Network Analysis, May/June 2006

SET 3

1. (a) For the given network (Fig. Set 3.1) graph, Construct the Basic cutest incidence matrix, tracking elements 1,6,8,3 as tree branches. Express the link branch Voltage in terms of tree branch voltages.

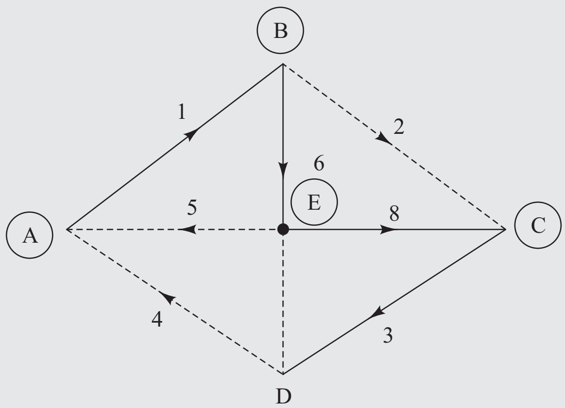

Fig. Set 3.1

Solution:

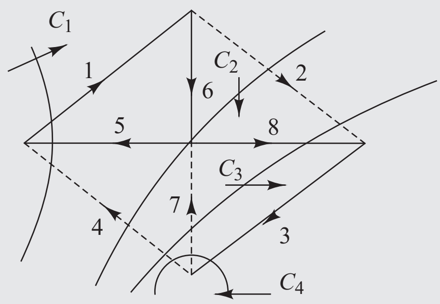

Fig. Set 3.2

Cut set incidence matrix is

$$Q = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

The link branch voltage in terms of tree branch voltages is given by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

1. (b) Using source Transformation, reduce the network between A and B into an equivalent voltage source. (Fig. Set 3.3)

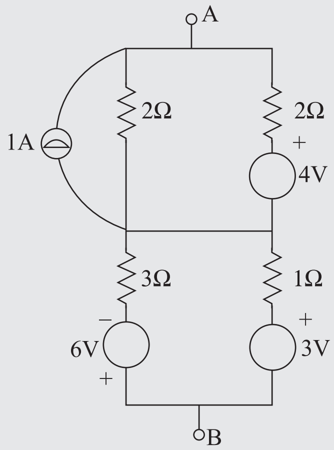

Fig. Set 3.3

Solution: Given circuit

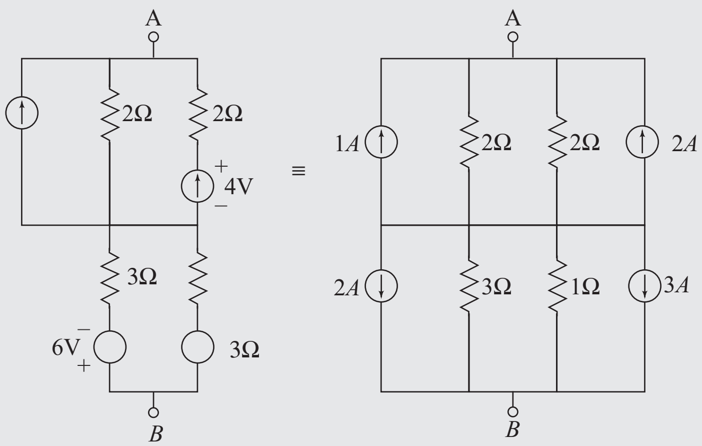

Fig. Set 3.4

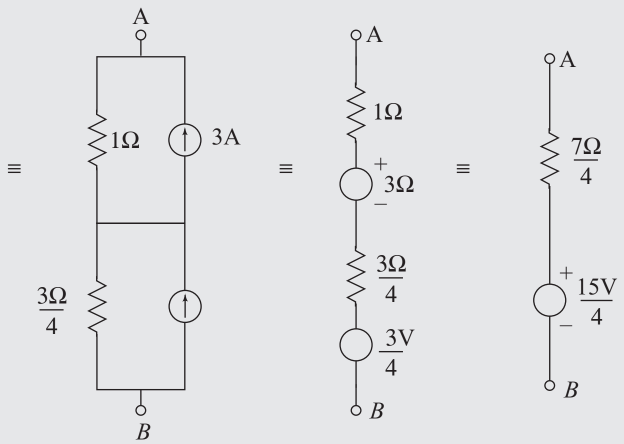

Fig. Set 3.5

1. (c) What is Duality? Explain the procedure for drawing the dual of given network with an example.

Solution: *Refer Duals and Duality; Section 3.16*

2. (a) Explain the Dot Convention for mutually coupled coils.

Solution *Refer Dot Convention; Section 9.14*

2. (b) Derive the Expression for coefficient coupling between pair of magnetically coupled coils.

Solution: *Refer coefficient of coupling; Section 9.5*

2. (c) Write the Loop Equations for the Coupled circuit shown in Fig. Set 3.6.

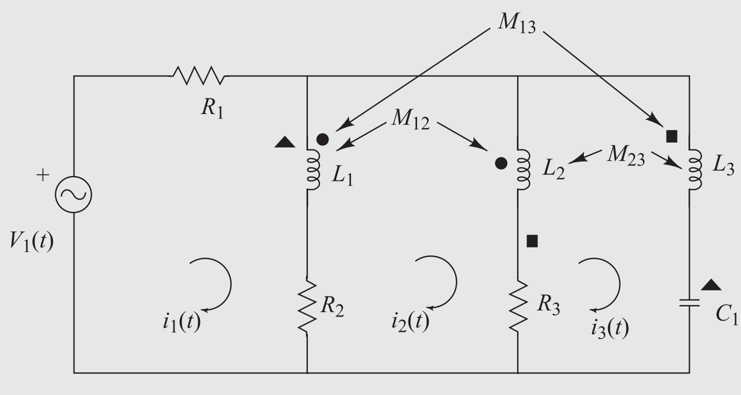

Fig. Set 3.6

Solution Given circuit is

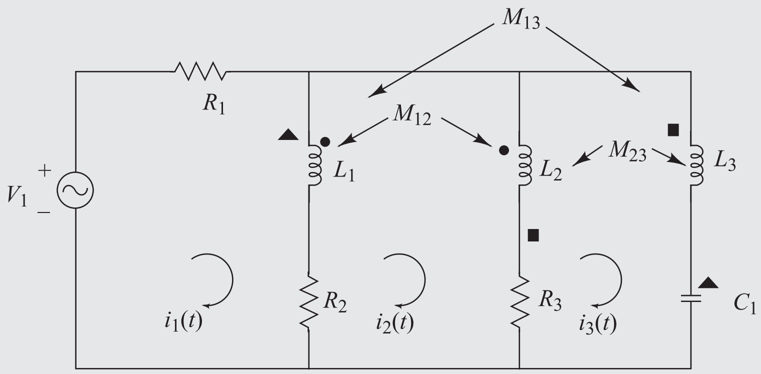

Fig. Set 3.7

The loop equations are

$$V_1(t) = R_1\, i_1(t) + L_1 \frac{d}{d\,t}\,[i_1(t) - i_2(t] - M_{12} \frac{d}{d\,t}\,[i_2(t) = i_3(t)]$$

$$- M_{13} \frac{d}{d\,t}\,[i_3(t)] + R_2[i_1(t) - i_2(t)] \qquad (1)$$

Loop 2

$$R_2\,[i_2(t) - i_1(t)] + L_1\left[\frac{di_2(t)}{d\,t} - \frac{di_i(t)}{d\,t}\right] - M_{12}\frac{d}{d\,t}\,[i_2(t) - i_3(t)]$$

$$+ M_{13}\frac{di_3(t)}{d\,t} + L_2\frac{d[i_2(t) - i_3(t)]}{d\,t} - M_{12}\left[\frac{di_2(t)}{d\,t} - \frac{di_i(t)}{d\,t}\right]$$

$$- M_{23}\frac{di_3(t)}{d\,t} + R_3(i_2 - i_3) = 0$$

Loop 3

$$R_3(i_3 - i_2) + L_2\frac{d\,(i_3 - i_2)}{d\,t} - M_{12}\frac{d\,(i_1 - i_2)}{d\,t} + M_{23}\frac{d\,i_3}{d\,t}$$

$$+ L_3\frac{d\,i_3}{d\,t} - M_{13}\frac{d}{d\,t} + M_{23}\frac{d\,(i_3 - i_2)}{d\,t} + \frac{1}{C_1}\int i_3\,dt = 0$$

3. (a) Explain clearly the significance of "Time Constant" in transient analysis of *R-L* and *R-C* Circuits.

Solution: *Refer 11.2, 11.3*

3. (b) In the following circuit (Fig. Set 3.8), when 220V A.C. is applied across A and B, Current drawn is 20 Amps and power input is 3000w. Find the value of Z and its parameters.

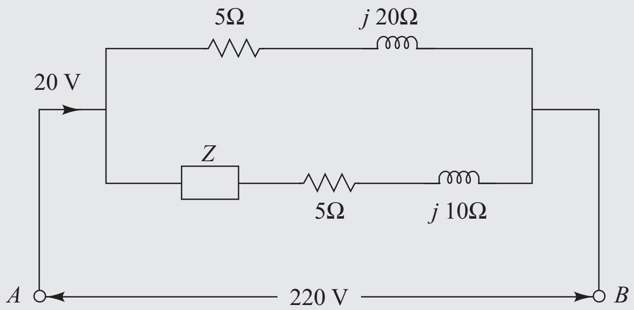

Fig. Set 3.8

Solution:

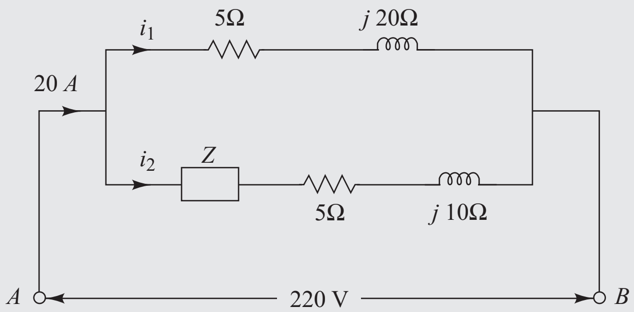

Fig. Set 3.9

$$i_1 = \frac{220}{5 + j20} \text{ A}$$

But $i_1 + i_2 = 20$ A

$$i_2 = 20 - \frac{220}{5 + j20} \qquad (1)$$

$$\text{Also, } i_2 = \frac{220}{Z + 5 + j10} \qquad (2)$$

From (1) and (2)

$$20 - \frac{220}{5 + j20} = \frac{220}{Z + 5 + j20}$$

$$\frac{-120+j400}{5+j20} = \frac{220}{5+Z+j20}$$

$$Z = \frac{5700+j3600}{-120+j400}$$

$$Z = -4.33 + j15.55$$

$$Z = 16.14\ \angle 105.56°$$

3. (c) Obtain the expression for resonant frequency for the circuit shown in Fig. Set 3.10.

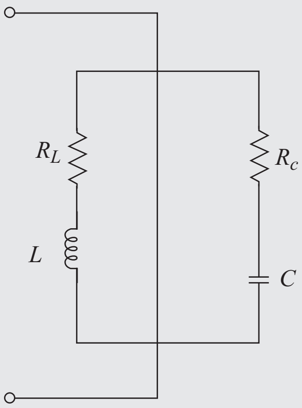

Fig. Set 3.10

Solution: *Refer Parallel Resonance* 8.10 *in Page* 8.34.

4. (a) Determine $V_C(t)$ and $i_L(t)$ in the circuit shown in the Fig. Set 3.6. Assume Zero initial conditions. Use Laplace Transform method.

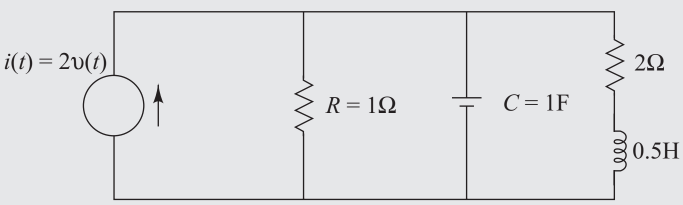

Fig. Set 3.11

Solution: Applying nodal analysis,

$$2u(t) = \frac{V_C}{R} + C\,\frac{dV_C}{dt} + i_L$$

$$\text{But, } i_L = \frac{V_C}{2+j0.5}$$

Applying Laplace transform on both sides,

$$\frac{2}{S} = \frac{V_C(S)}{R} + \frac{1}{SC}\,(SV_C(S) - V_C(0)) + \frac{V_C(S)}{2+0.5S}$$

Assuming zero initial conditions, $V_C(0) = 0$

$$\Rightarrow \qquad V_C(S)\left[1 + \frac{S}{S} + \frac{1}{2+(0.5)S}\right] = \frac{2}{S}$$

$$\Rightarrow \qquad V_C(S)\left(2 + \frac{1}{2+\frac{S}{2}}\right) = \frac{2}{S}$$

$$\Rightarrow \qquad V_C(S) = \frac{S+4}{S(S+5)}$$

$$\therefore \qquad i_L(S) = \frac{V_C(S)}{2+(0.5)S} = \frac{2}{S(S+5)}$$

Applying inverse Laplace transform for $V_C(S)$;

$$V_C(S) = \frac{S+4}{S(S+5)} - \frac{(4/5)}{S} + \frac{(1/5)}{S+5}$$

$$\therefore \qquad V_C(t) = \frac{4}{5}\,u(t) + \frac{1}{5}\,e^{-5t}\,u(t)$$

$$\Rightarrow \qquad V_C(t) = \frac{1}{5}\,u(t)\,[4 + e^{-5t}]$$

Similarly, $$i_L(S) = \frac{2}{S(S+5)} = \frac{(2/5)}{S} - \frac{(2/5)}{(S+5)}$$

Applying inverse Laplace transform for $i_L(S)$;

$$i_L(t) = \frac{2}{5}\,u(t)\,[1 - e^{-5t}]$$

4. (b) Obtain the S-Domain Equivalent for the following elements
   i. Resistance R
   ii. Inductance with initial current-I_0
   iii. Capacitors
   iv. Capacitors with initial Voltage V_0 give the relevant equations.

Solution *Refer circuit element in S-Domain* 13.1

5. (a) State and explain Nortion's theorem?

Solution *Refer Norton's theorem* 3.4

5. (b) Using Thevenin's theorem, find the current through 1 Ω resistor in the circuit shown in Fig. Set 3.12.

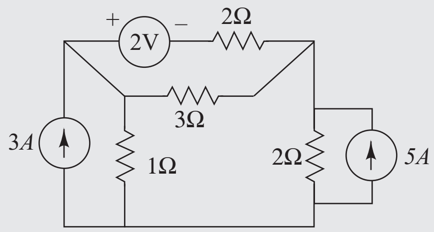

Fig. Set 3.12

Solution The given circuit is

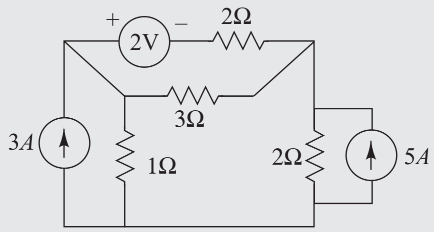

Fig. Set 3.13

To find R_{TH}

BY keeping all the series to zero the circuit reduces to

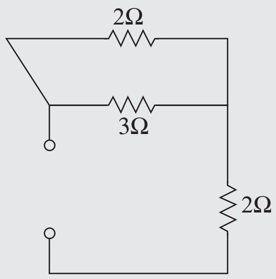

Fig. Set 3.14

$$R_{TH} = 2 \| 3 + 2$$

$$R_{TH} = \frac{6}{5} + 2$$

$$R_{TH} = \frac{16}{5}$$

To find V_{TH}

Transforming current source of 5A to voltage source the circuit reduces to

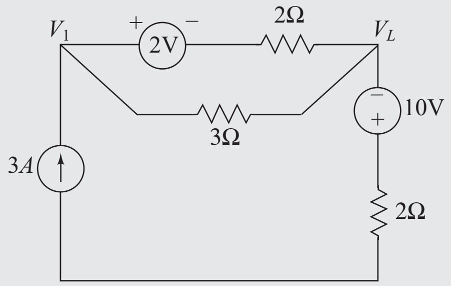

Fig. Set 3.15

Applying Nodal analysis

$$\frac{V_1 - V_2 - 2}{2} + \frac{V_1 - V_2}{3} = 3$$

$$V_1 - V_2 = \frac{24}{5} \qquad (1)$$

$$\frac{V_2 - V_1 + 2}{2} + \frac{V_2 - V_1}{3} + \frac{V_2 + 10}{2} = 0$$

$$(V_2 - V_1) + \frac{5}{6} + \frac{V_2}{2} + 6 = 0 \qquad (2)$$

From (1) and (2)

$$V_1 = -\frac{76}{5} \text{ V}$$

$$V_2 = -20 \text{ V}$$

6. (a) Why h-parameters are called as hybrid parameters?

Solution: *Refer Hybrid Parameters* 15.6

6. (b) Obtain the condition for a given network to be reciprocal as well as symmetrical network in terms of h-parameters?

Solution: *Refer* 15.20

6. (c) Obtain the z-parameters of the network shown in Fig. Set 3.16.

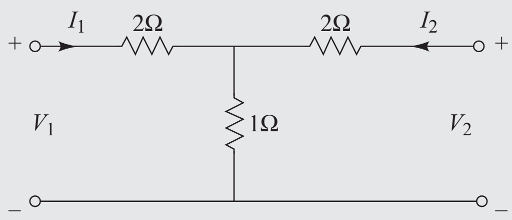

Fig. Set 3.16

Solution:

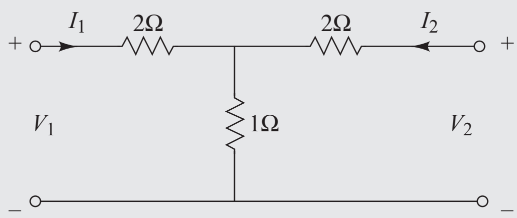

Fig. Set 3.17

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 3\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 1\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 3\ \Omega$$

7. (a) Fig. Set 3.18 shows a resistive T network and a resistive Π network connected in parallel. Find the overall y parameters of the combination.

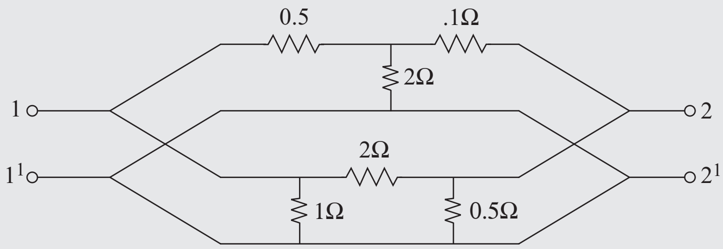

Fig. Set 3.18

Solution:

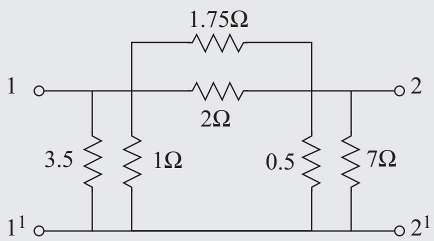

Fig. Set 3.19

The upper star connection is connected into π and the circuit is redrawn as follows

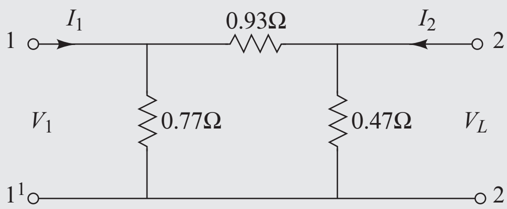

Fig. Set 3.20

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} = 1.7\ \mho$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} = -\,0.93\ \mho$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0} = 1.4\ \mho$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} = -\,0.93\ \mho$$

(b) Find the characteristic impedance of a symmetrical T network.

Solution: Refer Section 15.10

8. What is composite filter? Draw its circuit diagram? Give a general procedure for its design?

Solution In the m-derived filter sections, the stop band attenuation drastically reduces after f_∞ in low pass section and before f_∞ in high pass section. This drawback of m-derived filter can be overcome by connecting number of sections including prototype sections and m-derived sections with terminating half sections. Such a combination of different sections is called COMPOSITE FILTER.

The block diagram of the composite filter is shown in Fig. Set 3.21.

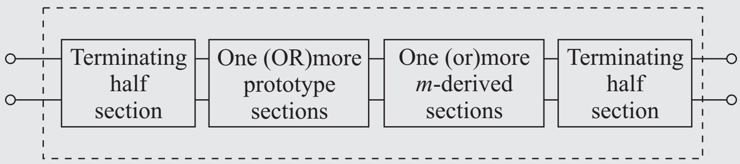

Fig. Set 3.21

In composite filter, cut off frequency and the design impedance are the two important design specifications. The number of various section in the composite filter totally depends on attenuation characteristics required. The typical value of m for attenuation at cut off is $m = 0.3$ to 0.35. If it is required to maintain the attenuation at a high value in attenuation band, we must connect either a prototype section in another m-derived section with comparatively larger value of m. To have proper impedance matching, and constant characteristic impedance throughout pass band, we must connect the terminating sections with $m = 0.6$.