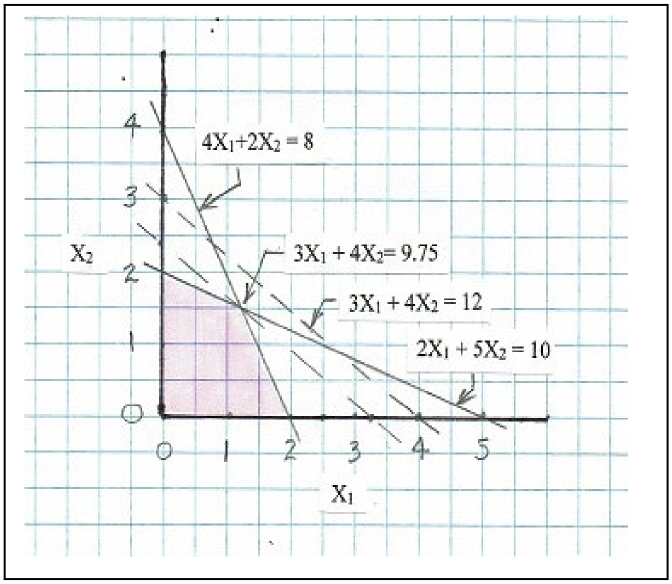
# Solutions to Technical Chapter 5: LP Solver

## Discussion Questions

1. **Define the following terms:**
   a. **Constraint:** A constraint limits or constraints actions. In Linear programming constraints often are limits on resources such as materials, labor or machines.
   b. **Objective function:** An objective is something desired or a goal. In LP an objective function is something desirable that is maximized or minimized.
   c. **Extreme point**. An extreme point in LP is a possible solution to the LP problem that is defined by the intersection of two or more constraints.
   d. **Optimal solution**. An optimal solution consists of the optimal values of the decision variables along with the optimal value of the objective function.
   e. **Feasible solution.** A feasible solution is any set of values of the decisions variables that satisfies all of the constraints simultaneously.
2. **What information is provided by shadow prices?** Shadow prices are the marginal change in the objective function for one unit change in the right-hand side of a constraint.
3. **The simplex method is an adjacent-extreme-point method. Explain.** What this means is that the simplex method moves from one extreme point to the immediate next one. It does not skip any extreme points as it moves from one to the next one.
4. **What is the purpose of the branch-and-bound method?** The purpose is to use the Simplex Method as a way of finding integer solutions. A solution cannot be found directly by moving from one extreme point to the next. As a result starting with the relaxed solution to the problem ignoring integer requirements, branch and bound will branch into two sub-problems if the solution isn't integer. Each sub-problem is solved by the Simplex method and branching continues to an integer solution is found to within tolerances of an optimal solution. Bounding is used to wall off or bound certain parts of the solution space so that it no longer needs to be searched.
5. **Define what is meant by a mixed-integer problem.** It has both continuous and integer variables, some of each type.
6. **What is the purpose of using binary variables?** Binary variables are used as indicator variables. They can indicate some decision, event or cost when equal to one and zero otherwise.
7. **How is the transportation problem different from other L.P. problems?** It only differs in the way the data is displayed in a network or in a table format with shipping from supply locations to demand locations. Otherwise, all the constraints are linear and the objective function is written in linear form.

1. Solve the following problem.

a. Graphical Solution




a. There are four extreme points (X1=0, X2=0; X1=0, X2=2; X1=1.25, X2=1.5; X1=2, X2=0. The adjacent extreme points that would be visited by Solver are starting at X1=0, X2=0 than move to X1 = 0, X2 =2, since X2 has the largest unit increase in the objective function (4 vs 3). Then Solver would move directly to the intersection of the two lines (X1 = 1.25 and X2 =1.5) since that extreme point has a positive increase in the objective function (from 8 to 9.75). No further increase is possible by moving to the next extreme point X1=0, X2=2 (at that point the objective function is only 6), so the optimal solution has been found.

b. The optimal solution using solver is $X_1 = 1.25$, $X_2 = 1.5$, max objective function = 9.75. This solution agrees with the graphical solution.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | name | | | | | |
| 2 | | units of product | | | | |
| 3 | **Variables** | X1 | X2 | | | |
| 4 | decision Variables | 1.25 | 1.5 | | | |
| 5 | coefficients | 3 | 4 | | | |
| 6 | | | | total profit | 9.75 | |
| 7 | | | | | | |
| 8 | **Constraints** | | | utilized | constraint | Capacity |
| 9 | | 4 | 2 | 8 | <= | 8 |
| 10 | | 2 | 5 | 10 | <= | 10 |

2. The optimal solution is $X_1 = 2$, $X_2 = 2$, min objective function = 14

3. Energex Problem

   a. Decision Variables: A = 12 oz cans of AwakeX and S = 12 ounce cans of SuperX

      Max 1A + 1.5 S

      Subject to:
      2A + 2.5 S <= 1000
      1A + 2 S <= 1500
      A >=0, S>=0

   b. The optimal values of A = 0, S= 400 and Objective function = 600

4. Toy manufacturing

   a. Maximize 1T + 1C
      Subject To:
      1T + 2C <= 300
      2T + 1C <= 400
      1T + 2C <= 400
      T>=0, C>=0

   b. Optimal Solution: T=166.7, C = 66.7, Profit = 233.3

   c. The binding constraints are molding and painting.

| | Shadow Price | Allowable increase | Allowable decrease |
|---|---|---|---|
| molding | .333 | 100 | 100 |
| painting | .333 | 200 | 250 |
| packaging | 0 | infinity | 100 |

   d. Yes increase the capacity of molding, because the cost (20 cents) is less than the shadow price (increase in profits) of $.333 per minute of molding capacity added.

5. Realistic Picture Frame

   a. maximize 1.5 Ru +1.25M + .95F + 1.1 Ro
      Subject to:
      1 Ru + 1.5 M + 2 F + 2 Ro <=1000
      1 Ru + .9M + .7F + .6Ro <= 500
      .3Ru + .2 M + .1F + .1 Ro <= 100
      $Ru \geq 0$ $M \geq 0$ $F \geq 0$ $Ro \geq 0$

b. Optimal Solution: Ru = 200, M = 0, F = 0, Ro = 400, Total Profit 740

c. The binding constraints in the optimal solution are wood and machine.

d. Amount to pay for additional Labor = 0, additional machine time 3.8 per hour up to 21.4 more hours and wood .36 per board foot up to 375 more board feet.

6. Berry Farm

a. In this problem we assume that the variables are the number of acres planted of each type (A, B, C and D) of strawberries. The sum of these variables should be 10 acres. The labor constraint and revenue coefficients must be calculated on a per acre basis to form the objective function and constraints as follows:

$$\begin{array}{lrrrrl} \text{Maximize} & 10000A + & 10000B + & 9000C + & 10800D & \\ \text{Subject to:} & 1000A + & 1200B + & 1200C + & 1200D & <= 12000 \text{ labor hours} \\ & 1A + & 1B + & 1C + & 1D & = 10 \text{ acres} \\ & 1A & & & & <=3 \\ & & 1B & & & <=3 \\ & & & 1C & & <=3 \\ & & & & 1D & <=3 \\ & A >=0 & B>=0 & C>=0 & D>=0 & \end{array}$$

b. Plant 3 acres of type A, 3 acres of type B, 1 acre of type C and 3 acres of type D berries for the maximum revenue.

c. You shouldn't hire any additional labor. The shadow price is zero, since you have excess labor in the optimal solution. The shadow price of acreage (increase in profit per acre) is \$9000 up to .5 additional acres. Yes, rent an additional .5 acre of land for \$8000 per acre or \$4000 total for the half acre.

7. Gasoline Blending

a.
$$\begin{array}{ll} \text{Minimize} & 1.2A + 1B + 1.1C \\ \text{Subject to:} & 15A - 10B + 5C = 0 \\ & A + B + C = 1000 \\ & A <= 500 \\ & B <= 900 \\ & C <= 600 \\ & A >= 0 \quad B >= 0 \quad C >= 0 \end{array}$$

b. Optimal Solution: A=40, B=360 and C=600, Min cost = 1068

c. How much would you pay for an additional gallon of Stock A, B or C?
Stock A =0, Stock B= 0, Stock C = .02 per gallon. These are the shadow prices for the stocks. In each case it is the increase in total cost for each additional gallon of stock added to the constraint.

8. Brand X Animal Feed

a. The blending constraints for this problem are:
Starch: .40W +.50C >=.45(W+C) which becomes -.05W+.05C >=0
Fiber: .50W +.35C >=.40(W+C) which becomes .1W-.05C > =0

b. The optimal solution is use 400 pounds of wheat and 400 pounds of corn. The minimum cost is 32

c. It will cost $.04 per pound to produce each additional pound of animal feed. This is the shadow price of the constraint requiring at least 800 pounds of animal feed to be produced.

9. **Transportation Problem.** Wholesaler ships to Zones.

a. The Equations for this problem are:

Minimize $6X_{11} + 4X_{12} + 10X_{21} + 3X_{22}$
Subject to:

$$X_{11} + X_{12} \le 1000$$
$$X_{21} + X_{22} \le 800$$
$$X_{11} + X_{21} = 600$$
$$X_{12} + X_{22} = 900$$

$$X_{11}, X_{12}, X_{21}, X_{22} \ge 0$$

b. The optimal solution is:
Ship 600 units from Chicago to East Coast
Ship 100 units from Chicago to South
Ship 800 units from Los Angeles to South.

Total cost is 6400

c. Shipped from Chicago = 700, Shipped from Los Angeles = 800

**Variables (shipments)**

| | East Coast | South | Available | Total Shipped |
|---|---|---|---|---|
| Chicago | 600 | 100 | 1000 | 700 |
| Los Angeles | 0 | 800 | 800 | 800 |
| Total Required | 600 | 900 | | |
| **Total Received** | 600 | 900 | | |
| | | **Total Cost** | 6400 | |

10. **Transportation Problem.** Small Electronics Company

a. Optimal Solution in Excel. Shipping 50 units from Atlanta to Dallas. Shipping 600 units from Chicago to Newark and 400 units to Dallas. Shipping 800 units form San Diego to Phoenix. The minimum cost is $10,550.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | **Unit Cost Matrix** | | | | | | |
| 3 | | | Newark | Dallas | Phoenix | Available | |
| 4 | | Atlanta | 6.5 | 7 | 8.5 | **900** | |
| 5 | | Chicago | 6 | 6.5 | 7.5 | **1000** | |
| 6 | | San Diego | 9 | 7.5 | 5 | **1200** | |
| 7 | | Required | **600** | **450** | **800** | | |
| 8 | | | | | | | |
| 9 | **Variables (shipments)** | | | | | | |
| 10 | | | Newark | Dallas | Phoenix | Capacity | **Shipped** |
| 11 | | Atlanta | 0 | 50 | 0 | **900** | 50 |
| 12 | | Chicago | 600 | 400 | 0 | **1000** | 1000 |
| 13 | | San Diego | 0 | 0 | 800 | **1200** | 800 |
| 14 | | Required | **600** | **450** | **800** | | |
| 15 | | **Received** | 600 | 450 | 800 | | |
| 16 | | | | | Total Cost | 10550 | |
| 17 | | | | | | | |

b. The shadow prices for shipping more to these thee destinations are:

| | Shadow Price | Allowable Increase | Allowable Decrease |
|---|---|---|---|
| Newark | 6.5 | 400 | 50 |
| Dallas | 7 | 850 | 50 |
| Phoenix | 5 | 400 | 800 |

c. Since the shadow price for shipping each additional unit to the Phoenix warehouse is $5, the added profit from charging $8 per unit is $3 per unit ($8-$5).

d. No, I would not use that route, since the reduced cost is 3.5 from 8.5 per unit shipped or $5 to begin using the Atlanta to Phoenix route.

11. **Transportation Problem:** Plumbing Supply Company

a. The optimal solution is to ship from Houston 50 units to New York and 150 units to Miami. Also, ship from Minneapolis 50 units to New York, 160 units to Los Angeles, and 90 units to Seattle. The total cost of transportation is 5430.

customers increases

**Unit Cost Matrix**

| | New York | Miami | Los Angeles | Seattle | Available |
|---|---|---|---|---|---|
| Houston | 10 | 9 | 15 | 16 | **400** |
| Minneapolis | 8 | 11 | 12 | 14 | **300** |
| Required | **100** | **150** | **160** | **90** | |

**Variables (shipments)**

| | New York | Miami | Los Angeles | Seattle | Available | Shipped |
|---|---|---|---|---|---|---|
| Houston | 50 | 150 | 0 | 0 | **400** | 200 |
| Minneapolis | 50 | 0 | 160 | 90 | **300** | 300 |
| Required | **100** | **150** | **160** | **90** | | |
| **Received** | 100 | 150 | 160 | 90 | | |
| | | | **Total Cost** | 5430 | | |

b. Increased cost of shipping one more unit to Miami is $9.

12. **Integer LP.**

a. The number of trucks can be rounded up to 167 trucks and 67 cars. However, this solution would not be a feasible solution. The molding capacity is now 301 hours (167+2*67 = 301 and exceeds the 300 hours available. We could round the solution up to 167 trucks and down to 66 cars to retain feasibility. This is exactly the solution we get when requiring trucks and cars to be integers in Solver.

b. The number of trucks and cars is not significantly different and is the same if the round-off is done appropriately.

c. Profit for the integer solution decreases to 233 from 233.3 in the original linear solution without the integer constraints on trucks and cars.

13. **Binary LP problem.** The candy store

a. Formulate this problem

Maximize Profit = $4x1 + 4.50\ x2 - 200Y1 - 300Y2$

$$\begin{array}{lccccccl} \text{Subject to:} & x1 & + & x2 & & & & =1000 \\ & x1 & & & -600 & & & <= 0 \\ & & & x2 & & -800 & & <=0 \\ & x1>=0 & & x2>=0 & \multicolumn{4}{l}{\text{Y1 and Y2 binary}} \end{array}$$

The Excel spreadsheet is shown below.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | candy store | | | | | | | |
| 2 | | units of product | | | | | | |
| 3 | **Variables** | regular | supreme | Y1 | Y2 | | | |
| 4 | decision Variables | 200 | 800 | 1 | 1 | | | |
| 5 | coefficients | 4 | 4.5 | -200 | -300 | | | |
| 6 | | | | | | total profit | 3900 | |
| 7 | | | | | | | | |
| 8 | **Constraints** | | | | | utilized | constraint | Capacity |
| 9 | produce 1000 units | 1 | 1 | | | 1000 | "=" | 1000 |
| 10 | max of 600 lbs | 1 | | -600 | | -400 | >= | 0 |
| 11 | max of 800 lbs | | 1 | | -800 | 0 | >= | 0 |
| 12 | | | | | | | | |
| 13 | | >=0 | >=0 | bin | bin | | | |
| 14 | | | | | | | | |

b. To maximize profit make 200 pounds of regular and 800 pounds of supreme

14. **Binary LP** Gadget Phone Company.

a. Problem formulation in the Excel spreadsheet

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Gadget Phone company | | | | | | | |
| 2 | | units produed on | | Fixed costs | | | | |
| 3 | **Variables** | machine 1 | machine 2 | Y1 | Y2 | | | |
| 4 | decision Variables | 1000 | 500 | 1 | 1 | | | |
| 5 | coefficients | 100 | 200 | 800 | 1200 | | | |
| 6 | | | | | | total cost | 202000 | |
| 7 | | | | | | | | |
| 8 | **Constraints** | | | | | utilized | constraint | Capacity |
| 9 | fill the order | 1 | 1 | | | 1500 | "=" | 1500 |
| 10 | capacity machine 1 | 1 | | -1000 | | 0 | <= | 0 |
| 11 | capacity machine 2 | | 1 | | -800 | -300 | <= | 0 |
| 12 | | | | | | | | |
| 13 | | x1>=0 | x2>=0 | binary | binary | | | |
| 14 | | | | | | | | |

b. The optimal solution is produce 1000 phones on machine 1 and 500 phones on machine 2. The total cost of production is 202,000.

c. The binding constraints are the requirements to fill the order of 1500 units and the capacity of machine 1. Machine 2 has excess capacity of 300 units.