SECTION B: WORKING WITH EQUATIONS, GRAPHS AND TABLES

Although many of the examples and most of the end-of-chapter problems in this book are quantitative, none requires mathematical skills beyond rudimentary high school algebra and geometry. In this brief section we review some of the skills you will need for dealing with these examples and problems.

One important skill is to be able to read simple verbal descriptions and translate the information they provide into the relevant equations or graphs. You will also need to be able to translate information given in tabular form into an equation or graph, and sometimes you will need to translate graphical information into a table or equation. The following examples illustrate all the tools you will need.

USING A VERBAL DESCRIPTION TO CONSTRUCT AN EQUATION

B.1

B.1 EXAMPLE

HOW TO CONSTRUCT A LONG-DISTANCE TELEPHONE BILLING EQUATION FROM A VERBAL DESCRIPTION OF THE BILLING PLAN

Your long-distance telephone plan charges you $5 per month plus 10 cents per minute for long-distance calls. Write an equation that describes your monthly telephone bill.

An **equation** is a simple mathematical expression that describes the relationship between two or more **variables**, or quantities that are free to assume different values in some range. The most common type of equation we will work with contains two types of variable: dependent variable and independent variable. In this example the dependent variable is the dollar amount of your monthly telephone bill, and the independent variable is the variable on which your bill depends, namely, the volume of long-distance calls you make during the month. Your bill also depends on the $5 monthly fee and the 10 cents per minute charge. But, in this example, those amounts are constants, not variables. A **constant**, also called a **parameter**, is a quantity in an equation that is fixed in value, not free to vary. As the terms suggest, the dependent variable describes an outcome that depends on the value taken by the independent variable.

Once you have identified the **dependent variable** and the **independent variable**, choose simple symbols to represent them. In algebra courses, X is typically used to represent the independent variable and Y the dependent variable. Many people find it easier to remember what the variables stand for, however, if they choose symbols that are linked in some straightforward way to the quantities that the variables represent. Thus, in this example, we might use B to represent your monthly *bill* in dollars and T to represent the total *time* in minutes you spent during the month on long-distance calls.

Having identified the relevant variables and chosen symbols to represent them, you are now in a position to write the equation that links them:

$$B = 5 + 0.10T \qquad \text{B.1}$$

where B is your monthly long-distance bill in dollars and T is your monthly total long-distance calling time in minutes. The fixed monthly fee (5) and the charge per minute (0.10) are parameters in this equation. Note the importance of being clear about the units of measure. Because B represents the monthly bill in dollars, we must also express the fixed monthly fee and the per-minute charge in dollars, which is why the latter number appears in Equation B.1 as 0.10 rather than 10. Equation B.1 follows the normal convention in which the dependent variable appears by itself on the left-hand side while the independent variable or variables and constants appear on the right-hand side.

Once we have the equation for the monthly bill, we can use it to calculate how much you will owe as a function of your monthly volume of long-distance calls. For example, if you make 32 minutes of calls, you can calculate your monthly bill by simply substituting 32 minutes for T in Equation B.1:

$$B = 5 + 0.10(32) = 8.20 \qquad \text{B.2}$$

Your monthly bill when you make 32 minutes of calls is thus equal to $8.20.

Equation
A mathematical expression that describes the relationship between two or more variables.

Variable
A quantity that is free to take a range of different values.

Constant (or parameter)
A quantity that is fixed in value.

Dependent variable
A variable in an equation whose value is determined by the value taken by another variable in the equation.

Independent variable
A variable in an equation whose value determines the value taken by another variable in the equation.

B.1 EXERCISE

Under the monthly billing plan described in Example B.1, how much would you owe for a month during which you made 45 minutes of long-distance calls?

B.2 GRAPHING THE EQUATION OF A STRAIGHT LINE

B.2 EXAMPLE

HOW TO PORTRAY THE BILLING PLAN AS A GRAPH

Construct a graph that portrays the monthly long-distance telephone billing plan described in Example B.1, putting your telephone charges, in dollars per month, on the vertical axis, and your total volume of calls, in minutes per month, on the horizontal axis.

The first step in responding to this instruction is the one we just took, namely, to translate the verbal description of the billing plan into an equation. When graphing an equation the normal convention is to use the vertical axis to represent the dependent variable and the horizontal axis to represent the independent variable. In Figure B.1 we therefore put B on the vertical axis and T on the horizontal axis. One way to construct the graph shown in the figure is to begin by plotting the monthly bill values that correspond to several different total amounts of long-distance calls. For example, someone who makes 10 minutes of calls during the month would have a bill of $B = 5 + 0.10(10) = \$6$. Thus, in Figure B.1 the value of 10 minutes per month on the horizontal axis corresponds to a bill of $6 per month on the vertical axis (point A). Someone who makes 30 minutes of long-distance calls during the month will have a monthly bill of $B = 5 + 0.10(30) = \$8$, so the value of 30 minutes per month on the horizontal axis corresponds to $8 per month on the vertical axis (point C). Similarly, someone who makes 70 minutes of long-distance calls during the month will have a monthly bill of $B = 5 + 0.10(70) = \$12$, so the value of 70 minutes on the horizontal axis corresponds to $12 on the vertical axis (point D). The line joining these points is the graph of the monthly billing Equation B.1.

As shown in Figure B.1, the graph of the equation $B = 5 + 0.10T$ is a straight line. The parameter 5 is the vertical intercept of the line—the value of B when $T = 0$, or the point at which the line intersects the vertical axis. The parameter 0.10 is the slope of the line, which is the ratio of the rise of the line to the corresponding run. The ratio rise/run is simply the vertical distance between any two points on the line divided by the horizontal distance between those points. For example, if we choose points A and C in Figure B.1, the rise is $8 - 6 = 2$ and the corresponding run is $30 - 10 = 20$, so rise/run $= 2/20 = 0.10$. More generally, for the graph of any equation $Y = a + bX$, the parameter a is the vertical intercept and the parameter b is the slope.

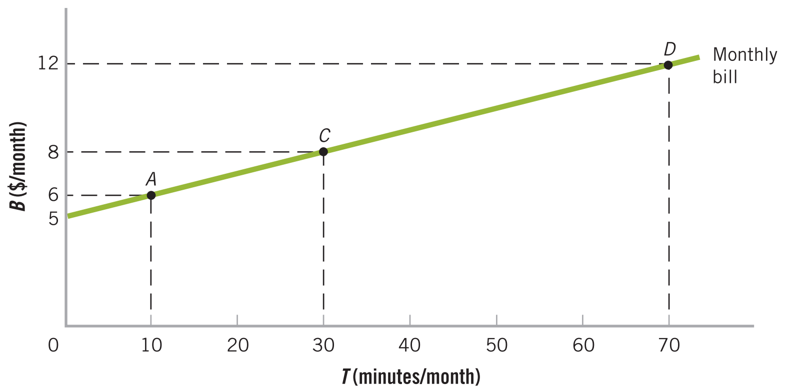

Figure B.1 **The monthly telephone bill in Example B.1** The graph of the equation $B = 5+0.10T$ is the straight line shown. Its vertical intercept is 5 and its slope is 0.10>.

DERIVING THE EQUATION OF A STRAIGHT LINE FROM ITS GRAPH

B.3

HOW TO DERIVE THE EQUATION FOR A STRAIGHT LINE FROM A GRAPH OF THE LINE

B.3 EXAMPLE

Figure B.2 shows the graph of the monthly billing plan for a new long-distance plan. What is the equation for this graph? How much is the fixed monthly fee under this plan? How much is the charge per minute?

The vertical distance between points *A* and *C* is 12 − 8 = 4 units, and the horizontal distance between points *A* and *C* is 40 − 20 = 20, so the slope of the line is 4/20 = 1/5 = 0.20. The vertical intercept (the value of *B* when *T* = 0) is 4. So the equation for the billing plan shown is *B* = 4 − 0.20*T*.

The slope of the line shown is the rise between any two points divided by the corresponding run. For points *A* and *C*, rise = 12 − 8 = 4, and run = 40 − 20 = 20, so the slope equals rise/run = 4/20 = 1/5 = 0.20. And since the horizontal intercept of the line is 4, its equation must be given by:

$$B = 4 + 0.20T \qquad \text{B.3}$$

Under this plan the fixed monthly fee is the value of the bill when *T* = 0, which is $4. The charge per minute is the slope of the billing line, 0.20, or 20 cents per minute.

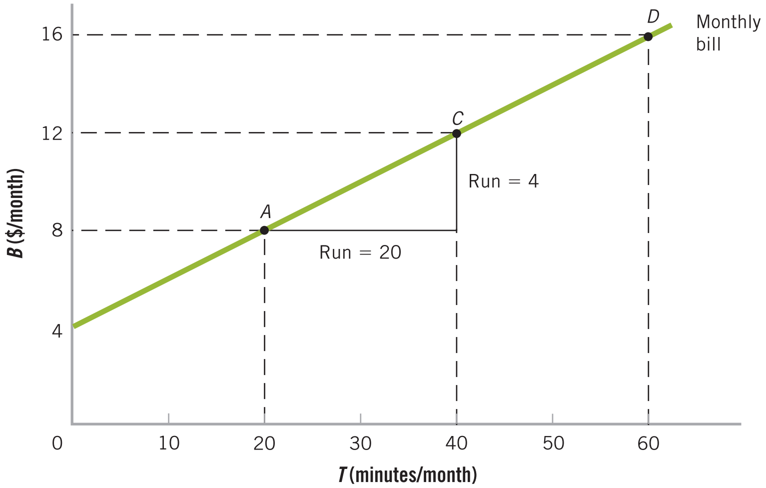

Figure B.2 Another monthly long-distance plan

B.2 EXERCISE

Write the equation for the billing plan shown in the accompanying graph. How much is its fixed monthly fee? Its charge per minute?

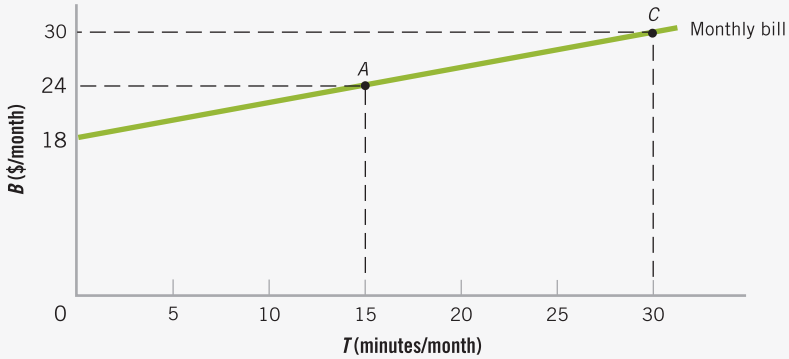

Figure B.3 Billing plan

B.4 CHANGES IN THE VERTICAL INTERCEPT AND SLOPE

Examples B.4 and B.5 and Exercises B.3 and B.4 provide practice in seeing how a line shifts with a change in its **vertical intercept** or **slope**.

B.4 EXAMPLE

Vertical intercept In a straight line, the value taken by the dependent variable when the independent variable equals zero.

Slope In a straight line, the ratio of the vertical distance the straight line travels between any two points (rise) to the corresponding horizontal distance (run).

Show how the billing plan whose graph is in Figure B.2 would change if the monthly fixed fee were increased from $4 to $8.

An increase in the monthly fixed fee from $4 to $8 would increase the vertical intercept of the billing plan by $4 but would leave its slope unchanged. An increase in the fixed fee thus leads to a parallel upward shift in the billing plan by $4, as shown in Figure B.3. For any given number of minutes of long-distance calls, the monthly charge on the new bill will be $4 higher than on the old bill. Thus 20 minutes of calls per month costs $8 under the original plan (point *A*) but $12 under the new plan (point *A*′). And 40 minutes costs $12 under the original plan (point *C*), $16 under the new plan (point *C*′); and 60 minutes costs $16 under the original plan (point *D*), $20 under the new plan (point *D*′).

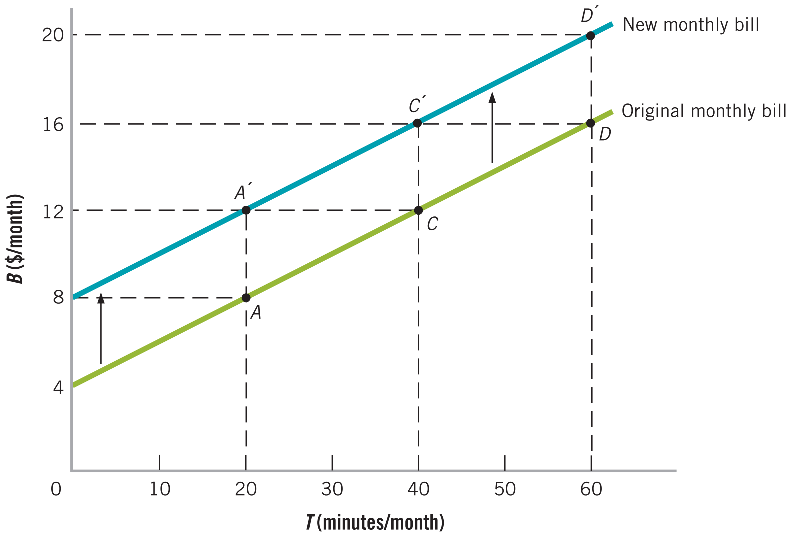

Figure B.4 **The effect of an increase in the vertical intercept** An increase in the vertical intercept of a straight line produces an upward parallel shift in the line.

B.3 EXERCISE

Show how the billing plan whose graph is in Figure B.2 would change if the monthly fixed fee were reduced from $4 to $2.

B.5 EXAMPLE

Show how the billing plan whose graph is in Figure B.2 would change if the charge per minute were increased from 20 cents to 40 cents.

Because the monthly fixed fee is unchanged, the vertical intercept of the new billing plan continues to be 4. But the slope of the new plan, shown in Figure B.4, is 0.40, or twice the slope of the original plan. More generally, in the equation $Y = a + bX$, an increase in b makes the slope of the graph of the equation steeper.

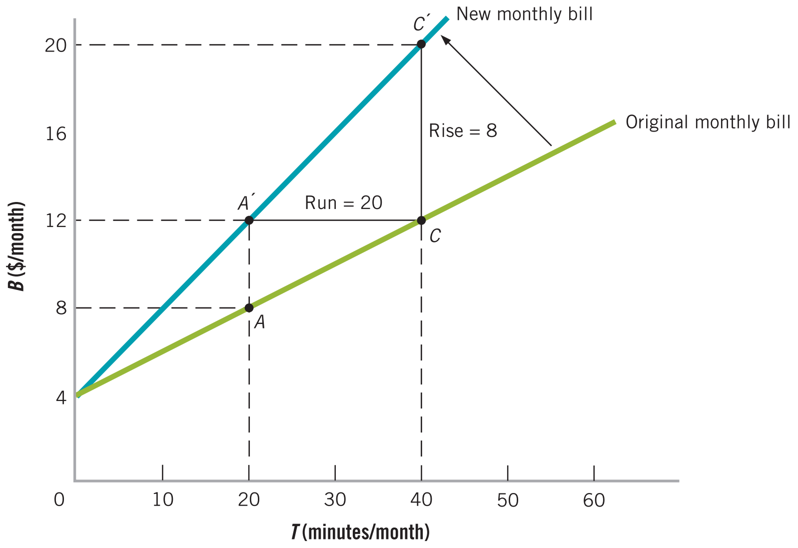

Figure B.5 **The effect of an increase in the charge per minute** Because the fixed monthly fee continues to be $4, the vertical intercept of the new plan is the same as that of the original plan. With the new charge per minute of 40 cents, the slope of the billing plan rises from 0.20 to 0.40.

Show how the billing plan whose graph is in Figure B.2 would change if the charge per minute were reduced from 20 cents to 10 cents.

B.4 EXERCISE

Exercise B.4 illustrates the general rule that in an equation $Y = a + bX$, a reduction in b makes the slope of the graph of the equation less steep.

CONSTRUCTING EQUATIONS AND GRAPHS FROM TABLES

B.5

Example B.6 and Exercise B.5 show how to transform tabular information into an equation or graph.

Table B.1 shows four points from a monthly long-distance telephone billing equation. If all points on this billing equation lie on a straight line, find the vertical intercept of the equation and graph it. What is the monthly fixed fee? What is the charge per minute? Calculate the total bill for a month with 1 hour of long-distance calls.

B.6 EXAMPLE

Table B.1 Points on a long-distance billing plan

LONG-DISTANCE BILL ($/MONTH)	TOTAL LONG-DISTANCE CALLS (MINUTES/MONTH)
10.50	10
11.00	20
11.50	30
12.00	40

continued ↳

↵ Example B.6 *continued*

One approach to this problem is simply to plot any two points from the table on a graph. Since we are told that the billing equation is a straight line, that line must be the one that passes through any two of its points. Thus, in Figure B.5 we use *A* to denote the point from Table B.1 for which a monthly bill of $11 corresponds to 20 minutes per month of calls (second row) and *C* to denote the point for which a monthly bill of $12 corresponds to 40 minutes per month of calls (fourth row). The straight line passing through these points is the graph of the billing equation.

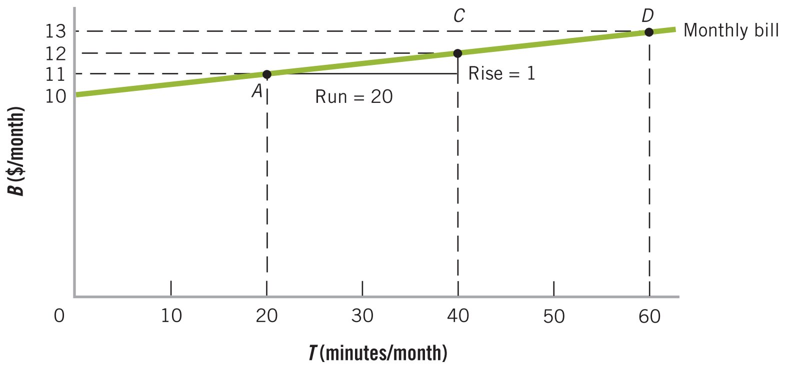

Figure B.6 **Plotting the monthly billing equation from a sample of points** Point *A* is taken from row 2, Table B.1, and point *C* from row 4. The monthly billing plan is the straight line that passes through these points.

Unless you have a steady hand, however, or use extremely large graph paper, the method of extending a line between two points on the billing plan is unlikely to be very accurate. An alternative approach is to calculate the equation for the billing plan directly. Since the equation is a straight line, we know that it takes the general form $B = f + sT$, where f is the fixed monthly fee and s is the slope. Our goal is to calculate the vertical intercept, f, and the slope, s. From the same two points we plotted earlier, *A* and *C*, we can calculate the slope of the billing plan as s = rise/run = 1/20 = 0.05.

So, all that remains is to calculate f, the fixed monthly fee. At point *C* on the billing plan, the total monthly bill is $12 for 40 minutes, so we can substitute $B = 12$, $s = 0.05$ and $T = 40$ into the general equation $B = f + sT$ to obtain:

$$12 = f + 0.05(40) \qquad \text{B.4}$$

or:

$$12 = f + 2 \qquad \text{B.5}$$

which solves for $f = 10$. So the monthly billing equation must be:

$$B = 10 + 0.05T \qquad \text{B.6}$$

For this billing equation, the fixed fee is $10 per month, the calling charge is 5 cents per minute ($0.05/minute) and the total bill for a month with 1 hour of long-distance calls is $B = 10 + 0.05(60) = \$13$, just as shown in Figure B.5.

B.5 EXERCISE

The following table shows four points from a monthly long-distance telephone billing plan.

LONG-DISTANCE BILL ($/MONTH)	TOTAL LONG-DISTANCE CALLS (MINUTES/MONTH)
20.00	10
30.00	20
40.00	30
50.00	40

If all points on this billing plan lie on a straight line, find the vertical intercept of the corresponding equation without graphing it. What is the monthly fixed fee? What is the charge per minute? How much would the charges be for 1 hour of long-distance calls per month?

KEY TERMS