CHAPTER

4

# TRANSIENT HEAT CONDUCTION

The temperature of a body, in general, varies with time as well as position. In rectangular coordinates, this variation is expressed as $T(x, y, z, t)$, where $(x, y, z)$ indicate variation in the $x$-, $y$-, and $z$-directions, and $t$ indicates variation with time. In the preceding chapter, we considered heat conduction under *steady* conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis, especially when the temperature varied in one direction only, and we were able to obtain analytical solutions. In this chapter, we consider the variation of temperature with *time* as well as *position* in one- and multidimensional systems.

We start this chapter with the analysis of *lumped systems* in which the temperature of a body varies with time but remains uniform throughout at any time. Then we consider the variation of temperature with time as well as position for one-dimensional heat conduction problems such as those associated with a large plane wall, a long cylinder, a sphere, and a semi-infinite medium using *transient temperature charts* and analytical solutions. Finally, we consider transient heat conduction in multidimensional systems by utilizing the *product solution.*

**OBJECTIVES**

When you finish studying this chapter, you should be able to:

- Assess when the spatial variation of temperature is negligible, and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable,
- Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables, and understand why a one-term solution is usually a reasonable approximation,
- Solve the transient conduction problem in large mediums using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface, and
- Construct solutions for multidimensional transient conduction problems using the product solution approach.

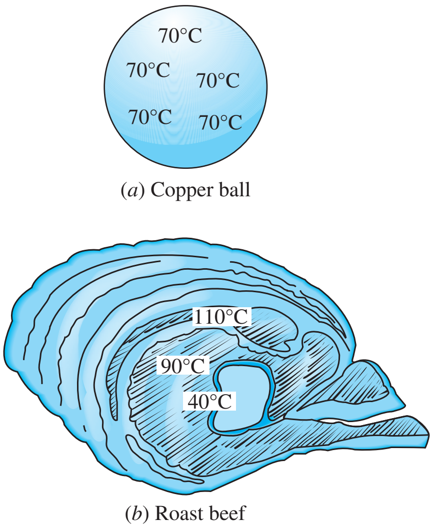



**FIGURE 4–1**
A small copper ball can be modeled as a lumped system, but a roast beef cannot.

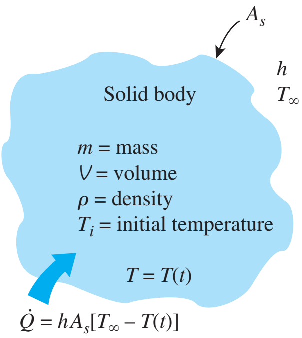



**FIGURE 4–2**
The geometry and parameters involved in the lumped system analysis.

## 4–1 ▪ LUMPED SYSTEM ANALYSIS

In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at any times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, $T(t)$. Heat transfer analysis that utilizes this idealization is known as **lumped system analysis**, which provides great simplification in certain classes of heat transfer problems without much sacrifice from accuracy.

Consider a small hot copper ball coming out of an oven (Fig. 4–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time. Thus the temperature of the ball remains nearly uniform at all times, and we can talk about the temperature of the ball with no reference to a specific location.

Now let us go to the other extreme and consider a large roast in an oven. If you have done any roasting, you must have noticed that the temperature distribution within the roast is not even close to being uniform. You can easily verify this by taking the roast out before it is completely done and cutting it in half. You will see that the outer parts of the roast are well done while the center part is barely warm. Thus, lumped system analysis is not applicable in this case. Before presenting a criterion about applicability of lumped system analysis, we develop the formulation associated with it.

Consider a body of arbitrary shape of mass $m$, volume $V$, surface area $A_s$, density $\rho$, and specific heat $c_p$ initially at a uniform temperature $T_i$ (Fig. 4–2). At time $t = 0$, the body is placed into a medium at temperature $T_\infty$, and heat transfer takes place between the body and its environment, with a heat transfer coefficient $h$. For the sake of discussion, we assume that $T_\infty > T_i$, but the analysis is equally valid for the opposite case. We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, $T = T(t)$.

During a differential time interval $dt$, the temperature of the body rises by a differential amount $dT$. An energy balance of the solid for the time interval $dt$ can be expressed as

$$\left(\begin{array}{c}\text{Heat transfer into the body}\\ \text{during } dt\end{array}\right) = \left(\begin{array}{c}\text{The increase in the}\\ \text{energy of the body}\\ \text{during } dt\end{array}\right)$$

or

$$hA_s(T_\infty - T)\,dt = mc_p\,dT \tag{4–1}$$

Noting that $m = \rho V$ and $dT = d(T - T_\infty)$ since $T_\infty$ = constant, Eq. 4–1 can be rearranged as

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\,dt \tag{4–2}$$

Integrating from $t = 0$, at which $T = T_i$, to any time $t$, at which $T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t \tag{4–3}$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \tag{4–4}$$

where

$$b = \frac{hA_s}{\rho V c_p} \quad (1/\text{s}) \tag{4–5}$$

is a positive quantity whose dimension is (time)$^{-1}$. The reciprocal of $b$ has time unit (usually s), and is called the **time constant**. Equation 4–4 is plotted in Fig. 4–3 for different values of $b$. There are two observations that can be made from this figure and the relation above:

1. Equation 4–4 enables us to determine the temperature $T(t)$ of a body at time $t$, or alternatively, the time $t$ required for the temperature to reach a specified value $T(t)$.
2. The temperature of a body approaches the ambient temperature $T_\infty$ exponentially. The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of $b$ indicates that the body approaches the environment temperature in a short time. The larger the value of the exponent $b$, the higher the rate of decay in temperature. Note that $b$ is proportional to the surface area, but inversely proportional to the mass and the specific heat of the body. This is not surprising since it takes longer to heat or cool a larger mass, especially when it has a large specific heat.

Once the temperature $T(t)$ at time $t$ is available from Eq. 4–4, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \quad (\text{W}) \tag{4–6}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to $t$ is simply the change in the energy content of the body:

$$Q = mc_p[T(t) - T_i] \quad (\text{kJ}) \tag{4–7}$$

The amount of heat transfer reaches its upper limit when the body reaches the surrounding temperature $T_\infty$. Therefore, the *maximum* heat transfer between the body and its surroundings is (Fig. 4–4)

$$Q_{\max} = mc_p(T_\infty - T_i) \quad (\text{kJ}) \tag{4–8}$$

We could also obtain this equation by substituting the $T(t)$ relation from Eq. 4–4 into the $\dot{Q}(t)$ relation in Eq. 4–6 and integrating it from $t = 0$ to $t \to \infty$.

## Criteria for Lumped System Analysis

The lumped system analysis certainly provides great convenience in heat transfer analysis, and naturally we would like to know when it is appropriate

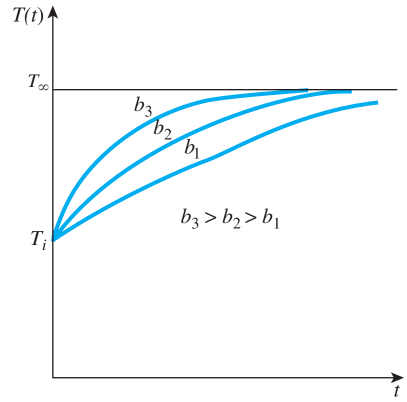



**FIGURE 4–3**
The temperature of a lumped system approaches the environment temperature as time gets larger.

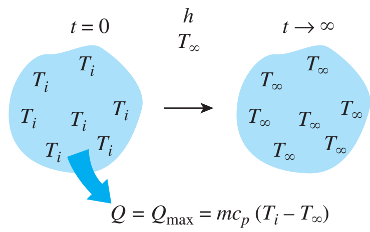



**FIGURE 4–4**
Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

**FIGURE 4–5**
**Jean-Baptiste Biot** (1774–1862) was a French physicist, astronomer, and mathematician born in Paris, France. Although younger, Biot worked on the analysis of heat conduction even earlier than Fourier did (1802 or 1803) and attempted, unsuccessfully, to deal with the problem of incorporating external convection effects in heat conduction analysis. Fourier read Biot's work and by 1807 had determined for himself how to solve the elusive problem. In 1804, Biot accompanied Gay Lussac on the first balloon ascent undertaken for scientific purposes. In 1820, with Felix Savart, he discovered the law known as "Biot and Savart's Law." He was especially interested in questions relating to the polarization of light, and for his achievements in this field he was awarded the Rumford Medal of the Royal Society in 1840. The dimensionless **Biot number (Bi)** used in transient heat transfer calculations is named after him.


to use it. The first step in establishing a criterion for the applicability of the lumped system analysis is to define a **characteristic length** as

$$L_c = \frac{V}{A_s}$$

and a dimensionless **Biot number** (Fig. 4–5) Bi as

$$\text{Bi} = \frac{hL_c}{k} \tag{4–9}$$

The characteristic length $L_c$ to be used in the Biot number for simple geometries in which heat transfer is one-dimensional, such as a large plane wall of thickness $2L$, a long cylinder of radius $r_o$, and a sphere of radius $r_o$, becomes $L$ (half thickness), $r_o/2$, and $r_o/3$, respectively. Equation 4–9 can also be expressed as (Fig. 4–6).

$$\text{Bi} = \frac{h}{k/L_c}\,\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

or

$$\text{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

When a solid body is being heated by the hotter fluid surrounding it (such as a potato being baked in an oven), heat is first *convected* to the body and subsequently *conducted* within the body. The Biot number is the *ratio* of the internal resistance of a body to *heat conduction* to its external resistance to *heat convection.* Therefore, a small Biot number represents small resistance to heat conduction, and thus small temperature gradients within the body.

Lumped system analysis assumes a *uniform* temperature distribution throughout the body, which is the case only when the thermal resistance of the body to heat conduction (the *conduction resistance*) is zero. Thus, lumped system analysis is *exact* when Bi $= 0$ and *approximate* when Bi $> 0$. Of course, the smaller the Bi number, the more accurate the lumped system analysis. Then the question we must answer is, how much accuracy are we willing to sacrifice for the convenience of the lumped system analysis?

Before answering this question, we should mention that a 15 percent uncertainty in the convection heat transfer coefficient $h$ in most cases is considered "normal" and "expected." Assuming $h$ to be *constant* and *uniform* is also an approximation of questionable validity, especially for irregular geometries. Therefore, in the absence of sufficient experimental data for the specific geometry under consideration, we cannot claim our results to be better than $\pm 15$ percent, even when Bi $= 0$. This being the case, introducing another source of uncertainty in the problem will not have much effect on the overall uncertainty, provided that it is minor. It is generally accepted that lumped system analysis is *applicable* if

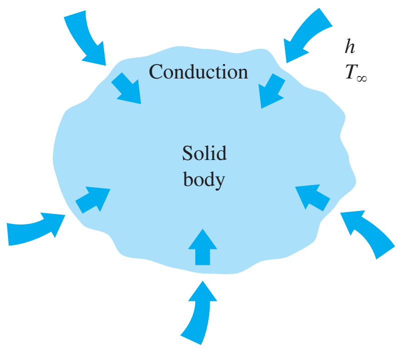
$$\text{Bi} \le 0.1$$

When this criterion is satisfied, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other even for well-rounded geometries such as a spherical ball. Thus, when Bi $< 0.1$,

the variation of temperature with location within the body is slight and can reasonably be approximated as being uniform.

The first step in the application of lumped system analysis is the calculation of the *Biot number,* and the assessment of the applicability of this approach. One may still wish to use lumped system analysis even when the criterion Bi $<$ 0.1 is not satisfied, if high accuracy is not a major concern.

Note that the Biot number is the ratio of the *convection* at the surface to *conduction* within the body, and this number should be as small as possible for lumped system analysis to be applicable. Therefore, *small bodies* with *high thermal conductivity* are good candidates for lumped system analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless. Thus, the hot small copper ball placed in quiescent air, discussed earlier, is most likely to satisfy the criterion for lumped system analysis (Fig. 4–7).




$$\text{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

**FIGURE 4–6**
The Biot number can be viewed as the ratio of the convection at the surface to conduction within the body.

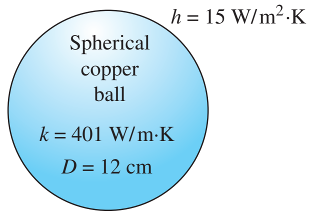
## Some Remarks on Heat Transfer in Lumped Systems

To understand the heat transfer mechanism during the heating or cooling of a solid by the fluid surrounding it, and the criterion for lumped system analysis, consider this analogy (Fig. 4–8). People from the mainland are to go *by boat* to an island whose entire shore is a harbor, and from the harbor to their destinations on the island *by bus.* The overcrowding of people at the harbor depends on the boat traffic to the island and the ground transportation system on the island. If there is an excellent ground transportation system with plenty of buses, there will be no overcrowding at the harbor, especially when the boat traffic is light. But when the opposite is true, there will be a huge overcrowding at the harbor, creating a large difference between the populations at the harbor and inland. The chance of overcrowding is much lower in a small island with plenty of fast buses.

In heat transfer, a poor ground transportation system corresponds to poor heat conduction in a body, and overcrowding at the harbor to the accumulation of thermal energy and the subsequent rise in temperature near the surface of the body relative to its inner parts. Lumped system analysis is obviously not applicable when there is overcrowding at the surface. Of course, we have disregarded radiation in this analogy and thus the air traffic to the island. Like passengers at the harbor, heat changes *vehicles* at the surface from *convection* to *conduction.* Noting that a surface has zero thickness and thus cannot store any energy, heat reaching the surface of a body by convection must continue its journey within the body by conduction.

Consider heat transfer from a hot body to its cooler surroundings. Heat is transferred from the body to the surrounding fluid as a result of a temperature difference. But this energy comes from the region near the surface, and thus the temperature of the body near the surface will drop. This creates a *temperature gradient* between the inner and outer regions of the body and initiates heat transfer by conduction from the interior of the body toward the outer surface.

When the convection heat transfer coefficient $h$ and thus the rate of convection from the body are high, the temperature of the body near the surface drops quickly (Fig. 4–9). This creates a larger temperature difference between the inner and outer regions unless the body is able to transfer heat from the inner to the outer regions just as fast. Thus, the magnitude of the maximum temperature




$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

**FIGURE 4–7**
Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

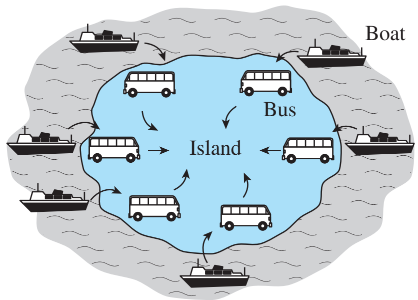



**FIGURE 4–8**
Analogy between heat transfer to a solid and passenger traffic to an island.

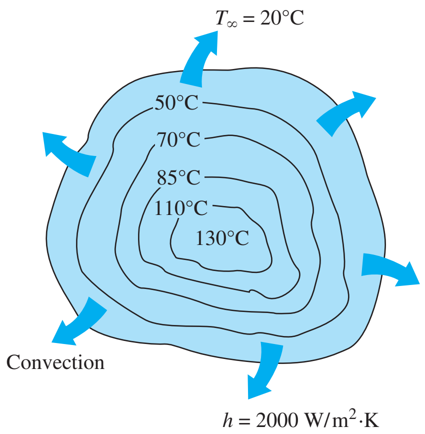



**FIGURE 4–9**
When the convection coefficient $h$ is high and $k$ is low, large temperature differences occur between the inner and outer regions of a large solid.

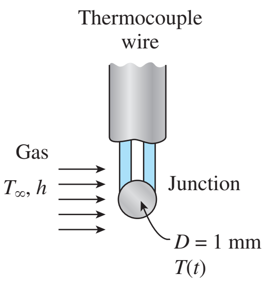



**FIGURE 4–10**
Schematic for Example 4–1.

difference within the body depends strongly on the ability of a body to conduct heat toward its surface relative to the ability of the surrounding medium to convect heat away from the surface. The Biot number is a measure of the relative magnitudes of these two competing effects.

Recall that heat conduction in a specified direction $n$ per unit surface area is expressed as $\dot{q} = -k\,\partial T/\partial n$, where $\partial T/\partial n$ is the temperature gradient and $k$ is the thermal conductivity of the solid. Thus, the temperature distribution in the body will be *uniform* only when its thermal conductivity is *infinite,* and no such material is known to exist. Therefore, temperature gradients and thus temperature differences must exist within the body, no matter how small, in order for heat conduction to take place. Of course, the temperature gradient and the thermal conductivity are inversely proportional for a given heat flux. Therefore, the larger the thermal conductivity, the smaller the temperature gradient.

### *EXAMPLE 4–1* Temperature Measurement by Thermocouples

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere, as shown in Fig. 4–10. The properties of the junction are $k$ = 35 W/m·K, $\rho$ = 8500 kg/m$^3$, and $c_p$ = 320 J/kg·K, and the convection heat transfer coefficient between the junction and the gas is $h$ = 210 W/m$^2$·K. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

**SOLUTION** The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial $\Delta T$ is to be determined.

***Assumptions*** **1** The junction is spherical in shape with a diameter of $D$ = 0.001 m. **2** The thermal properties of the junction and the heat transfer coefficient are constant. **3** Radiation effects are negligible.

***Properties*** The properties of the junction are given in the problem statement.

***Analysis*** The characteristic length of the junction is

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = \frac{1}{6}(0.001\text{ m}) = 1.67 \times 10^{-4}\text{ m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(210\text{ W/m}^2\cdot\text{K})(1.67 \times 10^{-4}\text{ m})}{35\text{ W/m}\cdot\text{K}} = 0.001 < 0.1$$

Therefore, lumped system analysis is applicable, and the error involved in this approximation is negligible.

In order to read 99 percent of the initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

For example, when $T_i$ = 0°C and $T_\infty$ = 100°C, a thermocouple is considered to have read 99 percent of this applied temperature difference when its reading indicates $T(t)$ = 99°C.

The value of the exponent $b$ is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{210 \text{ W/m}^2\cdot\text{K}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg}\cdot\text{ K})(1.67 \times 10^{-4} \text{ m})} = 0.462 \text{ s}^{-1}$$

We now substitute these values into Eq. 4–4 and obtain

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \longrightarrow \quad 0.01 = e^{-(0.462 \text{ s}^{-1})t}$$

which yields

$$t = \mathbf{10 \text{ s}}$$

Therefore, we must wait at least 10 s for the temperature of the thermocouple junction to approach within 99 percent of the initial junction-gas temperature difference.

***Discussion*** Note that conduction through the wires and radiation exchange with the surrounding surfaces affect the result, and should be considered in a more refined analysis.

### PtD *EXAMPLE 4–2* Air Cooling of Metal Plates

Metal plates ($k$ = 180 W/m·K, $\rho$ = 2800 kg/m$^3$, and $c_p$ = 880 J/kg·K) with a thickness of 2 cm exiting an oven are conveyed through a 10-m long cooling chamber at a speed of 4 cm/s (Fig. 4–11). The plates enter the cooling chamber at an initial temperature of 700°C. The air temperature in the cooling chamber is 15°C, and the plates are cooled with blowing air and the convection heat transfer coefficient is given as a function of the air velocity $h = 33V^{0.8}$, where $h$ is in W/m$^2$·K and $V$ is in m/s. To prevent any incident of thermal burn, it is necessary to design the cooling process such that the plates exit the cooling chamber at a relatively safe temperature of 50°C or less. Determine the air velocity and the heat transfer coefficient such that the temperature of the plates exiting the cooling chamber is at 50°C.

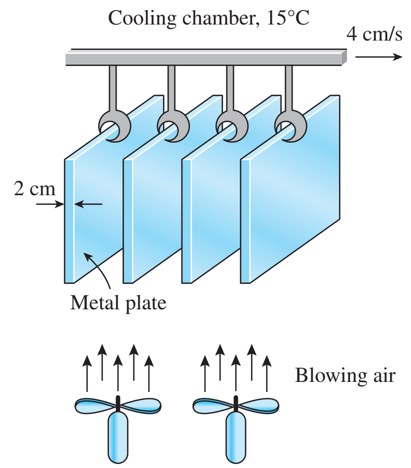



**FIGURE 4–11**
Schematic for Example 4–2.

**SOLUTION** In this example, the concepts of Prevention through Design (PtD) are applied in conjunction with lumped system analysis. Metal plates exiting an oven are being cooled by air in a cooling chamber. The air velocity and convection heat transfer coefficient that are required to cool the plates so that they exit the cooling chamber at 50°C are to be determined.

***Assumptions*** **1** The thermal properties of metal plates are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible. **4** The Biot number is Bi $<$ 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

***Properties*** The properties of the metal plates are given as $k$ = 180 W/m·K, $\rho$ = 2800 kg/m$^3$, and $c_p$ = 880 J/kg·K.

***Analysis*** The characteristic length and the Biot number of the metal plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{20 \text{ mm}}{2} = 10 \text{ mm}$$

Using the lumped system analysis,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{33V_{\text{air}}^{0.8}}{\rho c_p L_c} = \frac{33V_{\text{air}}^{0.8}}{(2800\ \text{kg/m}^3)(880\ \text{J/kg·K})(0.010\ \text{m})}$$

The duration of cooling can be determined from the cooling chamber length and the speed of the plates,

$$t = \frac{10\ \text{m}}{0.04\ \text{m/s}} = 250\ \text{s}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad b = -\frac{1}{t}\ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right]$$

$$= -\frac{1}{250\ \text{s}}\ln\left(\frac{50 - 15}{700 - 15}\right) = 0.0119\ \text{s}^{-1}$$

Thus, the air velocity and convection heat transfer coefficient necessary to cool the plates to 50°C as they exit the cooling chamber is

$$b = \frac{33V_{\text{air}}^{0.8}}{\rho c_p L_c} = 0.01195\ \text{s}^{-1}$$

$$V_{\text{air}} = \left[\frac{(0.0119\ \text{s}^{-1})(2800\ \text{kg/m}^3)(880\ \text{J/kg·K})(0.010\ \text{m})}{33}\right]^{1/0.8} = \mathbf{15.3\ m/s}$$

$$h = 33V_{\text{air}}^{0.8} = 33(15.3\ \text{m/s})^{0.8} = \mathbf{293\ W/m^2{\cdot}K}$$

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi $< 0.1$ needs to be satisfied

$$\text{Bi} = \frac{hL_c}{k} = \frac{(293\ \text{W/m}^2\text{·K})(0.010\ \text{m})}{180\ \text{W/m·K}} = 0.0163 < 0.1$$

***Discussion*** The effect of the air velocity on the temperature of the plates exiting the cooling chamber is plotted in Fig. 4–12. The figure shows that for air velocities less than 15.3 m/s the temperature of the plates stays well below 50°C which should prevent any incident of thermal burn.

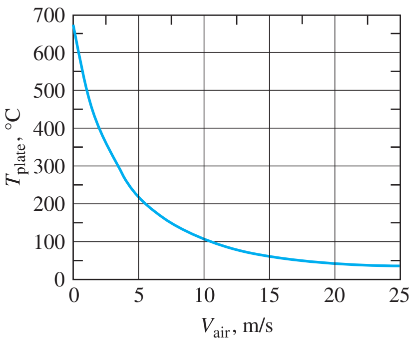



**FIGURE 4–12**
Variation of plate temperature with the air velocity at the exit of the cooling chamber.

## 4–2 ▪ TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

In Section 4–1, we considered bodies in which the variation of temperature within the body is negligible; that is, bodies that remain nearly *isothermal* during a process. Relatively small bodies of highly conductive materials approximate this behavior. In general, however, the temperature within a body changes from point to point as well as with time. In this section, we consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.

Consider a plane wall of thickness $2L$, a long cylinder of radius $r_o$, and a sphere of radius $r_o$ initially at a *uniform temperature* $T_i$, as shown in Fig. 4–13. At time $t = 0$, each geometry is placed in a large medium that is at a constant

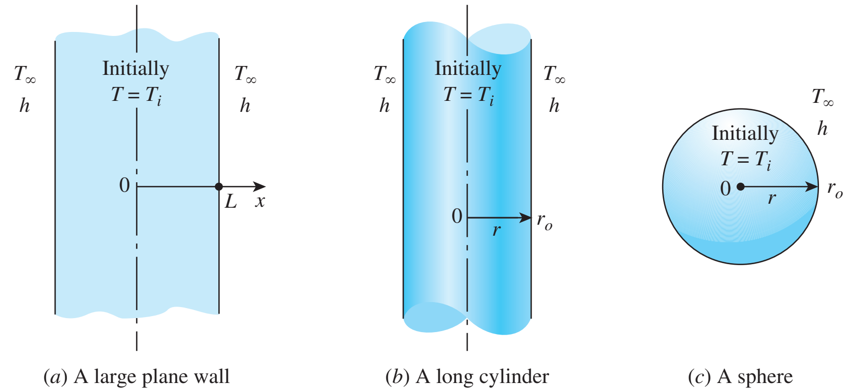



**FIGURE 4–13**
Schematic of the simple geometries in which heat transfer is one-dimensional.

temperature $T_\infty$ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a *uniform* and *constant* heat transfer coefficient $h$. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its *center plane* ($x = 0$), the cylinder is symmetric about its *centerline* ($r = 0$), and the sphere is symmetric about its *center point* ($r = 0$). We neglect *radiation* heat transfer between these bodies and their surrounding surfaces, or incorporate the radiation effect into the convection heat transfer coefficient $h$.

The variation of the temperature profile with *time* in the plane wall is illustrated in Fig. 4–14. When the wall is first exposed to the surrounding medium at $T_\infty < T_i$ at $t = 0$, the entire wall is at its initial temperature $T_i$. But the wall temperature at and near the surfaces starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a *temperature gradient* in the wall and initiates heat conduction from the inner parts of the wall toward its outer surfaces. Note that the temperature at the center of the wall remains at $T_i$ until $t = t_2$, and that the temperature profile within the wall remains symmetric at all times about the center plane. The temperature profile gets flatter and flatter as time passes as a result of heat transfer, and eventually becomes uniform at $T = T_\infty$. That is, the wall reaches *thermal equilibrium* with its surroundings. At that point, heat transfer stops since there is no longer a temperature difference. Similar discussions can be given for the long cylinder or sphere.

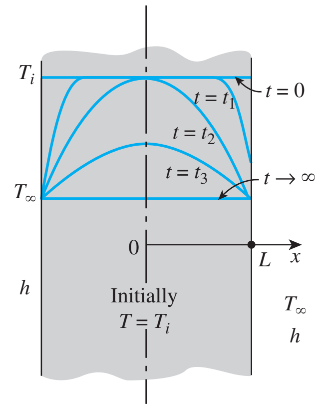



**FIGURE 4–14**
Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.

## Nondimensionalized One-Dimensional Transient Conduction Problem

The formulation of heat conduction problems for the determination of the one-dimensional transient temperature distribution in a plane wall, a cylinder, or a sphere results in a partial differential equation whose solution typically involves infinite series and transcendental equations, which are inconvenient to use. But the analytical solution provides valuable insight to the physical problem, and thus it is important to go through the steps involved. Below we demonstrate the solution procedure for the case of plane wall.

Consider a plane wall of thickness $2L$ initially at a uniform temperature of $T_i$, as shown in Fig. 4–13*a*. At time $t = 0$, the wall is immersed in a fluid at temperature $T_\infty$ and is subjected to convection heat transfer from both sides with a convection coefficient of $h$. The height and the width of the wall are large relative to its thickness, and thus heat conduction in the wall can be

approximated to be one-dimensional. Also, there is thermal symmetry about the midplane passing through $x = 0$, and thus the temperature distribution must be symmetrical about the midplane. Therefore, the value of temperature at any $-x$ value in $-L \leq x \leq 0$ at any time $t$ must be equal to the value at $+x$ in $0 \leq x \leq L$ at the same time. This means we can formulate and solve the heat conduction problem in the positive half domain $0 \leq x \leq L$, and then apply the solution to the other half.

Under the conditions of constant thermophysical properties, no heat generation, thermal symmetry about the midplane, uniform initial temperature, and constant convection coefficient, the one-dimensional transient heat conduction problem in the half-domain $0 \leq x \leq L$ of the plane wall can be expressed as (see Chapter 2)

*Differential equation:*
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4–10a}$$

*Boundary conditions:*
$$\frac{\partial T(0, t)}{\partial x} = 0 \quad \text{and} \quad -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty] \tag{4–10b}$$

*Initial condition:*
$$T(x, 0) = T_i \tag{4–10c}$$

where the property $\alpha = k/\rho c_p$ is the thermal diffusivity of the material.

We now attempt to nondimensionalize the problem by defining a dimensionless space variable $X = x/L$ and dimensionless temperature $\theta(x, t) = [T(x, t) - T_\infty]/[T_i - T_\infty]$. These are convenient choices since both $X$ and $\theta$ vary between 0 and 1. However, there is no clear guidance for the proper form of the dimensionless time variable and the $h/k$ ratio, so we will let the analysis indicate them. We note that

$$\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial (x/L)} = \frac{L}{T_i - T_\infty}\frac{\partial T}{\partial x}, \quad \frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{T_i - T_\infty}\frac{\partial^2 T}{\partial x^2} \quad \text{and} \quad \frac{\partial \theta}{\partial t} = \frac{1}{T_i - T_\infty}\frac{\partial T}{\partial t}$$

Substituting into Eqs. 4–10*a* and 4–10*b* and rearranging give

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{and} \quad \frac{\partial \theta(1, t)}{\partial X} = \frac{hL}{k}\theta(1, t) \tag{4–11}$$

Therefore, the proper form of the dimensionless time is $\tau = \alpha t/L^2$, which is called the **Fourier number** Fo (named after Jean Baptiste Joseph Fourier, *see* Fig. 1–27), and we recognize Bi $= k/hL$ as the Biot number defined in Section 4–1. Then the formulation of the one-dimensional transient heat conduction problem in a plane wall can be expressed in nondimensional form as

*Dimensionless differential equation:*
$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau} \tag{4–12a}$$

*Dimensionless BC's:*
$$\frac{\partial \theta(0, \tau)}{\partial X} = 0 \quad \text{and} \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau) \tag{4–12b}$$

*Dimensionless initial condition:*
$$\theta(X, 0) = 1 \tag{4–12c}$$

where

$$\theta(X, \tau) = \frac{T(x, t) - T_\infty}{T_i - T_\infty} \qquad \textit{Dimensionless temperature}$$

$$X = \frac{x}{L} \qquad \textit{Dimensionless distance from the center}$$

$$\text{Bi} = \frac{hL}{k} \qquad \textit{Dimensionless heat transfer coefficient} \text{ (Biot number)}$$

$$\tau = \frac{\alpha t}{L^2} = \text{Fo} \qquad \textit{Dimensionless time} \text{ (Fourier number)}$$

The heat conduction equation in cylindrical or spherical coordinates can be nondimensionalized in a similar way. Note that nondimensionalization reduces the number of independent variables and parameters from 8 to 3—from $x$, $L$, $t$, $k$, $\alpha$, $h$, $T_i$, and $T_\infty$ to $X$, Bi, and Fo (Fig. 4–15). That is,

$$\theta = f(X, \text{Bi}, \text{Fo}) \tag{4–13}$$

This makes it very practical to conduct parametric studies and avoid results in graphical form. Equation 4–13 is the generalized version of Eq. 4–4 for the lumped system analysis (no space variables). This can be shown by using the definitions of $\theta$, $\alpha$, $L_c$, Bi, and Fo in Eq. 4–4. The final result is

$$\theta = \frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} = e^{-\frac{hA_s t}{\rho V c_p}} = e^{-\text{BiFo}}$$

or $\theta = f(\text{Fo}, \text{Bi})$ which is the special case of Eq. 4–13 with no space variable.

(a) Original heat conduction problem:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial x}, \; T(x, 0) = T_i$$

$$\frac{\partial T(0, t)}{\partial x} = 0, \; -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

$$T = F(x, L, t, k, \alpha, h, T_i, T_\infty)$$

(b) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}, \; \theta(X, 0) = 1$$

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \; \frac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\theta(1, \tau)$$

$$\theta = f(X, \text{Bi}, \tau)$$

**FIGURE 4–15**
Nondimensionalization reduces the number of independent variables in one-dimensional transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

## Exact Solution of One-Dimensional Transient Conduction Problem*

The non-dimensionalized partial differential equation given in Eqs. 4–12 together with its boundary and initial conditions can be solved using several analytical and numerical techniques, including the Laplace or other transform methods, the method of separation of variables, the finite difference method, and the finite-element method. Here we use the method of **separation of variables** developed by J. Fourier in the 1820s and is based on expanding an arbitrary function (including a constant) in terms of Fourier series. The method is applied by assuming the dependent variable to be a product of a number of functions, each being a function of a single independent variable. This reduces the partial differential equation to a system of ordinary differential equations, each being a function of a single independent variable. In the case of transient conduction in a plane wall, for example, the dependent variable is the solution function $\theta(X, \tau)$, which is expressed as $\theta(X, \tau) = F(X)G(\tau)$, and the application of the method results in two ordinary differential equation, one in $X$ and the other in $\tau$.

The method is applicable if (1) the geometry is simple and finite (such as a rectangular block, a cylinder, or a sphere) so that the boundary surfaces can be described by simple mathematical functions, and (2) the differential equation and the boundary and initial conditions in their most simplified form are linear (no terms that involve products of the dependent variable or its derivatives) and involve only one nonhomogeneous term (a term without the dependent variable or its derivatives). If the formulation involves a number of nonhomogeneous terms, the problem can be split up into an equal number of simpler problems each involving only one nonhomogeneous term, and then combining the solutions by superposition.

Now we demonstrate the use of the method of separation of variables by applying it to the one-dimensional transient heat conduction problem given in

*This section can be skipped if desired without a loss of continuity.

Eqs. 4–12. First, we express the dimensionless temperature function $\theta(X, \tau)$ as a product of a function of $X$ only and a function of $\tau$ only as

$$\theta(X, \tau) = F(X)G(\tau) \tag{4–14}$$

Substituting Eq. 4–14 into Eq. 4–12*a* and dividing by the product $FG$ gives

$$\frac{1}{F}\frac{d^2F}{dX^2} = \frac{1}{G}\frac{dG}{d\tau} \tag{4–15}$$

Observe that all the terms that depend on $X$ are on the left-hand side of the equation and all the terms that depend on $\tau$ are on the right-hand side. That is, the terms that are function of different variables are *separated* (and thus the name *separation of variables*). The left-hand side of this equation is a function of $X$ only and the right-hand side is a function of only $\tau$. Considering that both $X$ and $\tau$ can be varied independently, the equality in Eq. 4–15 can hold for any value of $X$ and $\tau$ only if Eq. 4–15 is equal to a constant. Further, it must be a *negative* constant that we will indicate by $-\lambda^2$ since a positive constant will cause the function $G(\tau)$ to increase indefinitely with time (to be infinite), which is unphysical, and a value of zero for the constant means no time dependence, which is again inconsistent with the physical problem. Setting Eq. 4–15 equal to $-\lambda^2$ gives

$$\frac{d^2F}{dX^2} + \lambda^2 F = 0 \quad \text{and} \quad \frac{dG}{d\tau} + \lambda^2 G = 0 \tag{4–16}$$

whose general solutions are

$$F = C_1\cos(\lambda X) + C_2\sin(\lambda X) \quad \text{and} \quad G = C_3 e^{-\lambda^2\tau} \tag{4–17}$$

and

$$\theta = FG = C_3 e^{-\lambda^2\tau}[C_1\cos(\lambda X) + C_2\sin(\lambda X)] = e^{-\lambda^2\tau}[A\cos(\lambda X) + B\sin(\lambda X)] \tag{4–18}$$

where $A = C_1C_3$ and $B = C_2C_3$ are arbitrary constants. Note that we need to determine only $A$ and $B$ to obtain the solution of the problem.

Applying the boundary conditions in Eq. 4–12*b* gives

$$\frac{\partial\theta(0, \tau)}{\partial X} = 0 \rightarrow -e^{-\lambda^2\tau}(A\lambda\sin 0 + B\lambda\cos 0) = \rightarrow B = 0 \rightarrow \theta = Ae^{-\lambda^2\tau}\cos(\lambda X)$$

$$\frac{\partial\theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau) \rightarrow -Ae^{-\lambda^2\tau}\lambda\sin\lambda = -\text{Bi}Ae^{-\lambda^2\tau}\cos\lambda \rightarrow \lambda\tan\lambda = \text{Bi}$$

But tangent is a periodic function with a period of $\pi$, and the equation $\lambda\tan\lambda = \text{Bi}$ has the root $\lambda_1$ between 0 and $\pi$, the root $\lambda_2$ between $\pi$ and $2\pi$, the root $\lambda_n$ between $(n-1)\pi$ and $n\pi$, etc. To recognize that the transcendental equation $\lambda\tan\lambda = \text{Bi}$ has an infinite number of roots, it is expressed as

$$\lambda_n\tan\lambda_n = \text{Bi} \tag{4–19}$$

Eq. 4–19 is called the **characteristic equation** or **eigenfunction**, and its roots are called the **characteristic values** or **eigenvalues**. The characteristic equation is implicit in this case, and thus the characteristic values need to be determined numerically. Then it follows that there are an infinite number of solutions of the form $Ae^{-\lambda^2\tau}\cos(\lambda X)$, and the solution of this linear heat conduction problem is a linear combination of them,

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2\tau}\cos(\lambda_n X) \tag{4–20}$$

The constants $A_n$ are determined from the initial condition, Eq. 4–12*c*,

$$\theta(X, 0) = 1 \quad \rightarrow \quad 1 = \sum_{n=1}^{\infty} A_n \cos(\lambda_n X) \tag{4–21}$$

This is a Fourier series expansion that expresses a constant in terms of an infinite series of cosine functions. Now we multiply both sides of Eq. 4–21 by $\cos(\lambda_m X)$, and integrate from $X = 0$ to $X = 1$. The right-hand side involves an infinite number of integrals of the form $\int_0^1 \cos(\lambda_m X)\cos(\lambda_n X)dx$. It can be shown that all of these integrals vanish except when $n = m$, and the coefficient $A_n$ becomes

$$\int_0^1 \cos(\lambda_n X)dX = A_n \int_0^1 \cos^2(\lambda_n X)dX \quad \rightarrow \quad A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} \tag{4–22}$$

This completes the analysis for the solution of one-dimensional transient heat conduction problem in a plane wall. Solutions in other geometries such as a long cylinder and a sphere can be determined using the same approach. The long cylinder approximation allows the assumption of one-dimensional conduction in the radial direction. It is a reasonable approximation for cylinders having length ($L$) to radius ($r_0$) ratio, $L/r_0 \geq 10$. The results for all three geometries are summarized in Table 4–1. The solution for the plane wall is also applicable for a plane wall of thickness $L$ whose left surface at $x = 0$ is insulated and the right surface at $x = L$ is subjected to convection since this is precisely the mathematical problem we solved.

The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time. This may look intimidating at first, but there is no need to worry. As demonstrated in Fig. 4–16, the terms in the summation decline rapidly as $n$ and thus $\lambda_n$ increases because of the exponential decay function $e^{-\lambda_n^2 \tau}$. This is especially the case when the dimensionless time $\tau$ is large. Therefore, the evaluation of the first few terms of the infinite series (in this case just the first term) is usually adequate to determine the dimensionless temperature $\theta$.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \text{Bi}$$

For Bi = 5, $X = 1$, and $t = 0.2$:

| $n$ | $\lambda_n$ | $A_n$ | $\theta_n$ |
|---|---|---|---|
| 1 | 1.3138 | 1.2402 | 0.22321 |
| 2 | 4.0336 | –0.3442 | 0.00835 |
| 3 | 6.9096 | 0.1588 | 0.00001 |
| 4 | 9.8928 | –0.876 | 0.00000 |

**FIGURE 4–16**
The term in the series solution of transient conduction problems decline rapidly as $n$ and thus $\lambda_n$ increases because of the exponential decay function with the exponent $-\lambda_n \tau$.

**TABLE 4–1**

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius $r_o$ and a sphere of radius $r_o$ subjected to convention from all surfaces.*

| Geometry | Solution | $\lambda_n$'s are the roots of |
|---|---|---|
| Plane wall | $\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$ | $\lambda_n \tan \lambda_n = \text{Bi}$ |
| Cylinder | $\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$ | $\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$ |
| Sphere | $\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$ | $1 - \lambda_n \cot \lambda_n = \text{Bi}$ |

*Here $\theta = (T - T_\infty)/(T_i - T_\infty)$ is the dimensionless temperature, Bi $= hL/k$ or $hr_o/k$ is the Biot number, Fo $= \tau = \alpha t/L^2$ or $\alpha\tau/r_0^2$ is the Fourier number, and $J_0$ and $J_1$ are the Bessel functions of the first kind whose values are given in Table 4–3. Note that the characteristic length used for each geometry in the equations for the Biot and Fourier numbers is different for the exact (analytical) solution than the one used for the lumped system analysis.

## Approximate Analytical and Graphical Solutions

The analytical solution obtained above for one-dimensional transient heat conduction in a plane wall involves infinite series and implicit equations, which are difficult to evaluate. Therefore, there is clear motivation to simplify the analytical solutions and to present the solutions in *tabular* or *graphical* form using simple relations.

The dimensionless quantities defined above for a plane wall can also be used for a *cylinder* or *sphere* by replacing the space variable $x$ by $r$ and the half-thickness $L$ by the outer radius $r_o$. Note that the characteristic length in the definition of the Biot number is taken to be the *half-thickness* $L$ for the plane wall, and the *radius* $r_o$ for the long cylinder and sphere instead of $V/A$ used in lumped system analysis.

We mentioned earlier that the terms in the series solutions in Table 4–1 converge rapidly with increasing time, and for $\tau > 0.2$, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent. We are usually interested in the solution for times with $\tau > 0.2$, and thus it is very convenient to express the solution using this **one-term approximation**, given as

*Plane wall:*
$$\theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2 \tag{4–23}$$

*Cylinder:*
$$\theta_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2 \tag{4–24}$$

*Sphere:*
$$\theta_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2 \tag{4–25}$$

where the constants $A_1$ and $\lambda_1$ are functions of the Bi number only, and their values are listed in Table 4–2 against the Bi number for all three geometries. The function $J_0$ is the zeroth-order Bessel function of the first kind, whose value can be determined from Table 4–3. Noting that $\cos(0) = J_0(0) = 1$ and the limit of $(\sin x)/x$ is also 1, these relations simplify to the next ones at the center of a plane wall, cylinder, or sphere:

*Center of plane wall* ($x = 0$):
$$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4–26}$$

*Center of cylinder* ($r = 0$):
$$\theta_{0,\text{ cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4–27}$$

*Center of sphere* ($r = 0$):
$$\theta_{0,\text{ sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{4–28}$$

Comparing the two sets of equations above, we notice that the dimensionless temperatures anywhere in a plane wall, cylinder, and sphere are related to the center temperature by

$$\frac{\theta_{\text{wall}}}{\theta_{0,\text{ wall}}} = \cos\left(\frac{\lambda_1 x}{L}\right), \quad \frac{\theta_{\text{cyl}}}{\theta_{0,\text{ cyl}}} = J_0\left(\frac{\lambda_1 r}{r_o}\right), \quad \text{and} \quad \frac{\theta_{\text{sph}}}{\theta_{0,\text{ sph}}} = \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \tag{4–29}$$

which shows that time dependence of dimensionless temperature within a given geometry is the same throughout. That is, if the dimensionless center

**TABLE 4–2**

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = $hL/k$ for a plane wall of thickness $2L$, and Bi = $hr_o/k$ for a cylinder or sphere of radius $r_o$)

| | *Plane Wall* | | *Cylinder* | | *Sphere* | |
|---|---|---|---|---|---|---|
| **Bi** | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ |
| 0.01 | 0.0998 | 1.0017 | 0.1412 | 1.0025 | 0.1730 | 1.0030 |
| 0.02 | 0.1410 | 1.0033 | 0.1995 | 1.0050 | 0.2445 | 1.0060 |
| 0.04 | 0.1987 | 1.0066 | 0.2814 | 1.0099 | 0.3450 | 1.0120 |
| 0.06 | 0.2425 | 1.0098 | 0.3438 | 1.0148 | 0.4217 | 1.0179 |
| 0.08 | 0.2791 | 1.0130 | 0.3960 | 1.0197 | 0.4860 | 1.0239 |
| 0.1 | 0.3111 | 1.0161 | 0.4417 | 1.0246 | 0.5423 | 1.0298 |
| 0.2 | 0.4328 | 1.0311 | 0.6170 | 1.0483 | 0.7593 | 1.0592 |
| 0.3 | 0.5218 | 1.0450 | 0.7465 | 1.0712 | 0.9208 | 1.0880 |
| 0.4 | 0.5932 | 1.0580 | 0.8516 | 1.0931 | 1.0528 | 1.1164 |
| 0.5 | 0.6533 | 1.0701 | 0.9408 | 1.1143 | 1.1656 | 1.1441 |
| 0.6 | 0.7051 | 1.0814 | 1.0184 | 1.1345 | 1.2644 | 1.1713 |
| 0.7 | 0.7506 | 1.0918 | 1.0873 | 1.1539 | 1.3525 | 1.1978 |
| 0.8 | 0.7910 | 1.1016 | 1.1490 | 1.1724 | 1.4320 | 1.2236 |
| 0.9 | 0.8274 | 1.1107 | 1.2048 | 1.1902 | 1.5044 | 1.2488 |
| 1.0 | 0.8603 | 1.1191 | 1.2558 | 1.2071 | 1.5708 | 1.2732 |
| 2.0 | 1.0769 | 1.1785 | 1.5995 | 1.3384 | 2.0288 | 1.4793 |
| 3.0 | 1.1925 | 1.2102 | 1.7887 | 1.4191 | 2.2889 | 1.6227 |
| 4.0 | 1.2646 | 1.2287 | 1.9081 | 1.4698 | 2.4556 | 1.7202 |
| 5.0 | 1.3138 | 1.2403 | 1.9898 | 1.5029 | 2.5704 | 1.7870 |
| 6.0 | 1.3496 | 1.2479 | 2.0490 | 1.5253 | 2.6537 | 1.8338 |
| 7.0 | 1.3766 | 1.2532 | 2.0937 | 1.5411 | 2.7165 | 1.8673 |
| 8.0 | 1.3978 | 1.2570 | 2.1286 | 1.5526 | 2.7654 | 1.8920 |
| 9.0 | 1.4149 | 1.2598 | 2.1566 | 1.5611 | 2.8044 | 1.9106 |
| 10.0 | 1.4289 | 1.2620 | 2.1795 | 1.5677 | 2.8363 | 1.9249 |
| 20.0 | 1.4961 | 1.2699 | 2.2880 | 1.5919 | 2.9857 | 1.9781 |
| 30.0 | 1.5202 | 1.2717 | 2.3261 | 1.5973 | 3.0372 | 1.9898 |
| 40.0 | 1.5325 | 1.2723 | 2.3455 | 1.5993 | 3.0632 | 1.9942 |
| 50.0 | 1.5400 | 1.2727 | 2.3572 | 1.6002 | 3.0788 | 1.9962 |
| 100.0 | 1.5552 | 1.2731 | 2.3809 | 1.6015 | 3.1102 | 1.9990 |
| ∞ | 1.5708 | 1.2732 | 2.4048 | 1.6021 | 3.1416 | 2.0000 |

**TABLE 4–3**

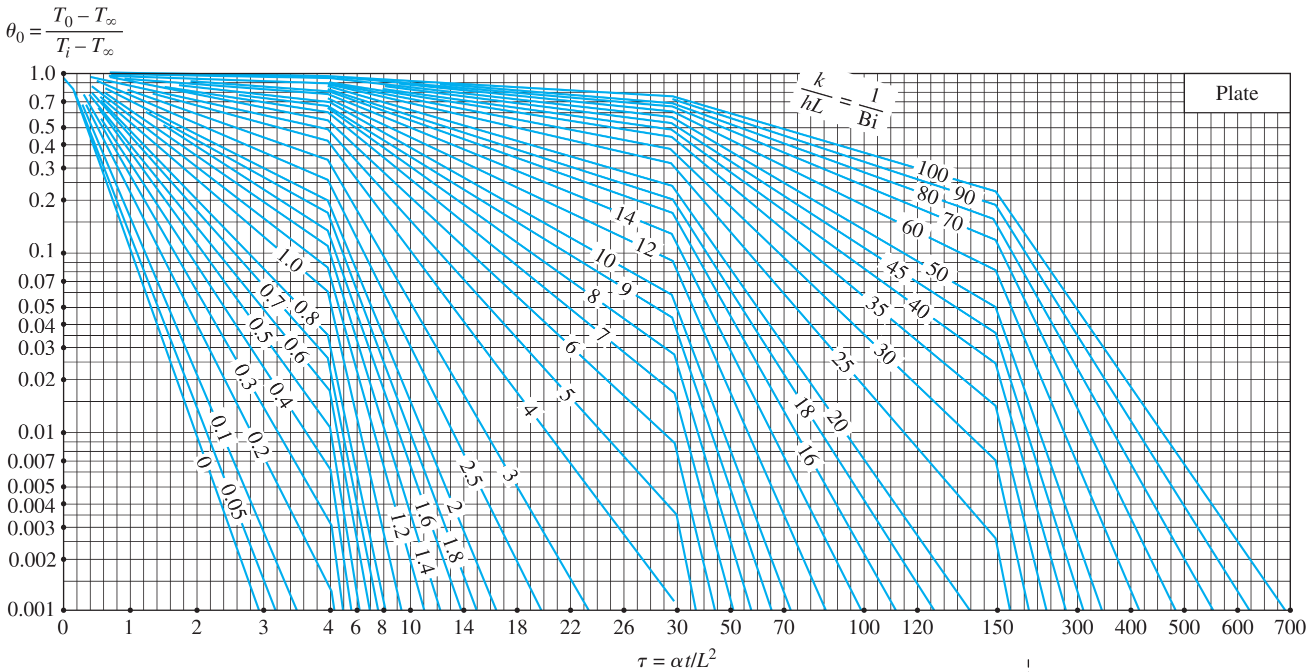
The zeroth- and first-order Bessel functions of the first kind

| $\eta$ | $J_0(\eta)$ | $J_1(\eta)$ |
|---|---|---|
| 0.0 | 1.0000 | 0.0000 |
| 0.1 | 0.9975 | 0.0499 |
| 0.2 | 0.9900 | 0.0995 |
| 0.3 | 0.9776 | 0.1483 |
| 0.4 | 0.9604 | 0.1960 |
| 0.5 | 0.9385 | 0.2423 |
| 0.6 | 0.9120 | 0.2867 |
| 0.7 | 0.8812 | 0.3290 |
| 0.8 | 0.8463 | 0.3688 |
| 0.9 | 0.8075 | 0.4059 |
| 1.0 | 0.7652 | 0.4400 |
| 1.1 | 0.7196 | 0.4709 |
| 1.2 | 0.6711 | 0.4983 |
| 1.3 | 0.6201 | 0.5220 |
| 1.4 | 0.5669 | 0.5419 |
| 1.5 | 0.5118 | 0.5579 |
| 1.6 | 0.4554 | 0.5699 |
| 1.7 | 0.3980 | 0.5778 |
| 1.8 | 0.3400 | 0.5815 |
| 1.9 | 0.2818 | 0.5812 |
| 2.0 | 0.2239 | 0.5767 |
| 2.1 | 0.1666 | 0.5683 |
| 2.2 | 0.1104 | 0.5560 |
| 2.3 | 0.0555 | 0.5399 |
| 2.4 | 0.0025 | 0.5202 |
| 2.6 | −0.0968 | 0.4708 |
| 2.8 | −0.1850 | 0.4097 |
| 3.0 | −0.2601 | 0.3391 |
| 3.2 | −0.3202 | 0.2613 |

temperature $\theta_0$ drops by 20 percent at a specified time, so does the dimensionless temperature $\theta_0$ anywhere else in the medium at the same time.

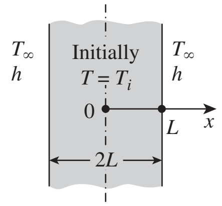
Once the Bi number is known, these relations can be used to determine the temperature anywhere in the medium. The determination of the constants $A_1$ and $\lambda_1$ usually requires interpolation. For those who prefer reading charts to interpolating, these relations are plotted and the one-term approximation solutions are presented in graphical form, known as the *transient temperature charts*. Note that the charts are sometimes difficult to read, and they are subject to reading errors. Therefore, the relations above should be preferred to the charts.

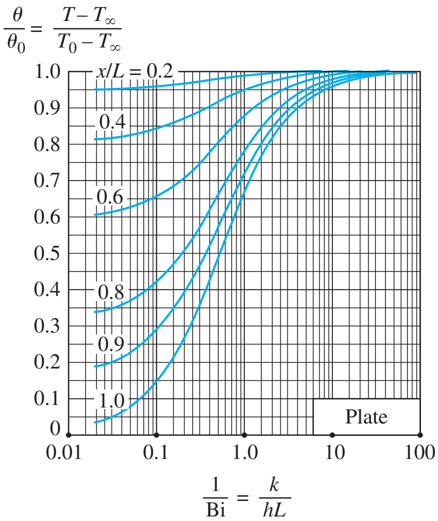
The transient temperature charts in Figs. 4–17, 4–18, and 4–19 for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called **Heisler charts**. They were supplemented in 1961 with transient




(*a*) Midplane temperature.

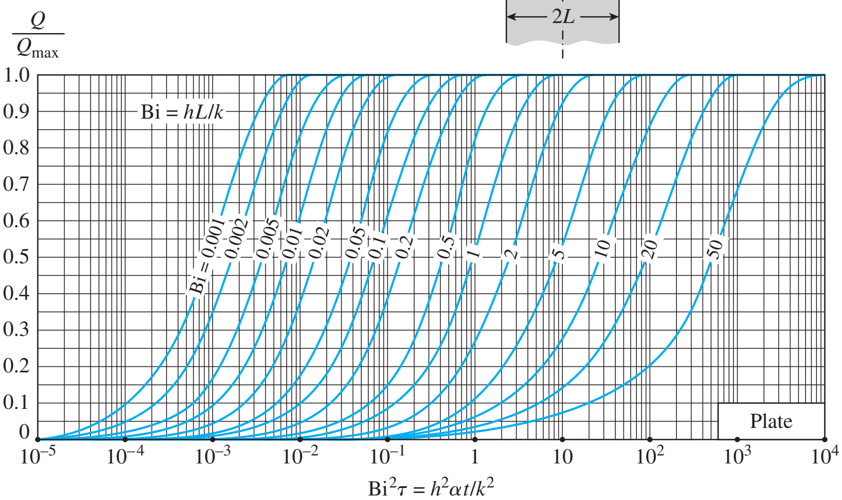
*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*




(*b*) Temperature distribution.

*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*

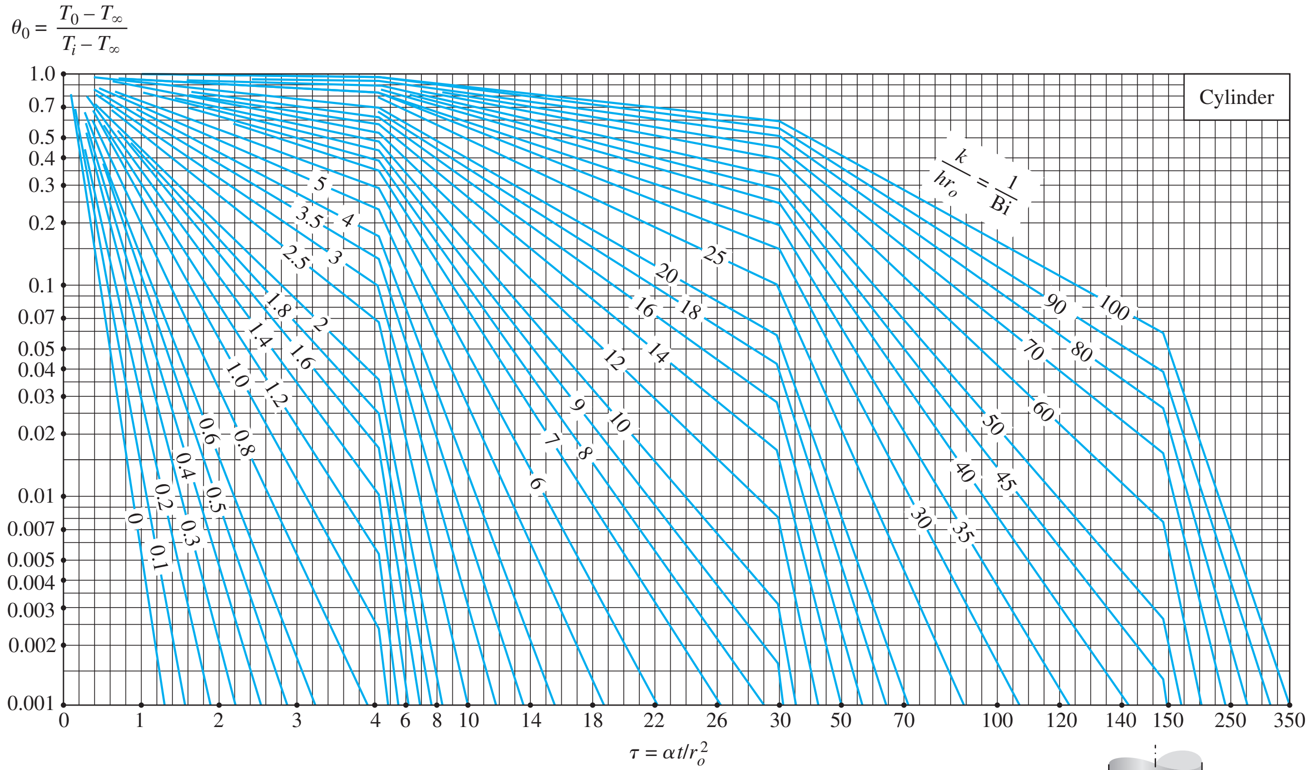



(*c*) Heat transfer.

*From H. Gröber et al.*

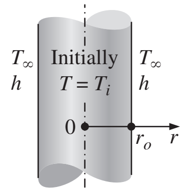
**FIGURE 4–17**

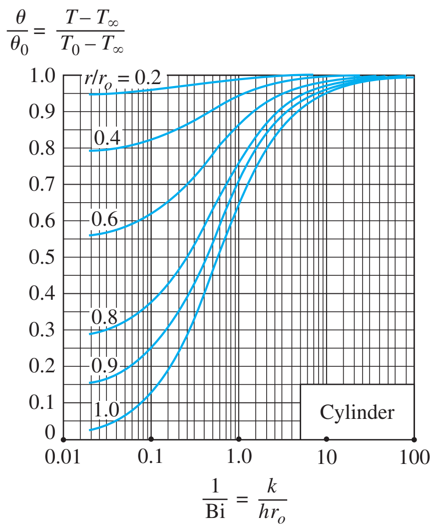
Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature $T_i$ subjected to convection from both sides to an environment at temperature $T_\infty$ with a convection coefficient of $h$.




(*a*) Centerline temperature.

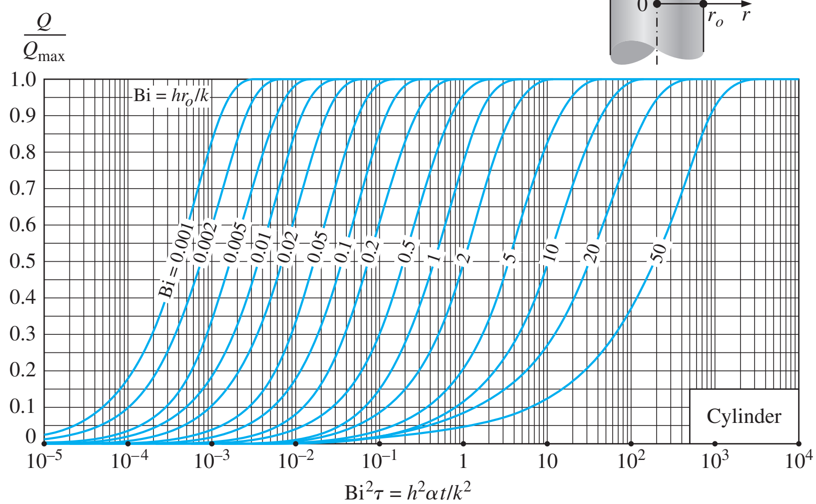
*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*




(*b*) Temperature distribution.

*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*

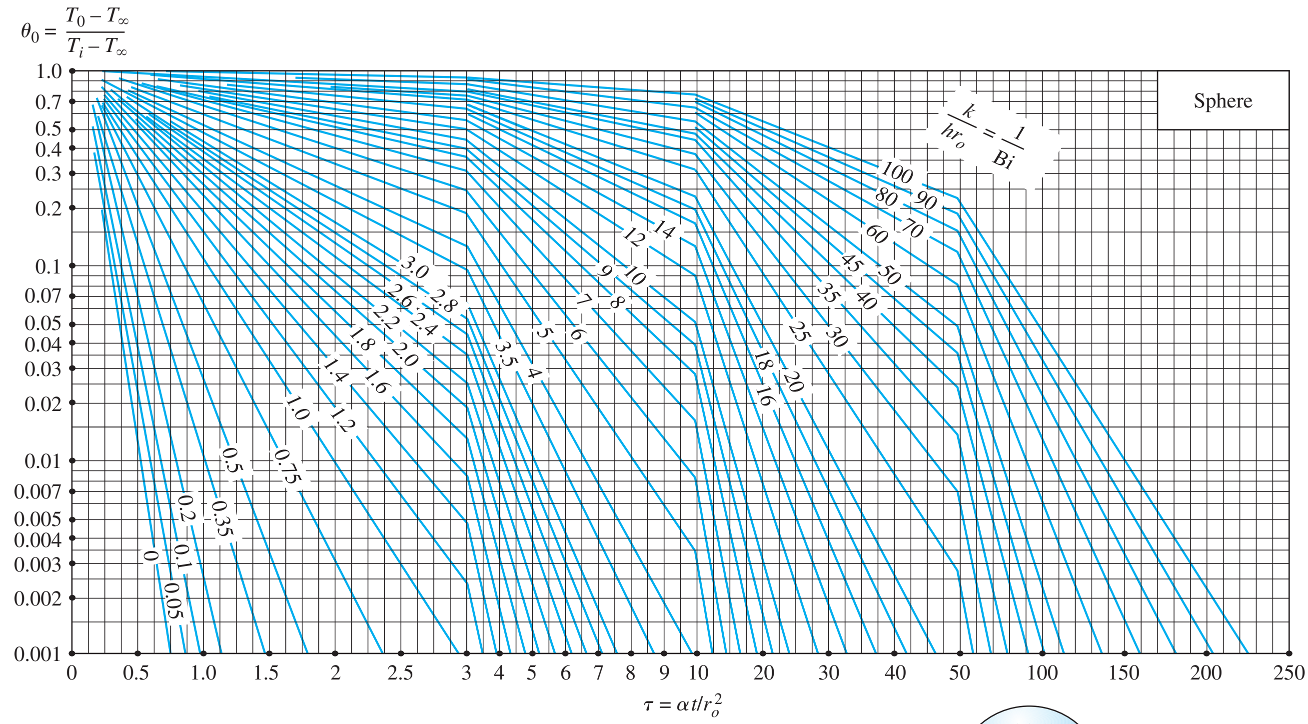


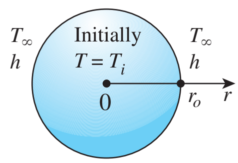

(*c*) Heat transfer.

*From H. Gröber et al.*

**FIGURE 4–18**

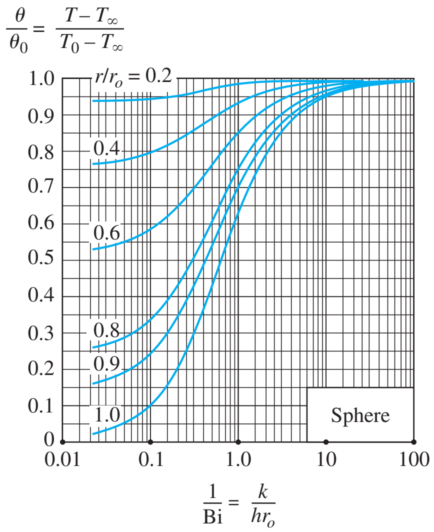
Transient temperature and heat transfer charts for a long cylinder of radius $r_o$ initially at a uniform temperature $T_i$ subjected to convection from all sides to an environment at temperature $T_\infty$ with a convection coefficient of $h$.

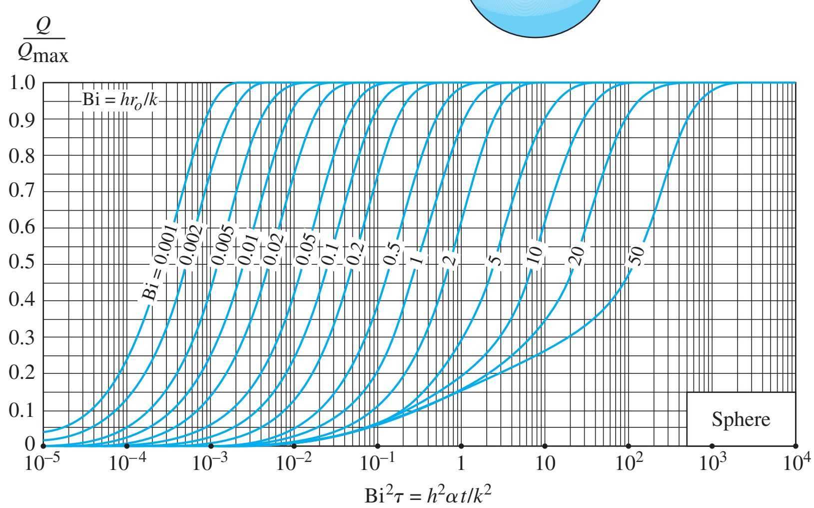



(*a*) Midpoint temperature.

*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*




(*b*) Temperature distribution.

*From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," Trans. ASME 69, 1947, pp. 227–36. Reprinted by permission of ASME International.*

(*c*) Heat transfer.

*From H. Gröber et al.*

**FIGURE 4–19**

Transient temperature and heat transfer charts for a sphere of radius $r_o$ initially at a uniform temperature $T_i$ subjected to convection from all sides to an environment at temperature $T_\infty$ with a convection coefficient of $h$.

heat transfer charts by H. Gröber. There are *three* charts associated with each geometry: the first chart is to determine the temperature $T_0$ at the *center* of the geometry at a given time $t$. The second chart is to determine the temperature at *other locations* at the same time in terms of $T_0$. The third chart is to determine the total amount of *heat transfer* up to the time $t$. These plots are valid for $\tau > 0.2$.

Note that the case $1/\mathrm{Bi} = k/hL = 0$ corresponds to $h \to \infty$, which corresponds to the case of *specified surface temperature* $T_\infty$. That is, the case in which the surfaces of the body are suddenly brought to the temperature $T_\infty$ at $t = 0$ and kept at $T_\infty$ at all times can be handled by setting $h$ to infinity (Fig. 4–20).

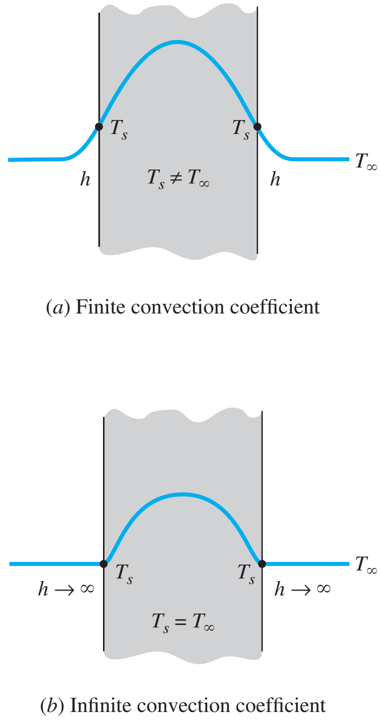



**FIGURE 4–20**
The specified surface temperature corresponds to the case of convection to an environment at $T_\infty$ with a convection coefficient $h$ that is *infinite*.

The temperature of the body changes from the initial temperature $T_i$ to the temperature of the surroundings $T_\infty$ at the end of the transient heat conduction process. Thus, the *maximum* amount of heat that a body can gain (or lose if $T_i > T_\infty$) is simply the *change* in the *energy content* of the body. That is,

$$Q_{\max} = mc_p(T_\infty - T_i) = \rho V c_p(T_\infty - T_i) \quad \text{(kJ)} \tag{4–30}$$

where $m$ is the mass, $V$ is the volume, $\rho$ is the density, and $c_p$ is the specific heat of the body. Thus, $Q_{\max}$ represents the amount of heat transfer for $t \to \infty$. The amount of heat transfer $Q$ at a finite time $t$ is obviously less than this maximum, and it can be expressed as the sum of the internal energy changes throughout the entire geometry as

$$Q = \int_V \rho c_p [T(x, t) - T_i] dV \tag{4–31}$$

where $T(x, t)$ is the temperature distribution in the medium at time $t$. Assuming constant properties, the ratio of $Q/Q_{\max}$ becomes

$$\frac{Q}{Q_{\max}} = \frac{\int_V \rho c_p [T(x, t) - T_i] dV}{\rho c_p (T_\infty - T_i) V} = \frac{1}{V}\int_V (1 - \theta) dV \tag{4–32}$$

Using the appropriate nondimensional temperature relations based on the one-term approximation for the plane wall, cylinder, and sphere, and performing the indicated integrations, we obtain the following relations for the fraction of heat transfer in those geometries:

*Plane wall:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\,\text{wall}} \frac{\sin \lambda_1}{\lambda_1} \tag{4–33}$$

*Cylinder:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1} \tag{4–34}$$

*Sphere:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3} \tag{4–35}$$

These $Q/Q_{\max}$ ratio relations based on the one-term approximation are also plotted in Figures 4–17*c*, 4–18*c*, and 4–19*c*, against the variables Bi and $h^2\alpha t/k^2$ for the large plane wall, long cylinder, and sphere, respectively. Note that once the *fraction* of heat transfer $Q/Q_{\max}$ has been determined from these charts or equations for the given $t$, the actual amount of heat transfer by that time can be evaluated by multiplying this fraction by $Q_{\max}$. A *negative* sign for $Q_{\max}$ indicates that the body is *rejecting* heat (Fig. 4–21).

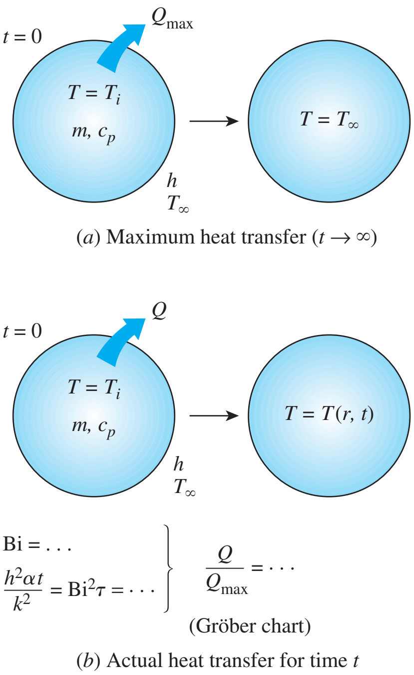



**FIGURE 4–21**
The fraction of total heat transfer $Q/Q_{max}$ up to a specified time $t$ is determined using the Gröber charts.

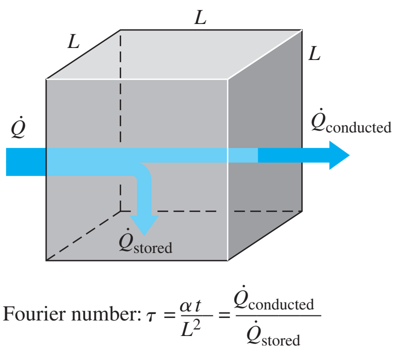



**FIGURE 4–22**
Fourier number at time $t$ can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.

The use of the Heisler/Gröber charts and the one-term solutions already discussed is limited to the conditions specified at the beginning of this section: the body is initially at a *uniform* temperature, the temperature of the medium surrounding the body and the convection heat transfer coefficient are *constant* and *uniform,* and there is no *heat generation* in the body.

We discussed the physical significance of the *Biot number* earlier and indicated that it is a measure of the relative magnitudes of the two heat transfer mechanisms: *convection* at the surface and *conduction* through the solid. A *small* value of Bi indicates that the inner resistance of the body to heat conduction is *small* relative to the resistance to convection between the surface and the fluid. As a result, the temperature distribution within the solid becomes fairly uniform, and lumped system analysis becomes applicable. Recall that when Bi $< 0.1$, the error in assuming the temperature within the body to be *uniform* is negligible.

To understand the physical significance of the *Fourier number* $\tau$ (or Fo), we express it as (Fig. 4–22)

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2\,(1/L)}{\rho c_p L^3/t}\,\frac{\Delta T}{\Delta T} = \frac{\text{The rate at which heat is } \textit{conducted} \text{ across a body of thikness } L \text{ and normal area } L^2 \text{ (and thus volume } L^3)}{\text{The rate at which heat is } \textit{stored} \text{ in a body of volume } L^3} \tag{4–36}$$

Therefore, the Fourier number is a measure of *heat conducted* through a body relative to *heat stored.* Thus, a large value of the Fourier number indicates faster propagation of heat through a body.

Perhaps you are wondering about what constitutes an infinitely large plate or an infinitely long cylinder. After all, nothing in this world is infinite. A plate whose thickness is small relative to the other dimensions can be modeled as an infinitely large plate, except very near the outer edges. But the edge effects on large bodies are usually negligible, and thus a large plane wall such as the wall of a house can be modeled as an infinitely large wall for heat transfer purposes. Similarly, a long cylinder whose diameter is small relative to its length can be analyzed as an infinitely long cylinder. The use of the transient temperature charts and the one-term solutions is illustrated in Examples 4–3, 4–4, and 4–5.

### *EXAMPLE 4–3* Boiling Eggs

An ordinary egg can be approximated as a 5-cm-diameter sphere (Fig. 4–23). The egg is initially at a uniform temperature of 5°C and is dropped into boiling water at 95°C. Taking the convection heat transfer coefficient to be $h$ = 1200 W/m²·K, determine how long it will take for the center of the egg to reach 70°C.

**SOLUTION** An egg is cooked in boiling water. The cooking time of the egg is to be determined.

***Assumptions*** **1** The egg is spherical in shape with a radius of $r_o$ = 2.5 cm. **2** Heat conduction in the egg is one-dimensional because of thermal symmetry about the midpoint. **3** The thermal properties of the egg and the heat transfer coefficient are constant. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

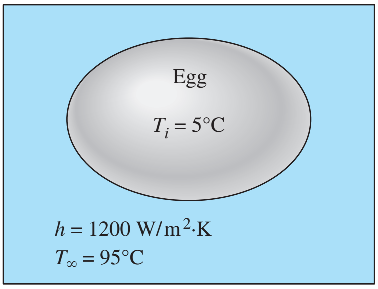



**FIGURE 4–23**
Schematic for Example 4–3.

***Properties*** The water content of eggs is about 74 percent, and thus the thermal conductivity and diffusivity of eggs can be approximated by those of water at the average temperature of (5 + 70)/2 = 37.5°C; $k = 0.627$ W/m·K and $\alpha = k/\rho c_p = 0.151 \times 10^{-6}$ m²/s (Table A–9).

***Analysis*** Egg white begins to thicken at 63°C and turns solid at 65°C. The yolk begins to thicken at 65°C and sets at 70°C. The whole egg sets at temperatures above 70°C. Therefore, the egg in this case will qualify as hard boiled. The temperature within the egg varies with radial distance as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts or the one-term solutions. Here we use the latter to demonstrate their use. The Biot number for this problem is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(1200 \text{ W/m}^2\cdot\text{K})(0.025 \text{ m})}{0.627 \text{ W/m}\cdot\text{K}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients $\lambda_1$ and $A_1$ for a sphere corresponding to this Bi are, from Table 4–2,

$$\lambda_1 = 3.0754, \quad A_1 = 1.9958$$

Substituting these and other values into Eq. 4–28 and solving for $\tau$ gives

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0754)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025 \text{ m})^2}{0.151 \times 10^{-6} \text{ m}^2\text{/s}} = 865 \text{ s} \approx \textbf{14.4 min}$$

Therefore, it will take about 15 min for the center of the egg to be heated from 5°C to 70°C.

***Discussion*** Note that the Biot number in lumped system analysis was defined differently as $\text{Bi} = hL_c/k = h(r_o/3)/k$. However, either definition can be used in determining the applicability of the lumped system analysis unless Bi ≈ 0.1.

Also note that the cooking time depends on many parameters such as the size of the egg, its temperature before cooking, the boiling temperature of water (and thus altitude), the heat transfer coefficient (and thus the level of bubble motion during boiling). Therefore, there is a considerable amount of science or a good amount of experience behind boiling eggs to the correct amount of doneness.

### *EXAMPLE 4–4* Heating of Brass Plates in an Oven

In a production facility, large brass plates of 4-cm thickness that are initially at a uniform temperature of 20°C are heated by passing them through an oven that is maintained at 500°C (Fig. 4–24). The plates remain in the oven for a period of 7 min. Taking the combined convection and radiation heat transfer coefficient to be $h = 120$ W/m²·K, determine the surface temperature of the plates when they come out of the oven.

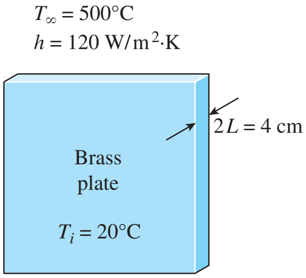



**FIGURE 4–24**
Schematic for Example 4–4.

**SOLUTION** Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

***Assumptions*** **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **2** The thermal properties of the plate and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

***Properties*** The properties of brass at room temperature are $k = 110$ W/m·K, $\rho = 8530$ kg/m$^3$, $c_p = 380$ J/kg·K, and $\alpha = 33.9 \times 10^{-6}$ m$^2$/s (Table A–3). More accurate results are obtained by using properties at average temperature.

***Analysis*** The temperature at a specified location at a given time can be determined from the Heisler charts or one-term solutions. Here we use the charts to demonstrate their use. Noting that the half-thickness of the plate is $L = 0.02$ m, from Fig. 4–17 we have

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110 \text{ W/m·K}}{(120 \text{ W/m}^2\text{·K})(0.02 \text{ m})} = 45.8 \\ \tau &= \frac{\alpha t}{L^2} = \frac{(33.9 \times 10^{-6} \text{ m}^2\text{/s})(7 \times 60 \text{ s})}{(0.02 \text{ m})^2} = 35.6 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.46$$

Also,

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hL} = 45.8 \\ \frac{x}{L} &= \frac{L}{L} = 1 \end{aligned}\right\} \frac{T - T_\infty}{T_0 - T_\infty} = 0.99$$

Therefore,

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{T - T_\infty}{T_0 - T_\infty}\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.46 \times 0.99 = 0.455$$

and

$$T = T_\infty + 0.455(T_i - T_\infty) = 500 + 0.455(20 - 500) = \mathbf{282°C}$$

Therefore, the surface temperature of the plates will be 282°C when they leave the oven.

***Discussion*** We notice that the Biot number in this case is Bi $= 1/45.8 = 0.022$, which is much less than 0.1. Therefore, we expect the lumped system analysis to be applicable. This is also evident from $(T - T_\infty)/(T_0 - T_\infty) = 0.99$, which indicates that the temperatures at the center and the surface of the plate relative to the surrounding temperature are within 1 percent of each other. Noting that the error involved in reading the Heisler charts is typically a few percent, the lumped system analysis in this case may yield just as accurate results with less effort.

The heat transfer surface area of the plate is $2A$, where $A$ is the face area of the plate (the plate transfers heat through both of its surfaces), and the volume of the plate is $V = (2L)A$, where $L$ is the half-thickness of the plate. The exponent $b$ used in the lumped system analysis is

$$\begin{aligned} b &= \frac{hA_s}{\rho c_p V} = \frac{h(2A)}{\rho c_p (2LA)} = \frac{h}{\rho c_p L} \\ &= \frac{120 \text{ W/m}^2\text{·K}}{(8530 \text{ kg/m}^3)(380 \text{ J/kg·K})(0.02 \text{ m})} = 0.00185 \text{ s}^{-1} \end{aligned}$$

Then the temperature of the plate at $t = 7$ min $= 420$ s is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 500}{20 - 500} = e^{-(0.00185\ \text{s}^{-1})(420\ \text{s})}$$

It yields

$$T(t) = 279°\text{C}$$

which is practically identical to the result obtained above using the Heisler charts. Therefore, we can use lumped system analysis with confidence when the Biot number is sufficiently small.

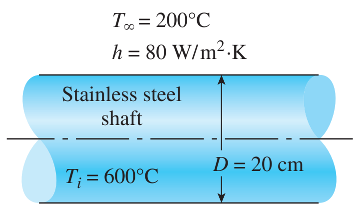



**FIGURE 4–25**
Schematic for Example 4–5.

### *EXAMPLE 4–5* Cooling of a Long Stainless Steel Cylindrical Shaft

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C (Fig. 4–25). The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of $h = 80$ W/m²·K. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

**SOLUTION** A long cylindrical shaft is allowed to cool slowly. The center temperature and the heat transfer per unit length are to be determined.

***Assumptions*** **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the centerline. **2** The thermal properties of the shaft and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

***Properties*** The properties of stainless steel 304 at room temperature are $k = 14.9$ W/m·K, $\rho = 7900$ kg/m³, $c_p = 477$ J/kg·K, and $\alpha = 3.95 \times 10^{-6}$ m²/s (Table A–3). More accurate results can be obtained by using properties at average temperature.

***Analysis*** The temperature within the shaft may vary with the radial distance $r$ as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts. Noting that the radius of the shaft is $r_o = 0.1$ m, from Fig. 4–18*a* we have

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{14.9\ \text{W/m·K}}{(80\ \text{W/m}^2\text{·K})(0.1\ \text{m})} = 1.86 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.95 \times 10^{-6}\ \text{m}^2\text{/s})(45 \times 60\ \text{s})}{(0.1\ \text{m})^2} = 1.07 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.40$$

and

$$T_0 = T_\infty + 0.4(T_i - T_\infty) = 200 + 0.4(600 - 200) = \mathbf{360°C}$$

Therefore, the center temperature of the shaft drops from 600°C to 360°C in 45 min.

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to its environment. Taking $L = 1$ m,

$$m = \rho V = \rho\pi r_o^2 L = (7900 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(1 \text{ m}) = 248.2 \text{ kg}$$
$$Q_{max} = mc_p(T_i - T_\infty) = (248.2 \text{ kg})(0.477 \text{ kJ/kg·K})(600 - 200)°\text{C}$$
$$= 47{,}350 \text{ kJ}$$

The dimensionless heat transfer ratio is determined from Fig. 4–18*c* for a long cylinder to be

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{1.86} = 0.537 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.537)^2(1.07) = 0.309 \end{aligned}\right\} \frac{Q}{Q_{max}} = 0.62$$

Therefore,

$$Q = 0.62Q_{max} = 0.62 \times (47{,}350 \text{ kJ}) = \mathbf{29{,}360 \text{ kJ}}$$

which is the total heat transfer from the shaft during the first 45 min of the cooling.

**Alternative solution** We could also solve this problem using the one-term solution relation instead of the transient charts. First we find the Biot number

$$\text{Bi} = \frac{hr_o}{k} = \frac{(80 \text{ W/m}^2\text{·K})(0.1 \text{ m})}{14.9 \text{ W/m·K}} = 0.537$$

The coefficients $\lambda_1$ and $A_1$ for a cylinder corresponding to this Bi are determined from Table 4–2 to be

$$\lambda_1 = 0.970, \quad A_1 = 1.122$$

Substituting these values into Eq. 4–27 gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = 1.122e^{-(0.970)^2(1.07)} = 0.41$$

and thus

$$T_0 = T_\infty + 0.41(T_i - T_\infty) = 200 + 0.41(600 - 200) = \mathbf{364°C}$$

The value of $J_1(\lambda_1)$ for $\lambda_1 = 0.970$ is determined from Table 4–3 to be 0.430. Then the fractional heat transfer is determined from Eq. 4–34 to be

$$\frac{Q}{Q_{max}} = 1 - 2\theta_0\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2 \times 0.41\frac{0.430}{0.970} = 0.636$$

and thus

$$Q = 0.636Q_{max} = 0.636 \times (47{,}350 \text{ kJ}) = \mathbf{30{,}120 \text{ kJ}}$$

***Discussion*** The slight difference between the two results is due to the reading error of the charts.

# 4–3 ▪ TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions, as shown in Figure 4–26. This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface. The earth, for example, can be considered to be a semi-infinite medium in determining the variation of temperature near its surface. Also, a thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation. The temperature in the core region of the wall remains unchanged in this case.

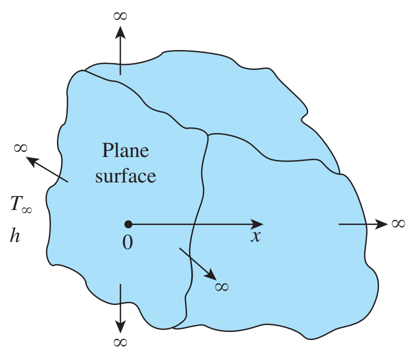



**FIGURE 4–26**
Schematic of a semi-infinite body.

For short periods of time, most bodies can be modeled as semi-infinite solids since heat does not have sufficient time to penetrate deep into the body, and the thickness of the body does not enter into the heat transfer analysis. A steel piece of any shape, for example, can be treated as a semi-infinite solid when it is quenched rapidly to harden its surface. A body whose surface is heated by a laser pulse can be treated the same way.

Consider a semi-infinite solid with constant thermophysical properties, no internal heat generation, uniform thermal conditions on its exposed surface, and initially a uniform temperature of $T_i$ throughout. Heat transfer in this case occurs only in the direction normal to the surface (the $x$ direction), and thus it is one-dimensional. Differential equations are independent of the boundary or initial conditions, and thus Eq. 4–10*a* for one-dimensional transient conduction in Cartesian coordinates applies. The depth of the solid is large ($x \to \infty$) compared to the depth that heat can penetrate, and these phenomena can be expressed mathematically as a boundary condition as $T(x \to \infty, t) = T_i$.

Heat conduction in a semi-infinite solid is governed by the thermal conditions imposed on the exposed surface, and thus the solution depends strongly on the boundary condition at $x = 0$. Below we present a detailed analytical solution for the case of constant temperature $T_s$ on the surface, and give the results for other more complicated boundary conditions. When the surface temperature is changed to $T_s$ at $t = 0$ and held constant at that value at all times, the formulation of the problem can be expressed as

*Differential equation:*
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4–37a}$$

*Boundary conditions:*
$$T(0, t) = T_s \quad \text{and} \quad T(x \to \infty, t) = T_i \tag{4–37b}$$

*Initial condition:*
$$T(x, 0) = T_i \tag{4–37c}$$

The separation of variables technique does not work in this case since the medium is infinite. But another clever approach that converts the partial differential equation into an ordinary differential equation by combining the two independent variables $x$ and $t$ into a single variable $\eta$, called the **similarity variable**, works well. For transient conduction in a semi-infinite medium, it is defined as

Similarity variable:
$$\eta = \frac{x}{\sqrt{4\alpha t}} \tag{4–38}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{and} \quad \eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = \frac{-x}{2t\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left(\frac{\partial T}{\partial x}\right)\frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t}\frac{d^2 T}{d\eta^2}$$

**FIGURE 4–27**
Transformation of variables in the derivatives of the heat conduction equation by the use of chain rule.

Assuming $T = T(\eta)$ (to be verified) and using the chain rule, all derivatives in the heat conduction equation can be transformed into the new variable, as shown in Fig. 4–27. Noting that $\eta = 0$ at $x = 0$ and $\eta \to \infty$ as $x \to \infty$ (and also at $t = 0$) and substituting into Eqs. 4–37 give, after simplification,

$$\frac{d^2 T}{d\eta^2} = -2\eta \frac{dT}{d\eta} \tag{4–39a}$$

$$T(0) = T_s \quad \text{and} \quad T(\eta \to \infty) = T_i \tag{4–39b}$$

Note that the second boundary condition and the initial condition result in the same boundary condition. Both the transformed equation and the boundary conditions depend on $\eta$ only and are independent of $x$ and $t$. Therefore, transformation is successful, and $\eta$ is indeed a similarity variable.

To solve the 2nd order ordinary differential equation in Eqs. 4–39, we define a new variable $w$ as $w = dT/d\eta$. This reduces Eq. 4–39$a$ into a first order differential equation than can be solved by separating variables,

$$\frac{dw}{d\eta} = -2\eta w \quad \rightarrow \quad \frac{dw}{w} = -2\eta d\eta \quad \rightarrow \quad \ln w = -\eta^2 + C_0 \rightarrow w = C_1 e^{-\eta^2}$$

where $C_1 = \ln C_0$. Back substituting $w = dT/d\eta$ and integrating again,

$$T = C_1 \int_0^{\eta} e^{-u^2}\, du + C_2 \tag{4–40}$$

where $u$ is a dummy integration variable. The boundary condition at $\eta = 0$ gives $C_2 = T_s$, and the one for $\eta \to \infty$ gives

$$T_i = C_1 \int_0^{\infty} e^{-u^2}\, du + C_2 = C_1 \frac{\sqrt{\pi}}{2} + T_S \rightarrow C_1 = \frac{2(T_i - T_S)}{\sqrt{\pi}} \tag{4–41}$$

Substituting the $C_1$ and $C_2$ expressions into Eq. 4–40 and rearranging, the variation of temperature becomes

$$\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-u^2} du = \text{erf}(\eta) = 1 - \text{erfc}(\eta) \tag{4–42}$$

where the mathematical functionsrom

$$\text{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-u^2} du \quad \text{and} \quad \text{erfc}(\eta) = 1 - \text{erf}(\eta) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-u^2} du \tag{4–43}$$

are called the **error function** and the **complementary error function**, respectively, of argument $\eta$ (Fig. 4–28). Despite its simple appearance, the integral in the definition of the error function cannot be performed analytically. Therefore, the function erfc($\eta$) is evaluated numerically for different values of $\eta$, and the results are listed in Table 4–4.

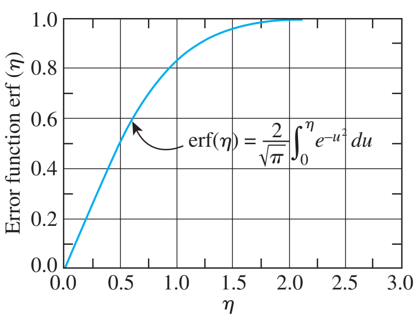



**FIGURE 4–28**
Error function is a standard mathematical function, just like the sinus and tangent functions, whose value varies between 0 and 1.

**TABLE 4–4**

The complementary error function

| $\eta$ | erfc ($\eta$) | $\eta$ | erfc ($\eta$) | $\eta$ | erfc ($\eta$) | $\eta$ | erfc ($\eta$) | $\eta$ | erfc ($\eta$) | $\eta$ | erfc ($\eta$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.00000 | 0.38 | 0.5910 | 0.76 | 0.2825 | 1.14 | 0.1069 | 1.52 | 0.03159 | 1.90 | 0.00721 |
| 0.02 | 0.9774 | 0.40 | 0.5716 | 0.78 | 0.2700 | 1.16 | 0.10090 | 1.54 | 0.02941 | 1.92 | 0.00662 |
| 0.04 | 0.9549 | 0.42 | 0.5525 | 0.80 | 0.2579 | 1.18 | 0.09516 | 1.56 | 0.02737 | 1.94 | 0.00608 |
| 0.06 | 0.9324 | 0.44 | 0.5338 | 0.82 | 0.2462 | 1.20 | 0.08969 | 1.58 | 0.02545 | 1.96 | 0.00557 |
| 0.08 | 0.9099 | 0.46 | 0.5153 | 0.84 | 0.2349 | 1.22 | 0.08447 | 1.60 | 0.02365 | 1.98 | 0.00511 |
| 0.10 | 0.8875 | 0.48 | 0.4973 | 0.86 | 0.2239 | 1.24 | 0.07950 | 1.62 | 0.02196 | 2.00 | 0.00468 |
| 0.12 | 0.8652 | 0.50 | 0.4795 | 0.88 | 0.2133 | 1.26 | 0.07476 | 1.64 | 0.02038 | 2.10 | 0.00298 |
| 0.14 | 0.8431 | 0.52 | 0.4621 | 0.90 | 0.2031 | 1.28 | 0.07027 | 1.66 | 0.01890 | 2.20 | 0.00186 |
| 0.16 | 0.8210 | 0.54 | 0.4451 | 0.92 | 0.1932 | 1.30 | 0.06599 | 1.68 | 0.01751 | 2.30 | 0.00114 |
| 0.18 | 0.7991 | 0.56 | 0.4284 | 0.94 | 0.1837 | 1.32 | 0.06194 | 1.70 | 0.01612 | 2.40 | 0.00069 |
| 0.20 | 0.7773 | 0.58 | 0.4121 | 0.96 | 0.1746 | 1.34 | 0.05809 | 1.72 | 0.01500 | 2.50 | 0.00041 |
| 0.22 | 0.7557 | 0.60 | 0.3961 | 0.98 | 0.1658 | 1.36 | 0.05444 | 1.74 | 0.01387 | 2.60 | 0.00024 |
| 0.24 | 0.7343 | 0.62 | 0.3806 | 1.00 | 0.1573 | 1.38 | 0.05098 | 1.76 | 0.01281 | 2.70 | 0.00013 |
| 0.26 | 0.7131 | 0.64 | 0.3654 | 1.02 | 0.1492 | 1.40 | 0.04772 | 1.78 | 0.01183 | 2.80 | 0.00008 |
| 0.28 | 0.6921 | 0.66 | 0.3506 | 1.04 | 0.1413 | 1.42 | 0.04462 | 1.80 | 0.01091 | 2.90 | 0.00004 |
| 0.30 | 0.6714 | 0.68 | 0.3362 | 1.06 | 0.1339 | 1.44 | 0.04170 | 1.82 | 0.01006 | 3.00 | 0.00002 |
| 0.32 | 0.6509 | 0.70 | 0.3222 | 1.08 | 0.1267 | 1.46 | 0.03895 | 1.84 | 0.00926 | 3.20 | 0.00001 |
| 0.34 | 0.6306 | 0.72 | 0.3086 | 1.10 | 0.1198 | 1.48 | 0.03635 | 1.86 | 0.00853 | 3.40 | 0.00000 |
| 0.36 | 0.6107 | 0.74 | 0.2953 | 1.12 | 0.1132 | 1.50 | 0.03390 | 1.88 | 0.00784 | 3.60 | 0.00000 |

Knowing the temperature distribution, the heat flux at the surface can be determined from the Fourier's law to be

$$\dot{q}_s = -k\frac{\partial T}{\partial x}\bigg|_{x=0} = -k\frac{dT}{d\eta}\frac{\partial \eta}{\partial x}\bigg|_{\eta=0} = -kC_1 e^{-\eta^2}\frac{1}{\sqrt{4\alpha t}}\bigg|_{\eta=0} = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \qquad \text{(4–44)}$$

The solutions in Eqs. 4–42 and 4–44 correspond to the case when the temperature of the exposed surface of the medium is suddenly raised (or lowered) to $T_s$ at $t = 0$ and is maintained at that value at all times. The specified surface temperature case is closely approximated in practice when condensation or boiling takes place on the surface. Using a similar approach or the Laplace transform technique, analytical solutions can be obtained for other boundary conditions on the surface, with the following results.

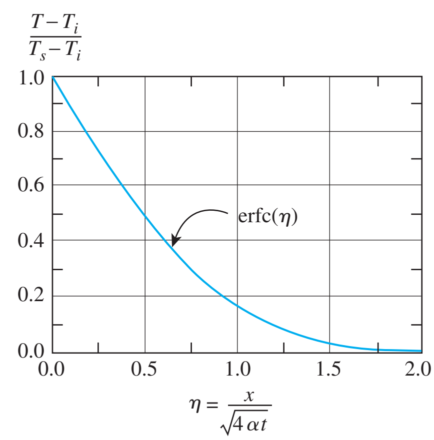



**FIGURE 4–29**

Dimensionless temperature distribution for transient conduction in a semi-infinite solid whose surface is maintained at a constant temperature $T_s$.

**Case 1: Specified Surface Temperature, $T_s$ = constant (Fig. 4–29).**

$$\frac{T(x, t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \qquad \text{(4–45)}$$

**Case 2: Specified Surface Heat Flux, $\dot{q}_s$ = constant.**

$$T(x, t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right] \qquad \text{(4–46)}$$

**Case 3: Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0, t)]$.**

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right) \tag{4–47}$$

**Case 4: Energy Pulse at Surface, $e_s$ = constant.**

Energy in the amount of $e_s$ per unit surface area (in J/m$^2$) is supplied to the semi-infinite body instantaneously at time $t = 0$ (by a laser pulse, for example), and the entire energy is assumed to enter the body, with no heat loss from the surface.

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}}\exp\left(-\frac{x^2}{4\alpha t}\right) \tag{4–48}$$

Note that Cases 1 and 3 are closely related. In Case 1, the surface $x = 0$ is brought to a temperature $T_s$ at time $t = 0$, and kept at that value at all times. In Case 3, the surface is exposed to convection by a fluid at a constant temperature $T_\infty$ with a heat transfer coefficient $h$.

The solutions for all four cases are plotted in Fig. 4–30 for a representative case using a large cast iron block initially at 0°C throughout. In Case 1, the surface temperature remains constant at the specified value of $T_s$, and temperature increases gradually within the medium as heat penetrates deeper into the solid. Note that during initial periods only a thin slice near the surface is affected by heat transfer. Also, the temperature gradient at the surface and thus the rate of heat transfer into the solid decreases with time. In Case 2, heat is continually supplied to the solid, and thus the temperature within the solid, including the surface, increases with time. This is also the case with convection (Case 3), except that the surrounding fluid temperature $T_\infty$ is the highest temperature that the solid body can rise to. In Case 4, the surface is subjected to an instant burst of heat supply at time $t = 0$, such as heating by a laser pulse, and then the surface is covered with insulation. The result is an instant rise in surface temperature, followed by a temperature drop as heat is conducted deeper into the solid. Note that the temperature profile is always normal to the surface at all times. (Why?)

The variation of temperature with position and time in a semi-infinite solid subjected to convection heat transfer is plotted in Fig. 4–31 for the nondimensionalized temperature against the dimensionless similarity variable $\eta = x/\sqrt{4\alpha t}$ for various values of the parameter $h\sqrt{\alpha t}/k$. Although the graphical solution given in Fig. 4–31 is simply a plot of the exact analytical solution, it is subject to reading errors, and thus is of limited accuracy compared to the analytical solution. Also, the values on the vertical axis of Fig. 4–31 correspond to $x = 0$, and thus represent the surface temperature. The curve $h\sqrt{\alpha t}/k = \infty$ corresponds to $h \to \infty$, which corresponds to the case of *specified temperature* $T_\infty$ at the surface at $x = 0$. That is, the case in which the surface of the semi-infinite body is suddenly brought to temperature $T_\infty$ at $t = 0$ and kept at $T_\infty$ at all times can be handled by setting $h$ to infinity. For a *finite* heat transfer coefficient $h$, the surface temperature approaches the fluid temperature $T_\infty$ as the time $t$ approaches infinity.

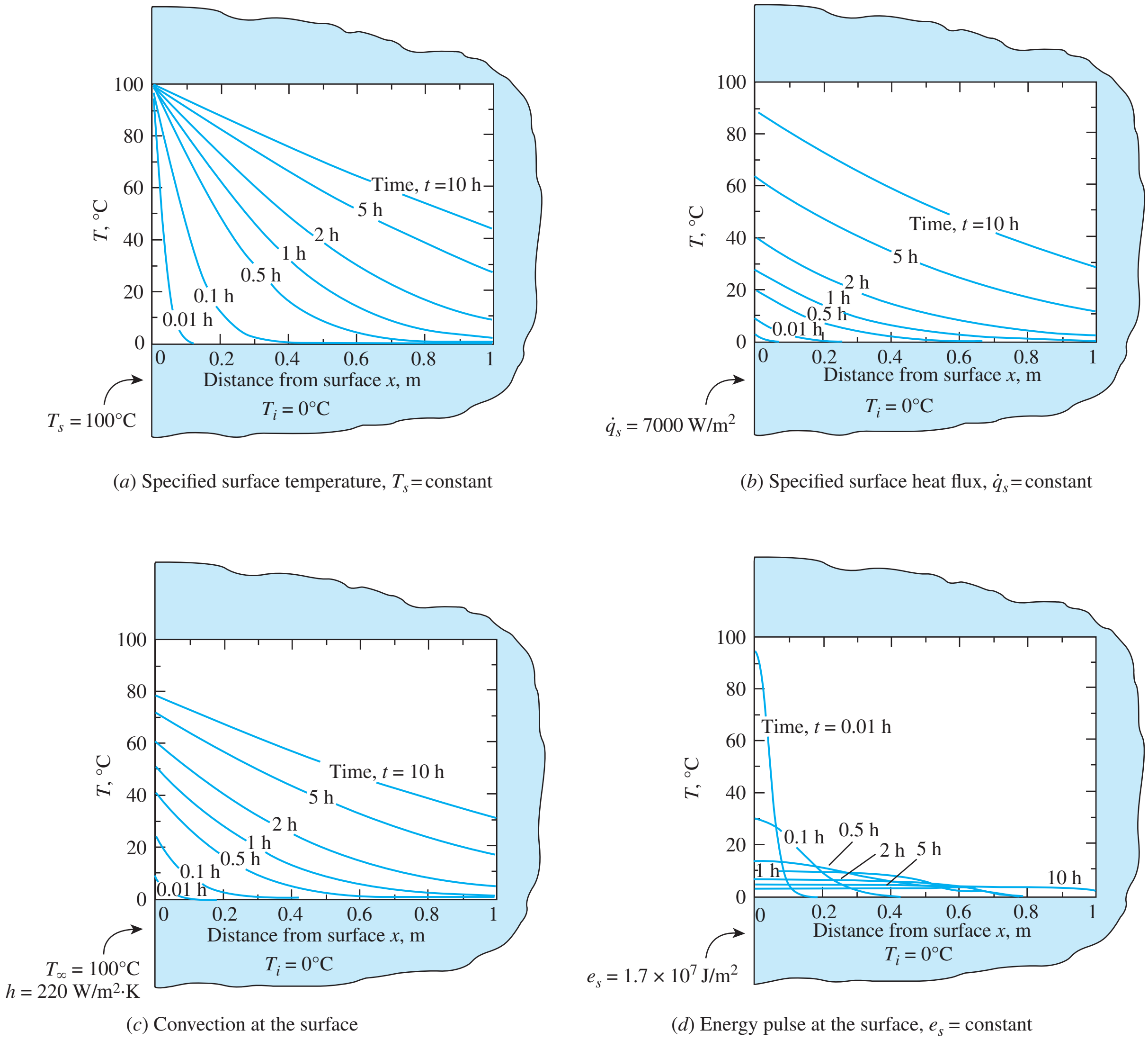



**FIGURE 4–30**
Variations of temperature with position and time in a large cast iron block ($\alpha = 2.31 \times 10^{-5}$ m$^2$/s, $k = 80.2$ W/m·K) initially at 0 °C under different thermal conditions on the surface.

## Contact of Two Semi-Infinite Solids

When two large bodies $A$ and $B$, initially at uniform temperatures $T_{A,i}$ and $T_{B,i}$ are brought into contact, they instantly achieve temperature equality at the contact surface (temperature equality is achieved over the entire surface if the contact resistance is negligible). If the two bodies are of the same material with constant properties, thermal symmetry requires the contact surface temperature to be the arithmetic average, $T_s = (T_{A,i} + T_{B,i})/2$ and to remain constant at that value at all times.

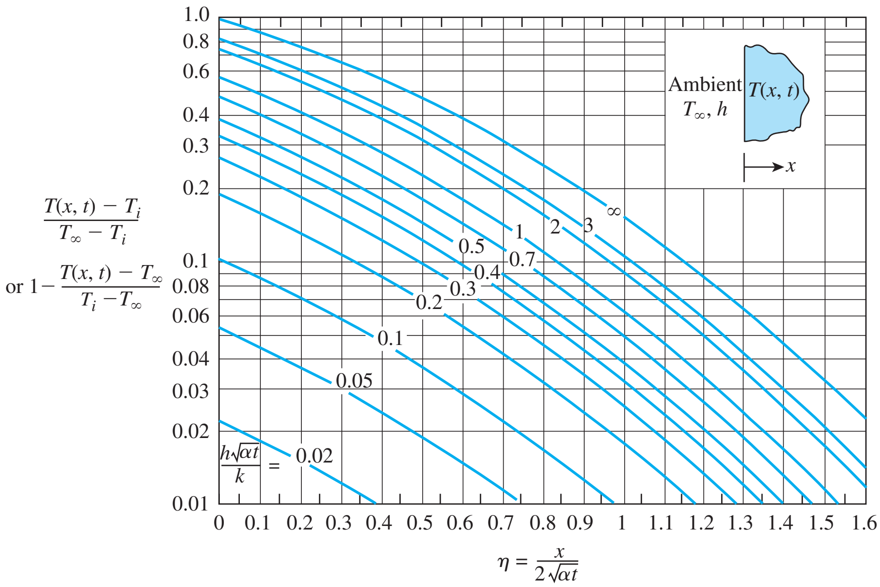



**FIGURE 4–31**
Variation of temperature with position and time in a semi-infinite solid initially at temperature $T_i$ subjected to convection to an environment at $T_\infty$ with a convection heat transfer coefficient of $h$ (plotted using EES).

If the bodies are of different materials, they still achieve a temperature equality, but the surface temperature $T_s$ in this case will be different than the arithmetic average. Noting that both bodies can be treated as semi-infinite solids with the same specified surface temperature, the energy balance on the contact surface gives, from Eq. 4–45,

$$\dot{q}_{s,A} = \dot{q}_{s,B} \rightarrow -\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi\alpha_A t}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi\alpha_B t}} \rightarrow \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \sqrt{\frac{(k\rho c_p)_B}{(k\rho c_p)_A}}$$

Then $T_s$ is determined to be (Fig. 4–32)

$$T_s = \frac{\sqrt{(k\rho c_p)_A}T_{A,i} + \sqrt{(k\rho c_p)_B}T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}} \tag{4–49}$$

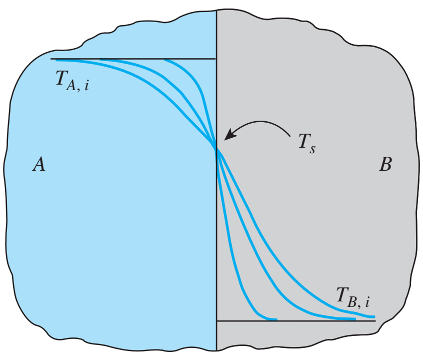



**FIGURE 4–32**
Contact of two semi-infinite solids of different initial temperatures.

Therefore, the interface temperature of two bodies brought into contact is dominated by the body with the larger $k\rho c_p$. This also explains why a metal at room temperature feels colder than wood at the same temperature. At room temperature, the $\sqrt{k\rho c_p}$ value is 24 kJ/m$^2$·K for aluminum, 0.38 kJ/m$^2$·K for wood, and 1.1 kJ/m$^2$·K for the human flesh. Using Eq. 4–49, it can be shown that when a person with a skin temperature of 35°C touches an aluminum block and then a wood block both at 15°C, the contact surface temperature will be 15.9°C in the case of aluminum and 30°C in the case of wood.

### EXAMPLE 4–6 Minimum Burial Depth of Water Pipes to Avoid Freezing

In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as an insulation to protect the water from subfreezing temperatures in winter.

The ground at a particular location is covered with snow pack at −10°C for a continuous period of three months, and the average soil properties at that location are $k = 0.4$ W/m·K and $\alpha = 0.15 \times 10^{-6}$ m²/s (Fig. 4–33). Assuming an initial uniform temperature of 15°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

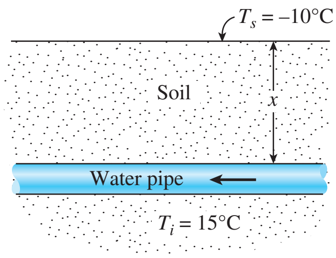



**FIGURE 4–33**
Schematic for Example 4–6.

**SOLUTION** The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

***Assumptions*** **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium. **2** The thermal properties of the soil are constant.

***Properties*** The properties of the soil are as given in the problem statement.

***Analysis*** The temperature of the soil surrounding the pipes will be 0°C after three months in the case of minimum burial depth. Therefore, from Fig. 4–31, we have

$$\left.\begin{aligned} &\frac{h\sqrt{\alpha t}}{k} = \infty \qquad \text{(since } h \to \infty\text{)} \\ &\frac{T(x, t) - T_i}{T_\infty - T_i} = \frac{0 - 15}{-10 - 15} = 0.6 \end{aligned}\right\} \eta = \frac{x}{2\sqrt{\alpha t}} = 0.36$$

We note that

$$t = (90 \text{ days})(24 \text{ h/day})(3600 \text{ s/h}) = 7.78 \times 10^6 \text{ s}$$

and thus

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.36\sqrt{(0.15 \times 10^{-6} \text{ m}^2\text{/s})(7.78 \times 10^6 \text{ s})} = \mathbf{0.78 \text{ m}}$$

Therefore, the water pipes must be buried to a depth of at least 78 cm to avoid freezing under the specified harsh winter conditions.

**ALTERNATIVE SOLUTION** The solution of this problem could also be determined from Eq. 4–45:

$$\frac{T(x, t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \longrightarrow \frac{0 - 15}{-10 - 15} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = 0.60$$

The argument that corresponds to this value of the complementary error function is determined from Table 4–4 to be $\eta = 0.37$. Therefore,

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.37\sqrt{(0.15 \times 10^{-6} \text{ m}^2\text{/s})(7.78 \times 10^6 \text{ s})} = \mathbf{0.80 \text{ m}}$$

Again, the slight difference is due to the reading error of the chart.

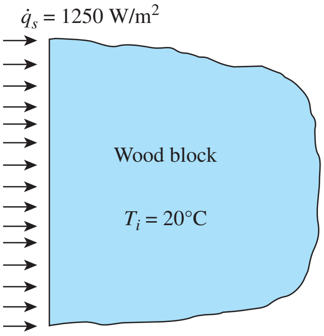



**FIGURE 4–34**
Schematic for Example 4–7.

### *EXAMPLE 4–7* Surface Temperature Rise of Heated Blocks

A thick black-painted wood block at 20°C is subjected to constant solar heat flux of 1250 W/m$^2$ (Fig. 4–34). Determine the exposed surface temperature of the block after 20 minutes. What would your answer be if the block were made of aluminum?

**SOLUTION** A wood block is subjected to solar heat flux. The surface temperature of the block is to be determined, and to be compared to the value for an aluminum block.

***Assumptions*** **1** All incident solar radiation is absorbed by the block. **2** Heat loss from the block is disregarded (and thus the result obtained is the maximum temperature). **3** The block is sufficiently thick to be treated as a semi-infinite solid, and the properties of the block are constant.

***Properties*** Thermal conductivity and diffusivity values at room temperature are $k = 0.159$ W/m·K and $\alpha = k/\rho c_p = 1.75 \times 10^{-7}$ m$^2$/s for hardwoods (Table A–5) and $k = 237$ W/m·K and $\alpha = 9.71 \times 10^{-5}$ m$^2$/s for pure aluminum (Table A–3).

***Analysis*** This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the surface temperature can be expressed from Eq. 4–46 as

$$T_s = T(0, t) = T_i + \frac{\dot{q}_s}{k}\sqrt{\frac{4\alpha t}{\pi}}$$

Substituting the given values, the surface temperatures for both the wood and aluminum blocks are determined to be

$$T_{s,\,\text{wood}} = 20°\text{C} + \frac{1250\,\text{W/m}^2}{0.159\,\text{W/m·K}}\sqrt{\frac{4(1.75 \times 10^{-7}\,\text{m}^2\text{/s})(20 \times 60\,\text{s})}{\pi}} = \mathbf{149°C}$$

$$T_{s,\,\text{Al}} = 20°\text{C} + \frac{1250\,\text{W/m}^2}{237\,\text{W/m·K}}\sqrt{\frac{4(9.71 \times 10^{-5}\,\text{m}^2\text{/s})(20 \times 60\,\text{s})}{\pi}} = \mathbf{22.0°C}$$

Note that thermal energy supplied to the wood accumulates near the surface because of the low conductivity and diffusivity of wood, causing the surface temperature to rise to high values. Metals, on the other hand, conduct the heat they receive to inner parts of the block because of their high conductivity and diffusivity, resulting in minimal temperature rise at the surface. In reality, both temperatures will be lower because of heat losses.

***Discussion*** The temperature profiles for both wood and aluminum blocks at $t$ = 20 min are evaluated and plotted in Fig. 4–35 using EES. At a depth of $x$ = 0.044 m the temperature in both blocks is 21.8°C. At a depth of 0.08 m, the temperatures become 20.0°C for wood and 21.6°C for aluminum block, which confirms that heat penetrates faster and further in metals compared to nonmetals.

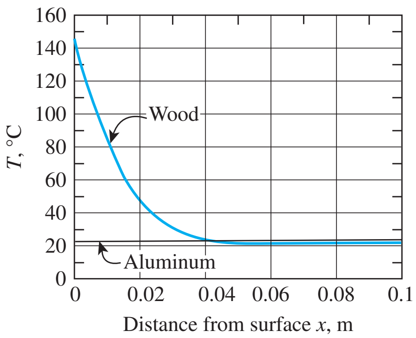



**FIGURE 4–35**
Variation of temperature within the wood and aluminum blocks at $t$ = 20 min.

## 4–4 ▪ TRANSIENT HEAT CONDUCTION IN MULTIDIMENSIONAL SYSTEMS

The transient temperature charts and analytical solutions presented earlier can be used to determine the temperature distribution and heat transfer in *one-dimensional* heat conduction problems associated with a large plane wall, a

long cylinder, a sphere, and a semi-infinite medium. Using a superposition approach called the **product solution**, these charts and solutions can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that *all* surfaces of the solid are subjected to convection to the *same* fluid at temperature $T_\infty$, with the *same* heat transfer coefficient $h$, and the body involves no heat generation (Fig. 4–36). The solution in such multidimensional geometries can be expressed as the *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.

Consider a *short cylinder* of height $a$ and radius $r_o$ initially at a uniform temperature $T_i$. There is no heat generation in the cylinder. At time $t = 0$, the cylinder is subjected to convection from all surfaces to a medium at temperature $T_\infty$ with a heat transfer coefficient $h$. The temperature within the cylinder will change with $x$ as well as $r$ and time $t$ since heat transfer occurs from the top and bottom of the cylinder as well as its side surfaces. That is, $T = T(r, x, t)$ and thus this is a two-dimensional transient heat conduction problem. When the properties are assumed to be constant, it can be shown that the solution of this two-dimensional problem can be expressed as

$$\left(\frac{T(r, x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}} \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}} \tag{4–50}$$

That is, the solution for the two-dimensional short cylinder of height $a$ and radius $r_o$ is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness $a$ and the long cylinder of radius $r_o$, which are the two geometries whose intersection is the short cylinder, as shown in Fig. 4–37. We generalize this as follows: *the solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.*

For convenience, the one-dimensional solutions are denoted by

$$\theta_{\text{wall}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\\text{wall}}}$$

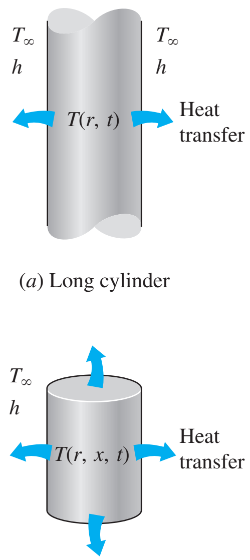
$$\theta_{\text{cyl}}(r, t) = \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\\text{cylinder}}}$$

$$\theta_{\text{semi-inf}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi infinite}\\\text{solid}}} \tag{4–51}$$

For example, the solution for a long solid bar whose cross section is an $a \times b$ rectangle is the intersection of the two infinite plane walls of thicknesses $a$ and $b$, as shown in Fig. 4–38, and thus the transient temperature distribution for this rectangular bar can be expressed as

$$\left(\frac{T(x, y, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{rectangular}\\\text{bar}}} = \theta_{\text{wall}}(x, t)\theta_{\text{wall}}(y, t) \tag{4–52}$$

The proper forms of the product solutions for some other geometries are given in Table 4–5. It is important to note that the $x$-coordinate is measured from the




(*a*) Long cylinder

(*b*) Short cylinder (two-dimensional)

**FIGURE 4–36**
The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions.

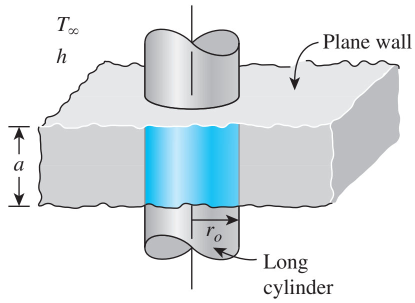
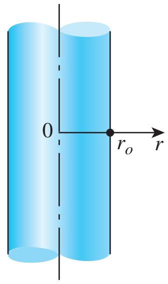



**FIGURE 4–37**
A short cylinder of radius $r_o$ and height $a$ is the *intersection* of a long cylinder of radius $r_o$ and a plane wall of thickness $a$.

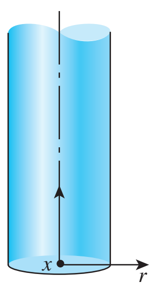
## TABLE 4–5

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature $T_i$ and exposed to convection from all surfaces to a medium at $T_\infty$



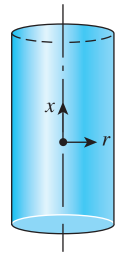

$\theta(r, t) = \theta_{cyl}(r, t)$

**Infinite cylinder**



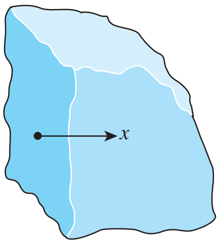

$\theta(x, r, t) = \theta_{cyl}(r, t)\, \theta_{semi\text{-}inf}(x, t)$

**Semi-infinite cylinder**



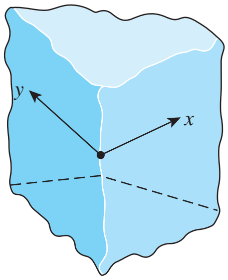

$\theta(x, r, t) = \theta_{cyl}(r, t)\, \theta_{wall}(x, t)$

**Short cylinder**



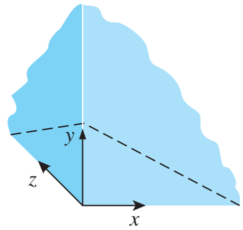

$\theta(x, t) = \theta_{semi\text{-}inf}(x, t)$

**Semi-infinite medium**



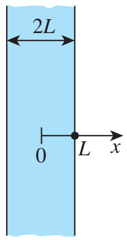

$\theta(x, y, t) = \theta_{semi\text{-}inf}(x, t)\, \theta_{semi\text{-}inf}(y, t)$

**Quarter-infinite medium**



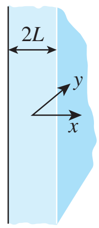

$\theta(x, y, z, t) =$
$\theta_{semi\text{-}inf}(x, t)\, \theta_{semi\text{-}inf}(y, t)\, \theta_{semi\text{-}inf}(z, t)$

**Corner region of a large medium**



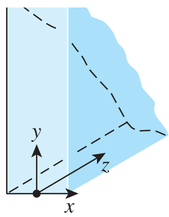

$\theta(x, t) = \theta_{wall}(x, t)$

**Infinite plate (or plane wall)**



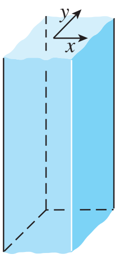

$\theta(x, y, t) = \theta_{wall}(x, t)\, \theta_{semi\text{-}inf}(y, t)$

**Semi-infinite plate**



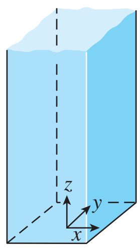

$\theta(x, y, z, t) =$
$\theta_{wall}(x, t)\, \theta_{semi\text{-}inf}(y, t)\, \theta_{semi\text{-}inf}(z, t)$

**Quarter-infinite plate**



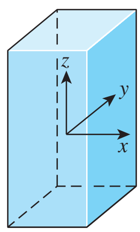

$\theta(x, y, t) = \theta_{wall}(x, t)\, \theta_{wall}(y, t)$

**Infinite rectangular bar**




$\theta(x, y, z, t) =$
$\theta_{wall}(x, t)\, \theta_{wall}(y, t)\, \theta_{semi\text{-}inf}(z, t)$

**Semi-infinite rectangular bar**




$\theta(x, y, z, t) =$
$\theta_{wall}(x, t)\, \theta_{wall}(y, t)\, \theta_{wall}(z, t)$

**Rectangular parallelepiped**

*surface* in a semi-infinite solid, and from the *midplane* in a plane wall. The radial distance $r$ is always measured from the centerline.

Note that the solution of a *two-dimensional* problem involves the product of *two* one-dimensional solutions, whereas the solution of a *three-dimensional* problem involves the product of *three* one-dimensional solutions.

A modified form of the product solution can also be used to determine the total transient heat transfer to or from a multidimensional geometry by using the one-dimensional values, as shown by L. S. Langston in 1982. The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] \tag{4–53}$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right] \tag{4–54}$$

The use of the product solution in transient two- and three-dimensional heat conduction problems is illustrated in the following examples.

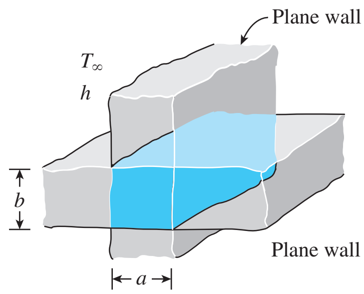



**FIGURE 4–38**
A long solid bar of rectangular profile $a \times b$ is the *intersection* of two plane walls of thicknesses $a$ and $b$.

### *EXAMPLE 4–8* Cooling of a Short Brass Cylinder

A short brass cylinder of diameter $D = 10$ cm and height $H = 12$ cm is initially at a uniform temperature $T_i = 120°C$. The cylinder is now placed in atmospheric air at 25°C, where heat transfer takes place by convection, with a heat transfer coefficient of $h = 60$ W/m²·K. Calculate the temperature at (*a*) the center of the cylinder and (*b*) the center of the top surface of the cylinder 15 min after the start of the cooling.

**SOLUTION** A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface are to be determined.

***Assumptions*** **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial $x$- and the radial $r$-directions. **2** The thermal properties of the cylinder and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

***Properties*** The properties of brass at room temperature are $k = 110$ W/m·K and $\alpha = 33.9 \times 10^{-6}$ m²/s (Table A–3). More accurate results can be obtained by using properties at average temperature.

***Analysis*** (*a*) This short cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = 5$ cm and a plane wall of thickness $2L = 12$ cm, as

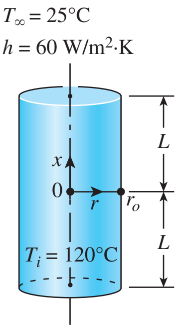



**FIGURE 4–39**
Schematic for Example 4–8.

shown in Fig. 4–39. The dimensionless temperature at the center of the plane wall is determined from Fig. 4–17*a* to be

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{L^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2/\text{s})(900\ \text{s})}{(0.06\ \text{m})^2} = 8.48 \\ \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110\ \text{W/m}\cdot\text{K}}{(60\ \text{W/m}^2\cdot\text{K})(0.06\ \text{m})} = 30.6 \end{aligned}\right\} \theta_{\text{wall}}(0, t) = \frac{T(0, t) - T_\infty}{T_i - T_\infty} = 0.8$$

Similarly, at the center of the cylinder, we have

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.39 \times 10^{-5}\ \text{m}^2/\text{s})(900\ \text{s})}{(0.05\ \text{m})^2} = 12.2 \\ \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{110\ \text{W/m}\cdot\text{K}}{(60\ \text{W/m}^2\cdot\text{K})(0.05\ \text{m})} = 36.7 \end{aligned}\right\} \theta_{\text{cyl}}(0, t) = \frac{T(0, t) - T_\infty}{T_i - T_\infty} = 0.5$$

Therefore,

$$\left(\frac{T(0, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\\text{cylinder}}} = \theta_{\text{wall}}(0, t) \times \theta_{\text{cyl}}(0, t) = 0.8 \times 0.5 = 0.4$$

and

$$T(0, 0, t) = T_\infty + 0.4(T_i - T_\infty) = 25 + 0.4(120 - 25) = \mathbf{63°C}$$

This is the temperature at the center of the short cylinder, which is also the center of both the long cylinder and the plate.

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder ($r = 0$), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to find the surface temperature of the wall. Noting that $x = L = 0.06$ m,

$$\left.\begin{aligned} \frac{x}{L} &= \frac{0.06\ \text{m}}{0.06\ \text{m}} = 1 \\ \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110\ \text{W/m}\cdot\text{K}}{(60\ \text{W/m}^2\cdot\text{K})(0.06\ \text{m})} = 30.6 \end{aligned}\right\} \frac{T(L, t) - T_\infty}{T_0 - T_\infty} = 0.98$$

Then

$$\theta_{\text{wall}}(L, t) = \frac{T(L, t) - T_\infty}{T_i - T_\infty} = \left(\frac{T(L, t) - T_\infty}{T_0 - T_\infty}\right)\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right) = 0.98 \times 0.8 = 0.784$$

Therefore,

$$\left(\frac{T(L, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\\text{cylinder}}} = \theta_{\text{wall}}(L, t)\theta_{\text{cyl}}(0, t) = 0.784 \times 0.5 = 0.392$$

and

$$T(L, 0, t) = T_\infty + 0.392(T_i - T_\infty) = 25 + 0.392(120 - 25) = \mathbf{62.2°C}$$

which is the temperature at the center of the top surface of the cylinder.

### EXAMPLE 4–9 Heat Transfer from a Short Cylinder

Determine the total heat transfer from the short brass cylinder ($\rho$ = 8530 kg/m$^3$, $c_p$ = 0.380 kJ/kg·K) discussed in Example 4–8.

**SOLUTION** We first determine the maximum heat that can be transferred from the cylinder, which is the sensible energy content of the cylinder relative to its environment:

$$m = \rho V = \rho \pi r_o^2 H = (8530 \text{ kg/m}^3)\pi(0.05 \text{ m})^2(0.12 \text{ m}) = 8.04 \text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (8.04 \text{ kg})(0.380 \text{ kJ/kg·K})(120 - 25)°\text{C} = 290.2 \text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries. For the plane wall, it is determined from Fig. 4–17*c* to be

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{30.6} = 0.0327 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.0327)^2(8.48) = 0.0091 \end{aligned}\right\} \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}} = 0.23$$

Similarly, for the cylinder, we have

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{36.7} = 0.0272 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.0272)^2(12.2) = 0.0090 \end{aligned}\right\} \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{infinite}\\\text{cylinder}}} = 0.47$$

Then the heat transfer ratio for the short cylinder is, from Eq. 4–53,

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{short cyl}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

$$= 0.23 + 0.47(1 - 0.23) = 0.592$$

Therefore, the total heat transfer from the cylinder during the first 15 min of cooling is

$$Q = 0.592Q_{\max} = 0.592 \times (290.2 \text{ kJ}) = \mathbf{172 \text{ kJ}}$$

### EXAMPLE 4–10 Cooling of a Long Cylinder by Water

A semi-infinite aluminum cylinder of diameter $D$ = 20 cm is initially at a uniform temperature $T_i$ = 200°C. The cylinder is now placed in water at 15°C where heat transfer takes place by convection, with a heat transfer coefficient of $h$ = 120 W/m$^2$·K. Determine the temperature at the center of the cylinder 15 cm from the end surface 5 min after the start of the cooling.

**SOLUTION** A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 15 cm from the end surface is to be determined.

***Assumptions*** **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial $x$- and the

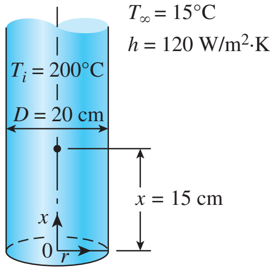



**FIGURE 4–40**
Schematic for Example 4–10.

radial $r$-directions. **2** The thermal properties of the cylinder and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

***Properties*** The properties of aluminum at room temperature are $k = 237$ W/m·K and $\alpha = 9.71 \times 10^{-5}$ m²/s (Table A–3). More accurate results can be obtained by using properties at average temperature.

***Analysis*** This semi-infinite cylinder can physically be formed by the intersection of an infinite cylinder of radius $r_o = 10$ cm and a semi-infinite medium, as shown in Fig. 4–40.

We solve this problem using the one-term solution relation for the cylinder and the analytic solution for the semi-infinite medium. First we consider the infinitely long cylinder and evaluate the Biot number:

$$\text{Bi} = \frac{hr_o}{k} = \frac{(120\ \text{W/m}^2\cdot\text{K})(0.1\ \text{m})}{237\ \text{W/m}\cdot\text{K}} = 0.05$$

The coefficients $\lambda_1$ and $A_1$ for a cylinder corresponding to this Bi are determined from Table 4–2 to be $\lambda_1 = 0.3126$ and $A_1 = 1.0124$. The Fourier number in this case is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71 \times 10^{-5}\ \text{m}^2/\text{s})(5 \times 60\ \text{s})}{(0.1\ \text{m})^2} = 2.91 > 0.2$$

and thus the one-term approximation is applicable. Substituting these values into Eq. 4–27 gives

$$\theta_0 = \theta_{\text{cyl}}(0, t) = A_1 e^{-\lambda_1^2 \tau} = 1.0124 e^{-(0.3126)^2(2.91)} = 0.762$$

The solution for the semi-infinite solid can be determined from

$$1 - \theta_{\text{semi-inf}}(x, t) = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

First we determine the various quantities in parentheses:

$$\eta = \frac{x}{2\sqrt{\alpha t}} = \frac{0.15\ \text{m}}{2\sqrt{(9.71 \times 10^{-5}\ \text{m}^2/\text{s})(5 \times 60\ \text{s})}} = 0.44$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(120\ \text{W/m}^2\cdot\text{K})\sqrt{(9.71 \times 10^{-5}\ \text{m}^2/\text{s})(300\ \text{s})}}{237\ \text{W/m}\cdot\text{K}} = 0.086$$

$$\frac{hx}{k} = \frac{(120\ \text{W/m}^2\cdot\text{K})(0.15\ \text{m})}{237\ \text{W/m}\cdot\text{K}} = 0.0759$$

$$\frac{h^2 \alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = (0.086)^2 = 0.0074$$

Substituting and evaluating the complementary error functions from Table 4–4,

$$\begin{aligned}\theta_{\text{semi-inf}}(x, t) &= 1 - \text{erfc}\ (0.44) + \exp\ (0.0759 + 0.0074)\ \text{erfc}\ (0.44 + 0.086)\\ &= 1 - 0.5338 + \exp\ (0.0833) \times 0.457\\ &= 0.963\end{aligned}$$

Now we apply the product solution to get

$$\left(\frac{T(x, 0, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = \theta_{\text{semi-inf}}(x, t)\theta_{\text{cyl}}(0, t) = 0.963 \times 0.762 = 0.734$$

and

$$T(x, 0, t) = T_\infty + 0.734(T_i - T_\infty) = 15 + 0.734(200 - 15) = \mathbf{151°C}$$

which is the temperature at the center of the cylinder 15 cm from the exposed bottom surface.

### *EXAMPLE 4–11* Refrigerating Steaks while Avoiding Frostbite

In a meat processing plant, 1-in-thick steaks initially at 75°F are to be cooled in the racks of a large refrigerator that is maintained at 5°F (Fig. 4–41). The steaks are placed close to each other, so that heat transfer from the 1-in-thick edges is negligible. The entire steak is to be cooled below 45°F, but its temperature is not to drop below 35°F at any point during refrigeration to avoid "frostbite." The convection heat transfer coefficient and thus the rate of heat transfer from the steak can be controlled by varying the speed of a circulating fan inside. Determine the heat transfer coefficient $h$ that will enable us to meet both temperature constraints while keeping the refrigeration time to a minimum. The steak can be treated as a homogeneous layer having the properties $\rho = 74.9$ lbm/ft$^3$, $c_p = 0.98$ Btu/lbm·°F, $k = 0.26$ Btu/h·ft·°F, and $\alpha = 0.0035$ ft$^2$/h.

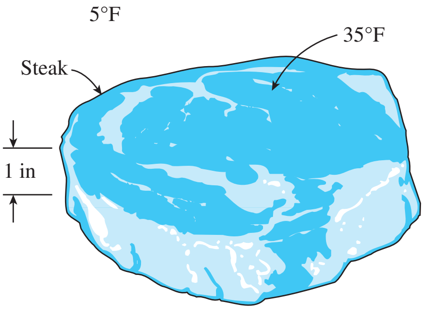



**FIGURE 4–41**
Schematic for Example 4–11.

**SOLUTION** Steaks are to be cooled in a refrigerator maintained at 5°F. The heat transfer coefficient that allows cooling the steaks below 45°F while avoiding frostbite is to be determined.

***Assumptions*** **1** Heat conduction through the steaks is one-dimensional since the steaks form a large layer relative to their thickness and there is thermal symmetry about the center plane. **2** The thermal properties of the steaks and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

***Properties*** The properties of the steaks are as given in the problem statement.

***Analysis*** The lowest temperature in the steak occurs at the surfaces and the highest temperature at the center at a given time, since the inner part is the last place to be cooled. In the limiting case, the surface temperature at $x = L = 0.5$ in from the center will be 35°F, while the midplane temperature is 45°F in an environment at 5°F. Then, from Fig. 4–17*b*, we obtain

$$\left.\begin{aligned} \frac{x}{L} &= \frac{0.5\text{ in}}{0.5\text{ in}} = 1 \\ \frac{T(L, t) - T_\infty}{T_0 - T_\infty} &= \frac{35 - 5}{45 - 5} = 0.75 \end{aligned}\right\} \quad \frac{1}{\text{Bi}} = \frac{k}{hL} = 1.5$$

which gives

$$h = \frac{1}{1.5}\frac{k}{L} = \frac{0.26\text{ Btu/h·ft·°F}}{1.5(0.5/12\text{ ft})} = \mathbf{4.16\ Btu/h·ft^2·°F}$$

***Discussion*** The convection heat transfer coefficient should be kept below this value to satisfy the constraints on the temperature of the steak during refrigeration. We can also meet the constraints by using a lower heat transfer coefficient, but doing so would extend the refrigeration time unnecessarily.

The restrictions that are inherent in the use of Heisler charts and the one-term solutions (or any other analytical solutions) can be lifted by using the numerical methods discussed in Chapter 5.

## TOPIC OF SPECIAL INTEREST*

*Refrigeration and Freezing of Foods*

### Control of Microorganisms in Foods

**Microorganisms** such as *bacteria, yeasts, molds,* and *viruses* are widely encountered in air, water, soil, living organisms, and unprocessed food items, and cause *off-flavors* and *odors, slime production, changes* in the texture and appearances, and the eventual *spoilage* of foods. Holding perishable foods at warm temperatures is the primary cause of spoilage, and the prevention of food spoilage and the premature degradation of quality due to microorganisms is the largest application area of refrigeration. The first step in controlling microorganisms is to understand what they are and the factors that affect their transmission, growth, and destruction.

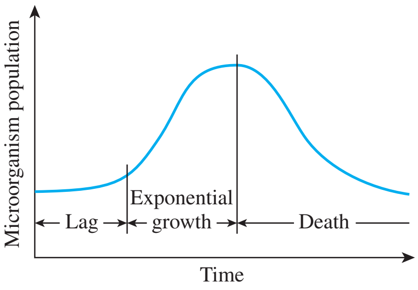



**FIGURE 4–42**
Typical growth curve of microorganisms.

Of the various kinds of microorganisms, *bacteria* are the prime cause for the spoilage of foods, especially moist foods. Dry and acidic foods create an undesirable environment for the growth of bacteria, but not for the growth of yeasts and molds. *Molds* are also encountered on moist surfaces, cheese, and spoiled foods. Specific *viruses* are encountered in certain animals and humans, and poor sanitation practices such as keeping processed foods in the same area as the uncooked ones and being careless about handwashing can cause the contamination of food products.

When *contamination* occurs, the microorganisms start to adapt to the new environmental conditions. This initial slow or no-growth period is called the **lag phase**, and the shelf life of a food item is directly proportional to the length of this phase (Fig. 4–42). The adaptation period is followed by an *exponential growth* period during which the population of microorganisms can double two or more times every hour under favorable conditions unless drastic sanitation measures are taken. The depletion of nutrients and the accumulation of toxins slow down the growth and start the *death* period.

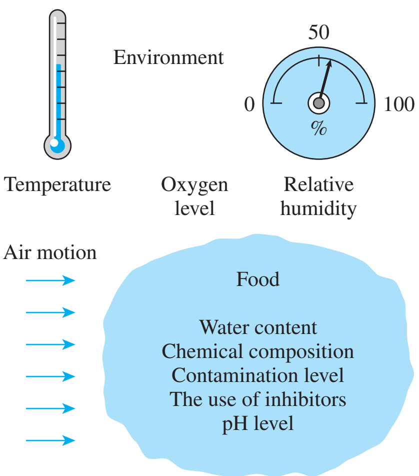



**FIGURE 4–43**
The factors that affect the rate of growth of microorganisms.

The *rate of growth* of microorganisms in a food item depends on the characteristics of the food itself such as the chemical structure, pH level, presence of inhibitors and competing microorganisms, and water activity as well as the environmental conditions such as the temperature and relative humidity of the environment and the air motion (Fig. 4–43).

*This section can be skipped without a loss of continuity.

Microorganisms need *food* to grow and multiply, and their nutritional needs are readily provided by the carbohydrates, proteins, minerals, and vitamins in a food. Different types of microorganisms have different nutritional needs, and the types of nutrients in a food determine the types of microorganisms that may dwell on them. The preservatives added to the food may also inhibit the growth of certain microorganisms. Different kinds of microorganisms that exist compete for the same food supply, and thus the composition of microorganisms in a food at any time depends on the *initial make-up* of the microorganisms.

All living organisms need *water* to grow, and microorganisms cannot grow in foods that are not sufficiently moist. Microbiological growth in refrigerated foods such as fresh fruits, vegetables, and meats starts at the *exposed surfaces* where contamination is most likely to occur. Fresh meat in a package left in a room will spoil quickly, as you may have noticed. A meat carcass hung in a controlled environment, on the other hand, will age healthily as a result of *dehydration* on the outer surface, which inhibits microbiological growth there and protects the carcass.

Microorganism growth in a food item is governed by the combined effects of the *characteristics of the food* and the *environmental factors*. We cannot do much about the characteristics of the food, but we certainly can alter the environmental conditions to more desirable levels through *heating, cooling, ventilating, humidification, dehumidification,* and control of the *oxygen* levels. The growth rate of microorganisms in foods is a strong function of temperature, and temperature control is the single most effective mechanism for controlling the growth rate.

Microorganisms grow best at "warm" temperatures, usually between 20 and 60°C. The growth rate *declines* at high temperatures, and *death* occurs at still higher temperatures, usually above 70°C for most microorganisms. *Cooling* is an effective and practical way of reducing the growth rate of microorganisms and thus extending the *shelf life* of perishable foods. A temperature of 4°C or lower is considered to be a safe refrigeration temperature. Sometimes a small increase in refrigeration temperature may cause a large increase in the growth rate, and thus a considerable decrease in shelf life of the food (Fig. 4–44). The growth rate of some microorganisms, for example, doubles for each 3°C rise in temperature.

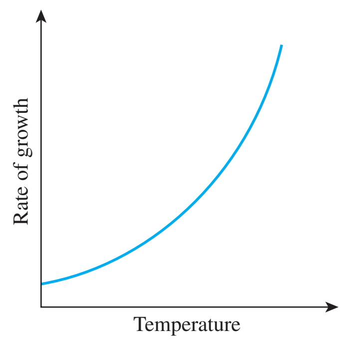



**FIGURE 4–44**
The rate of growth of microorganisms in a food product increases exponentially with increasing environmental temperature.

Another factor that affects microbiological growth and transmission is the *relative humidity* of the environment, which is a measure of the water content of the air. High humidity in *cold rooms* should be avoided since condensation that forms on the walls and ceiling creates the proper environment for *mold growth* and buildups. The drip of contaminated condensate onto food products in the room poses a potential health hazard.

Different microorganisms react differently to the presence of oxygen in the environment. Some microorganisms such as molds require oxygen for growth, while some others cannot grow in the presence of oxygen. Some grow best in low-oxygen environments, while others grow in environments regardless of the amount of oxygen. Therefore, the growth of certain microorganisms can be controlled by controlling the *amount of oxygen* in the environment. For example, vacuum packaging inhibits the growth of

microorganisms that require oxygen. Also, the storage life of some fruits can be extended by reducing the oxygen level in the storage room.

Microorganisms in food products can be controlled by (1) *preventing* contamination by following strict sanitation practices, (2) *inhibiting* growth by altering the environmental conditions, and (3) *destroying* the organisms by heat treatment or chemicals. The best way to minimize contamination in food processing areas is to use fine air filters in ventilation systems to capture the *dust particles* that transport the bacteria in the air. Of course, the filters must remain dry since microorganisms can grow in wet filters. Also, the ventilation system must maintain a positive pressure in the food processing areas to prevent any airborne contaminants from entering inside by infiltration. The elimination of *condensation* on the walls and the ceiling of the facility and the diversion of *plumbing* condensation drip pans of refrigerators to the drain system are two other preventive measures against contamination. Drip systems must be cleaned regularly to prevent microbiological growth in them. Also, any *contact* between raw and cooked food products should be minimized, and cooked products must be stored in rooms with positive pressures. Frozen foods must be kept at −18°C or below, and utmost care should be exercised when food products are packaged after they are frozen to avoid contamination during packaging.

The growth of microorganisms is best controlled by keeping the *temperature* and *relative humidity* of the environment in the desirable range. Keeping the relative humidity below 60 percent, for example, prevents the growth of all microorganisms on the surfaces. Microorganisms can be destroyed by *heating* the food product to high temperatures (usually above 70°C), by treating them with *chemicals,* or by exposing them to *ultraviolet light* or solar radiation.

Distinction should be made between *survival* and *growth* of microorganisms. A particular microorganism that may not grow at some low temperature may be able to survive at that temperature for a very long time (Fig. 4–45). Therefore, freezing is not an effective way of killing microorganisms. In fact, some microorganism cultures are preserved by freezing them at very low temperatures. The *rate of freezing* is also an important consideration in the refrigeration of foods since some microorganisms adapt to low temperatures and grow at those temperatures when the cooling rate is very low.

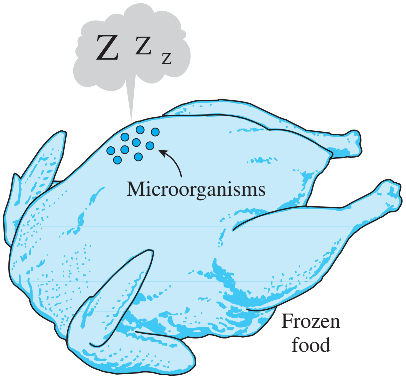



**FIGURE 4–45**
Freezing may stop the growth of microorganisms, but it may not necessarily kill them.

## Refrigeration and Freezing of Foods

The *storage life* of fresh perishable foods such as meats, fish, vegetables, and fruits can be extended by several days by storing them at temperatures just above freezing, usually between 1 and 4°C. The storage life of foods can be extended by several months by freezing and storing them at subfreezing temperatures, usually between −18 and −35°C, depending on the particular food (Fig. 4–46).

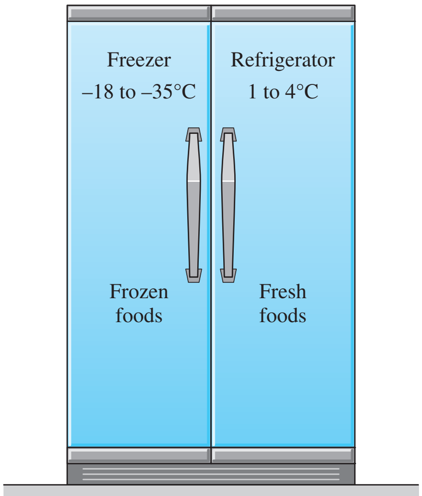



**FIGURE 4–46**
Recommended refrigeration and freezing temperatures for most perishable foods.

Refrigeration *slows down* the chemical and biological processes in foods, and the accompanying deterioration and loss of quality and nutrients. Sweet corn, for example, may lose half of its initial sugar content in one day at 21°C, but only 5 percent of it at 0°C. Fresh asparagus may lose 50 percent of its vitamin C content in one day at 20°C, but in 12 days

at 0°C. Refrigeration also extends the shelf life of products. The first appearance of unsightly yellowing of broccoli, for example, may be delayed by three or more days by refrigeration.

Early attempts to freeze food items resulted in poor-quality products because of the large ice crystals that formed. It was determined that the *rate of freezing* has a major effect on the size of ice crystals and the quality, texture, and nutritional and sensory properties of many foods. During *slow freezing,* ice crystals can grow to a large size, whereas during *fast freezing* a large number of ice crystals start forming at once and are much smaller in size. Large ice crystals are not desirable since they can *puncture* the walls of the cells, causing a degradation of texture and a loss of natural juices during thawing. A *crust* forms rapidly on the outer layer of the product and seals in the juices, aromatics, and flavoring agents. The product quality is also affected adversely by temperature fluctuations of the storage room.

The ordinary refrigeration of foods involves *cooling* only without any phase change. The *freezing* of foods, on the other hand, involves three stages: *cooling* to the freezing point (removing the sensible heat), *freezing* (removing the latent heat), and *further cooling* to the desired subfreezing temperature (removing the sensible heat of frozen food), as shown in Figure 4–47.

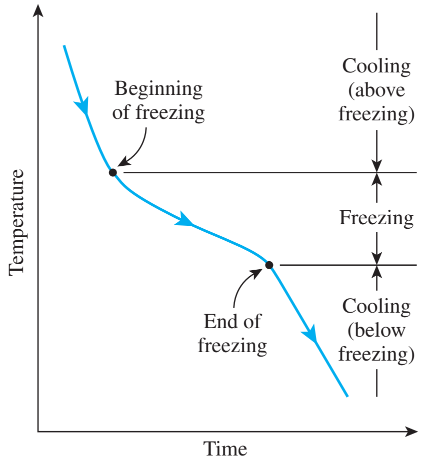



**FIGURE 4–47**
Typical freezing curve of a food item.

## Beef Products

Meat carcasses in slaughterhouses should be cooled *as fast as possible* to a uniform temperature of about 1.7°C to reduce the growth rate of microorganisms that may be present on carcass surfaces, and thus minimize spoilage. The right level of *temperature, humidity,* and *air motion* should be selected to prevent excessive shrinkage, toughening, and discoloration.

The deep body temperature of an animal is about 39°C, but this temperature tends to rise a couple of degrees in the midsections after slaughter as a result of the *heat generated* during the biological reactions that occur in the cells. The temperature of the exposed surfaces, on the other hand, tends to drop as a result of heat losses. The thickest part of the carcass is the *round,* and the center of the round is the last place to cool during chilling. Therefore, the cooling of the carcass can best be monitored by inserting a thermometer deep into the central part of the round.

About 70 percent of the beef carcass is water, and the carcass is cooled mostly by *evaporative cooling* as a result of moisture migration toward the surface where evaporation occurs. But this shrinking translates into a loss of salable mass that can amount to 2 percent of the total mass during an overnight chilling. To prevent *excessive* loss of mass, carcasses are usually washed or sprayed with water prior to cooling. With adequate care, spray chilling can eliminate carcass cooling shrinkage almost entirely.

The average total mass of dressed beef, which is normally split into two sides, is about 300 kg, and the average specific heat of the carcass is about 3.14 kJ/kg·K (Table 4–6). The *chilling room* must have a capacity equal to the daily kill of the slaughterhouse, which may be several hundred. A beef carcass is washed before it enters the chilling room and absorbs a large amount of water (about 3.6 kg) at its surface during the washing process. This does not represent a net mass gain, however, since it is lost

**TABLE 4–6**
Thermal properties of beef

| Quantity | Typical value |
|---|---|
| Average density | 1070 kg/m$^3$ |
| Specific heat: | |
| Above freezing | 3.14 kJ/kg·K |
| Below freezing | 1.70 kJ/kg·K |
| Freezing point | −2.7°C |
| Latent heat of fusion | 249 kJ/kg |
| Thermal conductivity | 0.41 W/m·K (at 6°C) |

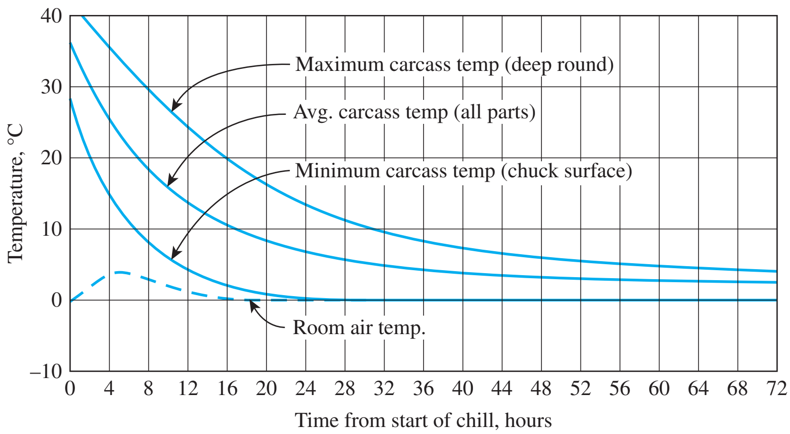



**FIGURE 4–48**
Typical cooling curve of a beef carcass in the chilling and holding rooms at an average temperature of 0°C.
*From ASHRAE, Handbook: Refrigeration, Chap. 11, Fig. 2.*

by dripping or evaporation in the chilling room during cooling. Ideally, the carcass does not lose or gain any net weight as it is cooled in the chilling room. However, it does lose about 0.5 percent of the total mass in the *holding room* as it continues to cool. The actual product loss is determined by first weighing the dry carcass before washing and then weighing it again after it is cooled.

The refrigerated air temperature in the chilling room of beef carcasses must be sufficiently high to avoid *freezing* and *discoloration* on the outer surfaces of the carcass. This means a long residence time for the massive beef carcasses in the chilling room to cool to the desired temperature. Beef carcasses are only partially cooled at the end of an overnight stay in the chilling room. The temperature of a beef carcass drops to 1.7 to 7°C at the surface and to about 15°C in mid parts of the round in 10 h. It takes another day or two in the *holding room* maintained at 1 to 2°C to complete *chilling* and *temperature equalization.* But hog carcasses are fully chilled during that period because of their smaller size. The *air circulation* in the holding room is kept at minimum levels to avoid excessive moisture loss and discoloration. The refrigeration load of the holding room is much smaller than that of the chilling room, and thus it requires a smaller refrigeration system.

Beef carcasses intended for distant markets are shipped the day after slaughter in refrigerated trucks, where the rest of the cooling is done. This practice makes it possible to deliver fresh meat long distances in a timely manner.

The variation in temperature of the beef carcass during cooling is given in Figure 4–48. Initially, the cooling process is dominated by *sensible* heat transfer. Note that the average temperature of the carcass is reduced by about 28°C (from 36 to 8°C) in 20 h. The cooling rate of the carcass could be increased by *lowering* the refrigerated air temperature and *increasing* the air velocity, but such measures also increase the risk of *surface freezing.*

Most meats are judged on their **tenderness**, and the preservation of tenderness is an important consideration in the refrigeration and freezing of meats. Meat consists primarily of bundles of tiny *muscle fibers* bundled together inside long strings of *connective tissues* that hold it together.

The tenderness of a certain cut of beef depends on the location of the cut, the age, and the activity level of the animal. Cuts from the relatively inactive mid-backbone section of the animal such as short loins, sirloin, and prime ribs are more tender than the cuts from the active parts such as the legs and the neck (Fig. 4–49). The more active the animal, the more the connective tissue, and the tougher the meat. The meat of an older animal is more flavorful, however, and is preferred for stewing since the toughness of the meat does not pose a problem for moist-heat cooking such as boiling. The protein *collagen,* which is the main component of the connective tissue, softens and dissolves in hot and moist environments and gradually transforms into *gelatin,* and tenderizes the meat.

The old saying "one should either cook an animal immediately after slaughter or wait at least two days" has a lot of truth in it. The biomechanical reactions in the muscle continue after the slaughter until the energy supplied to the muscle to do work diminishes. The muscle then stiffens and goes into *rigor mortis.* This process begins several hours after the animal is slaughtered and continues for 12 to 36 h until an enzymatic action sets in and tenderizes the connective tissue, as shown in Figure 4–50. It takes about seven days to complete tenderization naturally in storage facilities maintained at 2°C. Electrical stimulation also causes the meat to be tender. To avoid toughness, fresh meat should not be frozen before rigor mortis has passed.

You have probably noticed that steaks are tender and rather tasty when they are hot but toughen as they cool. This is because the gelatin that formed during cooking thickens as it cools, and meat loses its tenderness. So it is no surprise that first-class restaurants serve their steak on hot thick plates that keep the steaks warm for a long time. Also, cooking *softens* the connective tissue but *toughens* the tender muscle fibers. Therefore, barbecuing on *low heat* for a long time results in a tough steak.

Variety meats intended for long-term storage must be frozen rapidly to reduce spoilage and preserve quality. Perhaps the first thought that comes to mind to freeze meat is to place the meat packages into the *freezer* and wait. But the freezing time is *too long* in this case, especially for large boxes. For example, the core temperature of a 4-cm-deep box containing 32 kg of variety meat can be as high as 16°C 24 h after it is placed into a −30°C freezer. The freezing time of large boxes can be shortened considerably by adding some *dry ice* into it.

A more effective method of freezing, called *quick chilling,* involves the use of lower air temperatures, −40 to −30°C, with higher velocities of 2.5 m/s to 5 m/s over the product (Fig. 4–51). The internal temperature should be lowered to −4°C for products to be transferred to a storage freezer and to −18°C for products to be shipped immediately. The *rate of freezing* depends on the *package material* and its insulating properties, the *thickness* of the largest box, the *type* of meat, and the *capacity* of the refrigeration system. Note that the air temperature will rise excessively during initial stages of freezing and increase the freezing time if the capacity of the system is inadequate. A smaller refrigeration system will be adequate if dry ice is to be used in packages. Shrinkage during freezing varies from about 0.5 to 1 percent.

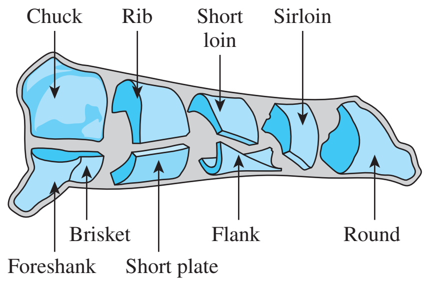



**FIGURE 4–49**
Various cuts of beef.
*From National Livestock and Meat Board.*

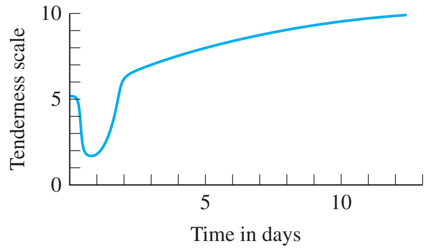



**FIGURE 4–50**
Variation of tenderness of meat stored at 2°C with time after slaughter.

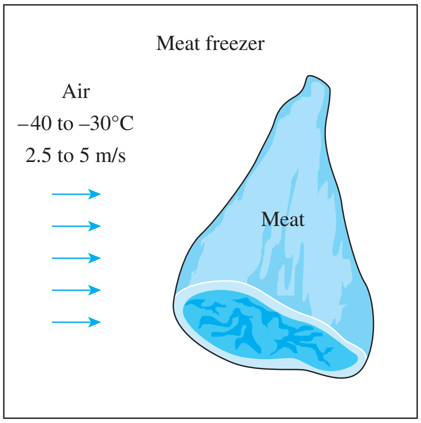



**FIGURE 4–51**
The freezing time of meat can be reduced considerably by using low temperature air at high velocity.

**TABLE 4–7**

Storage life of frozen meat products at different storage temperatures (from ASHRAE *Handbook: Refrigeration*, Chap. 10, Table 7)

| | *Storage Life, Months* | | |
|---|---|---|---|
| | *Temperature* | | |
| Product | −12°C | −18°C | −23°C |
| Beef | 4–12 | 6–18 | 12–24 |
| Lamb | 3–8 | 6–16 | 12–18 |
| Veal | 3–4 | 4–14 | 8 |
| Pork | 2–6 | 4–12 | 8–15 |
| Chopped beef | 3–4 | 4–6 | 8 |
| Cooked foods | 2–3 | 2–4 | |

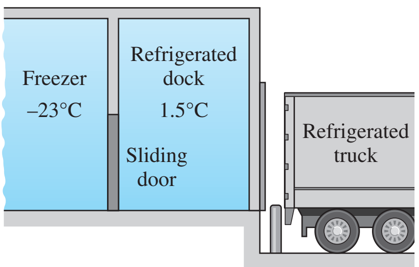



**FIGURE 4–52**
A refrigerated truck dock for loading frozen items to a refrigerated truck.

Although the average freezing point of lean meat can be taken to be −2°C with a latent heat of 249 kJ/kg, it should be remembered that freezing occurs over a *temperature range,* with most freezing occurring between −1 and −4°C. Therefore, cooling the meat through this temperature range and removing the latent heat takes the most time during freezing.

Meat can be kept at an internal temperature of −2 to −1°C for local use and storage for *under a week.* Meat must be frozen and stored at much lower temperatures for *long-term storage.* The lower the storage temperature, the longer the storage life of meat products, as shown in Table 4–7.

The *internal temperature* of carcasses entering the cooling sections varies from 38 to 41°C for hogs and from 37 to 39°C for lambs and calves. It takes about 15 h to cool the hogs and calves to the recommended temperature of 3 to 4°C. The cooling-room temperature is maintained at −1 to 0°C and the temperature difference between the refrigerant and the cooling air is kept at about 6°C. Air is circulated at a rate of about 7 to 12 air changes per hour. *Lamb carcasses* are cooled to an internal temperature of 1 to 2°C, which takes about 12 to 14 h, and are held at that temperature with 85 to 90 percent relative humidity until shipped or processed. The recommended rate of *air circulation* is 50 to 60 air changes per hour during the first 4 to 6 h, which is reduced to 10 to 12 changes per hour afterward.

*Freezing* does not seem to affect the *flavor* of meat much, but it affects the *quality* in several ways. The *rate* and *temperature* of freezing may influence color, tenderness, and drip. Rapid freezing increases tenderness and reduces the tissue damage and the amount of drip after thawing. Storage at low freezing temperatures causes significant changes in *animal fat.* Frozen pork experiences more undesirable changes during storage because of its fat structure, and thus its acceptable storage period is shorter than that of beef, veal, or lamb.

Meat storage facilities usually have a *refrigerated shipping dock* where the orders are assembled and shipped out. Such docks save valuable storage space from being used for shipping purposes and provide a more acceptable working environment for the employees. Packing plants that ship whole or half carcasses in bulk quantities may not need a shipping dock; a load-out door is often adequate for such cases.

A refrigerated *shipping dock,* as shown in Figure 4–52, reduces the *refrigeration load* of freezers or coolers and prevents *temperature fluctuations* in the storage area. It is often adequate to maintain the shipping docks at 4 to 7°C for the coolers and about 1.5°C for the freezers. The dew point of the dock air should be below the product temperature to avoid condensation on the surface of the products and loss of quality. The rate of *airflow* through the loading doors and other openings is proportional to the *square root* of the temperature difference, and thus reducing the temperature difference at the opening by half by keeping the shipping dock at the average temperature reduces the rate of airflow into the dock and thus into the freezer by $1 - \sqrt{0.5} \cong 0.3$, or 30 percent. Also, the air that flows into the freezer is already cooled to about 1.5°C by the refrigeration unit of the dock, which represents about 50 percent of the cooling load of the incoming air. Thus, the net effect of the refrigerated shipping dock is a reduction of the *infiltration load* of the freezer by about 65 percent since

$1 - 0.7 \times 0.5 = 0.65$. The net gain is equal to the difference between the reduction of the infiltration load of the freezer and the refrigeration load of the shipping dock. Note that the dock refrigerators operate at much higher temperatures (1.5°C instead of about −23°C), and thus they consume much less power for the same amount of cooling.

## Poultry Products

Poultry products can be preserved by *ice-chilling* to 1 to 2°C or *deep chilling* to about −2°C for short-term storage, or by *freezing* them to −18°C or below for long-term storage. Poultry processing plants are completely *automated,* and the small size of the birds makes continuous conveyor line operation feasible.

The birds are first electrically stunned before cutting to prevent struggling. Following a 90- to 120-s bleeding time, the birds are *scalded* by immersing them into a tank of warm water, usually at 51 to 55°C, for up to 120 s to loosen the feathers. Then the feathers are removed by feather-picking machines, and the eviscerated carcass is *washed* thoroughly before chilling. The internal temperature of the birds ranges from 24 to 35°C after washing, depending on the temperatures of the ambient air and the washing water as well as the extent of washing.

To control the microbial growth, the USDA regulations require that poultry be chilled to 4°C or below in less than 4 h for carcasses of less than 1.8 kg, in less than 6 h for carcasses of 1.8 to 3.6 kg. and in less than 8 h for carcasses more than 3.6 kg. Meeting these requirements today is not difficult since the slow *air chilling* is largely replaced by the rapid *immersion chilling* in tanks of slush ice. Immersion chilling has the added benefit that it not only prevents dehydration, but it causes a *net absorption of water* and thus increases the mass of salable product. Cool air chilling of unpacked poultry can cause a moisture loss of 1 to 2 percent, while water immersion chilling can cause a moisture absorption of 4 to 15 percent (Fig. 4–53). Water spray chilling can cause a moisture absorption of up to 4 percent. Most water absorbed is held between the flesh and the skin and the connective tissues in the skin. In immersion chilling, some soluble solids are lost from the carcass to the water, but the loss has no significant effect on flavor.

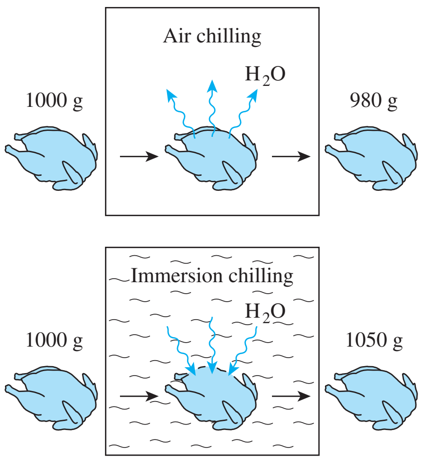



**FIGURE 4–53**
Air chilling causes dehydration and thus weight loss for poultry, whereas immersion chilling causes a weight gain as a result of water absorption.

Many slush ice tank chillers today are replaced by *continuous* flow-type immersion slush ice chillers. Continuous slush ice-chillers can reduce the internal temperature of poultry from 32 to 4°C in about 30 minutes at a rate up to 10, 000 birds per hour. Ice requirements depend on the inlet and exit temperatures of the carcass and the water, but 0.25 kg of ice per kg of carcass is usually adequate. However, *bacterial contamination* such as salmonella remains a concern with this method, and it may be necessary to chloride the water to control contamination.

*Tenderness* is an important consideration for poultry products just as it is for red meat, and preserving tenderness is an important consideration in the cooling and freezing of poultry. Birds cooked or frozen before passing through rigor mortis remain very tough. Natural tenderization begins soon after slaughter and is completed within 24 h when birds are held at 4°C.

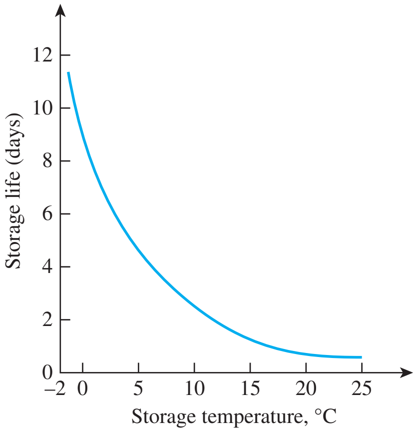



**FIGURE 4–54**
The storage life of fresh poultry decreases exponentially with increasing storage temperature.

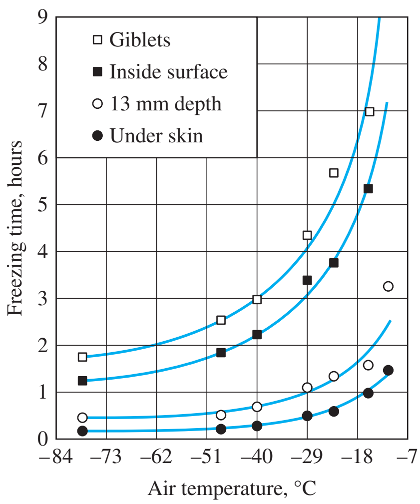
Tenderization is rapid during the first three hours and slows down thereafter. Immersion in hot water and cutting into the muscle adversely affect tenderization. Increasing the *scalding temperature* or the scalding time has been observed to increase toughness, and decreasing the scalding time has been observed to increase tenderness. The *beating action* of mechanical feather-picking machines causes considerable toughening. Therefore, it is recommended that any cutting be done after tenderization. *Cutting up* the bird into pieces before natural tenderization is completed reduces tenderness considerably. Therefore, it is recommended that any cutting be done after tenderization. *Rapid chilling* of poultry can also have a toughening effect. It is found that the tenderization process can be speeded up considerably by a patented *electrical stunning* process.

Poultry products are *highly perishable,* and thus they should be kept at the *lowest* possible temperature to maximize their shelf life. Studies have shown that the populations of certain bacteria double every 36 h at −2°C, 14 h at 0°C, 7 h at 5°C, and less than 1 h at 25°C (Fig. 4–54). Studies have also shown that the total bacterial counts on birds held at 2°C for 14 days are equivalent to those held at 10°C for 5 days or 24°C for 1 day. It has also been found that birds held at −1°C had 8 days of additional shelf life over those held at 4°C.

The growth of microorganisms on the *surfaces* of the poultry causes the development of an *off-odor* and *bacterial slime.* The higher the initial amount of bacterial contamination, the faster the sliming occurs. Therefore, good sanitation practices during processing such as cleaning the equipment frequently and washing the carcasses are as important as the storage temperature in extending shelf life.

Poultry must be frozen *rapidly* to ensure a light, pleasing appearance. Poultry that is frozen slowly appears dark and develops large ice crystals that damage the tissue. The ice crystals formed during rapid freezing are small. Delaying freezing of poultry causes the ice crystals to become larger. Rapid freezing can be accomplished by forced air at temperatures of −23 to −40°C and velocities of 1.5 to 5 m/s in *air-blast tunnel freezers.* Most poultry is frozen this way. Also, the packaged birds freeze much faster on open shelves than they do in boxes. If poultry packages must be frozen in boxes, then it is very desirable to leave the boxes open or to cut holes on the boxes in the direction of airflow during freezing. For best results, the blast tunnel should be fully loaded across its cross-section with even spacing between the products to assure uniform airflow around all sides of the packages. The freezing time of poultry as a function of refrigerated air temperature is given in Figure 4–55. Thermal properties of poultry are given in Table 4–8.

Other freezing methods for poultry include sandwiching between *cold plates, immersion* into a refrigerated liquid such as glycol or calcium chloride brine, and *cryogenic cooling* with liquid nitrogen. Poultry can be frozen in several hours by cold plates. Very high freezing rates can be obtained by *immersing* the packaged birds into a low-temperature brine. The freezing time of birds in −29°C brine can be as low as 20 min, depending on the size of the bird (Fig. 4–56). Also, immersion freezing produces a very appealing light appearance, and the high rates of heat transfer make




Note: Freezing time is the time required for temperature to fall from 0 to –4°C. The values are for 2.3 to 3.6 kg chickens with initial temperature of 0 to 2°C and with air velocity of 2.3 to 2.8 m/s.

**FIGURE 4–55**
The variation of freezing time of poultry with air temperature.

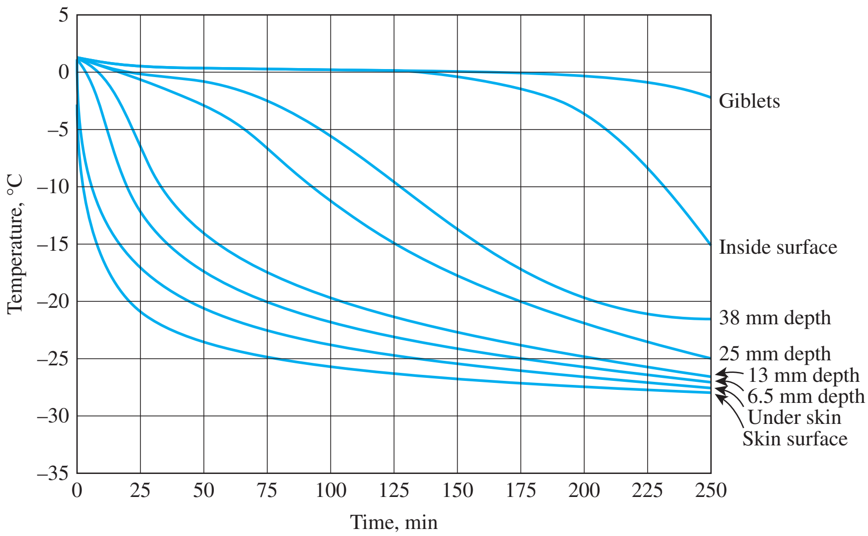



**FIGURE 4–56**

The variation of temperature of the breast of 6.8-kg turkeys initially at 1°C with depth during immersion cooling at −29°C.

*From van der Berg and Lentz, 1958.*

continuous line operation feasible. It also has lower initial and maintenance costs than forced air, but *leaks* into the packages through some small holes or cracks remain a concern. The convection heat transfer coefficient is 17 W/m$^2$·K for air at −29°C and 2.5 m/s whereas it is 170 W/m$^2$·K for sodium chloride brine at −18°C and a velocity of 0.02 m/s. Sometimes liquid nitrogen is used to crust freeze the poultry products to −73°C. The freezing is then completed with air in a holding room at −23°C.

Properly packaged poultry products can be *stored* frozen for up to about a year at temperatures of −18°C or lower. The storage life drops considerably at higher (but still below-freezing) temperatures. Significant changes occur in flavor and juiciness when poultry is frozen for too long, and a stale rancid odor develops. Frozen poultry may become dehydrated and experience **freezer burn**, which may reduce the eye appeal of the product and cause toughening of the affected area. Dehydration and thus freezer burn can be controlled by *humidification, lowering* the storage temperature, and packaging the product in essentially *impermeable* film. The storage life can be extended by packing the poultry in an *oxygen-free* environment. The bacterial counts in precooked frozen products must be kept at safe levels since bacteria may not be destroyed completely during the reheating process at home.

Frozen poultry can be *thawed* in ambient air, water, refrigerator, or oven without any significant difference in taste. Big birds like turkey should be thawed safely by holding it in a refrigerator at 2 to 4°C for two to four days, depending on the size of the bird. They can also be thawed by immersing them into cool water in a large container for 4 to 6 h, or holding them in a paper bag. Care must be exercised to keep the bird's surface *cool* to minimize *microbiological growth* when thawing in air or water.

**TABLE 4–8**

Thermal properties of poultry

| Quantity | Typical value |
|---|---|
| Average density: | |
| Muscle | 1070 kg/m$^3$ |
| Skin | 1030 kg/m$^3$ |
| Specific heat: | |
| Above freezing | 2.94 kJ/kg·K |
| Below freezing | 1.55 kJ/kg·K |
| Freezing point | −2.8°C |
| Latent heat of fusion | 247 kJ/kg |
| Thermal conductivity: | (in W/m·K) |
| Breast muscle | 0.502 at 20°C |
| | 1.384 at −20°C |
| | 1.506 at −40°C |
| Dark muscle | 1.557 at −40°C |

## *EXAMPLE 4–12* **Chilling of Beef Carcasses in a Meat Plant**

The chilling room of a meat plant is 18 m × 20 m × 5.5 m in size and has a capacity of 450 beef carcasses. The power consumed by the fans and the lights of the chilling room are 26 and 3 kW, respectively, and the room gains heat through its envelope at a rate of 13 kW. The average mass of beef carcasses is 285 kg. The carcasses enter the chilling room at 36°C after they are washed to facilitate evaporative cooling and are cooled to 15°C in 10 h. The water is expected to evaporate at a rate of 0.080 kg/s. The air enters the evaporator section of the refrigeration system at 0.7°C and leaves at −2°C. The air side of the evaporator is heavily finned, and the overall heat transfer coefficient of the evaporator based on the air side is 20 W/m$^2$·K. Also, the average temperature difference between the air and the refrigerant in the evaporator is 5.5°C. Determine (*a*) the refrigeration load of the chilling room, (*b*) the volume flow rate of air, and (*c*) the heat transfer surface area of the evaporator on the air side, assuming all the vapor and the fog in the air freezes in the evaporator.

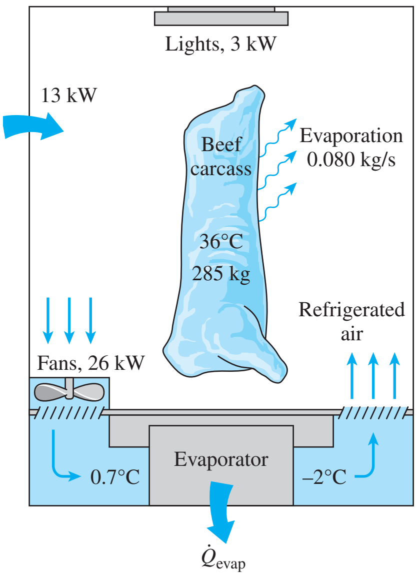



**FIGURE 4–57**
Schematic for Example 4–12.

**SOLUTION** The chilling room of a meat plant with a capacity of 450 beef carcasses is considered. The cooling load, the airflow rate, and the heat transfer area of the evaporator are to be determined.

***Assumptions*** **1** Water evaporates at a rate of 0.080 kg/s. **2** All the moisture in the air freezes in the evaporator.

***Properties*** The heat of fusion and the heat of vaporization of water at 0°C are 333.7 kJ/kg and 2501 kJ/kg (Table A–9). The density and specific heat of air at 0°C are 1.292 kg/m$^3$ and 1.006 kJ/kg·K (Table A–15). Also, the specific heat of beef carcass is determined from the relation in Table A–7*b* to be

$$c_p = 1.68 + 2.51 \times (\text{water content}) = 1.68 + 2.51 \times 0.58 = 3.14 \text{ kJ/kg·K}$$

***Analysis*** (a) A sketch of the chilling room is given in Figure 4–57. The amount of beef mass that needs to be cooled per unit time is

$$\begin{aligned}\dot{m}_{\text{beef}} &= (\text{Total beef mass cooled})/(\text{Cooling time})\\ &= (450 \text{ carcasses})(285 \text{ kg/carcass})/(10 \times 3600 \text{ s}) = 3.56 \text{ kg/s}\end{aligned}$$

The product refrigeration load can be viewed as the energy that needs to be removed from the beef carcass as it is cooled from 36 to 15°C at a rate of 3.56 kg/s and is determined to be

$$\dot{Q}_{\text{beef}} = (\dot{m}c_p\Delta T)_{\text{beef}} = (3.56 \text{ kg/s})(3.14 \text{ kJ/kg·K})(36 - 15)°\text{C} = 235 \text{ kW}$$

Then the total refrigeration load of the chilling room becomes

$$\begin{aligned}\dot{Q}_{\text{total, chillroom}} &= \dot{Q}_{\text{beef}} + \dot{Q}_{\text{fan}} + \dot{Q}_{\text{lights}} + \dot{Q}_{\text{heat gain}} = 235 + 26 + 3 + 13\\ &= \mathbf{277 \text{ kW}}\end{aligned}$$

The amount of carcass cooling due to evaporative cooling of water is

$$\dot{Q}_{\text{beef, evaporative}} = (\dot{m}h_{fg})_{\text{water}} = (0.080 \text{ kg/s})(2501 \text{ kJ/kg}) = 200 \text{ kW}$$

which is 200/235 = 0.85 = 85 percent of the total product cooling load. The remaining 15 percent of the heat is transferred by convection and radiation.

(*b*) Heat is transferred to air at the rate determined above, and the temperature of the air rises from −2°C to 0.7°C as a result. Therefore, the mass flow rate of air is

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{air}}}{(c_p \Delta T_{\text{air}})} = \frac{277 \text{ kW}}{(1.006 \text{ kJ/kg·K})[0.7 - (-2)°\text{C}]} = 102.0 \text{ kg/s}$$

Then the volume flow rate of air becomes

$$\dot{V}_{\text{air}} = \frac{\dot{m}_{\text{air}}}{\rho_{\text{air}}} = \frac{102 \text{ kg/s}}{1.292 \text{ kg/m}^3} = \mathbf{78.9 \text{ m}^3/\text{s}}$$

(*c*) Normally the heat transfer load of the evaporator is the same as the refrigeration load. But in this case the water that enters the evaporator as a liquid is frozen as the temperature drops to −2°C, and the evaporator must also remove the latent heat of freezing, which is determined from

$$\dot{Q}_{\text{freezing}} = (\dot{m} h_{\text{latent}})_{\text{water}} = (0.080 \text{ kg/s})(333.7 \text{ kJ/kg}) = 27 \text{ kW}$$

Therefore, the total rate of heat removal at the evaporator is

$$\dot{Q}_{\text{evaporator}} = \dot{Q}_{\text{total, chill room}} + \dot{Q}_{\text{freezing}} = 277 + 27 = 304 \text{ kW}$$

Then the heat transfer surface area of the evaporator on the air side is determined from $\dot{Q}_{\text{evaporator}} = (UA)_{\text{air side}} \Delta T$,

$$A = \frac{\dot{Q}_{\text{evaporator}}}{U \Delta T} = \frac{304{,}000 \text{ W}}{(20 \text{ W/m}^2\text{·K})(5.5°\text{C})} = \mathbf{2764 \text{ m}^2}$$

Obviously, a finned surface must be used to provide such a large surface area on the air side.

## SUMMARY

In this chapter, we considered the variation of temperature with time as well as position in one- or multidimensional systems. We first considered the *lumped systems* in which the temperature varies with time but remains uniform throughout the system at any time. The temperature of a lumped body of arbitrary shape of mass $m$, volume $V$, surface area $A_s$, density $\rho$, and specific heat $c_p$ initially at a uniform temperature $T_i$ that is exposed to convection at time $t = 0$ in a medium at temperature $T_\infty$ with a heat transfer coefficient $h$ is expressed as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

is a positive quantity whose dimension is (time)$^{-1}$. This relation can be used to determine the temperature $T(t)$ of a body at time $t$ or, alternatively, the time $t$ required for the temperature to reach a specified value $T(t)$. Once the temperature $T(t)$ at time $t$ is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty]$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to $t$ is simply the change in the energy content of the body,

$$Q = mc_p[T(t) - T_i]$$

The *maximum* heat transfer between the body and its surroundings is

$$Q_{\max} = mc_p(T_\infty - T_i)$$

The error involved in lumped system analysis is negligible when

$$\mathrm{Bi} = \frac{hL_c}{k} < 0.1$$

where Bi is the *Biot number* and $L_c = V/A_s$ is the *characteristic length.*

When the lumped system analysis is not applicable, the variation of temperature with position as well as time can be determined using the *transient temperature charts* given in Figs. 4–17, 4–18, 4–19, and 4–31 for a large plane wall, a long cylinder, a sphere, and a semi-infinite medium, respectively. These charts are applicable for one-dimensional heat transfer in those geometries. Therefore, their use is limited to situations in which the body is initially at a uniform temperature, all surfaces are subjected to the same thermal conditions, and the body does not involve any heat generation. These charts can also be used to determine the total heat transfer from the body up to a specified time *t*.

Using the *one-term approximation,* the solutions of one-dimensional transient heat conduction problems are expressed analytically as

*Plane wall:* $$\theta_{\text{wall}} = \frac{T(x, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L)$$

*Cylinder:* $$\theta_{\text{cyl}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o)$$

*Sphere:* $$\theta_{\text{sph}} = \frac{T(r, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

where the constants $A_1$ and $\lambda_1$ are functions of the Bi number only, and their values are listed in Table 4–2 against the Bi number for all three geometries. The error involved in one-term solutions is less than 2 percent when $\tau > 0.2$.

Using the one-term solutions, the fractional heat transfers in different geometries are expressed as

*Plane wall:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\,\text{wall}} \frac{\sin \lambda_1}{\lambda_1}$$

*Cylinder:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1}$$

*Sphere:* $$\left(\frac{Q}{Q_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

The solutions of transient heat conduction in a semi-infinite solid with constant properties under various boundary conditions at the surface are given as follows:

*Specified Surface Temperature, $T_s$ = constant:*

$$\frac{T(x, t) - T_i}{T_s - T_i} = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

*Specified Surface Heat Flux, $\dot{q}_s$ = constant:*

$$T(x, t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

*Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0, t)]$:*

$$\frac{T(x, t) - T_i}{T_\infty - T_i} = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \times \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

*Energy Pulse at Surface, $e_s$ = constant:*

$$T(x, t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$

where erfc($\eta$) is the *complementary error function* of argument $\eta$.

Using a superposition principle called the *product solution* these charts can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that all surfaces of the solid are subjected to convection to the same fluid at temperature $T_\infty$, with the same convection heat transfer coefficient *h*, and the body involves no heat generation. The solution in such multidimensional geometries can be expressed as the product of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.

The total heat transfer to or from a multidimensional geometry can also be determined by using the one-dimensional values. The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]$$

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## PROBLEMS*

### Lumped System Analysis

**4–1C** What is the physical significance of the Biot number? Is the Biot number more likely to be larger for highly conducting solids or poorly conducting ones?

**4–2C** What is lumped system analysis? When is it applicable?

**4–3C** In what medium is the lumped system analysis more likely to be applicable: in water or in air? Why?

**4–4C** For which solid is the lumped system analysis more likely to be applicable: an actual apple or a golden apple of the same size? Why?

**4–5C** For which kind of bodies made of the same material is the lumped system analysis more likely to be applicable: slender ones or well-rounded ones of the same volume? Why?

**4–6C** Consider heat transfer between two identical hot solid bodies and the air surrounding them. The first solid is being cooled by a fan while the second one is allowed to cool naturally. For which solid is the lumped system analysis more likely to be applicable? Why?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and complete solutions together with parametric studies are included on the text website. Problems with the icon are comprehensive in nature, and are intended to be solved with an equation solver such as EES. Problems with the icon PtD are Prevention through Design problems.

**4–7C** Consider heat transfer between two identical hot solid bodies and their environments. The first solid is dropped in a large container filled with water, while the second one is allowed to cool naturally in the air. For which solid is the lumped system analysis more likely to be applicable? Why?

**4–8C** Consider a hot baked potato on a plate. The temperature of the potato is observed to drop by 4°C during the first minute. Will the temperature drop during the second minute be less than, equal to, or more than 4°C? Why?

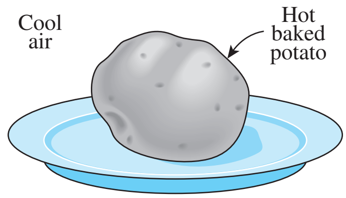


FIGURE P4–8C

**4–9C** Consider a potato being baked in an oven that is maintained at a constant temperature. The temperature of the potato is observed to rise by 5°C during the first minute. Will the temperature rise during the second minute be less than, equal to, or more than 5°C? Why?

**4–10C** Consider two identical 4-kg pieces of roast beef. The first piece is baked as a whole, while the second is baked after being cut into two equal pieces in the same oven. Will there be any difference between the cooking times of the whole and cut roasts? Why?

**4–11C** Consider a sphere and a cylinder of equal volume made of copper. Both the sphere and the cylinder are initially at the same temperature and are exposed to convection in the same environment. Which do you think will cool faster, the cylinder or the sphere? Why?

**4–12** Obtain relations for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius $r_o$, and a sphere of radius $r_o$.

**4–13** Obtain a relation for the time required for a lumped system to reach the average temperature $\frac{1}{2}(T_i + T_\infty)$, where $T_i$ is the initial temperature and $T_\infty$ is the temperature of the environment.

**4–14** A brick of 203 × 102 × 57 mm in dimension is being burned in a kiln to 1100°C, and then allowed to cool in a room with ambient air temperature of 30°C and convection heat transfer coefficient of 5 W/m²·K. If the brick has properties of $\rho$ = 1920 kg/m³, $c_p$ = 790 J/kg·K, and $k$ = 0.90 W/m·K, determine the time required to cool the brick to a temperature difference of 5°C from the ambient air temperature.

**4–15** Consider a 1000-W iron whose base plate is made of 0.5-cm-thick aluminum alloy 2024–T6 ($\rho$ = 2770 kg/m³, $c_p$ = 875 J/kg·K, $\alpha = 7.3 \times 10^{-5}$ m²/s). The base plate has a surface area of 0.03 m². Initially, the iron is in thermal equilibrium with the ambient air at 22°C. Taking the heat transfer coefficient at the surface of the base plate to be 12 W/m²·K and assuming 85 percent of the heat generated in the resistance wires is transferred to the plate, determine how long it will take for the plate temperature to reach 140°C. Is it realistic to assume the plate temperature to be uniform at all times?

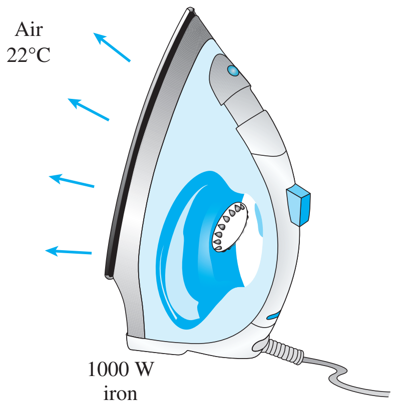


FIGURE P4–15

**4–16** Reconsider Prob. 4–15. Using EES (or other) software, investigate the effects of the heat transfer coefficient and the final plate temperature on the time it will take for the plate to reach this temperature. Let the heat transfer coefficient vary from 5 W/m²·K to 25 W/m²·K and the temperature from 30°C to 200°C. Plot the time as functions of the heat transfer coefficient and the temperature, and discuss the results.

**4–17** Metal plates ($k$ = 180 W/m·K, $\rho$ = 2800 kg/m³, and $c_p$ = 880 J/kg·K) with a thickness of 1 cm are being heated in an oven for 2 minutes. Air in the oven is maintained at 800°C with a convection heat transfer coefficient of 200 W/m²·K. If the initial temperature of the plates is 20°C, determine the temperature of the plates when they are removed from the oven.

**4–18** A 5-mm-thick stainless steel strip ($k$ = 21 W/m·K, $\rho$ = 8000 kg/m³, and $c_p$ = 570 J/kg·K) is being heat treated as it moves through a furnace at a speed of 1 cm/s. The air temperature in the furnace is maintained at 900°C with a convection heat transfer coefficient of 80 W/m²·K. If the furnace length is 3 m and the stainless steel strip enters it at 20°C, determine the temperature of the strip as it exits the furnace.

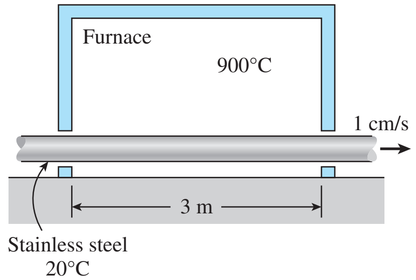


FIGURE P4–18

**4–19** A batch of 2-cm-thick stainless steel plates ($k$ = 21 W/m·K, $\rho$ = 8000 kg/m³, and $c_p$ = 570 J/kg·K) are conveyed through a furnace to be heat treated. The plates enter the furnace at 18°C, and travel a distance of 3 m inside the furnace. The air temperature in the furnace is maintained at 950°C with a convection heat transfer coefficient of 150 W/m²·K. Using EES (or other) software, determine how the velocity of the plates affects the temperature of the plates at the end of the heat treatment. Let the velocity of the plates vary from 5 to 60 mm/s, and plot the temperature of the plates at the furnace exit as a function of the velocity.

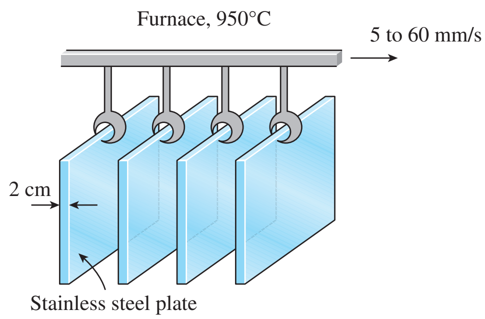


FIGURE P4–19

**4–20** A 6-mm-thick stainless steel strip ($k$ = 21 W/m·K, $\rho$ = 8000 kg/m³, and $c_p$ = 570 J/kg·K) exiting an oven at a temperature of 500°C is allowed to cool within a buffer zone distance of 5 m. To prevent thermal burn to workers

who are handling the strip at the end of the buffer zone, the surface temperature of the strip should be cooled to 45°C. If the air temperature in the buffer zone is 15°C and the convection heat transfer coefficient is 120 W/m²·K, determine the maximum speed of the stainless steel strip.

**4–21** After heat treatment, the 2-cm thick metal plates ($k$ = 180 W/m·K, $\rho$ = 2800 kg/m³, and $c_p$ = 880 J/kg·K) are conveyed through a cooling chamber with a length of 10 m. The plates enter the cooling chamber at an initial temperature of 500°C. The cooling chamber maintains a temperature of 10°C, and the convection heat transfer coefficient is given as a function of the air velocity blowing over the plates $h = 33V^{0.8}$, where $h$ is in W/m²·K and $V$ is in m/s. To prevent any incident of thermal burn, it is necessary for the plates to exit the cooling chamber at a temperature below 50°C. In designing the cooling process to meet this safety criteria, use the EES (or other) software to investigate the effect of the air velocity on the temperature of the plates at the exit of the cooling chamber. Let the air velocity vary from 0 to 40 m/s, and plot the temperatures of the plates exiting the cooling chamber as a function of air velocity at the moving plate speed of 2, 5, and 8 cm/s.

**4–22** A long copper rod of diameter 2.0 cm is initially at a uniform temperature of 100°C. It is now exposed to an air stream at 20°C with a heat transfer coefficient of 200 W/m²·K. How long would it take for the copper road to cool to an average temperature of 25°C?

**4–23** Springs in suspension system of automobiles are made of steel rods heated and wound into coils while ductile. Consider steel rods ($\rho$ = 7832 kg/m³, $c_p$ = 434 J/kg·K, and $k$ = 63.9 W/m·K) with diameter of 2.5 cm and length of 1.27 m. The steel rods are heated in an oven with a uniform convection heat transfer coefficient of 20 W/m²·K. The steel rods were heated from an initial temperature of 20°C to the desired temperature of 450°C before being wound into coils. Determine the ambient temperature in the oven, if the steel rods were to be heated to the desired temperature within 10 minutes.

**4–24** Steel rods ($\rho$ = 7832 kg/m³, $c_p$ = 434 J/kg·K, and $k$ = 63.9 W/m·K) are heated in a furnace to 850°C and then quenched in a water bath at 50°C for a period of 40 seconds as part of a hardening process. The convection heat transfer coefficient is 650 W/m²·K. If the steel rods have diameter of 40 mm and length of 2 m, determine their average temperature when they are taken out of the water bath.

**4–25** To warm up some milk for a baby, a mother pours milk into a thin-walled cylindrical container whose diameter is 6 cm. The height of the milk in the container is 7 cm. She then places the container into a large pan filled with hot water at 70°C. The milk is stirred constantly, so that its temperature is uniform at all times. If the heat transfer coefficient between the water and the container is 120 W/m²·K, determine how long it will take for the milk to warm up from 3°C to 38°C. Assume the entire surface area of the cylindrical container (including the top and bottom) is in thermal contact with the hot water. Take the properties of the milk to be the same as those of water. Can the milk in this case be treated as a lumped system? Why? *Answer:* 4.50 min

**4–26** A person is found dead at 5PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be 8 W/m²·K. Modeling the body as a 30-cm-diameter, 1.70-m-long cylinder and using the lumped system analysis as a rough approximation, estimate the time of death of that person.

**4–27** The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are $k$ = 35 W/m·K, $\rho$ = 8500 kg/m³, and $c_p$ = 320 J/kg·K, and the heat transfer coefficient between the junction and the gas is $h$ = 90 W/m²·K. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference. *Answer:* 27.8 s

**4–28** In an experiment, the temperature of a hot gas stream is to be measured by a thermocouple with a spherical junction. Due to the nature of this experiment, the response time of the thermocouple to register 99 percent of the initial temperature difference must be within 5 s. The properties of the thermocouple junction are $k$ = 35 W/m·K, $\rho$ = 8500 kg/m³, and $c_p$ = 320 J/kg·K. If the heat transfer coefficient between the thermocouple junction and the gas is 250 W/m²·K, determine the diameter of the junction.

**4–29** A thermocouple, with a spherical junction diameter of 0.5 mm, is used for measuring the temperature of hot air flow in a circular duct. The convection heat transfer coefficient of the air flow can be related with the diameter ($D$) of the duct and the average air flow velocity ($V$) as $h = 2.2(V/D)^{0.5}$, where $D$, $h$, and $V$ are in m, W/m²·K and m/s, respectively. The properties of the thermocouple junction are $k$ = 35 W/m·K, $\rho$ = 8500 kg/m³, and $c_p$ = 320 J/kg·K. Determine the minimum air flow velocity that the thermocouple can be used, if the maximum response time of the thermocouple to register 99 percent of the initial temperature difference is 5 s.

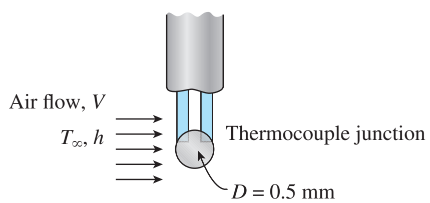


**FIGURE P4–29**

**4–30** Pulverized coal particles are used in oxy-fuel combustion power plants for electricity generation. Consider a situation where coal particles are suspended in hot air flowing through a heated tube, where the convection heat transfer coefficient is 100 W/m$^2$·K. If the average surface area and volume of the coal particles are 3.1 mm$^2$ and 0.5 mm$^3$, respectively, determine how much time it would take to heat the coal particles to two-thirds of the initial temperature difference.

**4–31** Oxy-fuel combustion power plants use pulverized coal particles as fuel to burn in a pure oxygen environment to generate electricity. Before entering the furnace, pulverized spherical coal particles with an average diameter of 300 μm, are being transported at 2 m/s through a 3-m long heated tube while suspended in hot air. The air temperature in the tube is 900°C and the average convection heat transfer coefficient is 250 W/m$^2$·K. Determine the temperature of the coal particles at the exit of the heated tube, if the initial temperature of the particles is 20°C.

**4–32** Plasma spraying is a process used for coating a material surface with a protective layer to prevent the material from degradation. In a plasma spraying process, the protective layer in powder form is injected into a plasma jet. The powder is then heated to molten droplets and propelled onto the material surface. Once deposited on the material surface, the molten droplets solidify and form a layer of protective coating. Consider a plasma spraying process using alumina ($k$ = 30 W/m·K, $\rho$ = 3970 kg/m$^3$, and $c_p$ = 800 J/kg·K) powder that is injected into a plasma jet at $T_\infty$ = 15,000°C and $h$ = 10,000 W/m$^2$·K. The alumina powder is made of particles that are spherical in shape with an average diameter of 60 μm and a melting point at 2300°C. Determine the amount of time it would take for the particles, with an initial temperature of 20°C, to reach their melting point from the moment they are injected into the plasma jet.

**4–33** Consider a spherical shell satellite with outer diameter of 4 m and shell thickness of 10 mm is reentering the atmosphere. The shell satellite is made of stainless steel with properties of $\rho$ = 8238 kg/m$^3$, $c_p$ = 468 J/kg·K, and $k$ = 13.4 W/m·K. During the reentry, the effective atmosphere temperature surrounding the satellite is 1250°C with convection heat transfer coefficient of 130 W/m$^2$·K. If the initial temperature of the shell is 10°C, determine the shell temperature after 5 minutes of reentry. Assume heat transfer occurs only on the satellite shell.

**4–34** Carbon steel balls ($\rho$ = 7833 kg/m$^3$, $k$ = 54 W/m·K, $c_p$ = 0.465 kJ/kg·°C, and $\alpha$ = 1.474 × 10$^{-6}$ m$^2$/s) 8 mm in diameter are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 75 W/m$^2$·K, determine how long the annealing process will take. If 2500 balls are to be annealed per hour, determine the total rate of heat transfer from the balls to the ambient air.

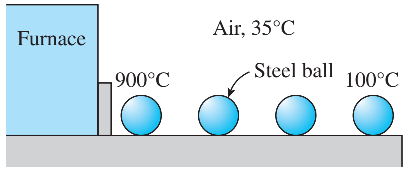


**FIGURE P4–34**

**4–35** Reconsider Prob. 4–34. Using EES (or other) software, investigate the effect of the initial temperature of the balls on the annealing time and the total rate of heat transfer. Let the temperature vary from 500°C to 1000°C. Plot the time and the total rate of heat transfer as a function of the initial temperature, and discuss the results.

**4–36E** In a manufacturing facility, 2-in-diameter brass balls ($k$ = 64.1 Btu/h·ft·°F, $\rho$ = 532 lbm/ft$^3$, and $c_p$ = 0.092 Btu/lbm·°F) initially at 250°F are quenched in a water bath at 120°F for a period of 2 min at a rate of 120 balls per minute. If the convection heat transfer coefficient is 42 Btu/h·ft$^2$·°F, determine (*a*) the temperature of the balls after quenching and (*b*) the rate at which heat needs to be removed from the water in order to keep its temperature constant at 120°F.

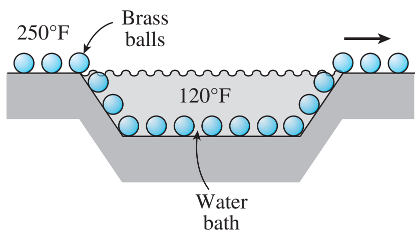


**FIGURE P4–36E**

**4–37** Consider a sphere of diameter 5 cm, a cube of side length 5 cm, and a rectangular prism of dimension 4 cm × 5 cm × 6 cm, all initially at 0°C and all made of silver ($k$ = 429 W/m·K, $\rho$ = 10,500 kg/m$^3$, $c_p$ = 0.235 kJ/kg·K). Now all three of these geometries are exposed to ambient air at 33°C on all of their surfaces with a heat transfer coefficient of 12 W/m$^2$·K. Determine how long it will take for the temperature of each geometry to rise to 25°C.

**4–38** An electronic device dissipating 20 W has a mass of 20 g, a specific heat of 850 J/kg·K, and a surface area of 4 cm$^2$. The device is lightly used, and it is on for 5 min and then off for several hours, during which it cools to the ambient temperature of 25°C. Taking the heat transfer coefficient to be 12 W/m$^2$·K, determine the temperature of the device at the end of the 5-min operating period. What would your answer be if the device were attached to an aluminum heat sink having a mass of 200 g and a surface area of 80 cm$^2$? Assume the device and the heat sink to be nearly isothermal.

## Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

**4–39C** An egg is to be cooked to a certain level of doneness by being dropped into boiling water. Can the cooking time be shortened by turning up the heat and bringing water to a more rapid boiling?

**4–40C** What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not? For example, is it proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder? Explain.

**4–41C** What is the physical significance of the Fourier number? Will the Fourier number for a specified heat transfer problem double when the time is doubled?

**4–42C** Why are the transient temperature charts prepared using nondimensionalized quantities such as the Biot and Fourier numbers instead of the actual variables such as thermal conductivity and time?

**4–43C** Can the transient temperature charts in Fig. 4–17 for a plane wall exposed to convection on both sides be used for a plane wall with one side exposed to convection while the other side is insulated? Explain.

**4–44C** How can we use the transient temperature charts when the surface temperature of the geometry is specified instead of the temperature of the surrounding medium and the convection heat transfer coefficient?

**4–45C** The Biot number during a heat transfer process between a sphere and its surroundings is determined to be 0.02. Would you use lumped system analysis or the transient temperature charts when determining the midpoint temperature of the sphere? Why?

**4–46C** A body at an initial temperature of $T_i$ is brought into a medium at a constant temperature of $T_\infty$. How can you determine the maximum possible amount of heat transfer between the body and the surrounding medium?

**4–47** A hot brass plate is having its upper surface cooled by impinging jet of air at temperature of 15°C and convection heat transfer coefficient of 220 W/m²·K. The 10-cm thick brass plate ($\rho$ = 8530 kg/m³, $c_p$ = 380 J/kg·K, $k$ = 110 W/m·K, and $\alpha = 33.9 \times 10^{-6}$ m²/s) has a uniform initial temperature of 650°C, and the bottom surface of the plate is insulated. Determine the temperature at the center plane of the brass plate after 3 minutes of cooling. Solve this problem using analytical one-term approximation method (not the Heisler charts).

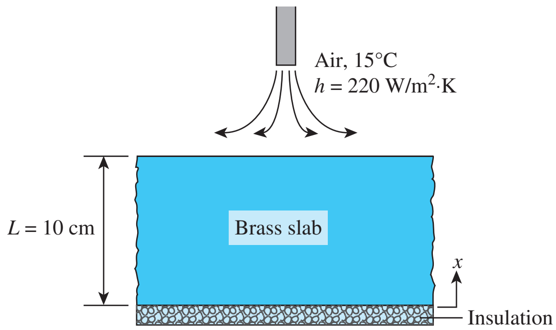


FIGURE P4–47

**4–48** In a meat processing plant, 2-cm-thick steaks ($k$ = 0.45 W/m·K and $\alpha = 0.91 \times 10^{-7}$ m²/s) that are initially at 25°C are to be cooled by passing them through a refrigeration room at −11°C. The heat transfer coefficient on both sides of the steaks is 9 W/m²·K. If both surfaces of the steaks are to be cooled to 2°C, determine how long the steaks should be kept in the refrigeration room. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–49** A 10-cm thick aluminum plate ($\rho$ = 2702 kg/m³, $c_p$ = 903 J/kg·K, $k$ = 237 W/m·K, and $\alpha = 97.1 \times 10^{-6}$ m²/s) is being heated in liquid with temperature of 500°C. The aluminum plate has a uniform initial temperature of 25°C. If the surface temperature of the aluminum plate is approximately the liquid temperature, determine the temperature at the center plane of the aluminum plate after 15 seconds of heating. Solve this problem using analytical one-term approximation method (not the Heisler charts).

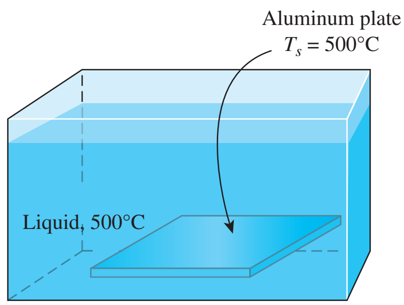


FIGURE P4–49

**4–50** In a production facility, 3-cm-thick large brass plates ($k$ = 110 W/m·K, $\rho$ = 8530 kg/m³, $c_p$ = 380 J/kg·K, and

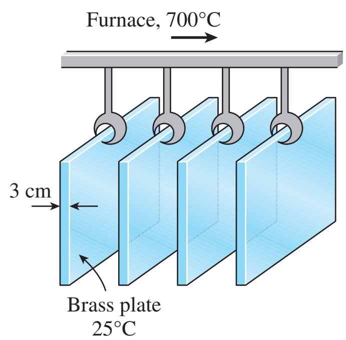


FIGURE P4–50

$\alpha = 33.9 \times 10^{-6}$ m²/s) that are initially at a uniform temperature of 25°C are heated by passing them through an oven maintained at 700°C. The plates remain in the oven for a period of 10 min. Taking the convection heat transfer coefficient to be $h$ = 80 W/m²·K, determine the surface temperature of the plates when they come out of the oven. Solve this problem using analytical one-term approximation method (not the Heisler charts). Can this problem be solved using lumped system analysis? Justify your answer.

**4–51** Reconsider Prob. 4–50. Using EES (or other) software, investigate the effects of the temperature of the oven and the heating time on the final surface temperature of the plates. Let the oven temperature vary from 500°C to 900°C and the time from 2 min to 30 min. Plot the surface temperature as the functions of the oven temperature and the time, and discuss the results.

**4–52** Layers of 23-cm-thick meat slabs ($k$ = 0.47 W/m·K and $\alpha = 0.13 \times 10^{-6}$ m²/s) initially at a uniform temperature of 7°C are to be frozen by refrigerated air at −30°C flowing at a velocity of 1.4 m/s. The average heat transfer coefficient between the meat and the air is 20 W/m²·K. Assuming the size of the meat slabs to be large relative to their thickness, determine how long it will take for the center temperature of the slabs to drop to −18°C. Also, determine the surface temperature of the meat slab at that time.

**4–53** In an annealing process, a 50-mm-thick stainless steel plate ($\rho$ = 8238 kg/m³, $c_p$ = 468 J/kg·K, $k$ = 13.4 W/m·K, and $\alpha = 3.48 \times 10^{-6}$ m²/s) was reheated in a furnace from an initial uniform temperature of 230°C. The ambient temperature inside the furnace is at a uniform temperature of 1000°C and has a convection heat transfer coefficient of 215 W/m²·K. If the entire stainless steel plate is to be heated to at least 600°C, determine the time that the plate should be heated in the furnace using ($a$) Table 4–2 and ($b$) the Heisler chart (Figure 4–17).

**4–54** A heated 6-mm-thick Pyroceram plate ($\rho$ = 2600 kg/m³, $c_p$ = 808 J/kg·K, $k$ = 3.98 W/m·K, and $\alpha = 1.89 \times 10^{-6}$ m²/s) is being cooled in a room with air temperature of 25°C and convection heat transfer coefficient of 13.3 W/m²·K. The heated Pyroceram plate had an initial temperature of 500°C, and is allowed to cool for 286 seconds. If the mass of the Pyroceram plate is 10 kg, determine the heat transfer from the Pyroceram plate during the cooling process using ($a$) Table 4–2 and ($b$) Figure 4–17.

**4–55E** Layers of 6-in-thick meat slabs ($k$ = 0.26 Btu/h·ft·°F and $\alpha = 1.4 \times 10^{-6}$ ft²/s) initially at a uniform temperature of 50°F are cooled by refrigerated air at 23°F to a temperature of 36°F at their center in 12 h. Estimate the average heat transfer coefficient during this cooling process. Solve this problem using the Heisler charts. *Answer:* 1.5 Btu/h·ft²·°F

**4–56** A long cylindrical wood log ($k$ = 0.17 W/m·K and $\alpha = 1.28 \times 10^{-7}$ m²/s) is 10 cm in diameter and is initially at a uniform temperature of 15°C. It is exposed to hot gases at 550°C in a fireplace with a heat transfer coefficient of 13.6 W/m²·K on the surface. If the ignition temperature of the wood is 420°C, determine how long it will be before the log ignites. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–57E** Long cylindrical AISI stainless steel rods ($k$ = 7.74 Btu/h·ft·°F and $\alpha$ = 0.135 ft²/h) of 4-in-diameter are heat treated by drawing them at a velocity of 7 ft/min through a 21-ft-long oven maintained at 1700°F. The heat transfer coefficient in the oven is 20 Btu/h·ft²·°F. If the rods enter the oven at 70°F, determine their centerline temperature when they leave. Solve this problem using analytical one-term approximation method (not the Heisler charts).

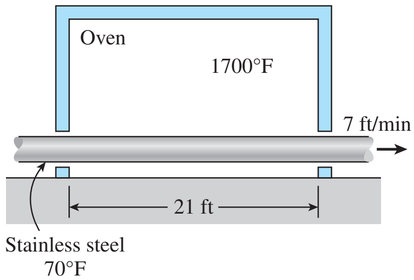


**FIGURE P4–57E**

**4–58** A long iron rod ($\rho$ = 7870 kg/m³, $c_p$ = 447 J/kg·K, $k$ = 80.2 W/m·K, and $\alpha = 23.1 \times 10^{-6}$ m²/s) with diameter of 25 mm is initially heated to a uniform temperature of 700°C. The iron rod is then quenched in a large water bath that is maintained at constant temperature of 50°C and convection heat transfer coefficient of 128 W/m²·K. Determine the time required for the iron rod surface temperature to cool to 200°C. Solve this problem using analytical one-term approximation method (not the Heisler charts).

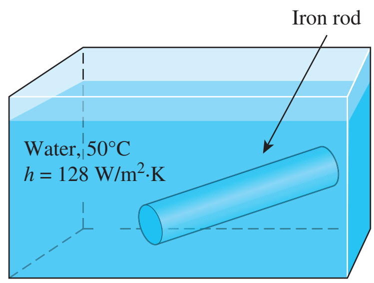


**FIGURE P4–58**

**4–59** A 30-cm-diameter, 4-m-high cylindrical column of a house made of concrete ($k$ = 0.79 W/m·K, $\alpha = 5.94 \times 10^{-7}$ m²/s, $\rho$ = 1600 kg/m³, and $c_p$ = 0.84 kJ/kg·K) cooled to 14°C during a cold night is heated again during the day by being exposed

to ambient air at an average temperature of 28°C with an average heat transfer coefficient of 14 W/m²·K. Using analytical one-term approximation method (not the Heisler charts), determine (*a*) how long it will take for the column surface temperature to rise to 27°C, (*b*) the amount of heat transfer until the center temperature reaches to 28°C, and (*c*) the amount of heat transfer until the surface temperature reaches to 27°C.

**4–60** A long 35-cm-diameter cylindrical shaft made of stainless steel 304 ($k$ = 14.9 W/m·K, $\rho$ = 7900 kg/m³, $c_p$ = 477 J/kg·K, and $\alpha = 3.95 \times 10^{-6}$ m²/s) comes out of an oven at a uniform temperature of 400°C. The shaft is then allowed to cool slowly in a chamber at 150°C with an average convection heat transfer coefficient of $h$ = 60 W/m²·K. Determine the temperature at the center of the shaft 20 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. Solve this problem using analytical one-term approximation method (not the Heisler charts). *Answers:* 390°C, 15,900 kJ

**4–61** Reconsider Prob. 4–60. Using EES (or other) software, investigate the effect of the cooling time on the final center temperature of the shaft and the amount of heat transfer. Let the time vary from 5 min to 60 min. Plot the center temperature and the heat transfer as a function of the time, and discuss the results.

**4–62** A 2-cm-diameter plastic rod has a thermocouple inserted to measure temperature at the center of the rod. The plastic rod ($\rho$ = 1190 kg/m³, $c_p$ = 1465 J/kg·K, and $k$ = 0.19 W/m·K) was initially heated to a uniform temperature of 70°C, and allowed to be cooled in ambient air temperature of 25°C. After 1388 s of cooling, the thermocouple measured the temperature at the center of the rod to be 30°C. Determine the convection heat transfer coefficient for this process. Solve this problem using analytical one-term approximation method (not the Heisler charts).

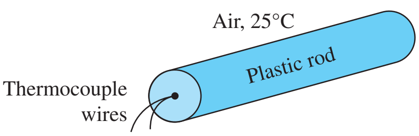


**FIGURE P4–62**

**4–63** A 65-kg beef carcass ($k$ = 0.47 W/m·K and $\alpha = 0.13 \times 10^{-6}$ m²/s) initially at a uniform temperature of 37°C is to be cooled by refrigerated air at −10°C flowing at a velocity of 1.2 m/s. The average heat transfer coefficient between the carcass and the air is 22 W/m²·K. Treating the carcass as a cylinder of diameter 24 cm and height 1.4 m and disregarding heat transfer from the base and top surfaces, determine how long it will take for the center temperature of the carcass to drop to 4°C. Also, determine if any part of the carcass will freeze during this process. *Answer:* 12.2 h

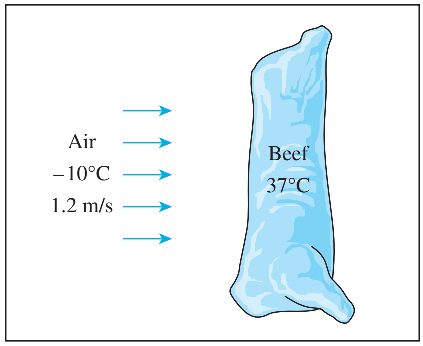


**FIGURE P4–63**

**4–64** A long Pyroceram rod ($\rho$ = 2600 kg/m³, $c_p$ = 808 J/kg·K, $k$ = 3.98 W/m·K, and $\alpha = 1.89 \times 10^{-6}$ m²/s) with diameter of 10 mm has an initial uniform temperature of 1000°C. The Pyroceram rod is allowed to cool in ambient temperature of 25°C and convection heat transfer coefficient of 80 W/m²·K. If the Pyroceram rod is allowed to cool for 3 minutes, determine the temperature at the center of the rod using (*a*) Table 4–2 and (*b*) the Heisler chart (Figure 4–18).

**4–65** Steel rods, 2 m in length and 60 mm in diameter, are being drawn through an oven that maintains a temperature of 800°C and convection heat transfer coefficient of 128 W/m²·K. The steel rods ($\rho$ = 7832 kg/m³, $c_p$ = 434 J/kg·K, $k$ = 63.9 W/m·K, and $\alpha = 18.8 \times 10^{-6}$ m²/s) were initially in uniform temperature of 30°C. Using (*a*) Table 4–2 and (*b*) Figure 4–18, determine the amount of heat is transferred to the steel rod after 133 s of heating.

**4–66** A father and son conducted the following simple experiment on a hot dog which measured 12.5 cm in length and 2.2 cm in diameter. They inserted one food thermometer into the midpoint of the hot dog and another one was placed just under the skin of the hot dog. The temperatures of the thermometers were monitored until both thermometers read 20°C, which is the ambient temperature. The hot dog was then placed in 94°C boiling water and after exactly 2 minutes they recorded the center temperature and the skin temperature of the hot dog to be 59°C and 88°C, respectively. Assuming the following properties for the hot dog: $\rho$ = 980 kg/m³ and $c_p$ = 3900 J/kg·K and using transient temperature charts, determine (*a*) the thermal diffusivity of the hot dog, (*b*) the thermal conductivity of the hot dog, and (*c*) the convection heat transfer coefficient.

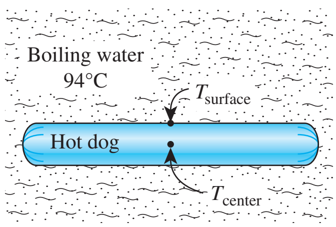


**FIGURE P4–66**

**4–67** An experiment is to be conducted to determine heat transfer coefficient on the surfaces of tomatoes that are placed in cold water at 7°C. The tomatoes ($k$ = 0.59 W/m·K, $\alpha$ = 0.141 $\times$ $10^{-6}$ m$^2$/s, $\rho$ = 999 kg/m$^3$, $c_p$ = 3.99 kJ/kg·K) with an initial uniform temperature of 30°C are spherical in shape with a diameter of 8 cm. After a period of 2 hours, the temperatures at the center and the surface of the tomatoes are measured to be 10.0°C and 7.1°C, respectively. Using analytical one-term approximation method (not the Heisler charts), determine the heat transfer coefficient and the amount of heat transfer during this period if there are eight such tomatoes in water.

**4–68** An ordinary egg can be approximated as a 5.5-cm-diameter sphere whose properties are roughly $k$ = 0.6 W/m·K and $\alpha$ = 0.14 $\times$ $10^{-6}$ m$^2$/s. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Taking the convection heat transfer coefficient to be $h$ = 1400 W/m$^2$·K, determine how long it will take for the center of the egg to reach 70°C. Solve this problem using analytical one-term approximation method (not the Heisler charts).

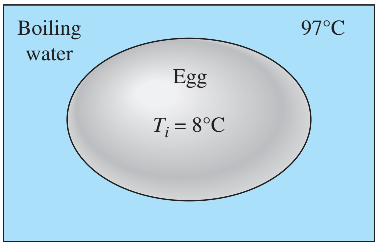


**FIGURE P4–68**

**4–69** Reconsider Prob. 4–68. Using EES (or other) software, investigate the effect of the final center temperature of the egg on the time it will take for the center to reach this temperature. Let the temperature vary from 50°C to 95°C. Plot the time versus the temperature, and discuss the results.

**4–70** For heat transfer purposes, an egg can be considered to be a 5.5-cm-diameter sphere having the properties of water. An egg that is initially at 8°C is dropped into the boiling water at 100°C. The heat transfer coefficient at the surface of the egg is estimated to be 800 W/m$^2$·K. If the egg is considered cooked when its center temperature reaches 60°C, determine how long the egg should be kept in the boiling water. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–71** Citrus fruits are very susceptible to cold weather, and extended exposure to subfreezing temperatures can destroy them. Consider an 8-cm-diameter orange that is initially at 15°C. A cold front moves in one night, and the ambient temperature suddenly drops to −6°C, with a heat transfer coefficient of 15 W/m$^2$·K. Using the properties of water for the orange and assuming the ambient conditions to remain constant for 4 h before the cold front moves out, determine if any part of the orange will freeze that night. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–72** A person puts a few apples into the freezer at −15°C to cool them quickly for guests who are about to arrive. Initially, the apples are at a uniform temperature of 20°C, and the heat transfer coefficient on the surfaces is 8 W/m$^2$·K. Treating the apples as 9-cm-diameter spheres and taking their properties to be $\rho$ = 840 kg/m$^3$, $c_p$ = 3.81 kJ/kg·K, $k$ = 0.418 W/m·K, and $\alpha$ = 1.3 $\times$ $10^{-7}$ m$^2$/s, determine the center and surface temperatures of the apples in 1 h. Also, determine the amount of heat transfer from each apple. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–73** Reconsider Prob. 4–72. Using EES (or other) software, investigate the effect of the initial temperature of the apples on the final center and surface temperatures and the amount of heat transfer. Let the initial temperature vary from 2°C to 30°C. Plot the center temperature, the surface temperature, and the amount of heat transfer as a function of the initial temperature, and discuss the results.

**4–74** A 9-cm-diameter potato ($\rho$ = 1100 kg/m$^3$, $c_p$ = 3900 J/kg·K, $k$ = 0.6 W/m·K, and $\alpha$ = 1.4 $\times$ $10^{-7}$ m$^2$/s) that is initially at a uniform temperature of 25°C is baked in an oven at 170°C until a temperature sensor inserted to the center of the potato indicates a reading of 70°C. The potato is then taken out of the oven and wrapped in thick towels so that almost no heat is lost from the baked potato. Assuming the heat transfer coefficient in the oven to be 40 W/m$^2$·K, determine (*a*) how long the potato is baked in the oven and (*b*) the final equilibrium temperature of the potato after it is wrapped. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–75** Chickens with an average mass of 1.7 kg ($k$ = 0.45 W/m·K and $\alpha$ = 0.13 $\times$ $10^{-6}$ m$^2$/s) initially at a uniform temperature of 15°C are to be chilled in agitated brine at −7°C. The average heat transfer coefficient between the chicken and the brine is determined experimentally to be 440 W/m$^2$·K. Taking the average density of the chicken to be 0.95 g/cm$^3$ and treating the chicken as a spherical lump, determine the center and the surface temperatures of the chicken in 2 h and 45 min. Also, determine if any part of the chicken will freeze during this process. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–76** Hailstones are formed in high altitude clouds at 253 K. Consider a hailstone with diameter of 20 mm and is falling through air at 15°C with convection heat transfer coefficient of 163 W/m$^2$·K. Assuming the hailstone can be modeled as a sphere and has properties of ice at 253 K, determine the duration it takes to reach melting point at the surface of the falling hailstone. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–77** In *Betty Crocker's Cookbook,* it is stated that it takes 2 h 45 min to roast a 3.2-kg rib initially at 4.5°C "rare" in an oven maintained at 163°C. It is recommended that a meat thermometer be used to monitor the cooking, and the rib is considered rare done when the thermometer inserted into the center

of the thickest part of the meat registers 60°C. The rib can be treated as a homogeneous spherical object with the properties $\rho$ = 1200 kg/m$^3$, $c_p$ = 4.1 kJ/kg·K, $k$ = 0.45 W/m·K, and $\alpha$ = 0.91 × 10$^{-7}$ m$^2$/s. Determine (*a*) the heat transfer coefficient at the surface of the rib; (*b*) the temperature of the outer surface of the rib when it is done; and (*c*) the amount of heat transferred to the rib. (*d*) Using the values obtained, predict how long it will take to roast this rib to "medium" level, which occurs when the innermost temperature of the rib reaches 71°C. Compare your result to the listed value of 3 h 20 min.

If the roast rib is to be set on the counter for about 15 min before it is sliced, it is recommended that the rib be taken out of the oven when the thermometer registers about 4°C below the indicated value because the rib will continue cooking even after it is taken out of the oven. Do you agree with this recommendation? Solve this problem using analytical one-term approximation method (not the Heisler charts).
*Answers: (a)* 156.9 W/m$^2$·K, *(b)* 159.5°C, *(c)* 1629 kJ, *(d)* 3.0 h

**4–78** Repeat Prob. 4–77 for a roast rib that is to be "well-done" instead of "rare." A rib is considered to be well-done when its center temperature reaches 77°C, and the roasting in this case takes about 4 h 15 min.

**4–79** White potatoes ($k$ = 0.50 W/m·K and $\alpha$ = 0.13 × 10$^{-6}$ m$^2$/s) that are initially at a uniform temperature of 25°C and have an average diameter of 6 cm are to be cooled by refrigerated air at 2°C flowing at a velocity of 4 m/s. The average heat transfer coefficient between the potatoes and the air is experimentally determined to be 19 W/m$^2$·K. Determine how long it will take for the center temperature of the potatoes to drop to 6°C. Also, determine if any part of the potatoes will experience chilling injury during this process.

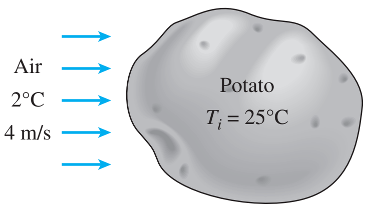


**FIGURE P4–79**

**4–80E** Oranges of 2.5-in-diameter ($k$ = 0.26 Btu/h·ft·°F and $\alpha$ = 1.4 × 10$^{-6}$ ft$^2$/s) initially at a uniform temperature of 78°F are to be cooled by refrigerated air at 25°F flowing at a velocity of 1 ft/s. The average heat transfer coefficient between the oranges and the air is experimentally determined to be 4.6 Btu/h·ft$^2$·°F. Determine how long it will take for the center temperature of the oranges to drop to 40°F. Also, determine if any part of the oranges will freeze during this process.

**4–81E** In a chicken processing plant, whole chickens averaging 5 lbm each and initially at 65°F are to be cooled in the racks of a large refrigerator that is maintained at 5°F. The entire chicken is to be cooled below 45°F, but the temperature of the chicken is not to drop below 35°F at any point during refrigeration. The convection heat transfer coefficient and thus the rate of heat transfer from the chicken can be controlled by varying the speed of a circulating fan inside. Determine the heat transfer-coefficient that will enable us to meet both temperature constraints while keeping the refrigeration time to a minimum. The chicken can be treated as a homogeneous spherical object having the properties $\rho$ = 74.9 lbm/ft$^3$, $c_p$ = 0.98 Btu/lbm·°F, $k$ = 0.26 Btu/h·ft·°F, and $\alpha$ = 0.0035 ft$^2$/h. Solve this problem using the Heisler charts.

## Transient Heat Conduction in Semi-Infinite Solids

**4–82C** Under what conditions can a plane wall be treated as a semi-infinite medium?

**4–83C** What is a semi-infinite medium? Give examples of solid bodies that can be treated as semi-infinite mediums for heat transfer purposes.

**4–84C** Consider a hot semi-infinite solid at an initial temperature of $T_i$ that is exposed to convection to a cooler medium at a constant temperature of $T_\infty$, with a heat transfer coefficient of $h$. Explain how you can determine the total amount of heat transfer from the solid up to a specified time $t_o$.

**4–85E** The walls of a furnace are made of 1.2-ft-thick concrete ($k$ = 0.64 Btu/h·ft·°F and $\alpha$ = 0.023 ft$^2$/h). Initially, the furnace and the surrounding air are in thermal equilibrium at 70°F. The furnace is then fired, and the inner surfaces of the furnace are subjected to hot gases at 1800°F with a very large heat transfer coefficient. Determine how long it will take for the temperature of the outer surface of the furnace walls to rise to 70.1°F. *Answer:* 116 min

**4–86** Consider a curing kiln whose walls are made of 30-cm-thick concrete with a thermal diffusivity of $\alpha$ = 0.23 × 10$^{-5}$ m$^2$/s. Initially, the kiln and its walls are in equilibrium with the surroundings at 6°C. Then all the doors are closed and the kiln is heated by steam so that the temperature of the inner surface of the walls is raised to 42°C and the temperature is maintained at that level for 2.5 h. The curing kiln is then opened and exposed to the atmospheric air after the steam flow is turned off. If the outer surfaces of the walls of the kiln were insulated, would it save any energy that day during the period the kiln was used for curing for 2.5 h only, or would it make no difference? Base your answer on calculations.

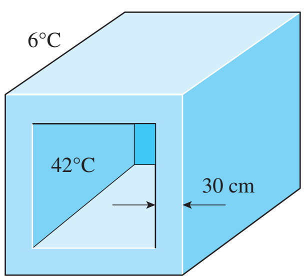


**FIGURE P4–86**

**4–87** In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as an insulation to protect the water from the freezing atmospheric temperatures in winter.

The ground at a particular location is covered with snow pack at −8°C for a continuous period of 60 days, and the average soil properties at that location are $k = 0.35$ W/m·K and $\alpha = 0.15 \times 10^{-6}$ m$^2$/s. Assuming an initial uniform temperature of 8°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

**4–88** A large cast iron container ($k = 52$ W/m·K and $\alpha = 1.70 \times 10^{-5}$ m$^2$/s) with 5-cm-thick walls is initially at a uniform temperature of 0°C and is filled with ice at 0°C. Now the outer surfaces of the container are exposed to hot water at 60°C with a very large heat transfer coefficient. Determine how long it will be before the ice inside the container starts melting. Also, taking the heat transfer coefficient on the inner surface of the container to be 250 W/m$^2$·K, determine the rate of heat transfer to the ice through a 1.2-m-wide and 2-m-high section of the wall when steady operating conditions are reached. Assume the ice starts melting when its inner surface temperature rises to 0.1°C.

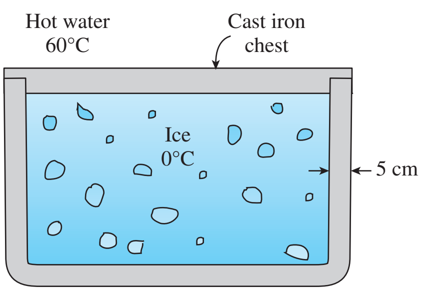


**FIGURE P4–88**

**4–89** A highway made of asphalt is initially at a uniform temperature of 55°C. Suddenly the highway surface temperature is reduced to 25°C by rain. Determine the temperature at the depth of 3 cm from the highway surface and the heat flux transferred from the highway after 60 minutes. Assume the highway surface temperature is maintained at 25°C. *Answers:* 53.6°C, 98 W/m$^2$

**4–90** A thick aluminum block initially at 20°C is subjected to constant heat flux of 4000 W/m$^2$ by an electric resistance heater whose top surface is insulated. Determine how much the surface temperature of the block will rise after 30 minutes.

**4–91** Refractory bricks are used as linings for furnaces, and they generally have low thermal conductivity to minimize heat loss through the furnace walls. Consider a thick furnace wall lining with refractory bricks ($k = 1.0$ W/m·K and $\alpha = 5.08 \times 10^{-7}$ m$^2$/s), where initially the wall has a uniform temperature of 15°C. If the wall surface is subjected to uniform heat flux of 20 kW/m$^2$, determine the temperature at the depth of 10 cm from the surface after an hour of heating time.

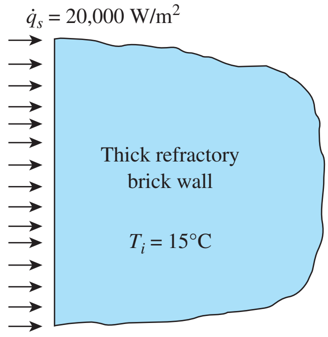


**FIGURE P4–91**

**4–92** A thick wall made of refractory bricks ($k = 1.0$ W/m·K and $\alpha = 5.08 \times 10^{-7}$ m$^2$/s) has a uniform initial temperature of 15°C. The wall surface is subjected to uniform heat flux of 20 kW/m$^2$. Using EES (or other) software, investigate the effect of heating time on the temperature at the wall surface and at $x = 1$ cm and $x = 5$ cm from the surface. Let the heating time vary from 10 to 3600 s, and plot the temperatures at $x = 0$, 1, and 5 cm from the wall surface as a function of heating time.

**4–93** A stainless steel slab ($k = 14.9$ W/m·K and $\alpha = 3.95 \times 10^{-6}$ m$^2$/s) and a copper slab ($k = 401$ W/m·K and $\alpha = 117 \times 10^{-6}$ m$^2$/s) are subjected to uniform heat flux of 10 kW/m$^2$ at the surface. Both slabs have a uniform initial temperature of 20°C. Using EES (or other) software, investigate the effect of time on the temperatures of both materials at the depth of 8 cm from the surface. By varying the time of exposure to the heat flux from 5 to 300 s, plot the temperatures at a depth of $x = 8$ cm from the surface as a function of time.

**4–94** Thick slabs of stainless steel ($k = 14.9$ W/m·K and $\alpha = 3.95 \times 10^{-6}$ m$^2$/s) and copper ($k = 401$ W/m·K and $\alpha = 117 \times 10^{-6}$ m$^2$/s) are subjected to uniform heat flux of 8 kW/m$^2$ at the surface. The two slabs have a uniform initial temperature of 20°C. Determine the temperatures of both slabs, at 1 cm from the surface, after 60 s of exposure to the heat flux.

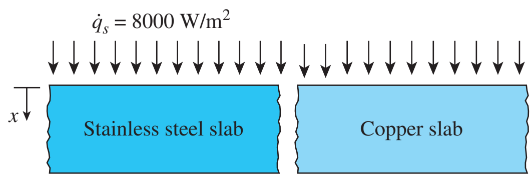


**FIGURE P4–94**

**4–95** A thick wood slab ($k$ = 0.17 W/m·K and $\alpha = 1.28 \times 10^{-7}$ m²/s) that is initially at a uniform temperature of 25°C is exposed to hot gases at 550°C for a period of 5 min. The heat transfer coefficient between the gases and the wood slab is 35 W/m²·K. If the ignition temperature of the wood is 450°C, determine if the wood will ignite.

**4–96** The soil temperature in the upper layers of the earth varies with the variations in the atmospheric conditions. Before a cold front moves in, the earth at a location is initially at a uniform temperature of 10°C. Then the area is subjected to a temperature of −10°C and high winds that resulted in a convection heat transfer coefficient of 40 W/m²·K on the earth's surface for a period of 10 h. Taking the properties of the soil at that location to be $k$ = 0.9 W/m·K and $\alpha = 1.6 \times 10^{-5}$ m²/s, determine the soil temperature at distances 0, 10, 20, and 50 cm from the earth's surface at the end of this 10-h period.

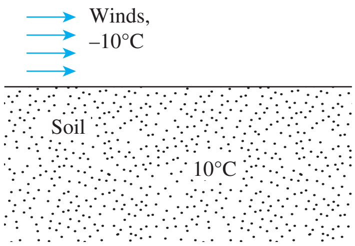


**FIGURE P4–96**

**4–97** Reconsider Prob. 4–96. Using EES (or other) software, plot the soil temperature as a function of the distance from the earth's surface as the distance varies from 0 m to 1m, and discuss the results.

**4–98** We often cut a watermelon in half and put it into the freezer to cool it quickly. But usually we forget to check on it and end up having a watermelon with a frozen layer on the top. To avoid this potential problem a person wants to set the timer such that it will go off when the temperature of the exposed surface of the watermelon drops to 3°C. Consider a 25-cm-diameter spherical watermelon that is cut into two equal parts and put into a freezer at −12°C. Initially, the entire watermelon is at a uniform temperature of 25°C, and the heat transfer coefficient on the surfaces is 22 W/m²·K. Assuming the watermelon to have the properties of water, determine how long it will take for the center of the exposed cut surfaces of the watermelon to drop to 3°C.

**4–99** In a vacuum chamber, a thick slab is placed under an array of laser diodes with an output constant pulse. A thermocouple is inserted inside the slab at 25 mm from the surface and the slab has an initial uniform temperature of 20°C. The known properties of the slab are $k$ = 63.9 W/m·K and $a = 18.8 \times 10^{-6}$ m²/s. If the thermocouple measured a temperature of 130°C after 30 s the slab surface has been exposed to the laser pulse, determine ($a$) the amount of energy per unit surface area directed on the slab surface and ($b$) the thermocouple reading after 60 s has elapsed.

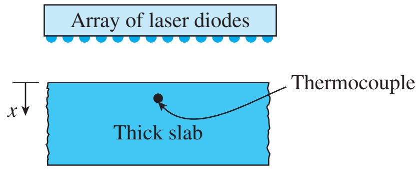


**FIGURE P4–99**

**4–100** Thick slabs of stainless steel ($k$ = 14.9 W/m·K and $\alpha = 3.95 \times 10^{-6}$ m²/s) and copper ($k$ = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m²/s) are placed under an array of laser diodes, which supply an energy pulse of $5 \times 10^7$ J/m² instantaneously at $t = 0$ to both materials. The two slabs have a uniform initial temperature of 20°C. Determine the temperatures of both slabs, at 5 cm from the surface and 60 s after receiving an energy pulse from the laser diodes.

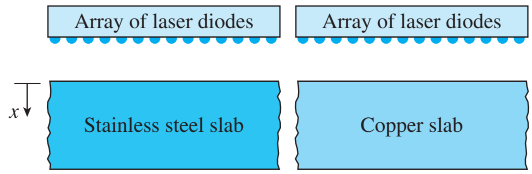


**FIGURE P4–100**

**4–101** A stainless steel slab ($k$ = 14.9 W/m·K and $\alpha = 3.95 \times 10^{-6}$ m²/s) and a copper slab ($k$ = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m²/s) are placed under an array of laser diodes, which supply an energy pulse of $5 \times 10^7$ J/m² instantaneously at $t = 0$ to both materials. The two slabs have a uniform initial temperature of 20°C. Using EES (or other) software, investigate the effect of time on the temperatures of both materials at the depth of 5 cm from the surface. By varying the time from 1 to 80 s after the slabs have received the energy pulse, plot the temperatures at 5 cm from the surface as a function of time.

**4–102** A bare-footed person whose feet are at 32°C steps on a large aluminum block at 20°C. Treating both the feet and the aluminum block as semi-infinite solids, determine the contact surface temperature. What would your answer be if the person stepped on a wood block instead? At room temperature, the $\sqrt{k\rho c_p}$ value is 24 kJ/m²·°C for aluminum, 0.38 kJ/m²·°C for wood, and 1.1 kJ/m²·°C for human flesh.

## Transient Heat Conduction in Multidimensional Systems

**4–103C** What is the product solution method? How is it used to determine the transient temperature distribution in a two-dimensional system?

**4–104C** How is the product solution used to determine the variation of temperature with time and position in three-dimensional systems?

**4–105C** A short cylinder initially at a uniform temperature $T_i$ is subjected to convection from all of its surfaces to a medium at temperature $T_\infty$. Explain how you can determine the temperature of the midpoint of the cylinder at a specified time $t$.

**4–106C** Consider a short cylinder whose top and bottom surfaces are insulated. The cylinder is initially at a uniform temperature $T_i$ and is subjected to convection from its side surface to a medium at temperature $T_\infty$ with a heat transfer coefficient of $h$. Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

**4–107** Consider a cubic block whose sides are 5 cm long and a cylindrical block whose height and diameter are also 5 cm. Both blocks are initially at 20°C and are made of granite ($k$ = 2.5 W/m·K and $\alpha = 1.15 \times 10^{-6}$ m²/s). Now both blocks are exposed to hot gases at 500°C in a furnace on all of their surfaces with a heat transfer coefficient of 40 W/m²·K. Determine the center temperature of each geometry after 10, 20, and 60 min.

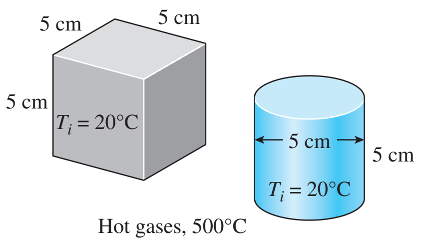


**FIGURE P4–107**

**4–108** Repeat Prob. 4–107 with the heat transfer coefficient at the top and the bottom surfaces of each block being doubled to 80 W/m²·K.

**4–109E** A hot dog can be considered to be a cylinder 5 in long and 0.8 in in diameter whose properties are $\rho$ = 61.2 lbm/ft³, $c_p$ = 0.93 Btu/lbm·°F, $k$ = 0.44 Btu/h·ft·°F, and $\alpha$ = 0.0077 ft²/h. A hot dog initially at 40°F is dropped into boiling water at 212°F. If the heat transfer coefficient at the surface of the hot dog is estimated to be 120 Btu/h·ft²·°F, determine the center temperature of the hot dog after 5, 10, and 15 min by treating the hot dog as (*a*) a finite cylinder and (*b*) an infinitely long cylinder.

**4–110** A 5-cm-high rectangular ice block ($k$ = 2.22 W/m·K and $\alpha = 0.124 \times 10^{-7}$ m²/s) initially at −20°C is placed on a table on its square base 4 cm × 4 cm in size in a room at 18°C. The heat transfer coefficient on the exposed surfaces of the ice block is 12 W/m²·K. Disregarding any heat transfer from the base to the table, determine how long it will be before the ice block starts melting. Where on the ice block will the first liquid droplets appear?

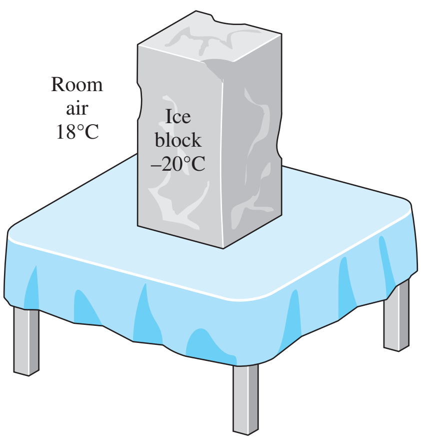


**FIGURE P4–110**

**4–111** Reconsider Prob. 4–110. Using EES (or other) software, investigate the effect of the initial temperature of the ice block on the time period before the ice block starts melting. Let the initial temperature vary from −26°C to −4°C. Plot the time versus the initial temperature, and discuss the results.

**4–112** A 2-cm-high cylindrical ice block ($k$ = 2.22 W/m·K and $\alpha = 0.124 \times 10^{-7}$ m²/s) is placed on a table on its base of diameter 2 cm in a room at 24°C. The heat transfer coefficient on the exposed surfaces of the ice block is 13 W/m²·K, and heat transfer from the base of the ice block to the table is negligible. If the ice block is not to start melting at any point for at least 3 h, determine what the initial temperature of the ice block should be.

**4–113** A short brass cylinder ($\rho$ = 8530 kg/m³, $c_p$ = 0.389 kJ/kg·K, $k$ = 110 W/m·K, and $\alpha = 3.39 \times 10^{-5}$ m²/s) of diameter 8 cm and height 15 cm is initially at a uniform temperature of 150°C. The cylinder is now placed in atmospheric air at 20°C, where heat transfer takes place by convection with a heat transfer coefficient of 40 W/m²·K. Calculate (*a*) the center temperature of the cylinder; (*b*) the center temperature of the top surface of the cylinder; and (*c*) the total heat transfer from the cylinder 15 min after the start of the cooling.

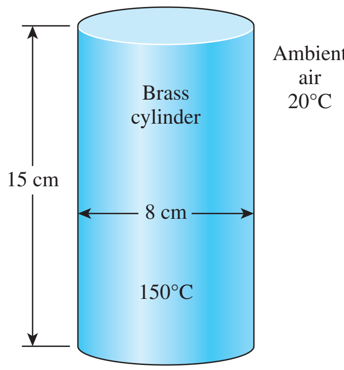


**FIGURE P4–113**

**4–114** Reconsider Prob. 4–113. Using EES (or other) software, investigate the effect of the cooling time on the center temperature of the cylinder, the center temperature of the top surface of the cylinder, and the total heat transfer. Let the time vary from 5 min to 60 min. Plot the center temperature of the cylinder, the center temperature of the top surface, and the total heat transfer as a function of the time, and discuss the results.

**4–115** A semi-infinite aluminum cylinder ($k$ = 237 W/m·K, $\alpha$ = 9.71 × $10^{-5}$ m$^2$/s) of diameter $D$ = 15 cm is initially at a uniform temperature of $T_i$ = 115°C. The cylinder is now placed in water at 10°C, where heat transfer takes place by convection with a heat transfer coefficient of $h$ = 140 W/m$^2$·K. Determine the temperature at the center of the cylinder 5 cm from the end surface 8 min after the start of cooling.

**4–116** A 20-cm-long cylindrical aluminum block ($\rho$ = 2702 kg/m$^3$, $c_p$ = 0.896 kJ/kg·K, $k$ = 236 W/m·K, and $\alpha$ = 9.75 × $10^{-5}$ m$^2$/s), 15 cm in diameter, is initially at a uniform temperature of 20°C. The block is to be heated in a furnace at 1200°C until its center temperature rises to 300°C. If the heat transfer coefficient on all surfaces of the block is 80 W/m$^2$·K, determine how long the block should be kept in the furnace. Also, determine the amount of heat transfer from the aluminum block if it is allowed to cool in the room until its temperature drops to 20°C throughout.

**4–117** Repeat Prob. 4–116 for the case where the aluminum block is inserted into the furnace on a low-conductivity material so that the heat transfer to or from the bottom surface of the block is negligible.

**4–118** EES Reconsider Prob. 4–116. Using EES (or other) software, investigate the effect of the final center temperature of the block on the heating time and the amount of heat transfer. Let the final center temperature vary from 50°C to 1000°C. Plot the time and the heat transfer as a function of the final center temperature, and discuss the results.

## Special Topic: Refrigeration and Freezing of Foods

**4–119C** What are the common kinds of microorganisms? What undesirable changes do microorganisms cause in foods?

**4–120C** How does refrigeration prevent or delay the spoilage of foods? Why does freezing extend the storage life of foods for months?

**4–121C** What are the environmental factors that affect the growth rate of microorganisms in foods?

**4–122C** What is the effect of cooking on the microorganisms in foods? Why is it important that the internal temperature of a roast in an oven be raised above 70°C?

**4–123C** How can the contamination of foods with microorganisms be prevented or minimized? How can the growth of microorganisms in foods be retarded? How can the microorganisms in foods be destroyed?

**4–124C** How does ($a$) the air motion and ($b$) the relative humidity of the environment affect the growth of microorganisms in foods?

**4–125C** The cooling of a beef carcass from 37°C to 5°C with refrigerated air at 0°C in a chilling room takes about 48 h. To reduce the cooling time, it is proposed to cool the carcass with refrigerated air at −10°C. How would you evaluate this proposal?

**4–126C** Consider the freezing of packaged meat in boxes with refrigerated air. How do ($a$) the temperature of air, ($b$) the velocity of air, ($c$) the capacity of the refrigeration system, and ($d$) the size of the meat boxes affect the freezing time?

**4–127C** How does the rate of freezing affect the tenderness, color, and the drip of meat during thawing?

**4–128C** It is claimed that beef can be stored for up to two years at –23°C but no more than one year at –12°C. Is this claim reasonable? Explain.

**4–129C** What is a refrigerated shipping dock? How does it reduce the refrigeration load of the cold storage rooms?

**4–130C** How does immersion chilling of poultry compare to forced-air chilling with respect to ($a$) cooling time, ($b$) moisture loss of poultry, and ($c$) microbial growth.

**4–131C** What is the proper storage temperature of frozen poultry? What are the primary methods of freezing for poultry?

**4–132C** What are the factors that affect the quality of frozen fish?

**4–133** The chilling room of a meat plant is 15 m × 18 m × 5.5 m in size and has a capacity of 350 beef carcasses. The power consumed by the fans and the lights in the chilling room are 22 and 2 kW, respectively, and the room gains heat through its envelope at a rate of 14 kW. The average mass of beef carcasses is 220 kg. The carcasses enter the chilling room at 35°C, after they are washed to facilitate evaporative cooling, and are cooled to 16°C in 12 h. The air enters the chilling room at −2.2°C and leaves at 0.5°C. Determine ($a$) the refrigeration load of the chilling room and ($b$) the volume flow rate of air. The average specific heats of beef carcasses and air are 3.14 and 1.0 kJ/kg·°C, respectively, and the density of air can be taken to be 1.28 kg/m$^3$.

**4–134** In a meat processing plant, 10-cm-thick beef slabs ($\rho$ = 1090 kg/m$^3$, $c_p$ = 3.54 kJ/kg·K, $k$ = 0.47 W/m·K, and $\alpha$ = 0.13 × $10^{-6}$ m$^2$/s) initially at 15°C are to be cooled in the racks of a large freezer that is maintained at −12°C. The meat slabs are placed close to each other so that heat transfer from the 10-cm-thick edges is negligible. The entire slab is to be cooled below 5°C, but the temperature of the steak is not

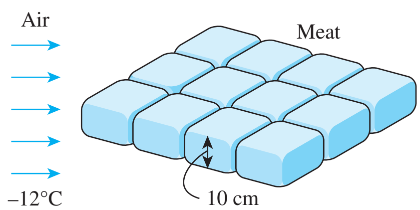


FIGURE P4–134

to drop below −1°C anywhere during refrigeration to avoid "frost bite." The convection heat transfer coefficient and thus the rate of heat transfer from the steak can be controlled by varying the speed of a circulating fan inside. Determine the

heat transfer coefficient $h$ that will enable us to meet both temperature constraints while keeping the refrigeration time to a minimum. *Answer:* 9.9 W/m²·K

**4–135** Chickens with an average mass of 2.2 kg and average specific heat of 3.54 kJ/kg·°C are to be cooled by chilled water that enters a continuous-flow-type immersion chiller at 0.5°C. Chickens are dropped into the chiller at a uniform temperature of 15°C at a rate of 500 chickens per hour and are cooled to an average temperature of 3°C before they are taken out. The chiller gains heat from the surroundings at a rate of 210 kJ/min. Determine (*a*) the rate of heat removal from the chicken, in kW, and (*b*) the mass flow rate of water, in kg/s, if the temperature rise of water is not to exceed 2°C.

**4–136E** Chickens with a water content of 74 percent, an initial temperature of 32°F, and a mass of about 7.5 lbm are to be frozen by refrigerated air at −40°F. Using Figure 4–55, determine how long it will take to reduce the inner surface temperature of chickens to 25°F. What would your answer be if the air temperature were −80°F?

**4–137** Turkeys with a water content of 64 percent that are initially at 1°C and have a mass of about 7 kg are to be frozen by submerging them into brine at −29°C. Using Figure 4–56, determine how long it will take to reduce the temperature of the turkey breast at a depth of 3.8 cm to −18°C. If the temperature at a depth of 3.8 cm in the breast represents the average temperature of the turkey, determine the amount of heat transfer per turkey assuming (*a*) the entire water content of the turkey is frozen and (*b*) only 90 percent of the water content of the turkey is frozen at −18°C. Take the specific heats of turkey to be 2.98 and 1.65 kJ/kg·°C above and below the freezing point of −2.8°C, respectively, and the latent heat of fusion of turkey to be 214 kJ/kg. *Answers: (a)* 1753 kJ, *(b)* 1617 kJ

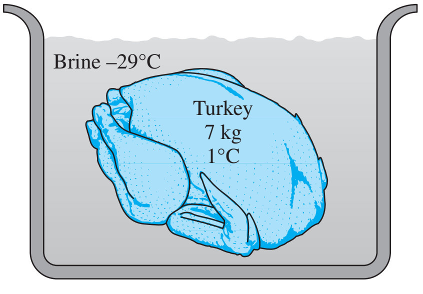


**FIGURE P4–137**

## Review Problems

**4–138** A long roll of 2-m-wide and 0.5-cm-thick 1-Mn manganese steel plate coming off a furnace at 820°C is to be quenched in an oil bath ($c_p$ = 2.0 kJ/kg·K) at 45°C. The metal sheet is moving at a steady velocity of 15 m/min, and the oil bath is 9 m long. Taking the convection heat transfer coefficient on both sides of the plate to be 860 W/m²·K, determine the temperature of the sheet metal when it leaves the oil bath. Also, determine the required rate of heat removal from the oil to keep its temperature constant at 45°C.

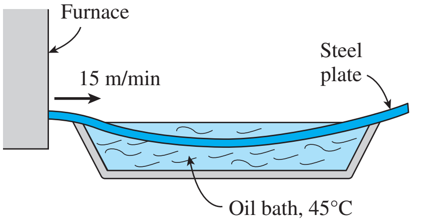


**FIGURE P4–138**

**4–139** Large steel plates 1.0-cm in thickness are quenched from 600°C to 100°C by submerging them in an oil reservoir held at 30°C. The average heat transfer coefficient for both faces of steel plates is 400 W/m²·K. Average steel properties are $k$ = 45 W/m·K, $\rho$ = 7800 kg/m³, and $c_p$ = 470 J/kg·K. Calculate the quench time for steel plates.

**4–140** Long aluminum wires of diameter 3 mm ($\rho$ = 2702 kg/m³, $c_p$ = 0.896 kJ/kg·K, $k$ = 236 W/m·K, and $\alpha$ = $9.75 \times 10^{-5}$ m²/s) are extruded at a temperature of 350°C and exposed to atmospheric air at 30°C with a heat transfer coefficient of 35 W/m²·K. (*a*) Determine how long it will take for the wire temperature to drop to 50°C. (*b*) If the wire is extruded at a velocity of 10 m/min, determine how far the wire travels after extrusion by the time its temperature drops to 50°C. What change in the cooling process would you propose to shorten this distance? (*c*) Assuming the aluminum wire leaves the extrusion room at 50°C, determine the rate of heat transfer from the wire to the extrusion room. *Answers: (a)* 144 s, *(b)* 24 m, *(c)* 856 W

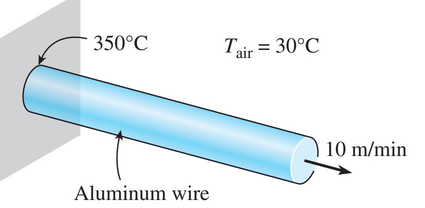


**FIGURE P4–140**

**4–141** Repeat Prob. 4–140 for a copper wire ($\rho$ = 8950 kg/m³, $c_p$ = 0.383 kJ/kg·K, $k$ = 386 W/m·K, and $\alpha$ = $1.13 \times 10^{-4}$ m²/s).

**4–142** Aluminum wires, 3 mm in diameter, are produced by extrusion. The wires leave the extruder at an average temperature of 350°C and at a linear rate of 10 m/min. Before leaving the extrusion room, the wires are cooled to an average temperature of 50°C by transferring heat to the surrounding air at 25°C with a heat transfer coefficient of 50 W/m²·K. Calculate the necessary length of the wire cooling section in the extrusion room.

**4–143E** During a picnic on a hot summer day, the only available drinks were those at the ambient temperature of 90°F.

In an effort to cool a 12-fluid-oz drink in a can, which is 5 in high and has a diameter of 2.5 in, a person grabs the can and starts shaking it in the iced water of the chest at 32°F. The temperature of the drink can be assumed to be uniform at all times, and the heat transfer coefficient between the iced water and the aluminum can is 30 Btu/h·ft$^2$·°F. Using the properties of water for the drink, estimate how long it will take for the canned drink to cool to 40°F.

**4–144** Two metal rods are being heated in an oven with uniform ambient temperature of 1000°C and convection heat transfer coefficient of 25 W/m$^2$·K. Rod A is made of aluminum ($\rho$ = 2702 kg/m$^3$, $c_p$ = 903 J/kg·K, and $k$ = 237 W/m·K) and rod B is made of stainless steel ($\rho$ = 8238 kg/m$^3$, $c_p$ = 468 J/kg·K, and $k$ = 13.4 W/m·K). Both rods have diameter of 25 mm and length of 1 m. If the initial temperature of both rods is 15°C, determine the average temperatures of both rods after 5 minutes elapsed time.

**4–145** Stainless steel ball bearings ($\rho$ = 8085 kg/m$^3$, $k$ = 15.1 W/m·°C, $c_p$ = 0.480 kJ/kg·°C, and $\alpha = 3.91 \times 10^{-6}$ m$^2$/s) having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water. If the temperature of the balls is not to fall below 850°C prior to quenching and the heat transfer coefficient in the air is 125 W/m$^2$·°C, determine how long they can stand in the air before being dropped into the water.

**4–146** During a fire, the trunks of some dry oak trees ($k$ = 0.17 W/m·K and $\alpha = 1.28 \times 10^{-7}$ m$^2$/s) that are initially at a uniform temperature of 30°C are exposed to hot gases at 520°C for a period of 5 h, with a heat transfer coefficient of 65 W/m$^2$·K on the surface. The ignition temperature of the trees is 410°C. Treating the trunks of the trees as long cylindrical rods of diameter 20 cm, determine if these dry trees will ignite as the fire sweeps through them. Solve this problem using analytical one-term approximation method (not the Heisler charts).

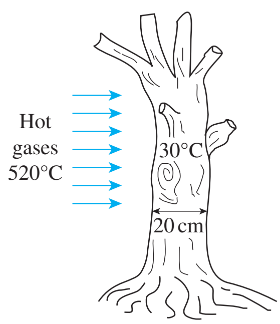


FIGURE P4–146

**4–147** The thermal conductivity of a solid whose density and specific heat are known can be determined from the relation $k = \alpha/\rho c_p$ after evaluating the thermal diffusivity $\alpha$.

Consider a 2-cm-diameter cylindrical rod made of a sample material whose density and specific heat are 3700 kg/m$^3$ and 920 J/kg·K, respectively. The sample is initially at a uniform temperature of 25°C. In order to measure the temperatures of the sample at its surface and its center, a thermocouple is inserted to the center of the sample along the centerline, and another thermocouple is welded into a small hole drilled on the surface. The sample is dropped into boiling water at 100°C. After 3 min, the surface and the center temperatures are recorded to be 93°C and 75°C, respectively. Determine the thermal diffusivity and the thermal conductivity of the material.

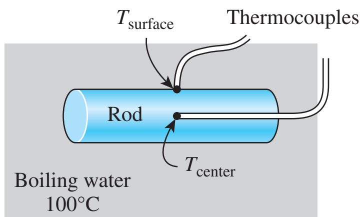


FIGURE P4–147

**4–148E** In *Betty Crocker's Cookbook,* it is stated that it takes 5 h to roast a 14-lb stuffed turkey initially at 40°F in an oven maintained at 325°F. It is recommended that a meat thermometer be used to monitor the cooking, and the turkey is considered done when the thermometer inserted deep into the thickest part of the breast or thigh without touching the bone registers 185°F. The turkey can be treated as a homogeneous spherical object with the properties $\rho$ = 75 lbm/ft$^3$, $c_p$ = 0.98 Btu/lbm·°F, $k$ = 0.26 Btu/h·ft·°F, and $\alpha$ = 0.0035 ft$^2$/h. Assuming the tip of the thermometer is at one-third radial distance from the center of the turkey, determine (*a*) the average heat transfer coefficient at the surface of the turkey, (*b*) the temperature of the skin of the turkey when it is done, and (*c*) the total amount of heat transferred to the turkey in the oven. Will the reading of the thermometer be more or less than 185°F 5 min after the turkey is taken out of the oven?

**4–149** A watermelon initially at 35°C is to be cooled by dropping it into a lake at 15°C. After 4 h and 40 min of cooling, the center temperature of watermelon is measured to be 20°C. Treating the watermelon as a 20-cm-diameter sphere and using the properties $k$ = 0.618 W/m·K, $\alpha = 0.15 \times 10^{-6}$ m$^2$/s, $\rho$ = 995 kg/m$^3$, and $c_p$ = 4.18 kJ/kg·K, determine the average heat transfer coefficient and the surface temperature of watermelon at the end of the cooling period. Solve this problem using analytical one-term approximation method (not the Heisler charts).

**4–150** Spherical glass beads coming out of a kiln are allowed to cool in a room temperature of 30°C. A glass bead with a diameter of 10 mm and an initial temperature of 400°C is allowed to cool for 3 minutes. If the convection heat transfer coefficient is 28 W/m$^2$·K, determine the temperature at the center of the glass bead using (*a*) Table 4–2 and (*b*) the Heisler chart (Figure 4–19). The glass bead has properties of $\rho$ = 2800 kg/m$^3$, $c_p$ = 750 J/kg·K, and $k$ = 0.7 W/m·K.

**4–151** The water main in the cities must be placed at sufficient depth below the earth's surface to avoid freezing during extended periods of subfreezing temperatures. Determine the minimum depth at which the water main must be placed at a location where the soil is initially at 15°C and the earth's surface temperature under the worst conditions is expected to remain at −10°C for a period of 75 days. Take the properties of soil at that location to be $k = 0.7$ W/m·K and $\alpha = 1.4 \times 10^{-5}$ m²/s. *Answer:* 7.05 m

**4–152** A 40-cm-thick brick wall ($k = 0.72$ W/m·K, and $\alpha = 1.6 \times 10^{-6}$ m²/s) is heated to an average temperature of 18°C by the heating system and the solar radiation incident on it during the day. During the night, the outer surface of the wall is exposed to cold air at −3°C with an average heat transfer coefficient of 20 W/m²·K, determine the wall temperatures at distances 15, 30, and 40 cm from the outer surface for a period of 2 h.

**4–153** In an event of a volcano eruption, lava at 1200°C is found flowing on the ground. The ground was initially at 15°C and the lava flow has a convection heat transfer coefficient of 3500 W/m²·K. Determine the ground surface (*a*) temperature and (*b*) heat flux after 2 s of lava flow. Assume the ground has properties of dry soil.

**4–154** A large heated steel block ($\rho = 7832$ kg/m³, $c_p = 434$ J/kg·K, $k = 63.9$ W/m·K, and $\alpha = 18.8 \times 10^{-6}$ m²/s) is allowed to cool in a room at 25°C. The steel block has an initial temperature of 450°C and the convection heat transfer coefficient is 25 W/m²·K. Assuming that the steel block can be treated as a quarter-infinite medium, determine the temperature at the edge of the steel block after 10 minutes of cooling.

**4–155** A large iron slab ($\rho = 7870$ kg/m³, $c_p = 447$ J/kg·K, and $k = 80.2$ W/m·K) was initially heated to a uniform temperature of 150°C and then placed on concrete floor ($\rho = 1600$ kg/m³, $c_p = 840$ J/kg·K, and $k = 0.79$ W/m·K). The concrete floor was initially at a uniform temperature of 30°C. Determine (*a*) The surface temperature between the iron slab and concrete floor and (*b*) The temperature of the concrete floor at the depth of 25 mm, if the surface temperature remains constant after 15 minutes.

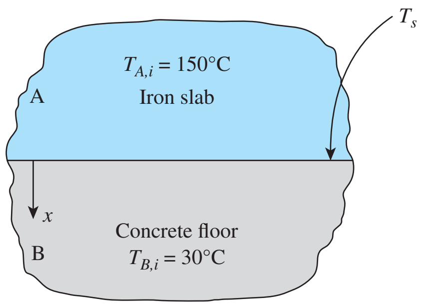


**FIGURE P4–155**

**4–156** A hot dog can be considered to be a 12-cm-long cylinder whose diameter is 2 cm and whose properties are $\rho = 980$ kg/m³, $c_p = 3.9$ kJ/kg·K, $k = 0.76$ W/m·K, and $\alpha = 2 \times 10^{-7}$ m²/s. A hot dog initially at 5°C is dropped into boiling water at 100°C. The heat transfer coefficient at the surface of the hot dog is estimated to be 600 W/m²·K. If the hot dog is considered cooked when its center temperature reaches 80°C, determine how long it will take to cook it in the boiling water.

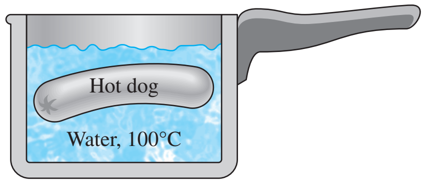


**FIGURE P4–156**

**4–157** Consider the engine block of a car made of cast iron ($k = 52$ W/m·K and $\alpha = 1.7 \times 10^{-5}$ m²/s). The engine can be considered to be a rectangular block whose sides are 80 cm, 40 cm, and 40 cm. The engine is at a temperature of 150°C when it is turned off. The engine is then exposed to atmospheric air at 17°C with a heat transfer coefficient of 6 W/m²·K. Determine (*a*) the center temperature of the top surface whose sides are 80 cm and 40 cm and (*b*) the corner temperature after 45 min of cooling.

**4–158** A man is found dead in a room at 16°C. The surface temperature on his waist is measured to be 23°C and the heat transfer coefficient is estimated to be 9 W/m²·K. Modeling the body as 28-cm diameter, 1.80-m-long cylinder, estimate how long it has been since he died. Take the properties of the body to be $k = 0.62$ W/m·K and $\alpha = 0.15 \times 10^{-6}$ m²/s, and assume the initial temperature of the body to be 36°C.

## Fundamentals of Engineering (FE) Exam Problems

**4–159** The Biot number can be thought of as the ratio of
(*a*) The conduction thermal resistance to the convective thermal resistance.
(*b*) The convective thermal resistance to the conduction thermal resistance.
(*c*) The thermal energy storage capacity to the conduction thermal resistance.
(*d*) The thermal energy storage capacity to the convection thermal resistance.
(*e*) None of the above.

**4–160** Lumped system analysis of transient heat conduction situations is valid when the Biot number is
(*a*) very small (*b*) approximately one
(*c*) very large (*d*) any real number
(*e*) cannot say unless the Fourier number is also known.

**4–161** Polyvinylchloride automotive body panels ($k = 0.092$ W/m·K, $c_p = 1.05$ kJ/kg·K, $\rho = 1714$ kg/m³), 3-mm thick, emerge from an injection molder at 120°C. They need to be cooled to 40°C by exposing both sides of the panels to 20°C air before they can be handled. If the convective heat transfer

coefficient is 30 W/m$^2$·K and radiation is not considered, the time that the panels must be exposed to air before they can be handled is

(*a*) 1.6 min (*b*) 2.4 min (*c*) 2.8 min
(*d*) 3.5 min (*e*) 4.2 min

**4–162** A steel casting cools to 90 percent of the original temperature difference in 30 min in still air. The time it takes to cool this same casting to 90 percent of the original temperature difference in a moving air stream whose convective heat transfer coefficient is 5 times that of still air is

(*a*) 3 min (*b*) 6 min (*c*) 9 min
(*d*) 12 min (*e*) 15 min

**4–163** An 18-cm-long, 16-cm-wide, and 12-cm-high hot iron block ($\rho$ = 7870 kg/m$^3$, $c_p$ = 447 J/kg·K) initially at 20°C is placed in an oven for heat treatment. The heat transfer coefficient on the surface of the block is 100 W/m$^2$·K. If it is required that the temperature of the block rises to 750°C in a 25-min period, the oven must be maintained at

(*a*) 750°C (*b*) 830°C (*c*) 875°C
(*d*) 910°C (*e*) 1000°C

**4–164** A 10-cm-inner diameter, 30-cm-long can filled with water initially at 25°C is put into a household refrigerator at 3°C. The heat transfer coefficient on the surface of the can is 14 W/m$^2$·K. Assuming that the temperature of the water remains uniform during the cooling process, the time it takes for the water temperature to drop to 5°C is

(*a*) 0.55 h (*b*) 1.17 h (*c*) 2.09 h
(*d*) 3.60 h (*e*) 4.97 h

**4–165** A 6-cm-diameter 13-cm-high canned drink ($\rho$ = 977 kg/m$^3$, $k$ = 0.607 W/m·K, $c_p$ = 4180 J/kg·K) initially at 25°C is to be cooled to 5°C by dropping it into iced water at 0°C. Total surface area and volume of the drink are $A_s$ = 301.6 cm$^2$ and $V$ = 367.6 cm$^3$. If the heat transfer coefficient is 120 W/m$^2$·K, determine how long it will take for the drink to cool to 5°C. Assume the can is agitated in water and thus the temperature of the drink changes uniformly with time.

(*a*) 1.5 min (*b*) 8.7 min (*c*) 11.1 min
(*d*) 26.6 min (*e*) 6.7 min

**4–166** Copper balls ($\rho$ = 8933 kg/m$^3$, $k$ = 401 W/m·K, $c_p$ = 385 J/kg·°C, $\alpha = 1.166 \times 10^{-4}$ m$^2$/s) initially at 200°C are allowed to cool in air at 30°C for a period of 2 minutes. If the balls have a diameter of 2 cm and the heat transfer coefficient is 80 W/m$^2$·K, the center temperature of the balls at the end of cooling is

(*a*) 104°C (*b*) 87°C (*c*) 198°C
(*d*) 126°C (*e*) 152°C

**4–167** Carbon steel balls ($\rho$ = 7830 kg/m$^3$, $k$ = 64 W/m·K, $c_p$ = 434 J/kg·K) initially at 150°C are quenched in an oil bath at 20°C for a period of 3 minutes. If the balls have a diameter of 5 cm and the convection heat transfer coefficient is 450 W/m$^2$·K. The center temperature of the balls after quenching will be (*Hint:* Check the Biot number).

(*a*) 27.4°C (*b*) 143°C (*c*) 12.7°C
(*d*) 48.2°C (*e*) 76.9°C

**4–168** In a production facility, large plates made of stainless steel ($k$ = 15 W/m·K, $\alpha = 3.91 \times 10^{-6}$ m$^2$/s) of 40 cm thickness are taken out of an oven at a uniform temperature of 750°C. The plates are placed in a water bath that is kept at a constant temperature of 20°C with a heat transfer coefficient of 600 W/m$^2$·K. The time it takes for the surface temperature of the plates to drop to 100°C is

(*a*) 0.28 h (*b*) 0.99 h (*c*) 2.05 h
(*d*) 3.55 h (*e*) 5.33 h

**4–169** A long 18-cm-diameter bar made of hardwood ($k$ = 0.159 W/m·K, $\alpha = 1.75 \times 10^{-7}$ m$^2$/s) is exposed to air at 30°C with a heat transfer coefficient of 8.83 W/m$^2$·K. If the center temperature of the bar is measured to be 15°C after a period of 3-hours, the initial temperature of the bar is

(*a*) 11.9°C (*b*) 4.9°C (*c*) 1.7°C
(*d*) 0°C (*e*) −9.2°C

**4–170** Consider a 7.6-cm-long and 3-cm-diameter cylindrical lamb meat chunk ($\rho$ = 1030 kg/m$^3$, $c_p$ = 3.49 kJ/kg·K, $k$ = 0.456 W/m·K, $\alpha = 1.3 \times 10^{-7}$ m$^2$/s). Fifteen such meat chunks initially at 2°C are dropped into boiling water at 95°C with a heat transfer coefficient of 1200 W/m$^2$·K. The amount of heat transfer during the first 8 minutes of cooking is

(*a*) 71 kJ (*b*) 227 kJ (*c*) 238 kJ
(*d*) 269 kJ (*e*) 307 kJ

**4–171** Consider a 7.6-cm-diameter cylindrical lamb meat chunk ($\rho$ = 1030 kg/m$^3$, $c_p$ = 3.49 kJ/kg·K, $k$ = 0.456 W/m·K, $\alpha = 1.3 \times 10^{-7}$ m$^2$/s). Such a meat chunk intially at 2°C is dropped into boiling water at 95°C with a heat transfer coefficient of 1200 W/m$^2$·K. The time it takes for the center temperature of the meat chunk to rise to 75 °C is

(*a*) 136 min (*b*) 21.2 min (*c*) 13.6 min
(*d*) 11.0 min (*e*) 8.5 min

**4–172** A potato that may be approximated as a 5.7-cm solid sphere with the properties $\rho$ = 910 kg/m$^3$, $c_p$ = 4.25 kJ/kg·K, $k$ = 0.68 W/m·K, and $\alpha = 1.76 \times 10^{-7}$ m$^2$/s. Twelve such potatoes initially at 25°C are to be cooked by placing them in an oven maintained at 250°C with a heat transfer coefficient of 95 W/m$^2$·K. The amount of heat transfer to the potatoes by the time the center temperature reaches 100°C is

(*a*) 56 kJ (*b*) 666 kJ (*c*) 838 kJ
(*d*) 940 kJ (*e*) 1088 kJ

**4–173** A small chicken ($k$ = 0.45 W/m·K, $\alpha = 0.15 \times 10^{-6}$ m$^2$/s) can be approximated as an 11.25-cm-diameter solid sphere. The chicken is initially at a uniform temperature of 8ºC and is to be cooked in an oven maintained at 220ºC with a heat transfer coefficient of 80 W/m$^2$·K. With this idealization, the temperature at the center of the chicken after a 90-min period is

(*a*) 25°C (*b*) 61°C (*c*) 89°C
(*d*) 122°C (*e*) 168°C

**4–174** A potato may be approximated as a 5.7-cm-diameter solid sphere with the properties $\rho$ = 910 kg/m$^3$, $c_p$ = 4.25 kJ/kg·K, $k$ = 0.68 W/m·K, and $\alpha$ = 1.76×10$^{-7}$ m$^2$/s. Twelve such potatoes initially at 25ºC are to be cooked by placing them in an oven maintained at 250ºC with a heat transfer coefficient of 95 W/m$^2$·K. The amount of heat transfer to the potatoes during a 30-min period is

(*a*) 77 kJ (*b*) 483 kJ (*c*) 927 kJ
(*d*) 970 kJ (*e*) 1012 kJ

**4–175** When water, as in a pond or lake, is heated by warm air above it, it remains stable, does not move, and forms a warm layer of water on top of a cold layer. Consider a deep lake ($k$ = 0.6 W/m·K, $c_p$ = 4.179 kJ/kg·K) that is initially at a uniform temperature of 2°C and has its surface temperature suddenly increased to 20°C by a spring weather front. The temperature of the water 1 m below the surface 400 hours after this change is

(*a*) 2.1°C (*b*) 4.2°C (*c*) 6.3°C
(*d*) 8.4°C (*e*) 10.2°C

**4–176** A large chunk of tissue at 35°C with a thermal diffusivity of 1 × 10$^{-7}$ m$^2$/s is dropped into iced water. The water is well-stirred so that the surface temperature of the tissue drops to 0 °C at time zero and remains at 0°C at all times. The temperature of the tissue after 4 minutes at a depth of 1 cm is

(*a*) 5°C (*b*) 30°C (*c*) 25°C
(*d*) 20°C (*e*) 10°C

**4–177** The 40-cm-thick roof of a large room made of concrete ($k$ = 0.79 W/m·K, $\alpha$ = 5.88 × 10$^{-7}$ m$^2$/s) is initially at a uniform temperature of 15°C. After a heavy snow storm, the outer surface of the roof remains covered with snow at −5°C. The roof temperature at 18.2 cm distance from the outer surface after a period of 2 hours is

(*a*) 14°C (*b*) 12.5°C (*c*) 7.8°C
(*d*) 0°C (*e*) −5°C

## Design and Essay Problems

**4–178** Conduct the following experiment at home to determine the combined convection and radiation heat transfer coefficient at the surface of an apple exposed to the room air. You will need two thermometers and a clock.

First, weigh the apple and measure its diameter. You can measure its volume by placing it in a large measuring cup halfway filled with water, and measuring the change in volume when it is completely immersed in the water. Refrigerate the apple overnight so that it is at a uniform temperature in the morning and measure the air temperature in the kitchen. Then take the apple out and stick one of the thermometers to its middle and the other just under the skin. Record both temperatures every 5 min for an hour. Using these two temperatures, calculate the heat transfer coefficient for each interval and take their average. The result is the combined convection and radiation heat transfer coefficient for this heat transfer process. Using your experimental data, also calculate the thermal conductivity and thermal diffusivity of the apple and compare them to the values given above.

**4–179** Repeat Prob. 4–178 using a banana instead of an apple. The thermal properties of bananas are practically the same as those of apples.

**4–180** Conduct the following experiment to determine the time constant for a can of soda and then predict the temperature of the soda at different times. Leave the soda in the refrigerator overnight. Measure the air temperature in the kitchen and the temperature of the soda while it is still in the refrigerator by taping the sensor of the thermometer to the outer surface of the can. Then take the soda out and measure its temperature again in 5 min. Using these values, calculate the exponent $b$. Using this $b$-value, predict the temperatures of the soda in 10, 15, 20, 30, and 60 min and compare the results with the actual temperature measurements. Do you think the lumped system analysis is valid in this case?

**4–181** Citrus trees are very susceptible to cold weather, and extended exposure to subfreezing temperatures can destroy the crop. In order to protect the trees from occasional cold fronts with subfreezing temperatures, tree growers in Florida usually install water sprinklers on the trees. When the temperature drops below a certain level, the sprinklers spray water on the trees and their fruits to protect them against the damage the subfreezing temperatures can cause. Explain the basic mechanism behind this protection measure and write an essay on how the system works in practice.