Network Analysis, May/June 2006

SET 4

1. (a) The following current wave form $i(t)$ is passed through a series R-L circuit with $R = 2\ \Omega$ and $L = 2$ mH. Find the Voltage across each element and sketch the same. (Fig. Set 4.1)

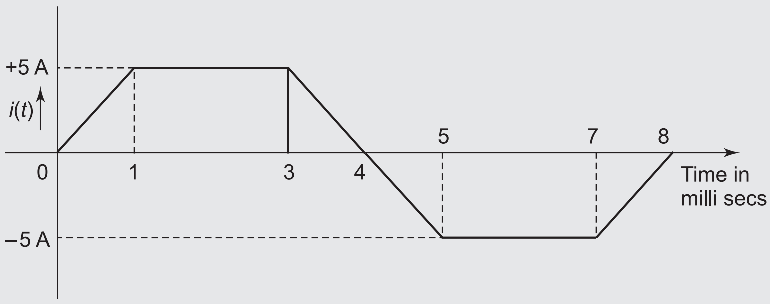


Fig. Set 4.1

Solution:

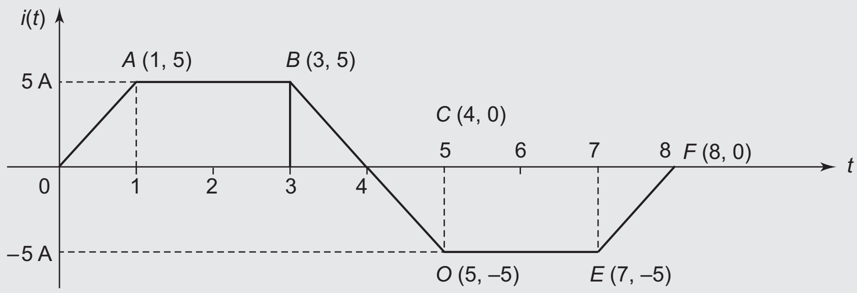


Fig. Set 4.2

For line OA, Slope $= \dfrac{5}{1} = 5$

line equation $i(t) - 0 = 5(t - 0)$

$\Rightarrow$ $i(t) = 5t$ $[\because\ y = mx]$

For line AB, $i(t) = 5$ (constant)

For line BD, $i(t) - 0 = \dfrac{10}{-2}\,(t - 4)$

(From line equation: $y - y_t = m(x - x_1)$

$\Rightarrow$ $i(t) = -\,5t + 20$

For line DE, $i(t) = -\,5$ (constant)

For line EF, $i(t) = 5(t - 8)$

$\Rightarrow$ $i(t) = 5t - 40$

Voltage induced in the inductor

Along OA

$$V_{OA} = \frac{Ldi}{dt} = 2 \times 10^{-3} \times \frac{d}{dt}(5t) = 10\ \mu\text{V}$$

($\because$ 't' in m sec)

Along AB,

$$V_{AB} = \frac{Ldi}{dt} = 0 \qquad (\because i(t) = \text{cons t} = 5\text{A})$$

Along BD

$$V_{BD} = \frac{Ldi}{dt} = 2 \times 10^{-3} \times \frac{d}{dt}(-5t + 20) = -10\ \mu V$$

Along DE

$$V_{DE} = \frac{Ldi}{dt} = 0 \qquad (\because i(t) = \text{const} = -5\text{A})$$

Along EF

$$V_{EF} = \frac{Ldi}{dt} = 2 \times 10^{-3} \times \frac{d}{dt}(5t - 40) = 10\ \mu V$$

Waveform:

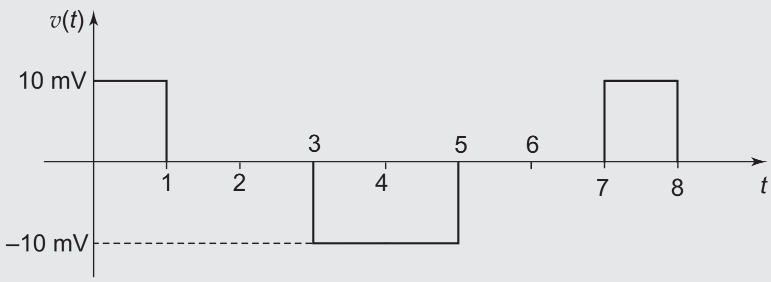


Fig. Set 4.3

Voltage across the resistor is same as current through the circuit multiplied by the resistance

$$N = IR = 2I$$

Waveform

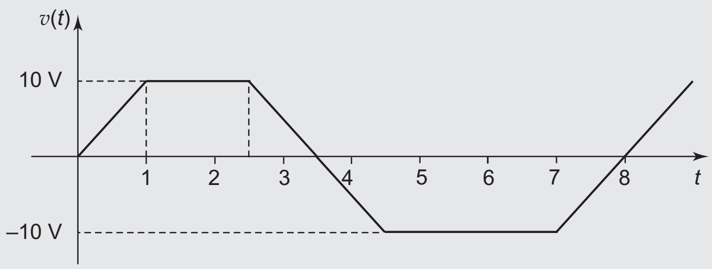


Fig. Set 4.4

1. (b) Using nodal analysis, determine the Power supplied by 8V Voltage source. (Fig. Set 4.5)

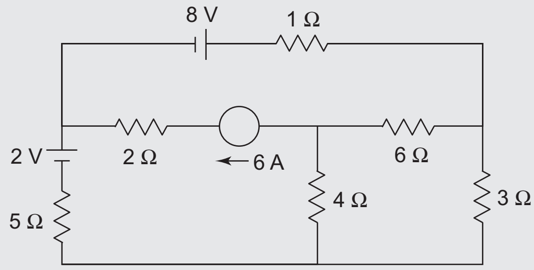


Fig. Set 4.5

Solution:

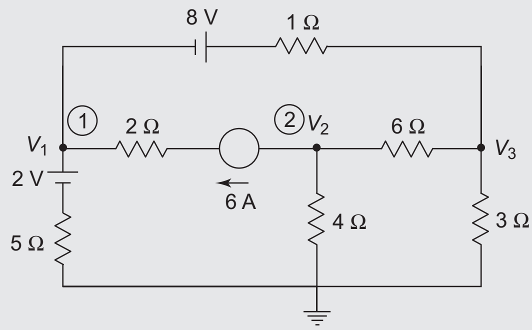


Fig. Set 4.6

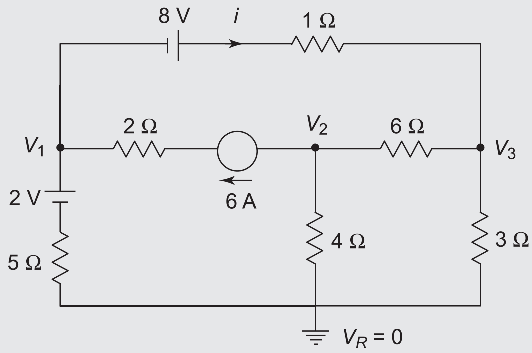


Fig. Set 4.7

Applying KCL at node (1);

$$\frac{V_1 - 2}{5} + \frac{V_1 - V_3 + 8}{1} = 6 \quad \Rightarrow \quad 5V_3 - 6V_1 = 8 \qquad (1)$$

Applying KCL at node (2);

$$6 + \frac{V_2}{4} + \frac{V_2 - V_3}{6} = 0 \quad \Rightarrow \quad 5V_2 - 2V_3 + 72 = 0 \qquad (2)$$

Applying KCL at node (3);

$$\frac{V_3 - V_2}{6} + \frac{V_3}{3} + \frac{V_3 - V_1 - 8}{1} = 0 \Rightarrow 9V_3 - V_2 - 6V_1 = 48$$

(3)

Solving (1), (2) and (3), we get

$$V_1 = -\,4.593 \text{ volts}$$
$$V_2 = 11.56 \text{ volts}$$
$$V_3 = -\,7.11 \text{ volts}$$

$$\text{From the circuit, } i = \frac{V_1 + 8 - V_3}{1} = 10.517 \text{ A}$$

Power supplied by 8 V source is (8×10.517)

$= 84.136$ Watts

1. (c) Write the Tieset matrix for the graph shown in Fig. Set 4.8, taking the tree consisting of branches 2,3,4.

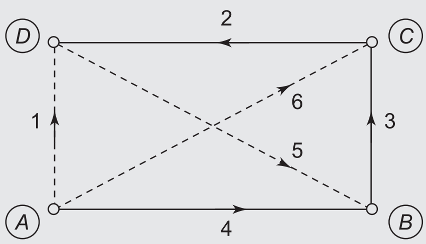


Fig. Set 4.8

Solution

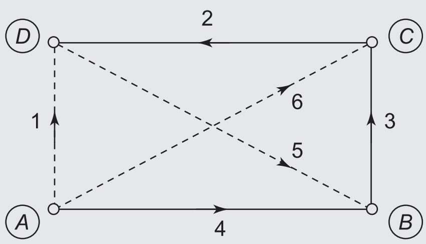


Fig. Set 4.9

Basictiesets \ e	1	2	3	4	5	6
(5, 3, 2)	0	–1	–1	0	1	0
(6, 3, 4)	0	0	–1	–1	0	1
(1, 2, 3, 4)	1	–1	–1	–1	0	0

2. (a) Obtain the Equivalent '*T*' for magnetically Coupled circuit shown in Fig. Set 4.10.

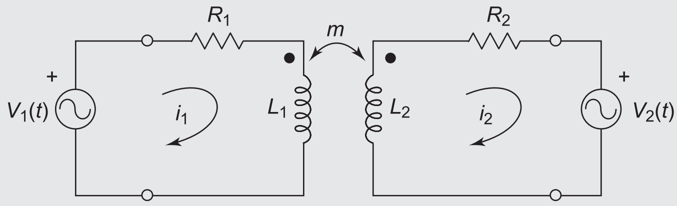


Fig. Set 4.10

Solution

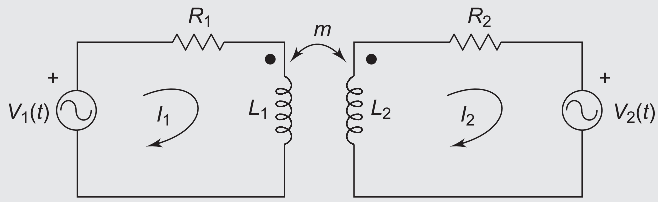


Fig. Set 4.11

$$V_1(t) = I_1F_2 + L_1\,\frac{d\,I_1}{d\,t} + M\,\frac{d\,I_2}{d\,t}$$

$$V_2(t) = I_2F_2 + L_2\,\frac{d\,I_2}{d\,t} + M\,\frac{d\,I_1}{d\,t}$$

The equivalent '*T*' for magnetically coupled circuit is

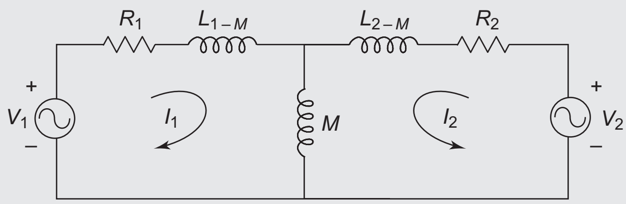


Fig. Set 4.12

2. (b) A coil of 500 turns is wound uniformly over a wooden ring having a mean circumference of 50 cms and a cross sectional area of 500 mm^2. If the current through the coil is 3 Amps, Calculate
   (i) The magnetic field strength
   (ii) The flux density and
   (iii) The total flux.

Solution Given $N = 500$, $I = 3\text{A}$

$$A = 500 \times 10^{-6}\ \text{m}^2$$

Mean circumference (Magnetic path)

$$l = 50 \times 10^{-2}\ \text{m}$$

(i) $H = \dfrac{mmt}{l}$

But mmf = NI = 1500 AT

and $l = 50 \times 10^{-2}$

Magnetic Field Strength, H = 3000 AT/m

(ii) $B = \mu_0 m = 4\pi \times 10^{-7} \times 3000 = 3.769\ mwb/\text{m}^2$

$\therefore$ flux density (B) = 3.769 mwb/m^2

(iii) $\phi = B \times A = 3.769 \times 10^{-3} \times 500 \times 10^{-6}$

$= 1.8845 \times 10^{-6}\ wb$

$\therefore$ Total flux (f) = $1.8845 \times 10^{-6}\ wb$

2. (c) Write down the Loop Equations for the network shown in Fig. Set 4.13.

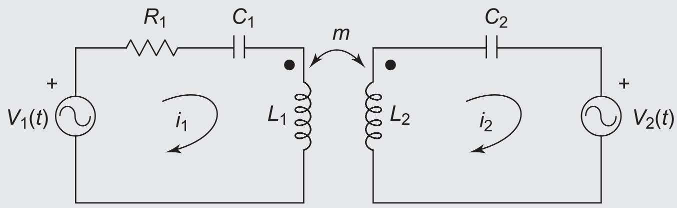


Fig. Set 4.13

Solution: As i_1 is entering at the dot terminal, and i_2 is leaving the dot terminal, sign of M (mutual inductance) is $-$ve

$$i_1(R_1 = j/\omega C_1 + jwL_1) - i_2\, jwM = V_1(t)$$

is loop equation for 1st mesh.

$$i_2(\,jwL_2 - j/wC_2) - i_1(\,jwM\,) = -\,V_2(t)$$

is loop equation for 2nd mesh.

3. (a) In the circuit (Fig. Set 4.14) shown, the switch is changes from position 1 to 2 at $t = 0$. Determine the initial conditions i, di/dt, d^2i/dt^2 at $t = 0^+$

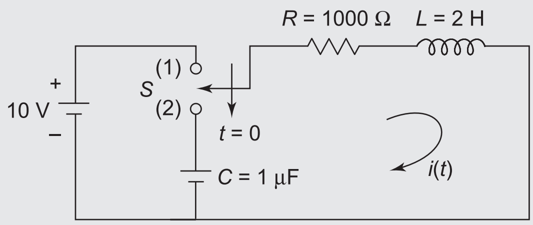


Fig. Set 4.14

Solution:

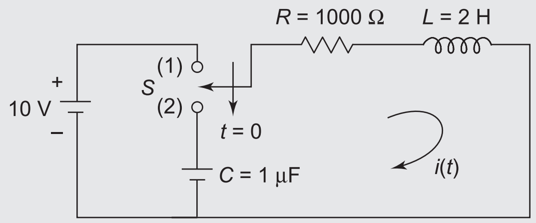


Fig. Set 4.15

Initially the voltage across ‘C’ = 0
Capacitor does not allow sudden charge in voltage.
Inductor does not allow sudden change in current
A position (2), using KVt

$$1000i(t) + \frac{1}{1\times10^{-6}}\int idt + 2\frac{di}{dt} = 0 \qquad (1)$$

At $t = 0$

$$i(t)|_{t=0} = \frac{10}{1000} = 10 \text{ mA}$$

Since inductor is short circuit at steady state i.e., when switch is at position ‘1’
By inductor property

$$i(t)|_{t=0^-} = i(t)|_{t=0^+} = 10 \text{ mA}$$

At $t = 0^+$, $\int i(t)\, dt = 0$ (Since voltage across capacitor is zero)

$$(1) \Rightarrow 1000\ i(t)|_{t=0^+} + 2\frac{di}{dt}\bigg|_{t=0^+} = 0 \qquad \text{(using 2)}$$

$$\frac{di}{dt}\bigg|_{t=0^+} = -5 \text{ A/S} \qquad (3)$$

Diff (1) once,

$$1000\frac{di}{dt} + \frac{1}{1\times10^{-6}}\ i(t) + 2\frac{d^2i}{dt^2} = 0$$

$t = 0^+$

$$1000\frac{di}{dt}\bigg|_{t=0^+} + \frac{1}{1\times10^{-6}}\, i(t)|_{t=0^+} + 2\frac{d^2i}{dt^2}\bigg|_{t=0^+} = 0$$

$$\Rightarrow \quad \left.\frac{d^2 i}{d t^2}\right|_{t=0^+} = \frac{1}{2}(+\,5000 - 10000) \qquad \text{(Using 3)}$$

$$\therefore \quad \left.\frac{d^2 i}{d t^2}\right|_{t=0^+} = -\,2500 \ {}_{\text{A/S}^2}$$

3. (b) In the parallel resonant circuit, determine the resonance frequency, dynamic resistance and Band width for the circuit (Fig. Set 4.16) shown.

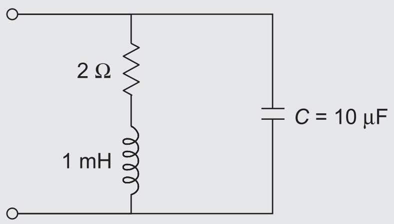


Fig. Set 4.16

Solution: Total admittance

(tan k ckt)

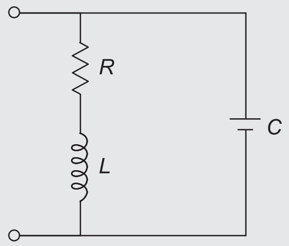


Fig. Set 4.17

$$Y = \frac{1}{R + jwL} + \frac{1}{-\,j/wc}$$

$$= \frac{R - jwL}{R^2 + w^2 L^2} + jwc$$

$$= \frac{R}{R^2 + w^2 C^2} + j\left(\omega C - \frac{wL}{R^2 + w^2 L^2}\right)$$

At resonance, the susceptance part (B) becomes zero.
Reactance

$Y = G + jB$ $\qquad$ $Z = R + jX$

Conductance $\quad$ Susceptance $\quad$ Resistance

$$\therefore \quad w_r\, C = \frac{w_r L}{R^2 t\, w_r^2\, L^2}$$

$$R^2 + w_r^2\, L^2 = \frac{L}{C} \quad \Rightarrow \quad w_r^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right)$$

$$\Rightarrow \quad w_r^2 = \frac{1}{LC} - \frac{R^2}{L^2} \quad \Rightarrow \quad w_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Here $R = 2\ \Omega$, $L = 1$ mH, $C = 10\ \mu F$

$$w_r = \sqrt{\frac{1}{10^{-8}} - \frac{4}{10^{-6}}} = \sqrt{10^6 \times 96}$$
$$= 9.79 \times 10^3 \text{ Hz}$$

$$\therefore \qquad \text{fr} = \frac{wr}{2\pi} = 1.559 \text{ kHz}$$

$$\text{Dynamic resistance } (R) = \frac{1}{6} = \frac{R^2 + w_r^2 L^2}{R}$$

$$= \left.\frac{R + w^2 L^2}{R}\right|_{w - wr} = 2 + \frac{96 \times 10^6 \times 10^{-6}}{2} = 50\ \Omega$$

$$\text{Bandwidth} = \frac{1}{RC} \quad \text{(for IId resonant ckt)}$$

$$= \frac{1}{50 \times 10 \mu f} = 2 \text{ kHz}$$

$$\text{(or)} \qquad \text{BW} = \frac{R}{L} = \frac{2}{1 mH} = 2 \text{ kHz}$$

3. (c) When an voltage of 220V A.C. supply connected across the AB terminals, the total power input is 3.25 kw and the current is 20 Amps. Find the current through Z_3. (Fig. Set 4.18)

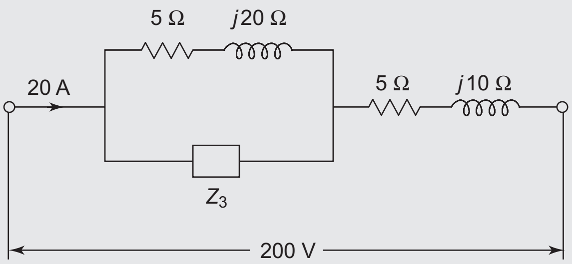


Fig. Set 4.18

Solution: Across IId branch

$$V = 20\ (5 + j10) = 223.6\ \angle 63.43° = 100 + j200$$
$$I(5 + j20) + 100 + j200 = 220.$$

(Let I be the current through $5 + j20\ \Omega$ branch)

$$I = \frac{120 - j200}{5 + j20} = -\ 8 - 8i$$

$$I_{Rs} = 20 - I = 28 + 8i = 29.12\ \angle 15.9°$$

4. (a) Find the Laplace Transform of single pulse shown in Fig. Set 4.19.

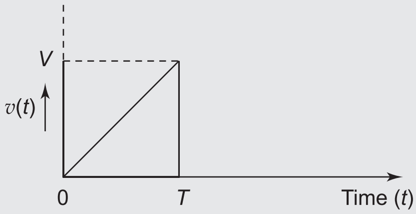


Fig. Set 4.19

Solution:

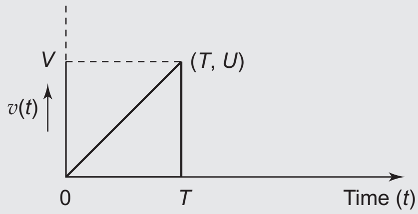


Fig. Set 4.20

$$f(t) = \begin{cases} \frac{V}{T}.t, 0 < t < T \\ 0, \text{elsewhere} \end{cases}$$

$$L(f(t)) = \int_0^\infty e^{-st} f(t)\,dt = \int_0^T e^{-st} \frac{V}{T} t\,dt$$

$$= \frac{V}{T} \int_0^T te^{-st}\,dt = \frac{V}{T}\left[\left.\frac{t\,e^{-st}}{-s}\right|_0^T - \left.\frac{e^{-st}}{s^2}\right|_0^T\right]$$

$$= \frac{V}{T}\left[\frac{-T_{e^{-sT}}}{s} - \left(\frac{e^{-sT}-1}{s^2}\right)\right] = \frac{V}{T}\left[\frac{1}{s^2} - e^{-sT}\left(\frac{1}{s^2} + \frac{T}{s}\right)\right]$$

$$= \frac{V}{T}\left[\left(\frac{1-e^{-sT}}{s^2}\right) - \frac{Te^{-sT}}{s}\right]$$

$$L[f(t)] = \frac{V}{T}\left[\frac{1-e^{-sT}(1+ST)}{s^2}\right]$$

4. (b) Define RMS value, Average value, Form factor of an alternating quantity. Also state the relationship between them.

Solution: Refer to Set-1 4(c) [AC is periodic]

4. (c) Find the RMS value of the voltage wave whose equation is $v(t) = 10 + 200 \sin (wt - 30°) + 100 \cos 3\,wt - 50 \sin (5wt + 60°)$.

Solution: $$V_{rms} = \sqrt{10^2 + \frac{(200)^2}{2} + \frac{(100)^2}{2} + \frac{(50)^2}{2}}$$
$$= \sqrt{100 + 20000 + 5000 + 1250}$$
$$= 162.327 \text{ V}$$

5. (a) What is complex power? Explain in detail.

Solution: *Complex power*

Active power (P):

The active power or real power in an a.c. circuit is given by the product of voltage, current and cosine of the phase angle. It is always positive

$$P = VI \cos \phi \text{ watts}$$

Reactive power (Q):

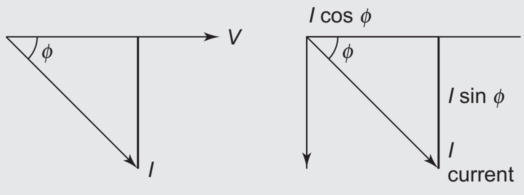
The reactive power in an a.c. circuit is given by the product of voltage, current and sine of the phase angle ϕ.
If ϕ is leading then reactive power is taken as +ve and it is capacitive.
If ϕ is lagging then reactive power is taken as −ve and it is inductive

$$Q = VI \sin \phi \text{ VARs.}$$

Apparent power:

The apparent power in an a.c. circuit is the product of voltage and current. It is measured in voltamps.

$$S = VI \text{ volt amps.}$$



(a) Phasor diagram (b) Current A^le

Fig. Set 4.21

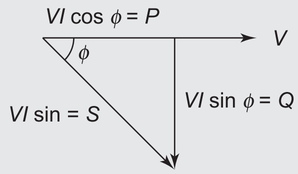


Fig. Set 4.22

The component $I \cos \phi$ = Active component or real component or in phase component of a current.
The product of voltage and the above component (active component) gives active power. The component $I \sin \phi$ = Reactive component or quadrature component of current)
The product of this component with voltage V gives the reactive power.

$$\text{Power factor } \cos \phi = \frac{\text{Read power}}{\text{Apparent power}}$$

The factor $\sin \phi$ is called the reactive factor.

Complex power = (Active power) + j (Reactive power)

5. (b) The current in a given circuit is $I = (12 - j5)$ A when the applied voltage is $V = (160 - j120)V$. Determine
   i. the complex expression for power
   ii. power factor of the circuit
   iii. the complex expression for impedance of the circuit
   iv. Draw the phasor diagram.

Solution: (i) $P_a = V_{\text{eff}} I_{\text{eff}}\ VA$

$$P_{ar} = V_{eff} I_{eff} \cos\theta \text{ watts}$$
$$P_r = V_{eff} I_{eff} \sin\theta \text{ VAR}$$
$$Z = \frac{V}{I} = \frac{160 - j120}{12 - j5} = \frac{14.91}{R} - \frac{3.786}{x} j$$
$$|I| = 13\ A$$
$$= 15.38\ \angle - 14.25°$$
$$\therefore \quad P_{\text{avg}} = I^2 R = 2519.79 \text{ W}$$
$$P_r = I^2 X = 639.834\ VAR$$
$$P_a = I^2 Z = 2599.22 \text{ W}$$

Complex power = $2519.79 + j639.834$

(ii) $Pf = \cos\phi = \cos(-14.25°) = 0.969$

(iii) $Z = 14.91 - 3.786j$

(iv) Power Δ^{le}

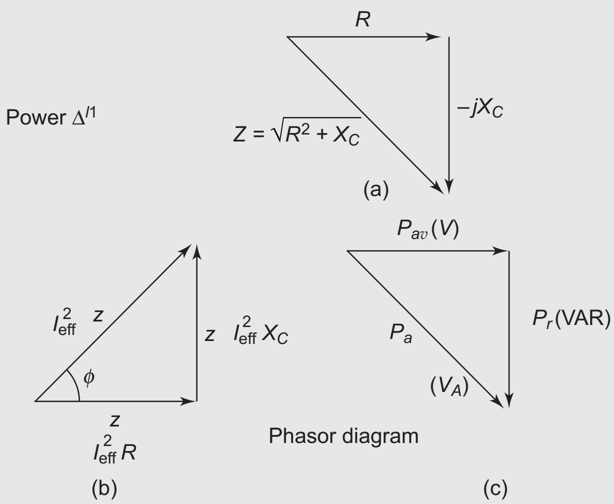


Fig. Set 4.23

6. (a) Why Z-parameters are known as open circuit parameters?

Solution:

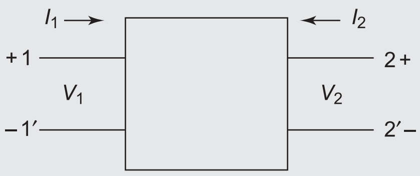


Fig. Set 4.24

The Z parameters of a two-port for the positive directions of voltages and currents may be defined by expressing the port voltages V_1 and V_2 in terms of the currents I_1 and I_2. Here V_1 and V_2 are dependent variables, and I_1, I_2 are independent variables. The voltages at port 1–1′ is the response produced by the two currents. I_1 and I_2

Thus

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

The individual Z-parameters for a given network can be defined by setting each of the port currents equal to zero. Suppose port 2–2′ is left open-circuited, then

$$I_2 = 0$$

Thus $Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$ driving point impedance at port 1–1′ with port 2–2′ open circuited. It is called open circuit input impedance

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

It is called open circuit forward transfer impedance

Suppose port 1–1′ is left open circuited then, $I_1 = 0$

Thus $Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$

It is called open circuit reverse transfer impedance

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

It is called open circuit output impedance.

It is observed that the individual parameters are specified only when the current in one of the ports is zero. This corresponds to one of the ports being open circuited from which Z-parameters also derive the name open circuit impedance parameters.

6. (b) What is meant by port? Explain two port network?

Solution: Port: A port is defined as any pair of terminals into which energy is supplied, or from which energy is withdrawn, or where the network variables may be measured.

Two port network

A two-port network is simply a network inside a black box, and the network has only two pairs of accessible terminals; usually one pair represents input and the other represents the output.

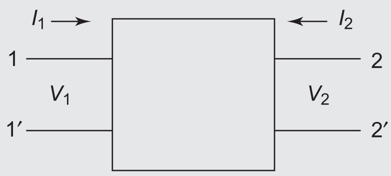


Fig. Set 4.25

Two ports containing no sources in their branches are called passive ports; among them are power transmission lines and transformers. Two ports containing sources in their branches are called active ports. Two of these are dependent variables and the other two are independent variables. The number of possible combinations generated by the four variables taken two at a time sin ($4C_2$). Thus, there are six possible sets of equations describing a two-port network.

6. (c) Find the Y-parameters for the network shown in Fig. Set 4.26.

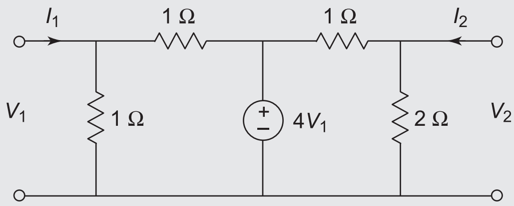


Fig. 4.26

Solution:

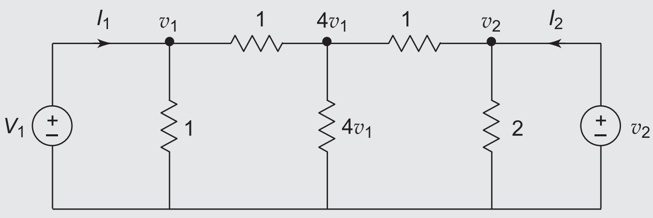


Fig. Set 4.27

Y-parameters are generally of the form

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

By nodal analysis

$$I_1 = \frac{V_1}{1} + \left(\frac{V_1 - 4V_1}{1}\right) = V_1 - 3V_1$$

$$I_1 = V_1 - 3V_1$$

$$\Rightarrow \quad I_1 = -2V_1 \qquad (1)$$

$$I_2 = \frac{V_2}{2} + V_2 - 4V_1$$

$$\Rightarrow \quad I_2 = -4V_1 + \frac{3}{2}V_2 \qquad (2)$$

$\therefore$ By comparing 1 and 2 with the above equations

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ -4 & 3/2 \end{pmatrix}$$

7. (a) Show that the propagation constant for Π network is $\gamma_A = \cosh^{-1}\left(1 + \frac{Z_1}{2Z_2}\right)$

Solution: π*-type attenuator*

$\Rightarrow$

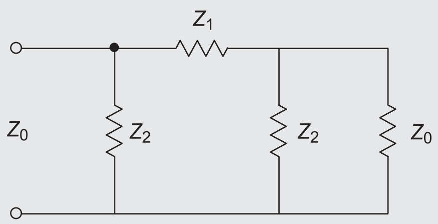


Fig. Set 4.28

From fundamental equations, we have

$$Z_1 = Z_0 \sinh \alpha$$

$$Z_2 = Z_0 \coth \alpha/2$$

$$\frac{Z_1}{Z_2} = \frac{\sinh\alpha}{\coth\alpha/2}$$

$$\frac{Z_1}{2Z_2} = \frac{\sinh\alpha \, . \, \sinh\alpha/2}{2\cosh\alpha/2} = \frac{2\sinh\alpha/2\cosh\alpha/2 \, . \, \sinh\alpha/2}{2\cosh\alpha/2}$$

($\because$ sin 2h x = 2 sinh x cosh x)

$$\frac{Z_1}{2Z_2} = \sin 2\text{ h } \alpha/2 \qquad (\text{Here } \gamma = \alpha)$$

since it is symmetric

$$\Rightarrow 1 + \frac{Z_1}{2Z_2} = (\cosh \alpha/2)^2 = \left(\frac{e^{\alpha/2} + e^{-\alpha/2}}{2}\right)^2$$

$$= \frac{e^{\alpha} + e^{-\alpha} + 2}{4} = \frac{e^{\alpha} + e^{-\alpha}}{4} + \frac{1}{2}$$

Multiplying by 2

$$2\left(1 + \frac{Z_1}{2Z_2}\right) = \frac{e^{\alpha} + e^{-\alpha}}{2} + 1$$

$$1 + \frac{Z_1}{Z_2} = \cosh\,\alpha \quad \Rightarrow \quad \gamma_A = \cosh^{-1}\left(1 + \frac{Z_1}{Z_2}\right)$$

If $Z_2 \longrightarrow 2Z_2$ (replaced then $\gamma_A = \cosh^{-1}\left(1 + \frac{Z_1}{2Z_2}\right)$

7. (b) Write short note on iterative and image impedances in symmetrical networks.

Solution: Two importance parameter for design of attenuators is image impedance for unsymmetrical attenuator and characteristic impedance for symmetrical attenuator and also attenuation constant for both types of attenuators.

Z_{11} and Z_{12} are two impedances such that when terminals 2–2′ are terminated in Z_{12} the input impedance at terminals marked 1–1′ is Z_{11}. Using ABCD parameters the two-terminal pair impedances and admittances and certain algebraic expression it can be shown that Z_{11}

$= \sqrt{Z_{OC_1} Zsc_1}$

where Z_{oc1} is the input impedance measured at terminals 1–1′ when the terminals 2–2′ are kept open circuited and $Z_{sc1}$is the short ckt impedance as measured at 1–1′
when terminals marked 22′ shorted

Similarly $Z_{12} = \sqrt{Z_{oc2}\, Z_{sc2}}$

If the network is symmetrical then

$$Z_{11} = Z_{12} = Z_0 = \sqrt{Z_{oc1}\, Z_{sc1}} = \sqrt{Z_{oc2}\, Z_{sc2}}$$

Where Z_0 is the characteristic impedance of the attenuating network. The characteristic impedance or iterative impedance is defined as the impedance of a network with which a network must be terminated so that the input and terminating resistances are equal.

If the attenuation network is asymmetric the network will have two different characteristic impedances known as image impedances.

The values of impedances ($Z_{11} \neq Z_{12}$) are different depending on which end is used as the input.

8. What is a half section? What is its main characteristic? Why it is used? Derive expression for impedances as seen from the two ports of an *m*-derived half section.

Solution: Refer to textbook (Chapter 16).