.6	1.5
$F_4$	6.5	5.63	3.25
		$\Sigma F_x = 0$	$\Sigma F_y = 0$



The sum of the horizontal components and the sum of the vertical components are zero.

The conclusion is that the system of forces is in equilibrium, as expected in a structure such as a truss.

Which equilibrant force can mathematically prove the system is in equilibrium?

Click the correct answer.

Force	$F_c$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	445	-252

Force	$F_c$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	445	252

Force	$F_c$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	-445	252

Force	$F_c$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	-445	-252

Do you know the answer?

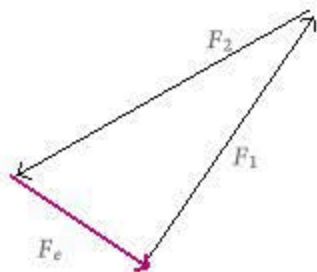
I KNOW IT

THINK SO

UNSURE

NO IDEA

Force	Magnitude	X-component	Y-component
$F_1$	828.6	212	801
$F_2$	856.2	-657	-549
Equilibrant	$F_e$	$F_x$	$F_y$



SMALL

MEDIUM

LARGE

Which equilibrant force can mathematically prove the system is in equilibrium?

Click the correct answer.

Force	$F_e$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	445	-252

Force	$F_e$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	445	252

Force	$F_e$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	-445	252

Force	$F_e$ Mag	$F_x$ X-com	$F_y$ Y-com
Equilibrant	511.4	-445	-252

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Which of the following statements are correct on proving forces in equilibrium mathematically?

---

Check **all** that apply.

- ☐ For a system in equilibrium, the resultant sum of all applied forces is equal to zero
- ☐ For a system in equilibrium, the resultant sum of forces in any direction must equal zero
- ☐ The applied forces must be shown as positive values
- ☐ The magnitude of the forces can be positive or negative

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA





In the event of an already balanced system of forces, the equilibrant force would be equal to zero.



#### Define the equilibrant force

If a system of concurrent forces is not balanced, i.e. not in equilibrium, it is possible to determine the additional force required to produce equilibrium.

Such a force is called the **equilibrant**.

The equilibrant of a system of concurrent forces is that force which, when added to the system, produces equilibrium.

GIVE FEEDBACK

OK

**Type your answer in the box.**

If a system of concurrent forces is not balanced, i.e. not in equilibrium, it is possible to determine the additional force required to produce equilibrium. Such a force is called the .

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

In the event of a balanced system of concurrent forces, the equilibrant force would be equal to:

---

**Click the correct answer.**

Zero

Unit vector force

Final force of the force polygon

Any other external force

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

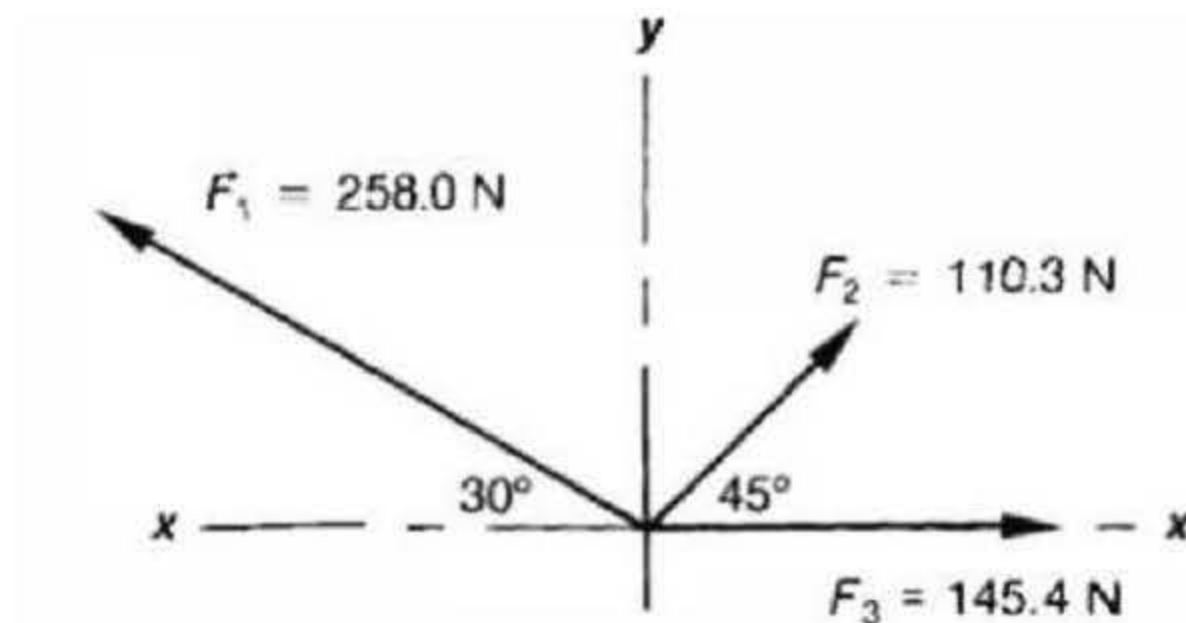
**NO IDEA**



The closed polygon of forces has all its forces, including the equilibrant force, follow the head-to-tail order.



Our task is to determine the equilibrant force ( $F_e$ ) for the system of forces shown in the following figure.



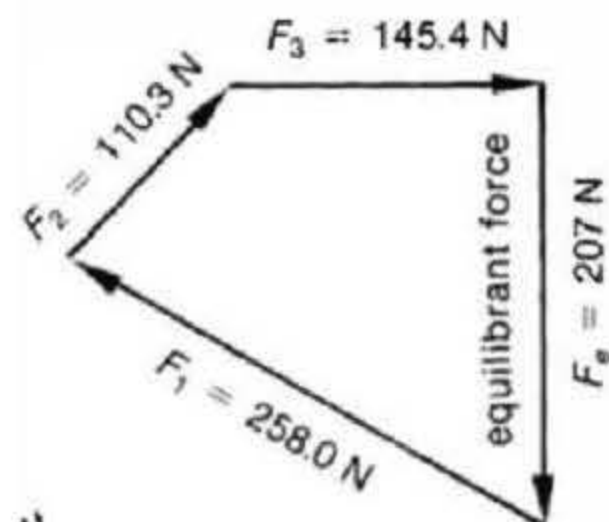


The force polygon enables us to determine the correct direction of the equilibrant force.



To solve this problem graphically, we must recall that for a system of forces in equilibrium, the force polygon must close. If we attempt to construct the force polygon using only the given forces  $F_1$ ,  $F_2$  and  $F_3$  and arranging them in head-to-tail order, we find that the first and last points do not coincide.

To close the gap an additional line is required, as shown in the following figure. This line will close the polygon and represent the required equilibrant force both in magnitude and direction.

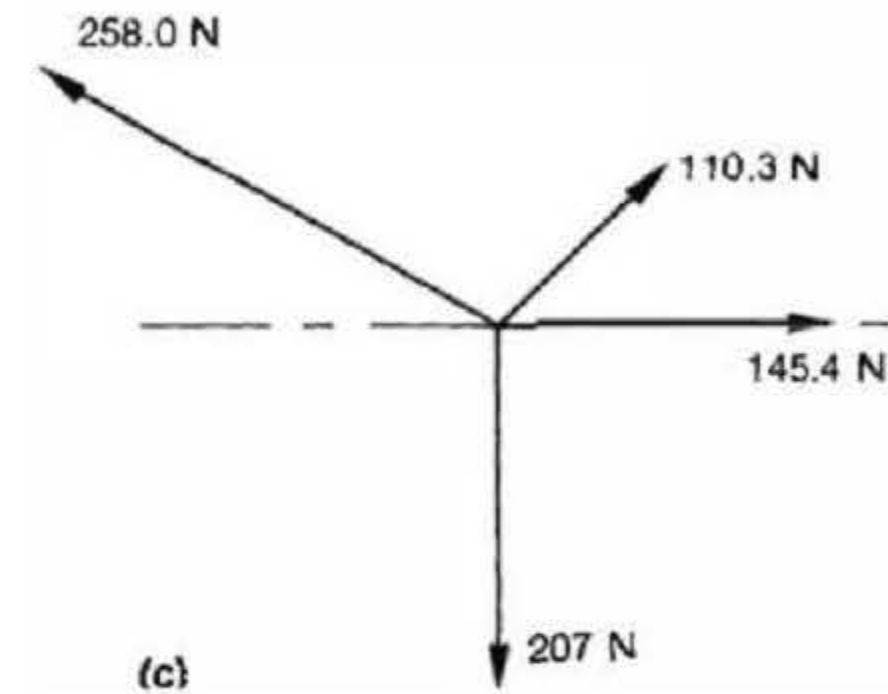




The equilibrant and the resultant forces are always equal in magnitude and opposite in direction.



The system of forces in equilibrium is shown below.



The closed polygon of forces has all its forces, including the equilibrant force, follow the head-to-tail order.

In this example the equilibrant is a vertical downward force.

Which of the following statements are correct in determining the equilibrant force?

---

Check **all** that apply.

- ☐ In an already balanced system of forces, the equilibrant forces would be equal to zero
- ☐ Equilibrant force can only be solved graphically
- ☐ Equilibrant force is the final force that closes the force polygon in a graphical solution
- ☐ Mathematically the equilibrant force and the existing resultant forces are always equal in magnitude and opposite in direction

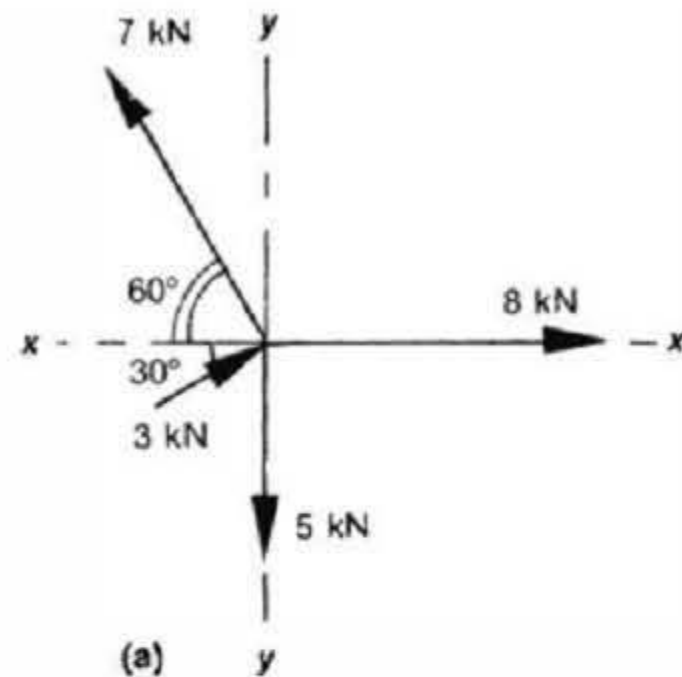
Do you know the answer?

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



SMALL

MEDIUM

LARGE

Which of the following is the correct equilibrant force for this diagram?

Click the correct answer.



Do you know the answer?

I KNOW IT

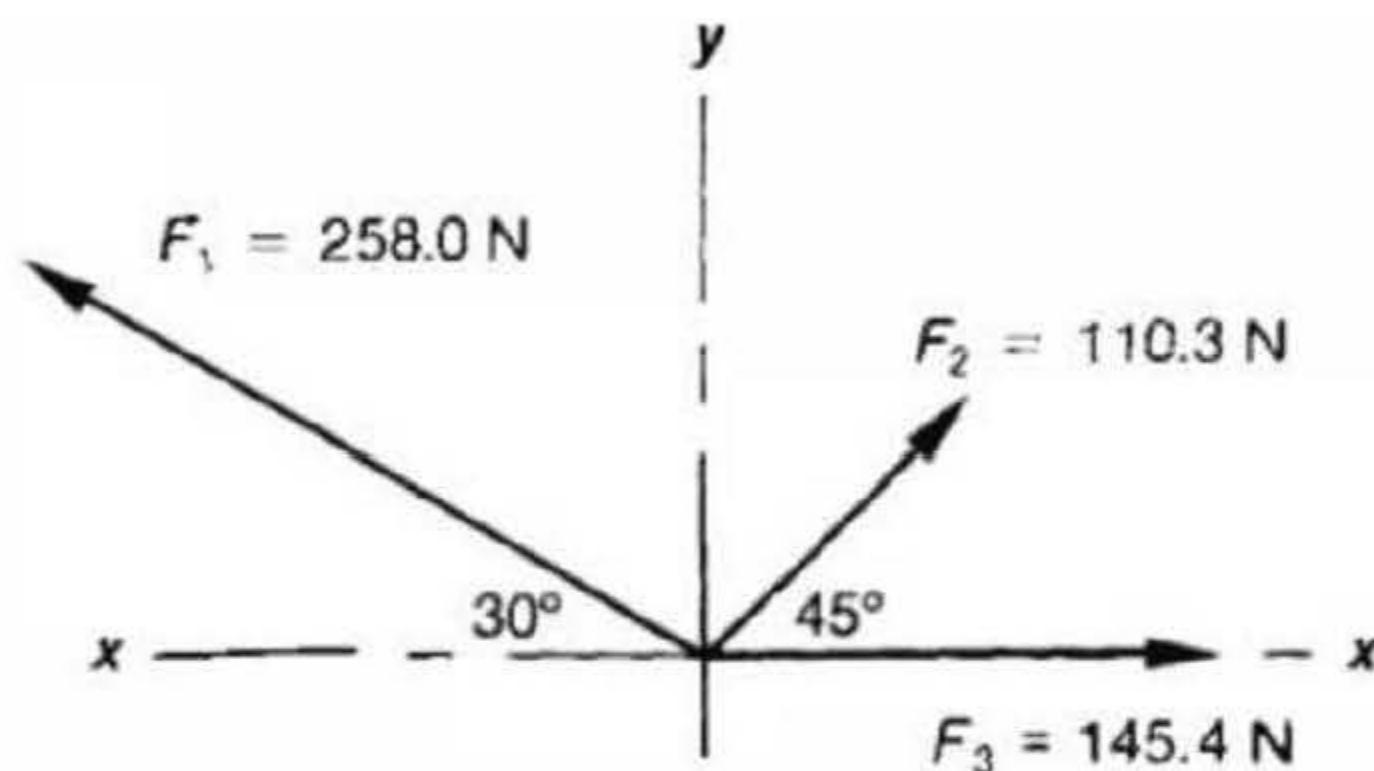
THINK SO

UNSURE

NO IDEA



Our task is to determine the equilibrant force ( $F_e$ ) for the system of forces shown in the following figure.



The equilibrant force can be found mathematically by the addition of rectangular components including those for the unknown equilibrant force.

The forces  $F_1$ ,  $F_2$  and  $F_3$  can be resolved into x and y components (usually horizontal and vertical) using  $F_x = F \cos \theta$  and  $F_y = F \sin \theta$ , where  $\theta$  is the angle between 0 to 360 degrees measured counter-clockwise between each force and positive x direction. From the previous diagram, we can see that:

$F_1$	$F_x = F \cos \theta$	$F_y = F \sin \theta$
	$F_x = 258 \text{ N} \cdot \cos 150$	$F_y = 258 \text{ N} \cdot \sin 150$
	$F_x = -223.4 \text{ N}$	$F_y = 129 \text{ N}$
$F_2$	$F_x = F \cos \theta$	$F_y = F \sin \theta$
	$F_x = 110.3 \text{ N} \cdot \cos 45$	$F_y = 110.3 \text{ N} \cdot \sin 45$
	$F_x = 78 \text{ N}$	$F_y = 78 \text{ N}$
$F_3$	$F_x = F \cos \theta$	$F_y = F \sin \theta$
	$F_x = 145.4 \text{ N} \cos 0$	$F_y = 145.4 \text{ N} \cdot \sin 0$
	$F_x = 145.4 \text{ N}$	$F_y = 0 \text{ N}$



There are three unknowns to solve for: the x-direction force, the y-direction force and the magnitude of overall force.



The solution is best set out in tabular form as in the following table.

Force	Magnitude	x-component	y-component
$F_1$	258.0	-223.4	129.0
$F_2$	110.3	78.0	78.0
$F_3$	145.4	145.4	0
equilibrant	$F_e$	$F_x$	$F_y$

&lt; BACK

GIVE FEEDBACK

CONTINUE &gt;

For equilibrium:

$$\begin{aligned}\Sigma F_x &= -223.4 + 78 + 145.4 + F_x = 0 \\ \therefore F_x &= 0\end{aligned}$$

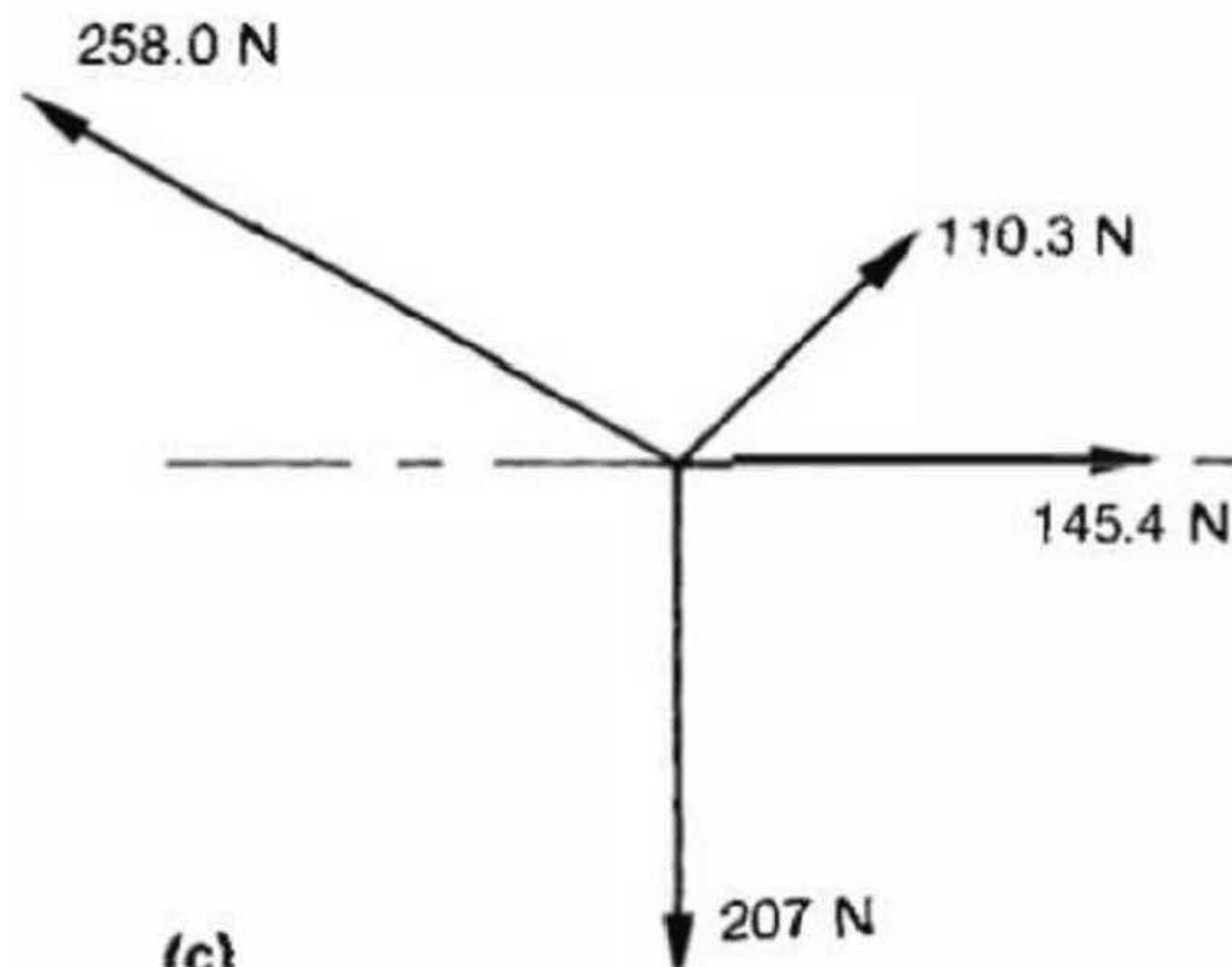
Also:

$$\begin{aligned}\Sigma F_y &= 129 + 78 + 0 + F_y = 0 \\ \therefore F_y &= -207 \text{ N}\end{aligned}$$

The magnitude of the equilibrant force is therefore:

$$\begin{aligned}F_e &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{0 + 207^2} \\ &= 207 \text{ N}\end{aligned}$$

Consideration of the directions of components  $F_x$  and  $F_y$  indicates that, in this case, the equilibrant is acting vertically downwards.



**Type your answer in the box.**

The angle the equilibrant force makes with the positive x axis is  degrees (rounded to 2 decimal places).

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

Type your answer in the box.

Fill in the correct values for unknown components of the given forces.

Force	Magnitude (kN)	x-component (kN)	y-component (kN)
$F_1$	5	<input type="text"/>	-4.70
$F_2$	10	-7.07	<input type="text"/>
$F_3$	8	<input type="text"/>	6.13
$F_4$	6	6	0

Do you know the answer?

I KNOW IT

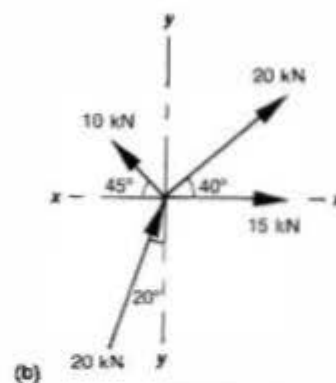
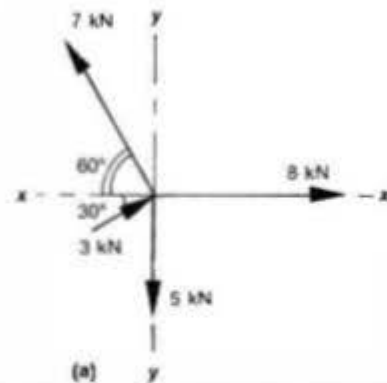
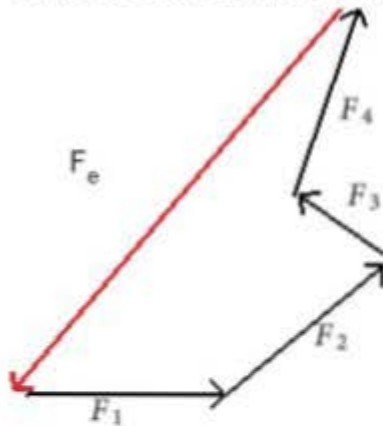
THINK SO

UNSURE

NO IDEA

Type your answer in the box.

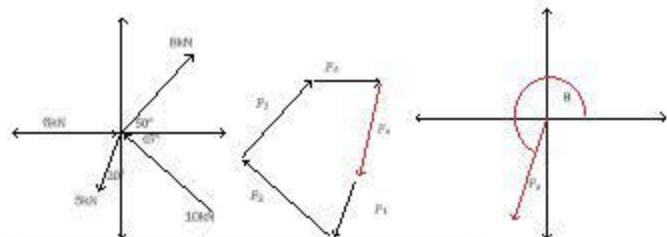
Fill in the correct value rounded to 2 decimal places.



Force	Magnitude	x-component	y-component
$F_1$	15	15	0
$F_2$	20	15.32	12.86
$F_3$	10	-7.07	7.07
$F_4$	20	6.84	18.79
$F_e$	<input type="text"/>	<input type="text"/>	<input type="text"/>







Force	Magnitude (kN)	x-component (kN)	y-component (kN)
$F_1$	5	—	-4.70
$F_2$	10	-7.07	—
$F_3$	8	—	6.13
$F_4$	6	6	0
$F_5$	—	—	—



SMALL

MEDIUM

LARGE

Type your answer in the box.

Fill in the correct values for unknown components of the given forces.

Force	Magnitude (kN)	x-component (kN)	y-component (kN)
$F_1$	5	<input type="text"/>	-4.70
$F_2$	10	-7.07	<input type="text"/>
$F_3$	8	<input type="text"/>	6.13
$F_4$	6	6	0

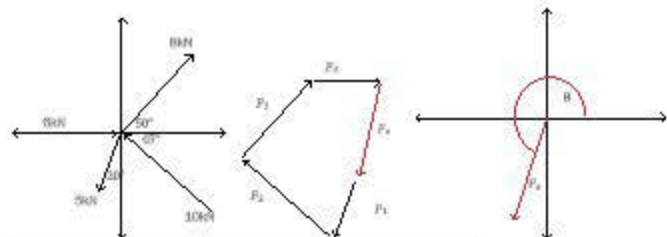
Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



Force	Magnitude (kN)	x-component (kN)	y-component (kN)
$F_1$	5	_____	-4.70
$F_2$	10	-7.07	_____
$F_3$	8	_____	6.33
$F_4$	6	6	0
$F_5$	_____	_____	_____



SMALL

MEDIUM

LARGE

Type your answer in the box.

The angle the equilibrant force makes with the positive x axis is  degrees (rounded to 2 decimal places).

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

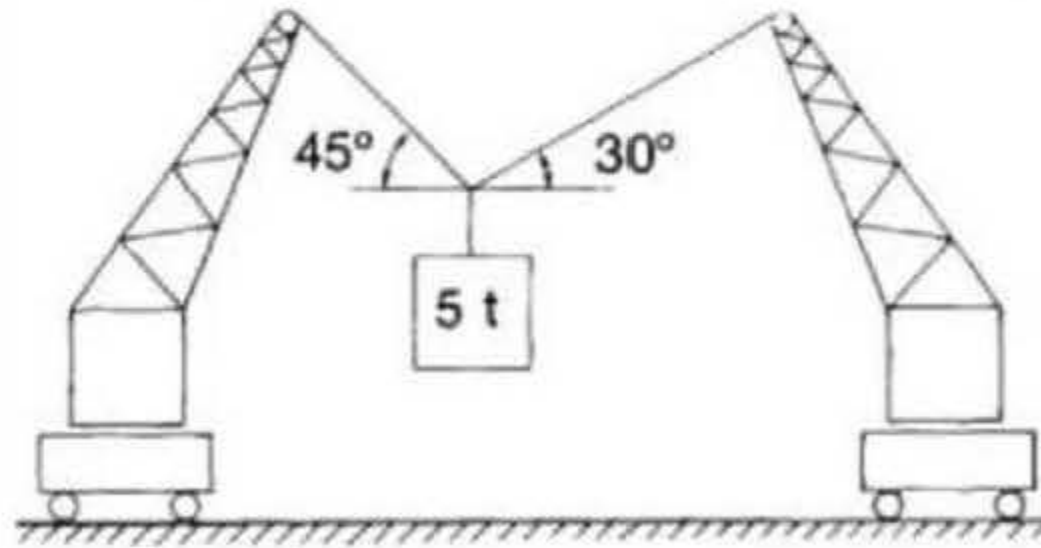
NO IDEA

### Define and analyse a space diagram

When problems involving the equilibrium of bodies under the action of force systems are being solved, a method of setting out the essential details in the form of a diagram is necessary.

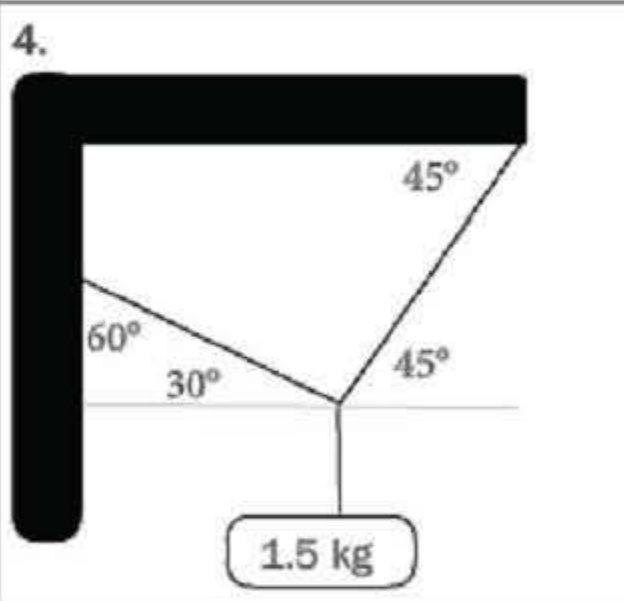
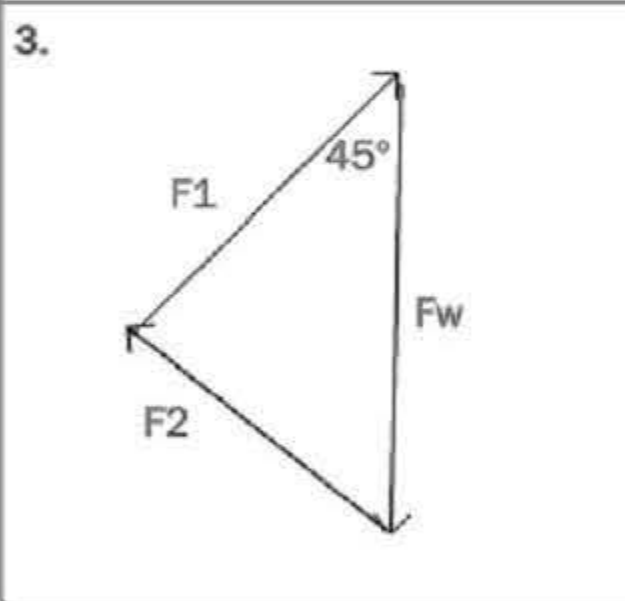
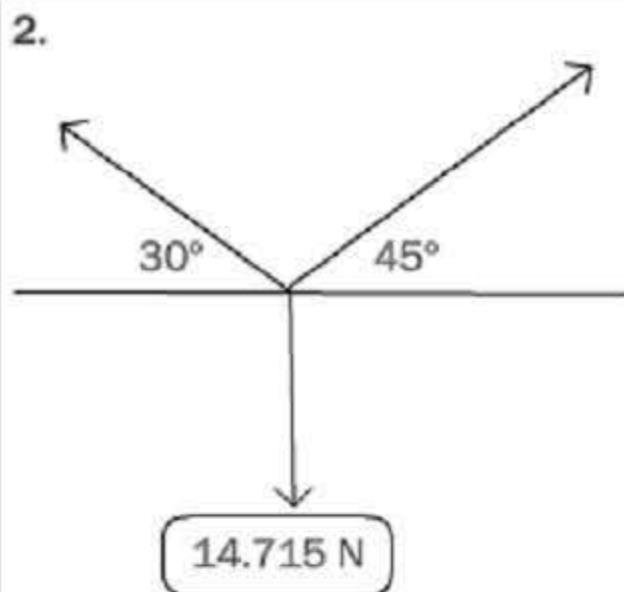
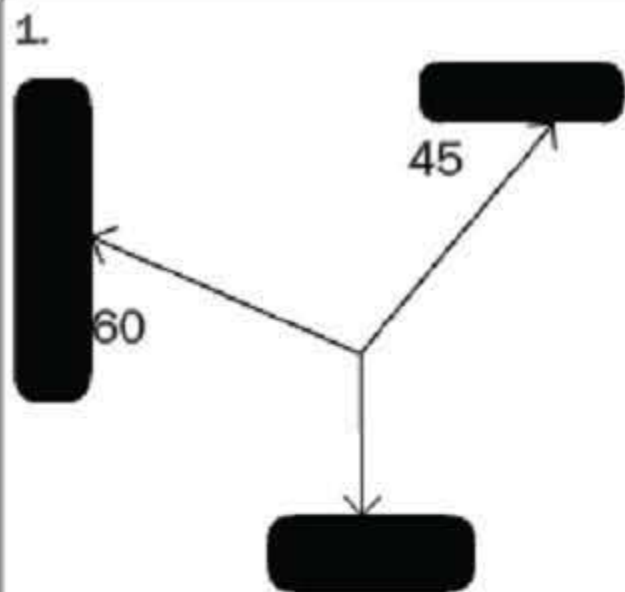
We often start with a semi-pictorial sketch or **space diagram** showing the physical conditions of the problem, i.e. a layout of the mechanical or structural components such as pulleys, supports, cables, rollers, etc.

This is an example of a space diagram showing two cranes supporting a 5 tonne mass.



GIVE FEEDBACK

OK

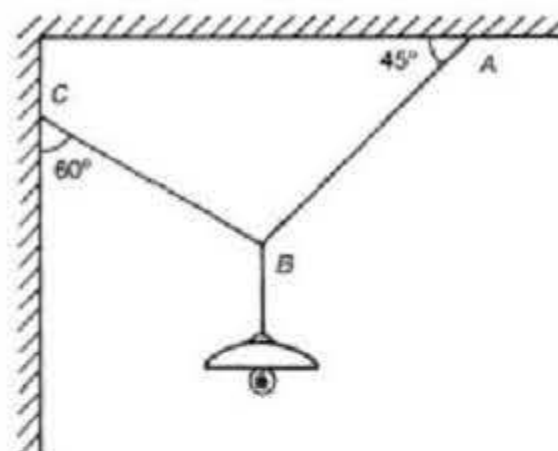


SMALL

MEDIUM

LARGE

A light fitting of mass 1.5 kg is hanging from the ceiling on wire AB and is tied to the wall by string BC. Which of the following is the correct space diagram for this structure?



Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

A space diagram is a diagram showing the physical conditions of the structure, i.e. a layout of the mechanical or structural components such as pulleys, supports, cables, rollers, etc.

---

**Click the correct answer.**

True

False

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

What is a space diagram?

---

Check **all** that apply.

- ☐ A semi-pictorial sketch showing the physical conditions of the force system
- ☐ A layout of the mechanical or structural components of the force system
- ☐ A sketch of the forces rearranged in head-to-tail form
- ☐ A sketch of the components and forces in a force system

Do you know the answer?

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

What is a space diagram?

---

**Click the correct answer.**

A diagram that displays the physical conditions of the problem and layout of the mechanical or structural components

A text-based diagram describing in detail the structural components such as pulleys, supports, cables, rollers, etc.

A three-dimensional diagram showing all the structural components in relation to the location

A force diagram where each component is represented as a vector

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

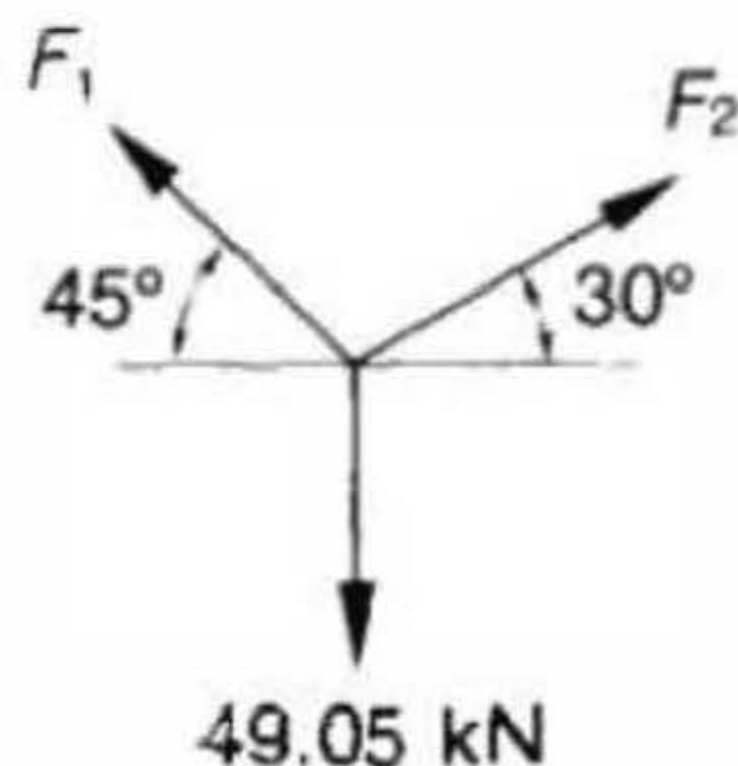
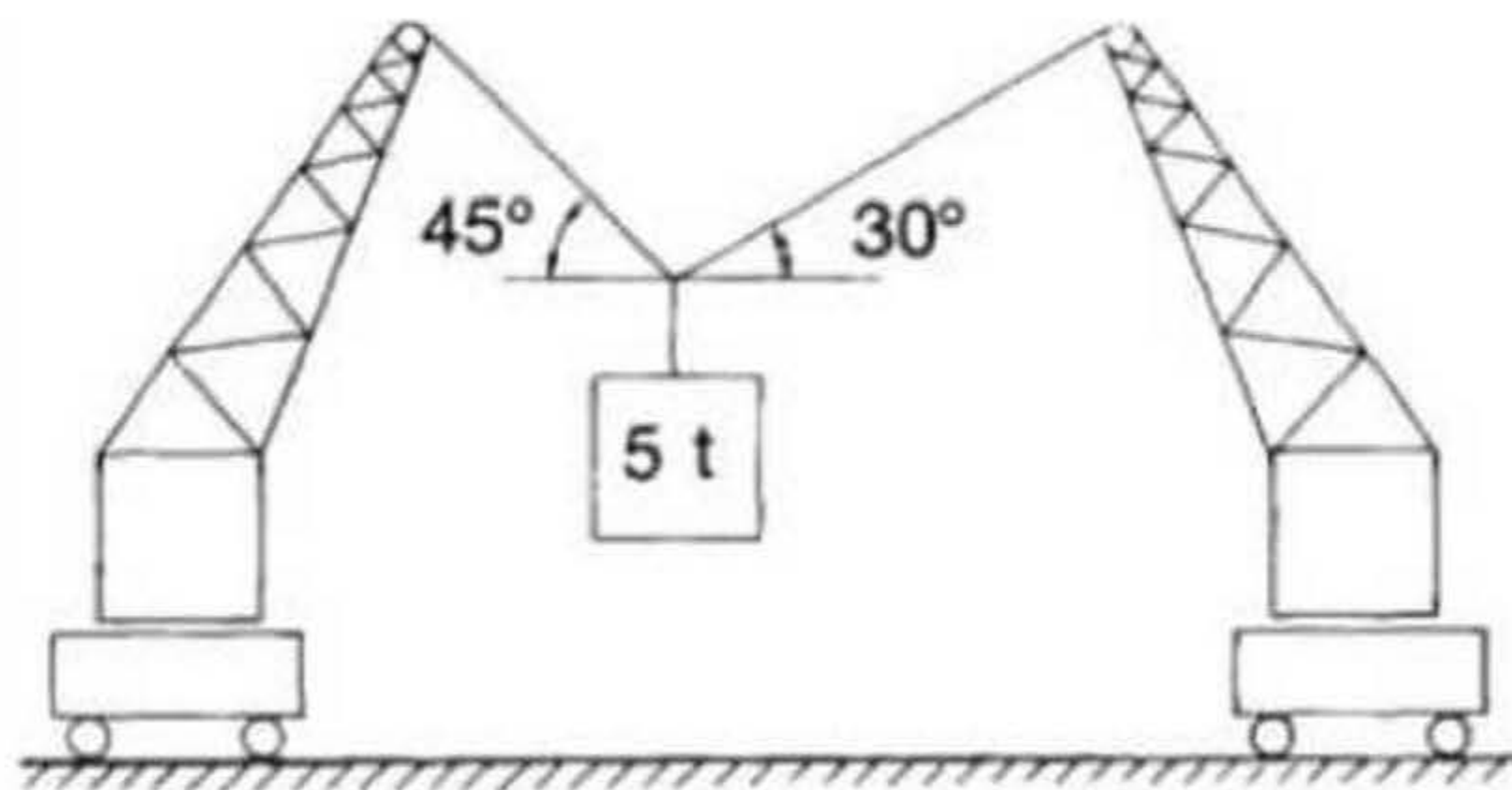


## Define free body diagram

Once you have the space diagram drawn, the next step is to isolate the essential facts about the forces involved and to draw a **free body diagram**.

This usually shows the point of concurrency acted on by all the forces, indicating the magnitudes and directions of the forces.

This is an example of a free body diagram produced using the space diagram.





What is a free body diagram?

---

**Click the correct answer.**

A diagram drawn to represent all forces (represented as vectors) acting on the point of concurrency

A force polygon based on the information obtained in the space diagram

A three-dimensional graph showing all the forces represented as vectors

A computer-generated diagram showing all forces to scale

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

A free body diagram usually shows the point of concurrency acted on by all the forces, ignoring the directions of the forces.

---

**Click the correct answer.**

False

True

**Do you know the answer?**

**I KNOW IT**

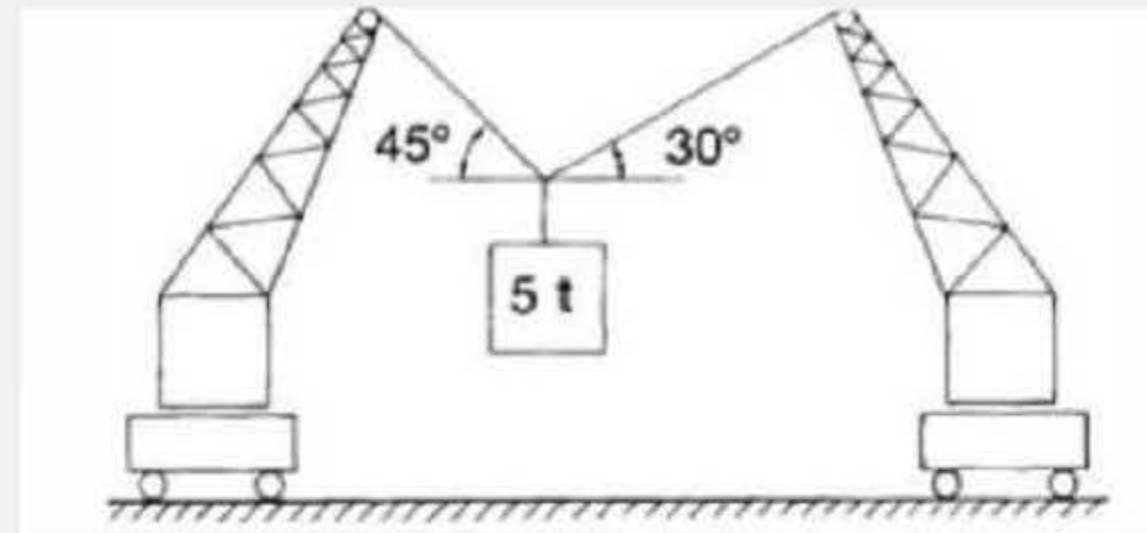
**THINK SO**

**UNSURE**

**NO IDEA**

## Produce a free body diagram after analysing a space diagram

Our task is to produce a free body diagram after analysing the following space diagram.



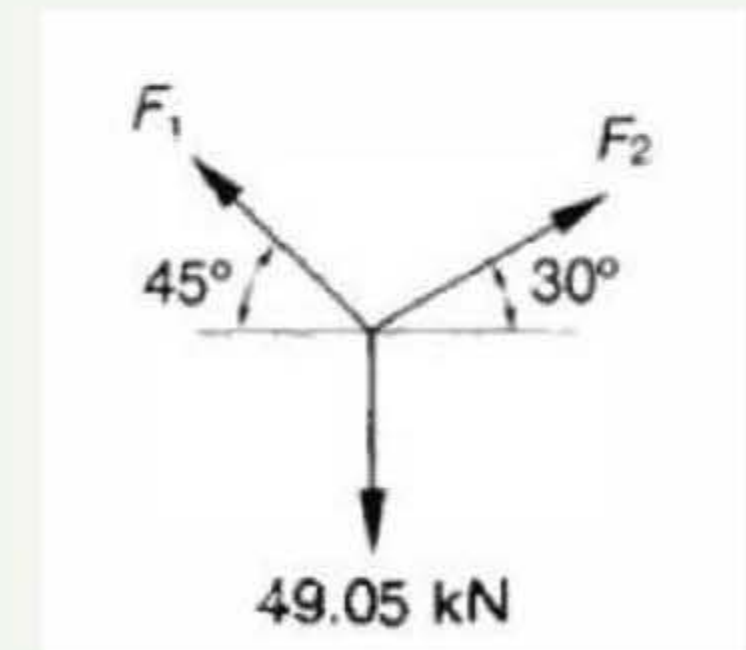
The weight of the load is:

$$\begin{aligned}F_w &= m g \\&= 5,000 \text{ kg} \cdot 9.81 \text{ N/kg} \\&= 49,050 \text{ N} \\&= 49.05 \text{ kN}\end{aligned}$$

You must make sure that the force arrow is drawn in the correct direction in the free body diagram.

The free body diagram is drawn to represent all forces acting on the point of concurrency.

Note that each force is represented as a vector, showing its magnitude and direction in relation to the point.



1.

2.

3.

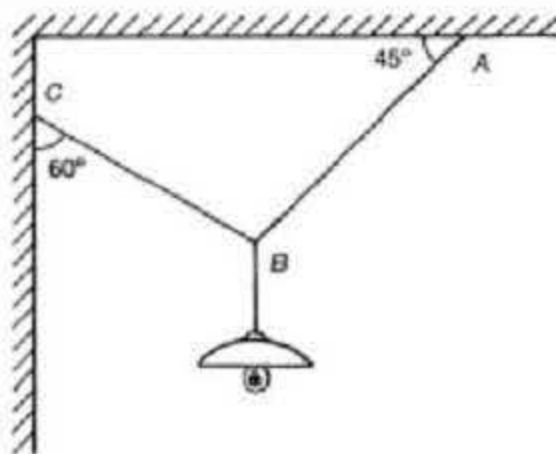
4.

SMALL

MEDIUM

LARGE

A light fitting of mass 1.5 kg is hanging from the ceiling on wire AB and is tied to the wall by string BC. Which of the following is the correct free body diagram for this structure?



Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

1.

2.

3.

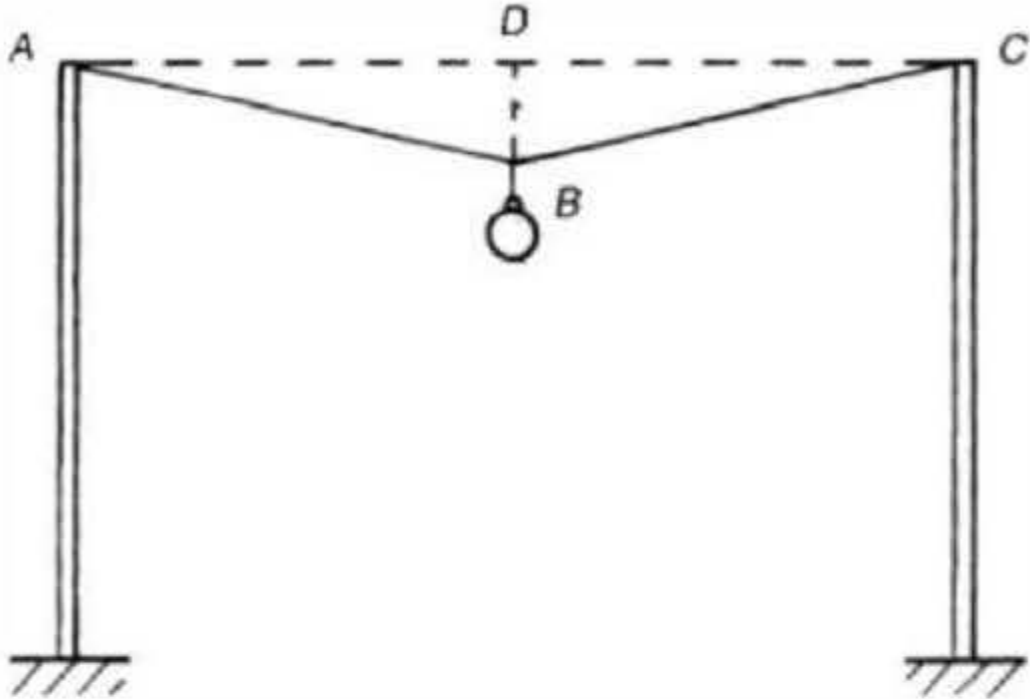
4.

SMALL

MEDIUM

LARGE

A streetlight of mass 15 kg is supported at midpoint between two poles by a cable ABC. If the length of the cable ABC is 20 m and distance BD at midpoint is 2 m, which of the following is the correct space diagram for this structure?



Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



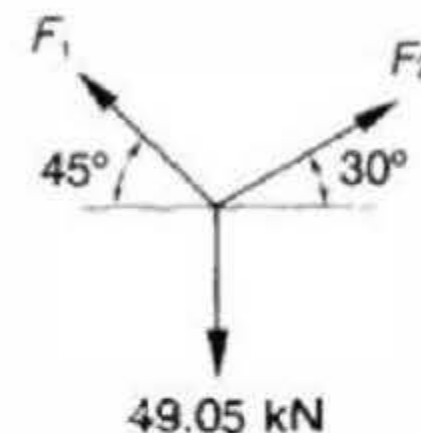
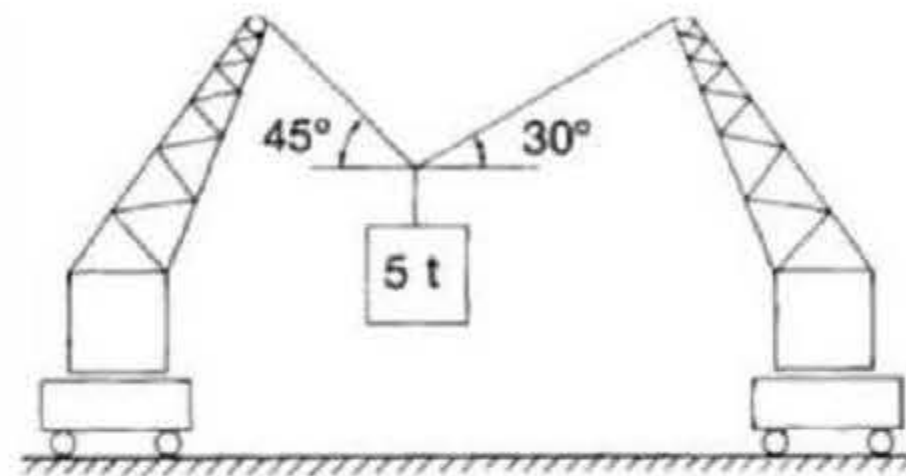


The diagram must be drawn in head-to-tail order with the force vector.

Note that a triangle is a special case of force polygon where the number of forces involved is three.



Construct a force triangle based on the information contained in the free body diagram <sup>1</sup>/<sub>4</sub>



Our task is to consider the equilibrium of forces from the free body diagram and hence determine the force in each cable.

Knowing one force (i.e. the weight of the 5 tonne load) and the direction of the other two forces enables us to construct the force triangle.

GIVE FEEDBACK

CONTINUE >



If the triangle of forces is drawn to scale, the answers can easily be scaled off.



## Construct a force triangle based on the information contained in the free body diagram<sup>2/4</sup>

The sine rule can be used to calculate the unknown forces as side lengths of the force triangle.

In order to achieve this, all three internal angles of the force triangle must be calculated first.

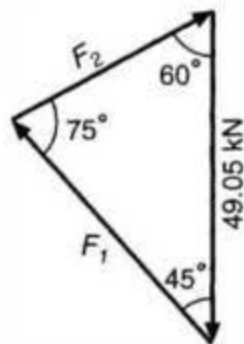
Internal angles of the force triangle are related to the angles shown on the free body diagram but are not directly equal to them.

Examine the following slide carefully to see how the internal angles are obtained.

[< BACK](#)[GIVE FEEDBACK](#)[CONTINUE >](#)

Construct a force triangle based on the information contained in the free body diagram

3/4



Using the sine rule,

$$\frac{49.05}{\sin 75^\circ} = \frac{F_1}{\sin 60^\circ} = \frac{F_2}{\sin 45^\circ}$$

< BACK

GIVE FEEDBACK

CONTINUE >





Notice the forces on the cable are different due to the angle of the cable. Instead of using the sine rule, you can use the x-y horizontal and vertical components to solve the forces.

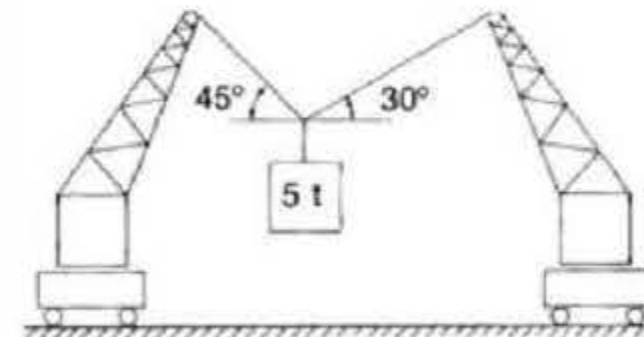


Construct a force triangle based on the information contained in the free body diagram 4/4

Then:

$$F_1 = 49.05 \cdot \frac{\sin 60^\circ}{\sin 75^\circ}$$
$$= 43.98 \text{ kN}$$

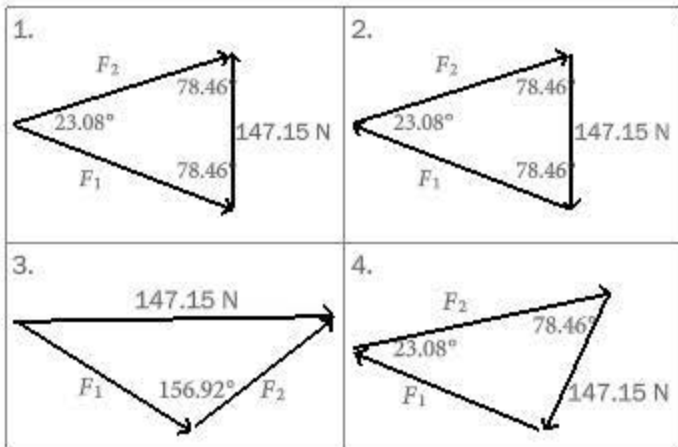
$$F_2 = 49.05 \cdot \frac{\sin 45^\circ}{\sin 75^\circ}$$
$$= 35.91 \text{ kN}$$



< BACK

GIVE FEEDBACK

OK

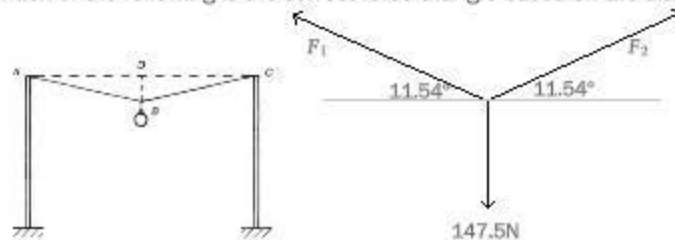


SMALL

MEDIUM

LARGE

Which of the following is the correct force triangle based on the diagrams given?



Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

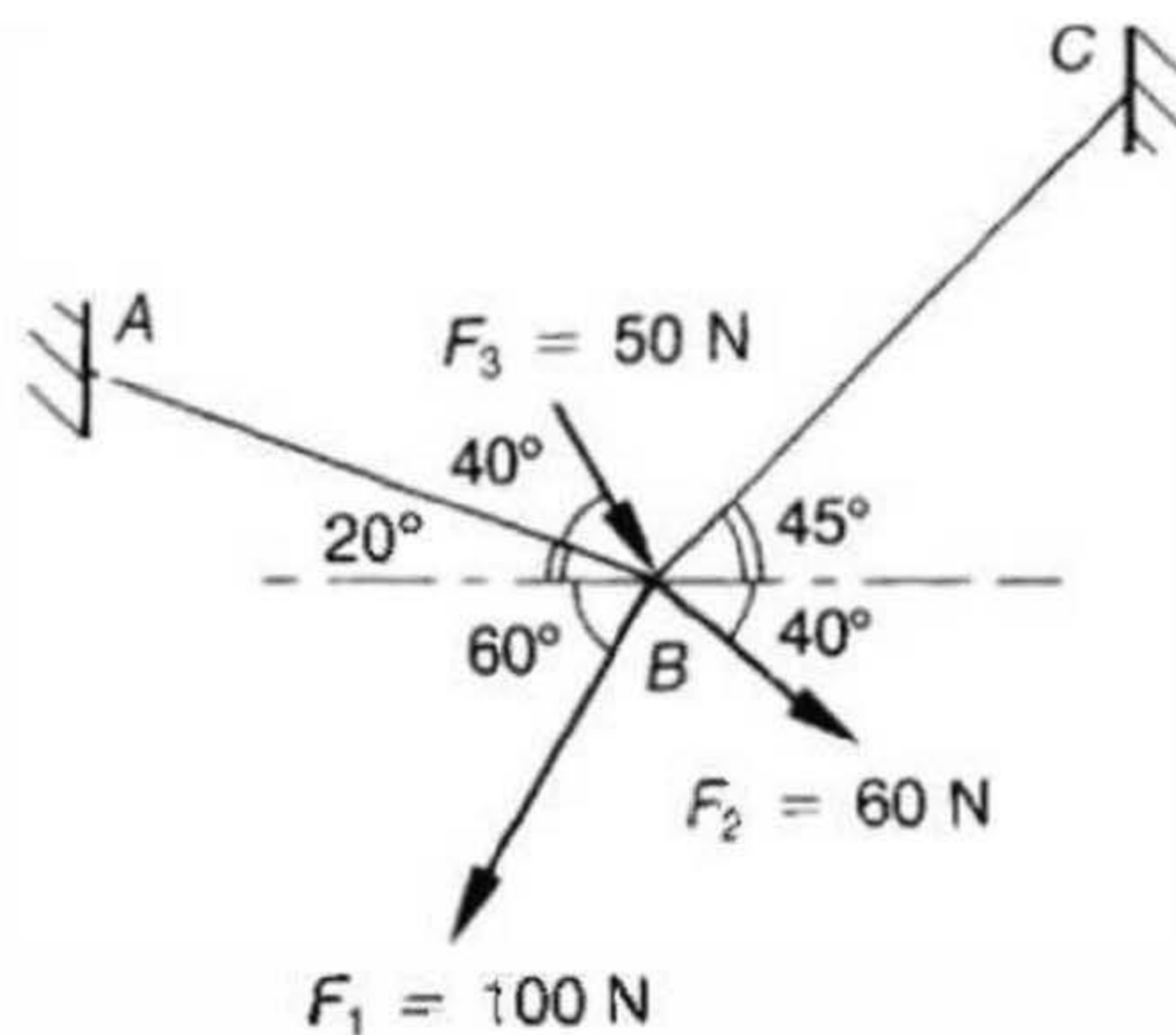
THINK SO

UNSURE

NO IDEA

An elastic member  $ABC$  is stretched as shown in the following figures by three forces:  $F_1$ ,  $F_2$  and  $F_3$ .

Our task is to determine the forces in  $AB$  and  $BC$ .





An alternative mathematical solution is also possible but is complex and involves two simultaneous equations involving components of unknown forces. It is not recommended.



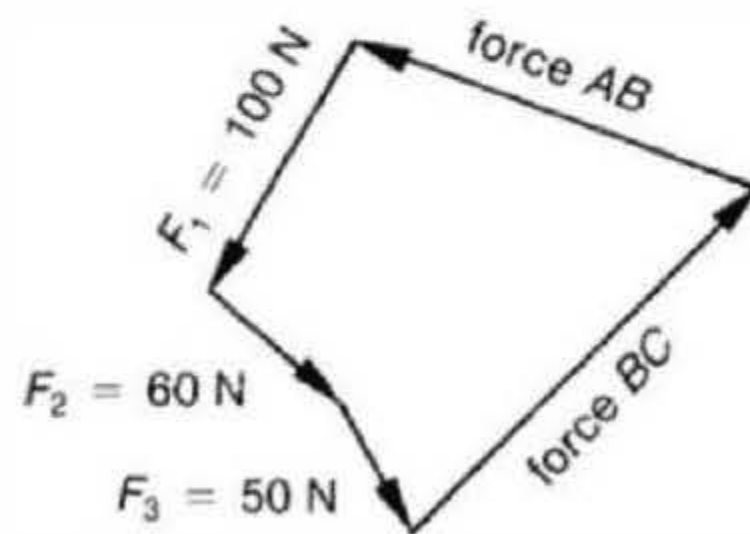
## Construct a force polygon based on the information contained in the free body diagram 2/2

In this example more than three forces are involved. Therefore, a polygon of forces, rather than a triangle, must be constructed.

The procedure is to construct as much of the polygon as possible using all known forces and then to complete it by drawing two lines, parallel to the unknown forces, through the first and last points. The answers are scaled off the force polygon and are:

$$F_{AB} = 148 \text{ N}$$

$$F_{BC} = 167 \text{ N}$$



< BACK

GIVE FEEDBACK

OK

1.

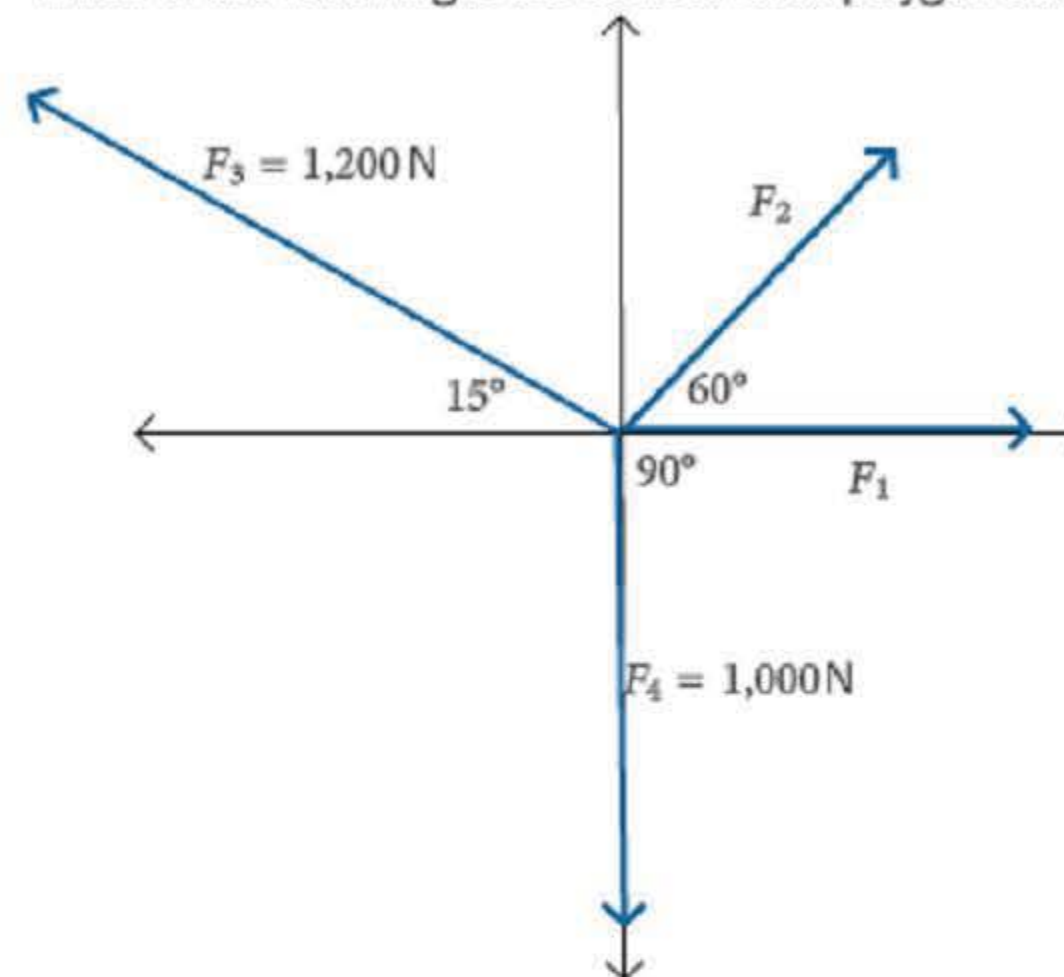
2.

3.

4.



Which of the following is the correct force polygon based on the given free body diagram?



Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA





The normal force to a line or surface is a direction perpendicular to it, e.g. normal force is perpendicular to the supporting surface.



### Classify the three categories of contact surfaces or supports

1/3

Different types of contact surfaces or supports can be discussed in relation to the kind of reaction force that may exist between a body, such as a beam or a truss, and its supporting surface.

There are two categories of supports: those at which the reaction force is always normal to the supporting surface and those which can support a force at any angle to the supporting surface.

The first category includes a smooth, frictionless surface contact between a body and a supporting surface such that the body can slide along the surface at the point of contact without any resistance.

A support on rollers, e.g. as seen under some long bridge sections, has a similar force action, i.e. it provides normal reaction only.

At a smooth surface or roller support, the reacting force is always perpendicular to the supporting surface.

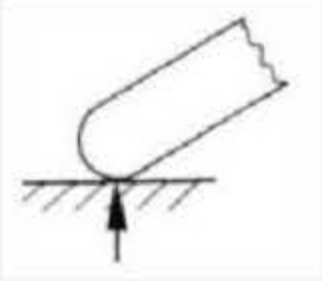

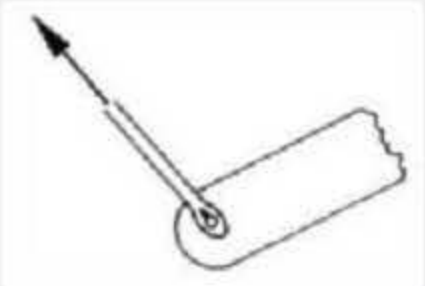

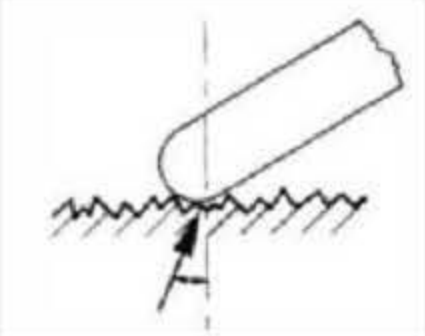

GIVE FEEDBACK

CONTINUE >

The second category includes rough surface contact between a body and a supporting surface in which friction prevents relative sliding motion, producing a reaction force in any direction.

A fixed hinge or bearing has a similar force action capable of supporting a force at any angle to the supporting surface.

In addition to these reaction forces, it is helpful to recognise that forces in links pivoted at each end and in cables connecting two points of a structure always act along the axes of such members.

<p><b>1 Normal reaction only</b> Smooth contact surface</p> 	<p>Ball or roller support</p> 
<p><b>2 Force along axis of member</b> Cable</p> 	<p>Link</p> 
<p><b>3 Reaction at any angle</b> Rough contact surface</p> 	<p>Pin or hinge</p> 



Which of the following statements about a rough surface are true?

---

Check **all** that apply.

- ☐ Friction will prevent a body from relative sliding motion
- ☐ A rough surface can provide a reaction force in only one direction
- ☐ A fixed hinge can be considered as a rough contact surface
- ☐ Hardwood is considered a rough surface

Do you know the answer?

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

Match the following support reactions to their correct definitions.



Drag statements on the right to match the left.

Normal reaction



Reaction force always normal or perpendicular to the support surface



Reaction to any angle



Reaction forces supported at any angle to the support surface



Force along axis of member



Linear reaction force acting in the direction of the member of the structure



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Match the contact surface with the correct type of support reaction.



Drag statements on the right to match the left.

Smooth contact surface



Normal reaction



Ball or roller support



Normal reaction



Cable



Force along the axis



Link



Force along the axis



Rough contact surface



Reaction at any angle



Pin or hinge



Reaction at any angle



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Which of the following statements about a smooth surface are true?

---

Check **all** that apply.

- ☐ Concrete floor is a smooth surface
- ☐ The surface offers no resistance at the point of contact
- ☐ Only ice is a smooth surface
- ☐ The smooth surface will provide normal reaction only

Do you know the answer?

**I KNOW IT**

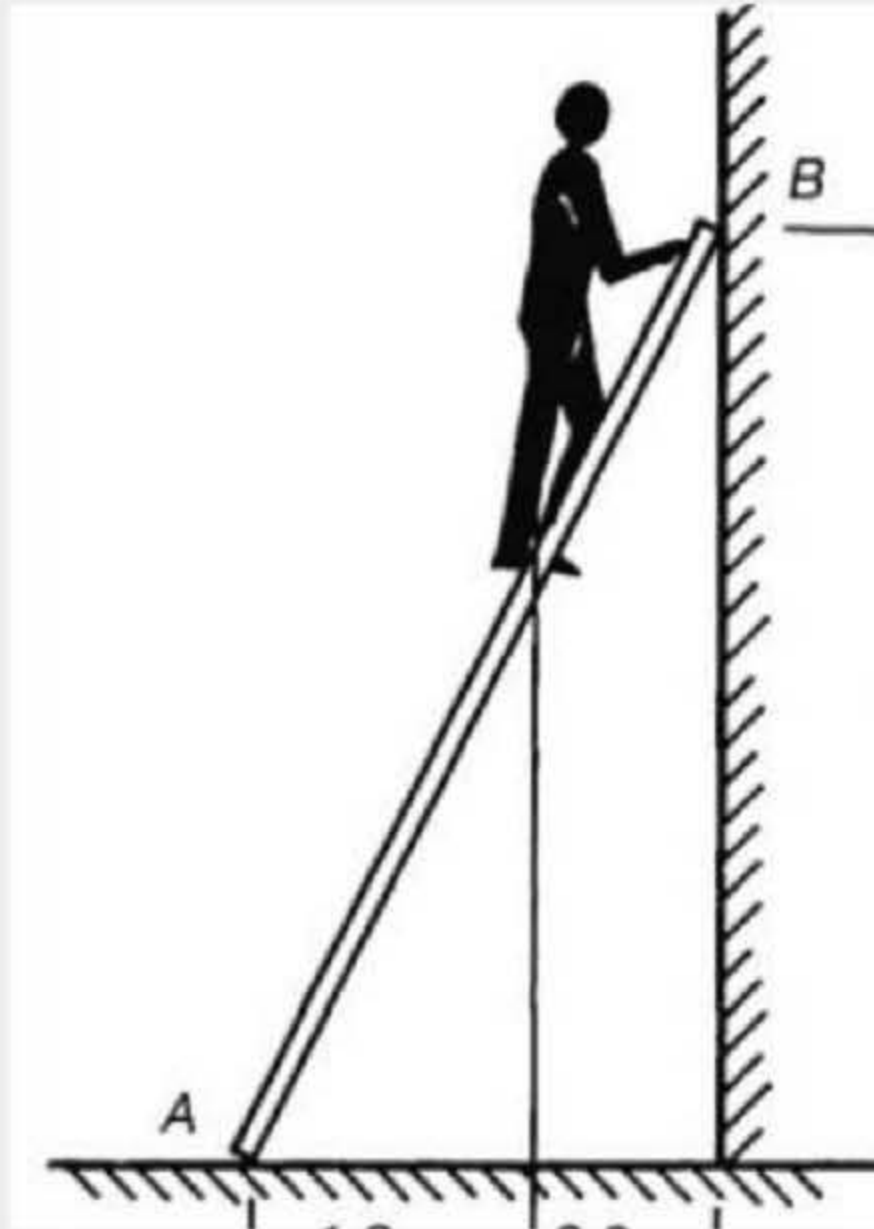
**THINK SO**

**UNSURE**

**NO IDEA**

## Define a three-force body

An interesting case of equilibrium is that of a body in equilibrium under the action of three non-parallel forces. Such a body is usually called a **three-force body**.



In the diagram shown, there are three forces acting on the ladder:

1. The man's mass
2. The reaction force on the floor surface
3. The reaction force against the wall

Given the ladder is at rest and the forces are not parallel, it can be said that this is a three-force body in equilibrium.

**Type your answer in the box.**

A body in equilibrium under the action of three   forces is called a three-force body.

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

**Type your answer in the box.**

A body in equilibrium under the action of three non-parallel forces is called a

body.

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**



The only exception to the three-force body principle is when the forces are parallel.



### The three-force body principle

It can be shown that if a three-force body is in equilibrium, the lines of action of the three forces must intersect at a common point.

This principle, known as the **three-force principle**, is useful in solving many engineering problems as it helps to determine easily the direction of an unknown force without recourse to more complex mathematical methods.



GIVE FEEDBACK

OK



**Type your answer in the box.**

The only exception to three-force body principle is when the forces are .

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

**Type your answer in the box.**

If a three-force body is in equilibrium, the lines of action of the three forces must intersect at a  point.

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

**UNSURE**

**NO IDEA**

## Solving problems using the three-force principle

When solving problems using the three-force principle, always follow this sequence of steps:

### Step 1

Draw a free body diagram of the structure or component in essential outline only.  
All dimensions and angles must be drawn to scale.

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

## Solving problems using the three-force principle

### Step 2

Draw the line of action of the known force and identify its magnitude and direction.  
(Force representing weight should always be drawn vertically downwards. Any other force must be drawn as given in the problem.)

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

## Solving problems using the three-force principle

### Step 3

Consider the nature of supports and draw the line of action of the second force either normal to the smooth surface or roller support surface, or along a link or cable (as the case may be).

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

## Solving problems using the three-force principle

### Step 4

Extend the two lines of action drawn in Steps 2 and 3 as far as necessary until they intersect at a point, i.e. locate the point of concurrency.

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

## Solving problems using the three-force principle

### Step 5

Join the point of concurrency with the point of application of the third force, which is usually at the other support. This defines the line of action of the third force and hence its angle to the horizontal.

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

## Solving problems using the three-force principle

### Step 6

Draw the force triangle as a separate diagram. (Sides representing forces must be to scale and parallel to the lines of action of the forces.)

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK



## Solving problems using the three-force principle

### Step 7

Scale off the force magnitudes from the force triangle. Directional sense is established by the head-to-tail sequence of arrows, starting with the known force.

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

GIVE FEEDBACK

OK

Rearrange the steps for solving a three-force principle problem in the correct order.

↑↓ Place these in the proper order.

Draw a free body diagram of the structure or component in essential outline only



Draw the line of action of the known force and identify its magnitude and direction



Consider the nature of supports and draw the line of action of the second force



Extend the lines of action drawn in the free body diagram until they intersect at a point (locate the point of concurrency)



Join the point of concurrency with the point of application of the third force; define the line of action of the third force and its angle to the horizontal x axis



Draw the force triangle as a separate diagram; scale off the force magnitudes from the force triangle



Do you know the answer?



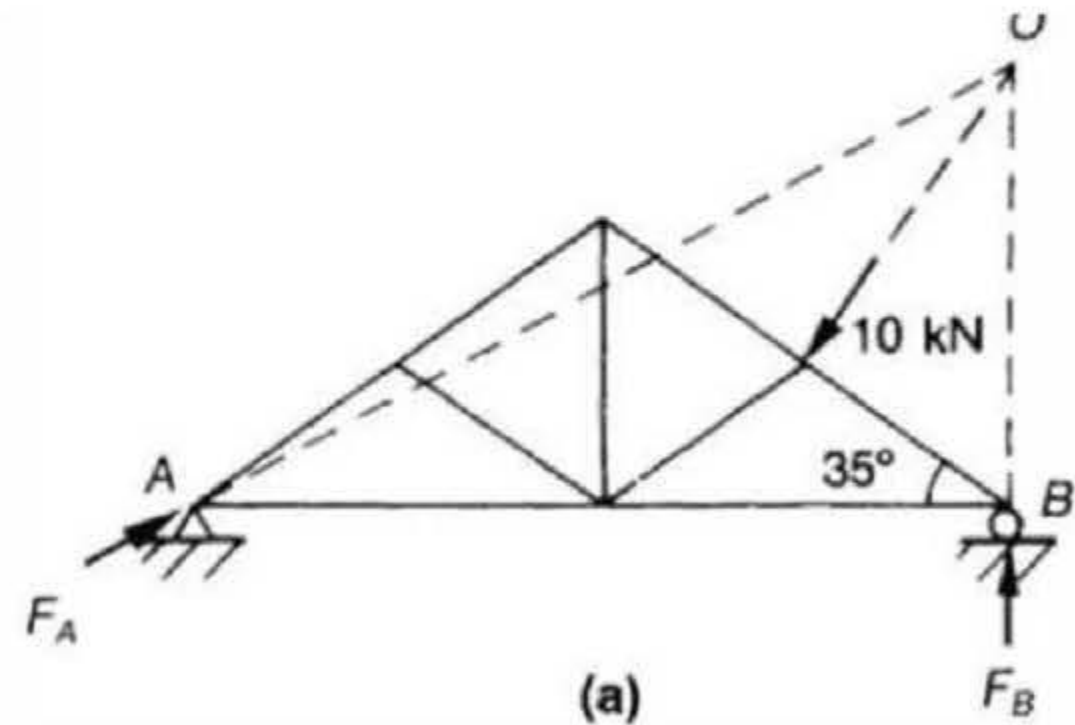
Always check the type of surface before determining the angle of the line of action.

Once you locate the point of concurrency you can construct the force polygon and determine unknown forces.



The truss shown in the figure is subjected to wind loads equivalent to a force of 10 kN applied at 90 degrees to the member at the joint as shown below.

Our task is to determine the point of concurrency in relation to point  $A$  and  $B$ .

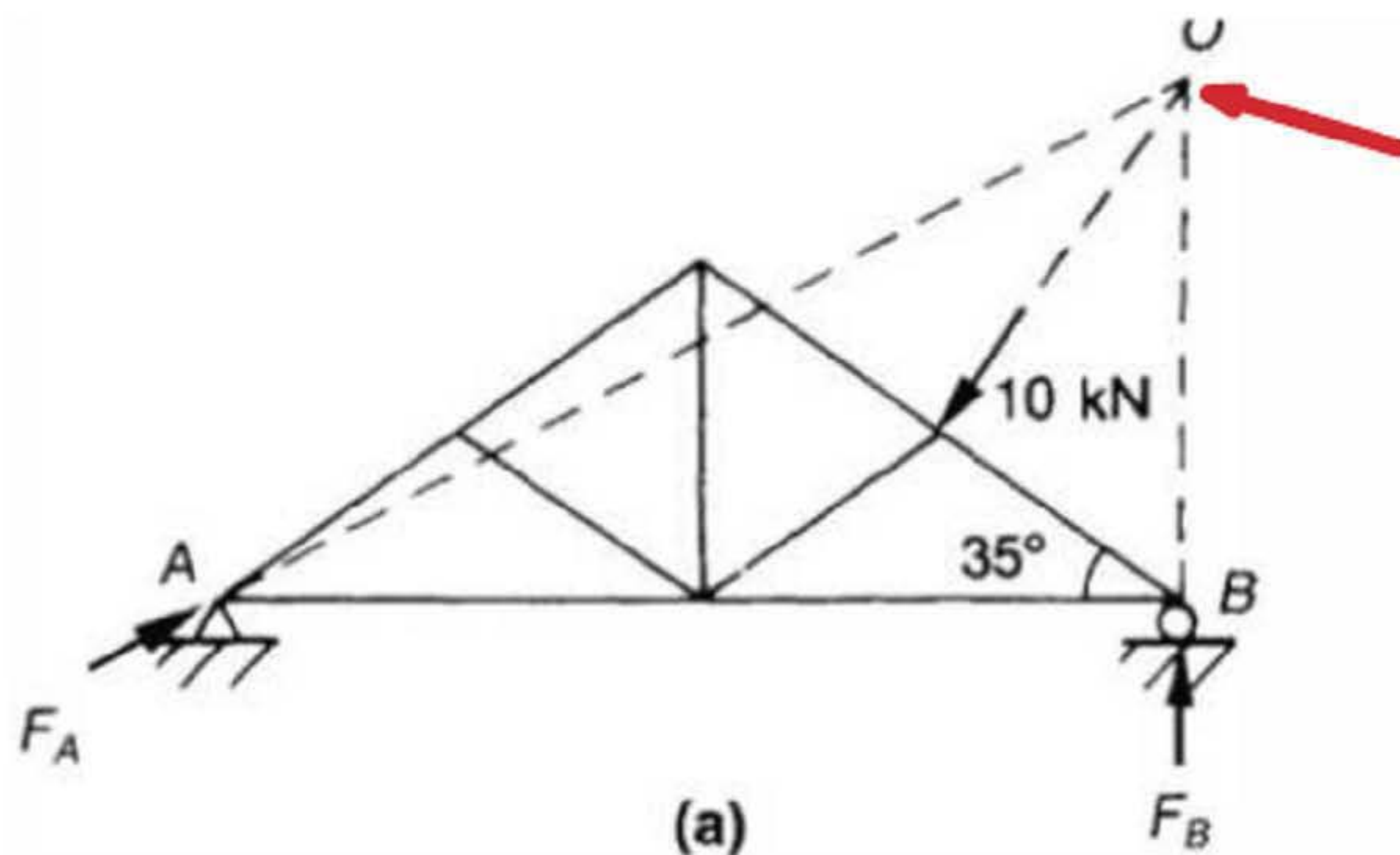


GIVE FEEDBACK

CONTINUE >

The normal reaction will be at the roller support at  $B$ . Therefore, draw a vertical line through  $B$  to intersect with the line of action of the load at point  $O$ . Join  $O$  and  $A$ .

The point of concurrency (point  $O$ ) is indicated by the red arrow.



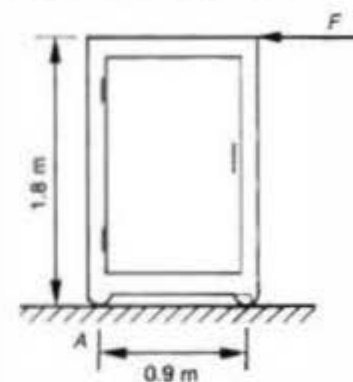
1. The point of concurrency

2. The point of concurrency

3. The point of concurrency

4. The point of concurrency

The mass of a cabinet is 50 kg. A force  $F$  is applied to tip the cabinet as shown.



Assuming the cabinet is stationary, which of the following free body diagrams shows the correct point of concurrency (with the correct line of action and forces)?



Click the correct answer.

1.

2.

3.

4.

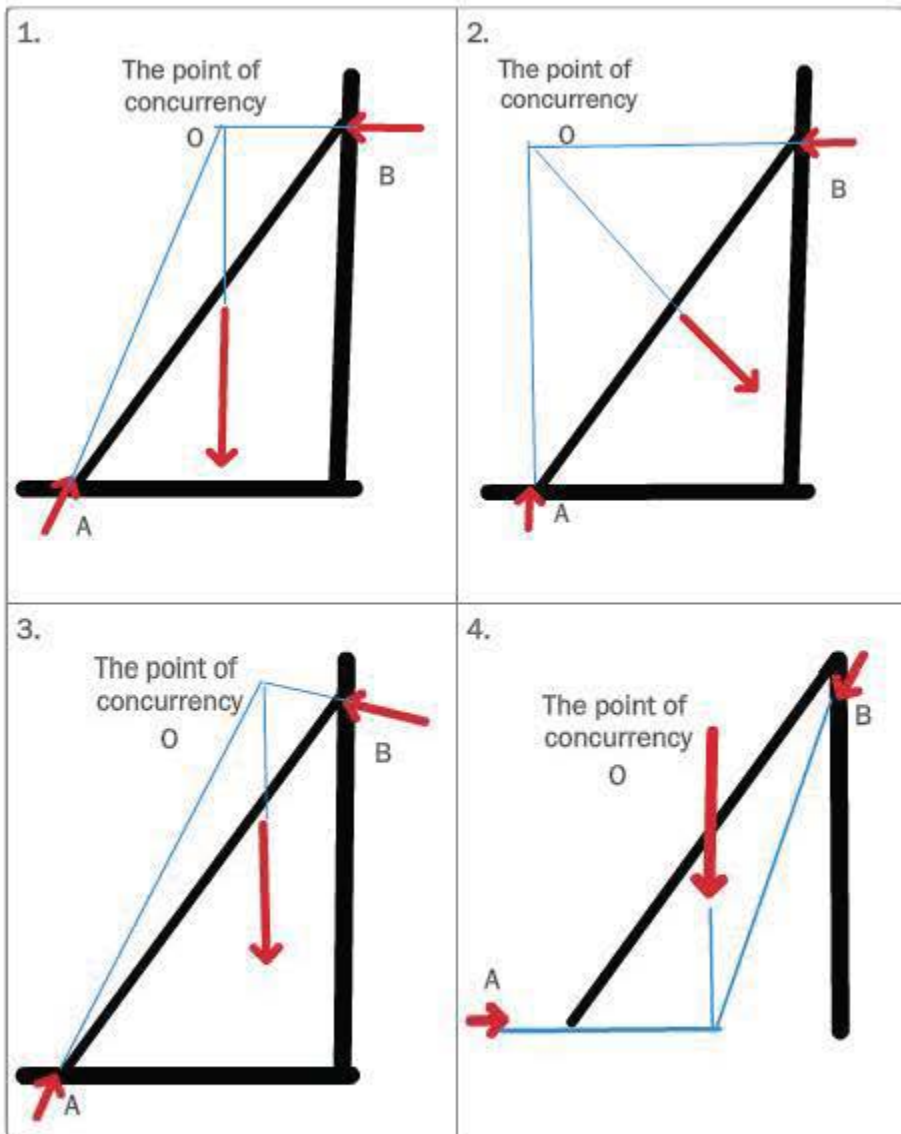
Do you know the answer?

I KNOW IT

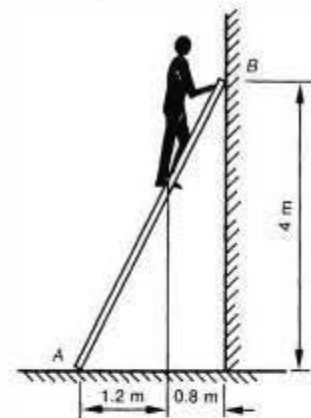
THINK SO

UNSURE

NO IDEA



A ladder rests on a rough floor surface and against a smooth wall as shown. Neglecting the mass of the ladder, a man of mass 85 kg climbs to a point on the ladder as shown.



Which of the following free body diagrams shows the correct point of concurrency (with the correct line of action and forces)?

Click the correct answer.

1.

2.

3.

4.

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA



**Type your answer in the box.**

The point of concurrency can be located by extending the lines of action of the forces until they  
 at one single point.

---

**Do you know the answer?**

**I KNOW IT**

**THINK SO**

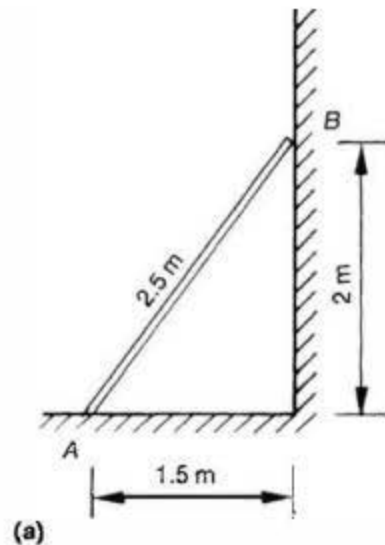
**UNSURE**

**NO IDEA**

### Determine unknown reaction forces in a system of concurrent forces using the three-force principle

A ladder 2.5 m in length and 10 kg in mass rests on the floor and against a smooth wall as shown.

Our task is to determine the reaction forces involved.



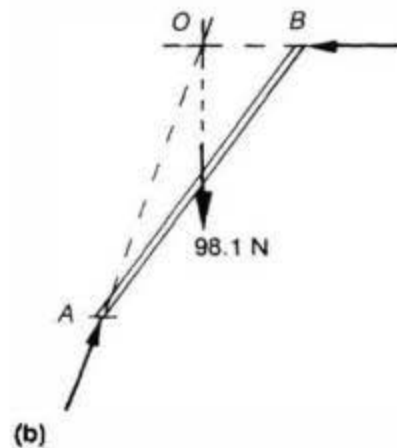
Problem statement	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
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### Determine unknown reaction forces in a system of concurrent forces using the three-force principle

Draw a free body diagram showing weight acting through the midpoint and a horizontal reaction on the smooth wall at  $B$ .

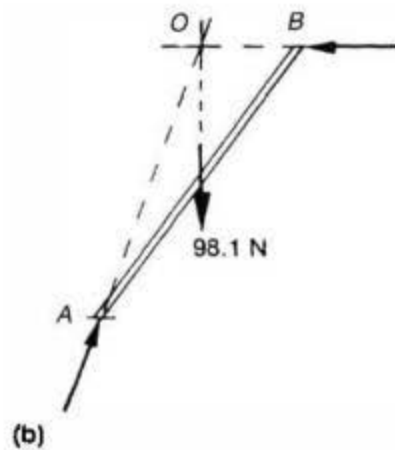
$$\begin{aligned}F_w &= m g \\&= 10 \cdot 9.81 \\&= 98.1 \text{ N}\end{aligned}$$



Problem statement	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
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### Determine unknown reaction forces in a system of concurrent forces using the three-force principle

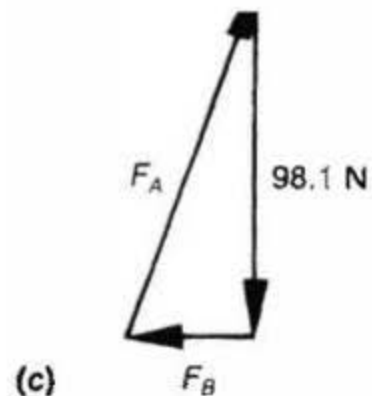
Extend their lines of action to intersect at point  $O$  and join point  $O$  with point  $A$ . This determines the direction of the reaction at  $A$ .



Problem statement	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
-------------------	--------	--------	--------	--------	--------	--------

### Determine unknown reaction forces in a system of concurrent forces using the three-force principle

Using the graphical solution method, a triangle of forces can now be constructed and the reaction forces found:



$$F_B = 36.8 \text{ N}$$

$$F_A = 105 \text{ N at } 69^\circ \text{ to the horizontal x axis.}$$

Problem statement	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
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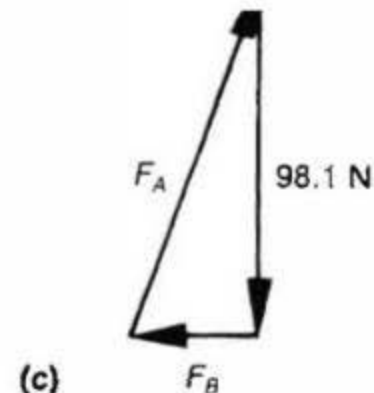
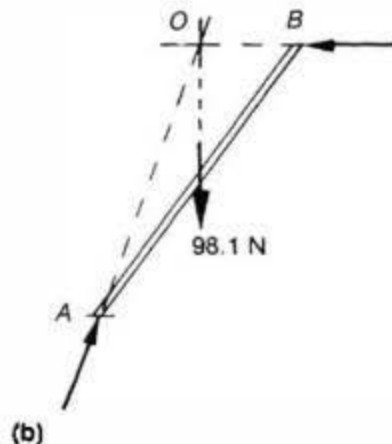
## Determine unknown reaction forces in a system of concurrent forces using the three-force principle



In this case, you can use the sine rule method or the horizontal vertical components method to solve the unknown forces.



Alternatively the mathematical solution method can be used. We can create the same free body diagram, locate the point of concurrency and construct the force triangle as before.



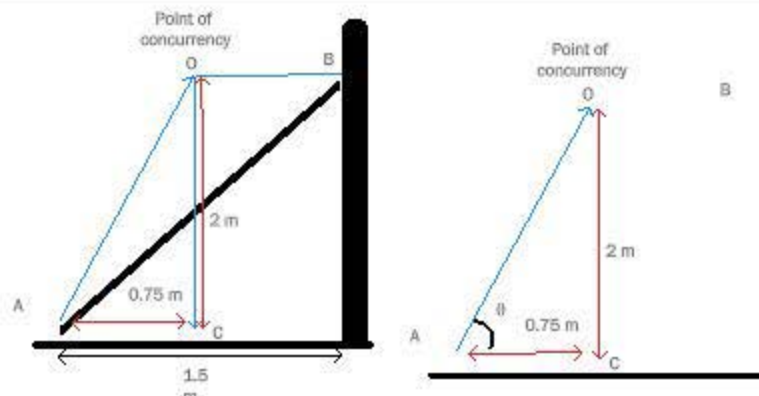
Using the mathematical solution method, sine rule and trigonometry will be used to work out the unknown forces and angles.

Problem statement	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
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## Determine unknown reaction forces in a system of concurrent forces using the three-force principle



Using the free body diagram and previous measurements, you can work out the angle of reaction  $F_A$  makes with the horizontal.



Note: It is established that the weight of the ladder is acting through the midpoint; the distance between the midpoint and point A is 0.75m.

$$\tan \theta = \frac{2}{0.75}$$

$$\theta = \tan^{-1} \left( \frac{2}{0.75} \right)$$

$$\theta = 69.44^\circ$$

Hence the angle of direction of the reaction force at point A is  $69.4^\circ$ .

Problem  
statement

Step 1

Step 2

Step 3

Step 4

Step 5

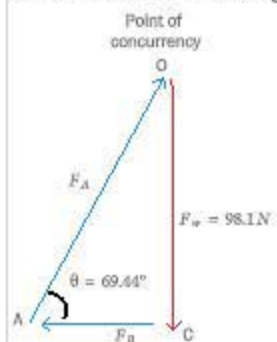
Step 6

GIVE FEEDBACK

OK

## Determine unknown reaction forces in a system of concurrent forces using the three-force principle

A more detailed triangle of forces can now be constructed.



$$\sin 69.44^\circ = \frac{F_w}{F_A}$$

$$F_A = \frac{(98.1 \text{ N})}{\sin 69.44^\circ} = 104.8 \text{ N}$$

$$\tan 69.44^\circ = \frac{F_w}{F_B}$$

$$F_B = \frac{(98.1 \text{ N})}{\tan 69.44^\circ} = 38.8 \text{ N}$$

Problem  
statement

Step 1

Step 2

Step 3

Step 4

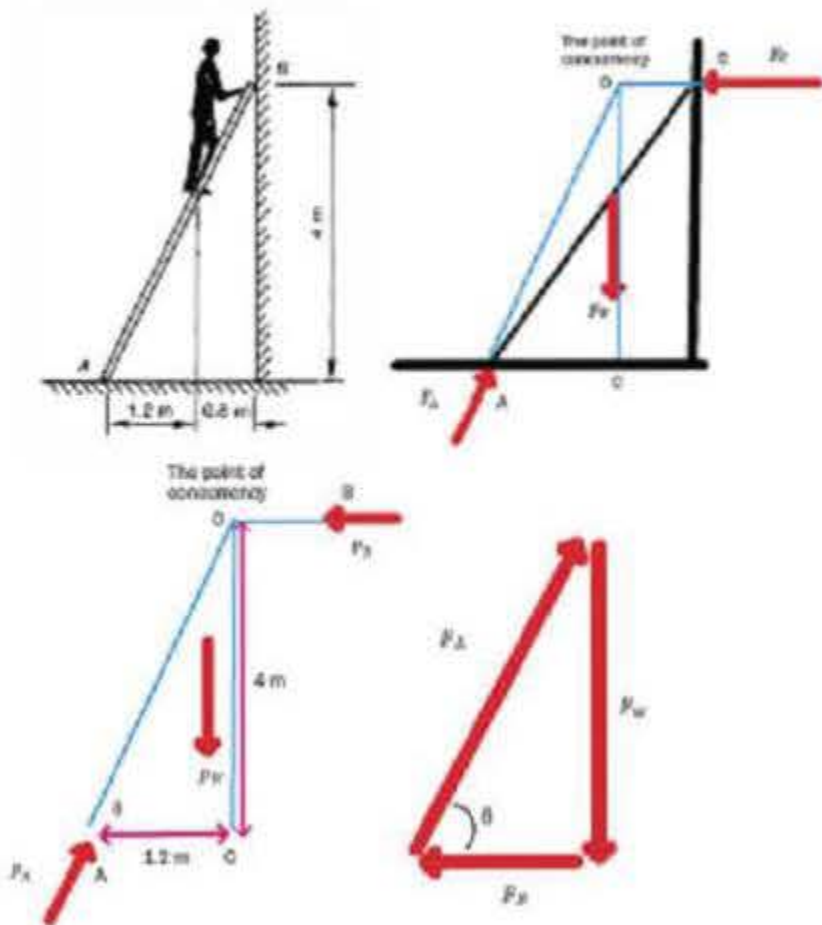
Step 5

Step 6

GIVE FEEDBACK

OK

A ladder rests on a rough floor surface and against a smooth wall as shown. Neglecting the mass of the ladder, a man of mass 85 kg climbs to a point on the ladder as shown. Based on the diagrams given, determine the weight  $F_W$ , the reaction forces  $F_A$  and  $F_B$ , and the angle  $\theta$ .



SMALL

MEDIUM

LARGE

Type your answer in the box.

The weight  $F_W$  is equal to  N (rounded to 2 decimal places).



Do you know the answer?

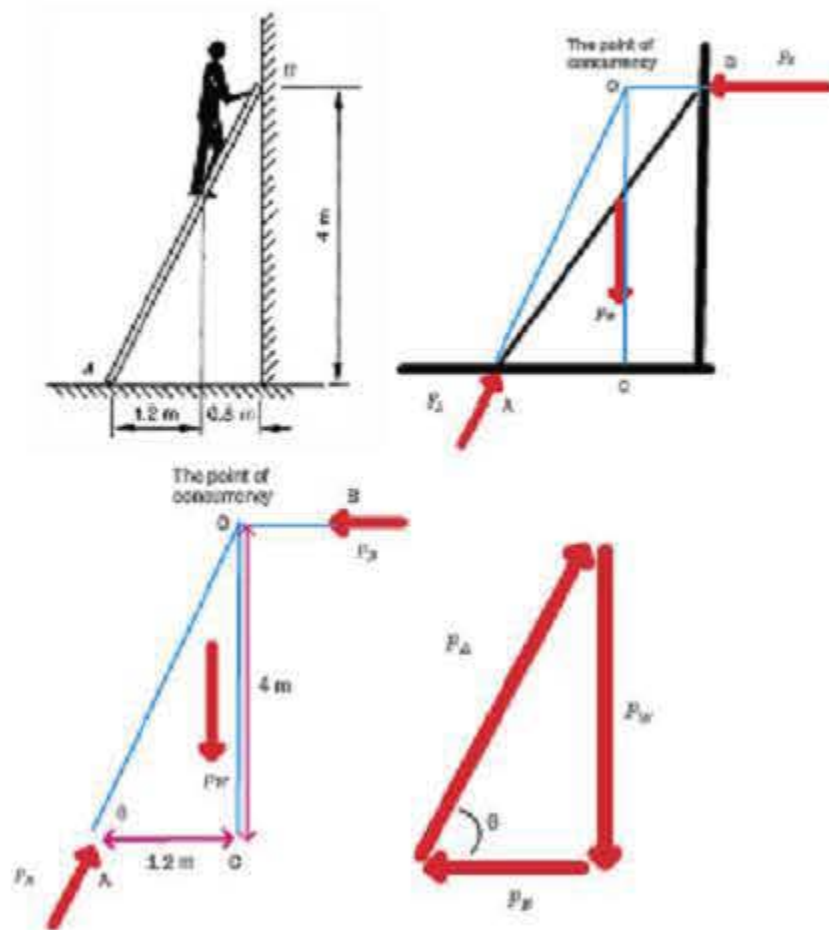
I KNOW IT

THINK SO

UNSURE

NO IDEA

A ladder rests on a rough floor surface and against a smooth wall as shown. Neglecting the mass of the ladder, a man of mass 85 kg climbs to a point on the ladder as shown. Based on the diagrams given, determine the weight  $F_W$ , the reaction forces  $F_A$  and  $F_B$ , and the angle  $\theta$ .



SMALL

MEDIUM

LARGE

Type your answer in the box.

The angle of the reaction force at point A is  degrees (rounded to 2 decimal places).



Do you know the answer?

I KNOW IT

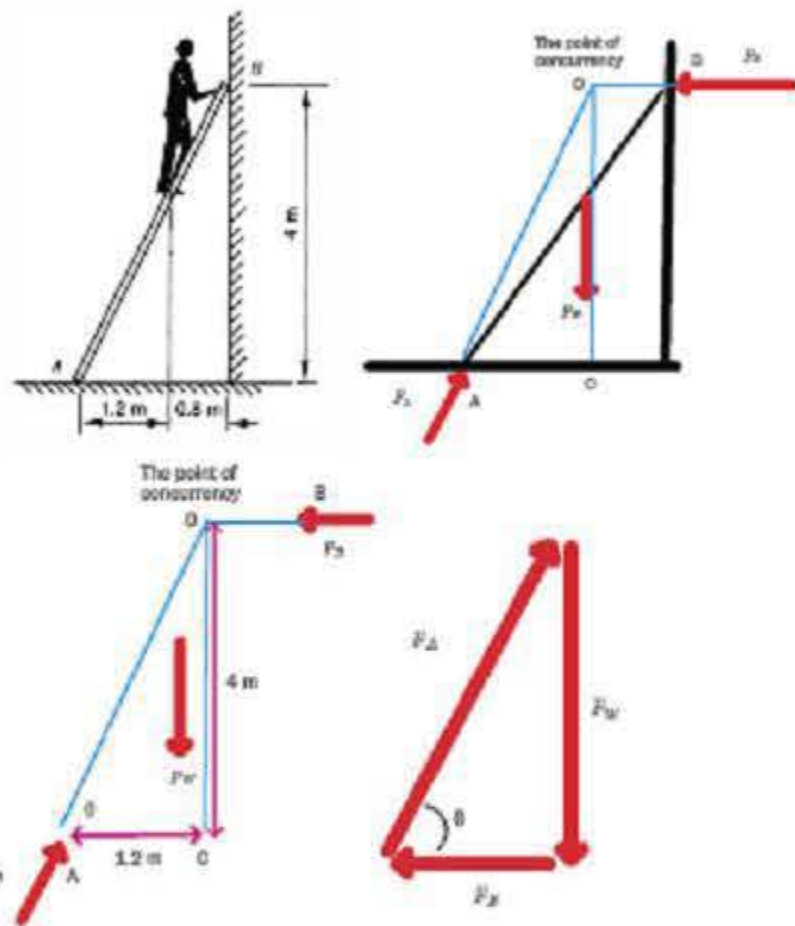
THINK SO

UNSURE

NO IDEA



A ladder rests on a rough floor surface and against a smooth wall as shown. Neglecting the mass of the ladder, a man of mass 85 kg climbs to a point on the ladder as shown. Based on the diagrams given, determine the weight  $F_W$ , the reaction forces  $F_A$  and  $F_B$ , and the angle  $\theta$ .



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight  $F_W$  is equal to 833.85 N and the angle of the reaction force at point A is 73.3 degrees, the reaction force to the smooth wall  $F_B$  is equal to  N (rounded to 2 decimal places).

$$\tan \theta = \frac{4}{1.2}$$

$$\theta = \tan^{-1} \left( \frac{4}{1.2} \right)$$



Do you know the answer?

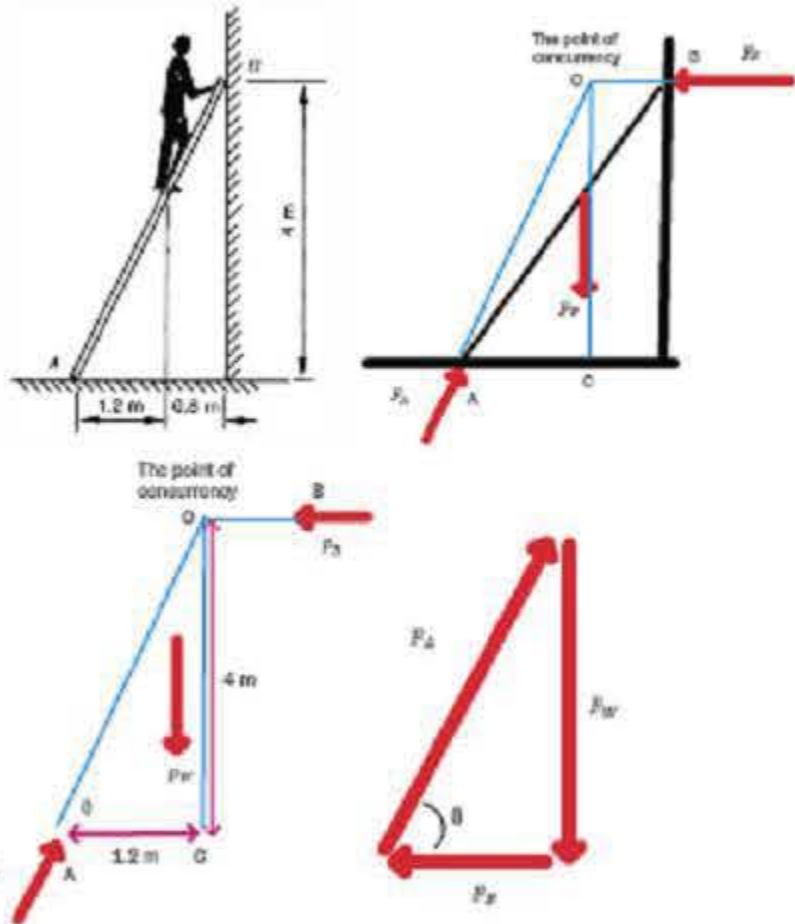
I KNOW IT

THINK SO

UNSURE

NO IDEA

A ladder rests on a rough floor surface and against a smooth wall as shown. Neglecting the mass of the ladder, a man of mass 85 kg climbs to a point on the ladder as shown. Based on the diagrams given, determine the weight  $F_W$ , the reaction forces  $F_A$  and  $F_B$ , and the angle  $\theta$ .



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight  $F_W$  is equal to 833.85 N and the angle of the reaction force at point A is 73.3 degrees, the reaction force to the rough floor  $F_A$  is equal to  N (rounded to 2 decimal places).



Do you know the answer?

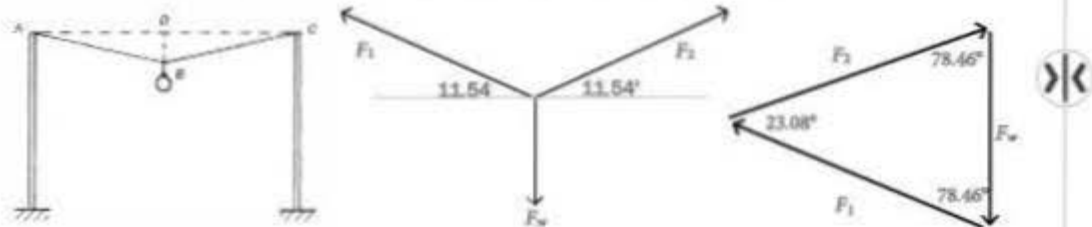
I KNOW IT

THINK SO

UNSURE

NO IDEA

A streetlight of mass 15 kg is supported at midpoint between two poles by a cable ABC. If the length of the cable ABC is 20 m and distance  $BD$  at midpoint is 2 m, determine the forces  $F_1$  and  $F_2$  in the cable based on the given diagrams.



SMALL

MEDIUM

LARGE

Type your answer in the box.

The weight of the streetlight  $F_w$  is equal to  N (rounded to 2 decimal places).



Do you know the answer?

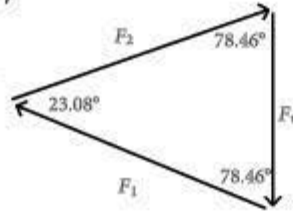
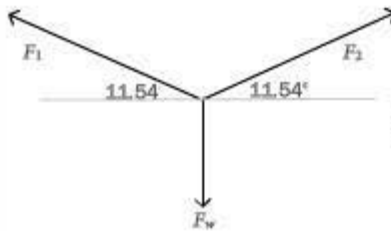
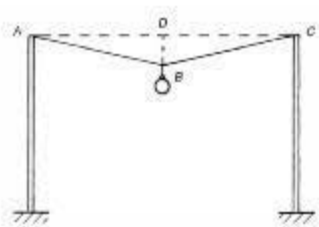
I KNOW IT

THINK SO

UNSURE

NO IDEA

A streetlight of mass 15 kg is supported at midpoint between two poles by a cable ABC. If the length of the cable ABC is 20 m and distance BD at midpoint is 2 m, determine the forces  $F_1$  and  $F_2$  in the cable based on the given diagrams.



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight of the streetlight  $F_w$  is equal to 147.15 N,  $F_1$  is equal to  N (rounded to 2 decimal places).

Hint: use the sine rule  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .



Do you know the answer?

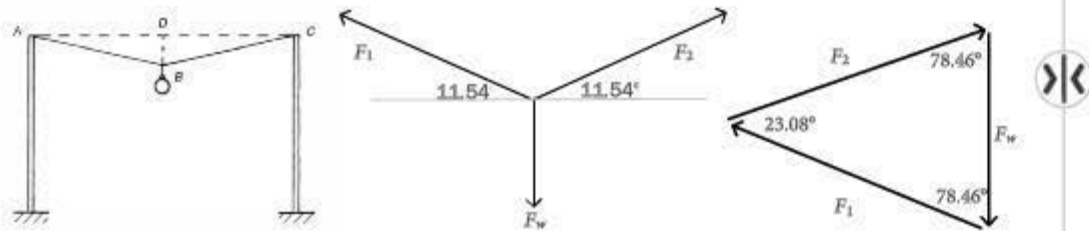
I KNOW IT

THINK SO

UNSURE

NO IDEA

A streetlight of mass 15 kg is supported at midpoint between two poles by a cable ABC. If the length of the cable ABC is 20 m and distance BD at midpoint is 2 m, determine the forces  $F_1$  and  $F_2$  in the cable based on the given diagrams.



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight of the streetlight  $F_w$  is equal to 147.15 N and  $F_1$  is equal to 367.78 N,  $F_2$  is equal to  N (rounded to 2 decimal places).

Hint: use the sine rule  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Type your answer in the box.

Given the weight  $F_W$  is equal to 833.85 N and the angle of the reaction force at point A is 73.3 degrees, the reaction force to the smooth wall  $F_B$  is equal to  N (rounded to 2 decimal places).

$$\tan \theta = \frac{4}{1.2}$$

$$\theta = \tan^{-1} \left( \frac{4}{1.2} \right)$$

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Do you know the answer?

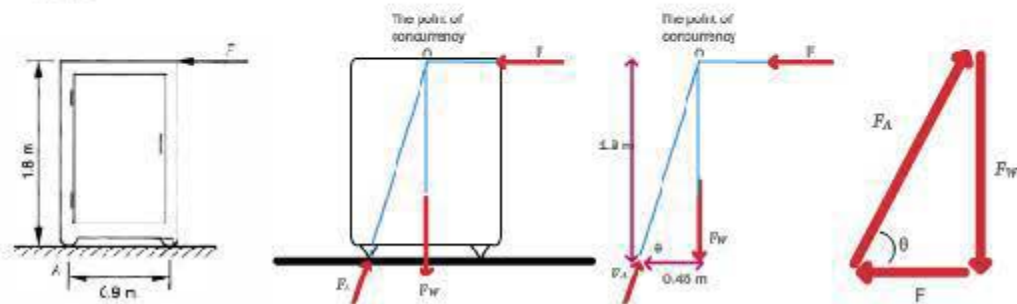
I KNOW IT

THINK SO

UNSURE

NO IDEA

The mass of a cabinet is 50 kg. A force  $F$  is applied to tip the cabinet as shown. Based on the given diagrams, determine the force  $F$  required to start to tip the cabinet and the angle and magnitude of reaction force at point A.



SMALL

MEDIUM

LARGE

Type your answer in the box.

The weight of the cabinet,  $F_W$ , is equal to  N (rounded to 2 decimal places).



Do you know the answer?

I KNOW IT

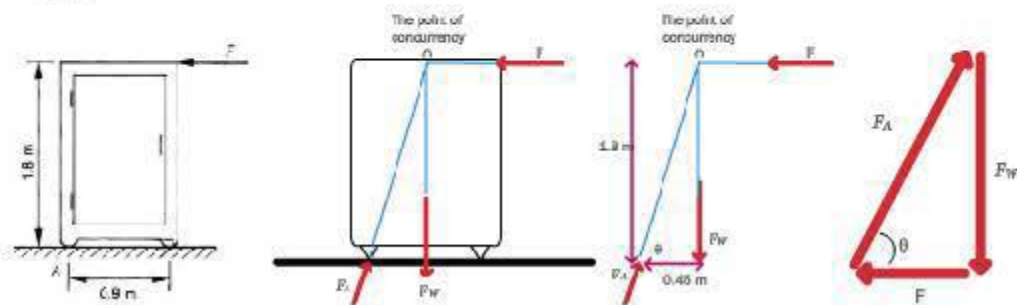
THINK SO

UNSURE

NO IDEA



The mass of a cabinet is 50 kg. A force  $F$  is applied to tip the cabinet as shown. Based on the given diagrams, determine the force  $F$  required to start to tip the cabinet and the angle and magnitude of reaction force at point A.



SMALL

MEDIUM

LARGE

Type your answer in the box.

The angle of the reaction force at point A is  degrees (rounded to 2 decimal places).



Do you know the answer?

I KNOW IT

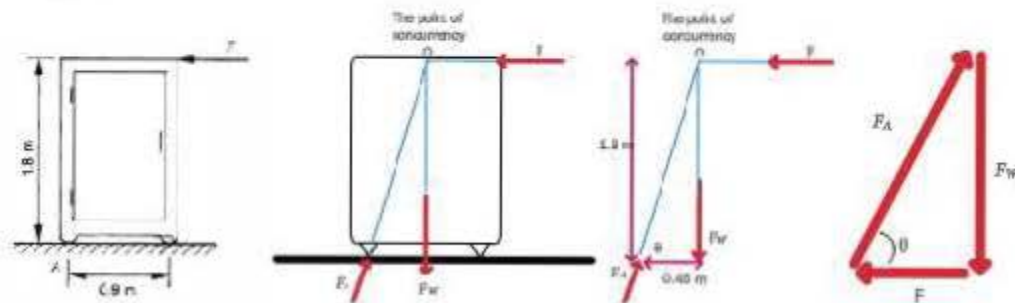
THINK SO

UNSURE

NO IDEA



The mass of a cabinet is 50 kg. A force  $F$  is applied to tip the cabinet as shown. Based on the given diagrams, determine the force  $F$  required to start to tip the cabinet and the angle and magnitude of reaction force at point A.



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight  $F_W$  is equal to 490.5 N and the angle of the reaction force at point A is 75.96 degrees, the force  $F$  required to start to tip the cabinet is equal to  N (rounded to 1 decimal place).



Do you know the answer?

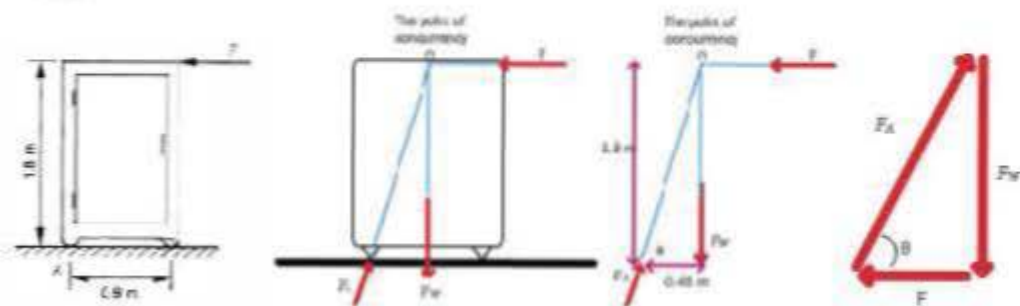
I KNOW IT

THINK SO

UNSURE

NO IDEA

The mass of a cabinet is 50 kg. A force  $F$  is applied to tip the cabinet as shown. Based on the given diagrams, determine the force  $F$  required to start to tip the cabinet and the angle and magnitude of reaction force at point A.



SMALL

MEDIUM

LARGE

Type your answer in the box.

Given the weight  $F_W$  is equal to 490.5 N and the angle of the reaction force at point A is 75.96 degrees, the reaction force to point A,  $F_A$ , is equal to  N (rounded to 2 decimal places).



Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA