Appendix **F**

Solved Question Papers

Network Analysis, May/June 2006

SET 1

1. (a) Describe the Volt-ampere relations for R, L and C Parameters.

Solution: *Volt-ampere Relations for R, L and C Parameters*

The passive elements R, L, C are defined by the way in which the current and voltage are related for individual element.

(i) If the current 'I' and voltage 'V' are related by a constant for a single element then the element is a resistance 'R'. The Resistance 'R' represents the constant of proportionality.

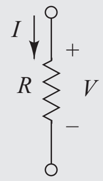


Fig. Set 1.1

$\therefore$ Voltage, $V = RI$ (ohms law)

Current, $I = \dfrac{V}{R}$

Power, $P = VI = I^2R$

The units of resistance 'R' is ohms (Ω).

(ii) If the current and voltage are related such that the voltage is the time derivative of current, then the element is an inductance 'L'. The inductance 'L' represents the constant of proportionality

$\therefore$ Voltage, $V = L\dfrac{dI}{dt}$

Current, $I = \dfrac{1}{L}\int V\,dt + K_1$ $[K_1 = \text{constant}]$

Power, $P = VI = LI\dfrac{dI}{dt}$

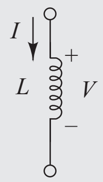


Fig. Set 1.2

The units of inductance '*L*' is Henry (H).

(iii) If the voltage and current are related such that the current is the time derivative of the voltage, then the element is a capacitance '*C*'. The capacitance '*C*' is the constant of proportionality

$\therefore$ Current, $I = C\dfrac{dV}{dt}$

Voltage, $V = \dfrac{1}{C}\int I\,dt + K_2$ $[K_2 = \text{constant}]$

Power, $P = VI = VC\dfrac{dV}{dt}$

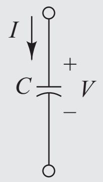


Fig. Set 1.3

The units of capacitance '*C*' is Farads (F).

1. (b) Derive the expression for the energy stored in an ideal inductor?

Solution: *Expression for Energy Stored in an ideal inductor*

Let '*L*' be the co-efficient of self inductance and *i* be the current flowing through it.

Let '*dw*' be the small amount of work to be expended to over come self induced emf.

$$\therefore \quad dw = Ei\,dt$$

$$dw = \frac{di}{dt}\,i\,dt \qquad \left[\because E = L\frac{di}{dt}\right]$$

from lenz law

$$dw = Li\,di \tag{1}$$

Hence total work to be done in establishing a maximum current i_0 is given by integrating (1) from 0 to i_0.

$$\therefore \quad w = \int_0^{i_0} dw = \int_0^{i_0} Li\,d\,i = L\int_0^{i_0} i\,d\,i$$

$$= L\left[\frac{1}{2}\frac{i_0^2}{1}\right]$$

$$w = \frac{1}{2}\ Li_0^2$$

$\therefore$ Energy stored in an inductor $w = \dfrac{1}{2}\ Li_0^2$

1. (c) Find the Current I_1 and I_2 using Nodal Analysis (Fig. Set 1.4)

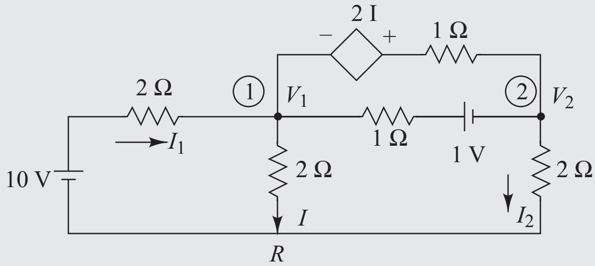


Fig. Set 1.4

Solution: At node (1):

$$\frac{V_1 - 10}{2} + \frac{V_1}{2} + \frac{V_1 - (1 + V_1)}{1} + \frac{(2I + V_1) - V_2}{1} = 0$$

$$\Rightarrow \quad V_1\left(\frac{1}{2} + \frac{1}{2} + 1 + 1\right) + (-1 - 1)V_2 = \frac{10}{2} + 1 - 2I = 6 - 2I$$

$$\Rightarrow \quad 3V_1 - 2V_2 = 6 - 2I \qquad (1)$$

At node 2:

$$\frac{V_2}{2} + \frac{(1 + V_2) - V_1}{1} + \frac{V_2\,(2I + V_1)}{1} = 0$$

$$\Rightarrow \quad (-1 - 1)V_1 + \left(\frac{1}{2} + 1 + 1\right)\ V_2 = -1 + 2I$$

$$\Rightarrow \quad -2V_1 + \frac{5}{2}\ V_2 = 2I - 1 \qquad (2)$$

But $\qquad I = \dfrac{V_1}{2} \qquad (3)$

From (3) $\qquad$ (1) $\Rightarrow\ 4V_1 - 2V_2 = 6$

$\qquad$ (2) $\Rightarrow\ 3V_1 - \dfrac{5}{2}\ V_2 = 1$

Solving,

$$V_1 = 3.25 \text{ V}$$
$$V_2 = 3.5 \text{ V}$$

$$\therefore \quad I_1 = \frac{10 - V_1}{2} = 3.375 \text{ A}; \qquad I_2 = \frac{V_2}{2} = 1.75 \text{ A}.$$

2. (a) Define Magneto Motive Force, Magnetic Flux, and Reluctance of a Magnetic circuit. Specify the unit for the above quantities, state the relation between the above quantities.

Solution: *Magneto Motive Force* (MMF)

Magneto Motive Force (MMF) is the measure of the ability of a coil to produce a flux.

As EMF is considered to be an electric pressure, MMF is also considered to be a magnetic pressure. A coil with N turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of 'NI' ampere turns.

$\therefore$ MMF = NI Ampere Turns.

The MMF is the source of flux (ϕ) in the magnetic circuit. The length of the circuit and the MMF determines the amount of flux produced in the circuit.

Units of MMF = Ampere Turns (AT)

<u>Reluctance (S)</u>

It is the property of the medium which opposes the passage of magnetic flux. The reluctance in the Magnetic circuit is similar to the resistance in the electric circuit.

$$\therefore \quad \text{Reluctance} = \frac{\text{MMF}}{\text{flux}}$$

$$\therefore \quad S = \frac{\text{MMF}}{\phi}$$

Units of Reluctance is AT/wb.

The reluctance is the measure of the opposing offered to the set up of the flux by a magnetic circuit.

$$\therefore \quad S = \frac{\text{MMF}}{\phi} = \frac{NI}{\phi} \qquad [\because \phi = B \times A]$$

$$\therefore \quad S = \frac{NI}{B \times A} = \frac{NI}{\mu_o \mu_r + 1 \times A} \qquad [\because B = \mu_0 \, \mu_r \, H]$$

$$\therefore \quad S = \frac{NI}{\mu_o \mu_r \dfrac{NI}{l} \times A} \qquad \left[\because H = \frac{NI}{L}\right]$$

$$\therefore \qquad S = \frac{l}{\mu_o \mu_r A} \text{ AT/wb}$$

$$\therefore \qquad S = \frac{L}{\mu A} \text{ AT/wb}$$

where l = length of Magnetic Path; A = Area of cross section of magnetic circuit; and $\mu = \mu_0 \mu_r$ = Permeability of Medium.

Magnetic Flux (ϕ)

The total number of lines of induction passing normally through a surface is called Magnetic flux (ϕ).

Flux does not actually flow in a magnetic circuit.

Magnetic flux is directly proportional to the pole strength of the magnet.

i.e. $\phi \, \alpha \, m$

(or) $\qquad \phi = \mu m$

where $\qquad \mu$ = Permeability of Medium.

Units of magnetic flux is weber (wb).

Relation between MMF, S and ϕ

The Relation between MMF, Magnetic flux and Reluctance of a magnetic circuit is given as

$$\text{Magnetic flux} = \frac{\text{Maneto Motive Force}}{\text{Reluctance}}$$

i.e. $$\phi = \frac{\text{MMF}}{S}$$

i.e. $$\boxed{\phi = \frac{NI}{\frac{L}{MA}}}$$

2. (b) Write down the voltage equation for the following Fig. Set 1.5, and determine the effective inductance.

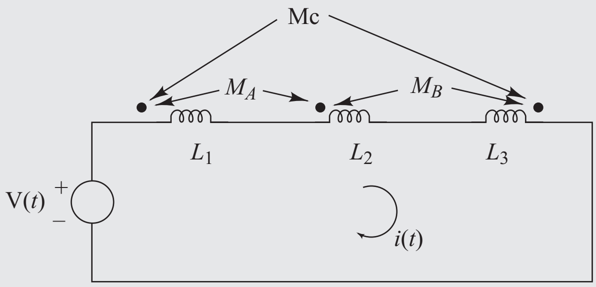


Fig. Set 1.5

Solution: Apply KVL in the given loop

$$V(t) = L_1 \frac{d\,i(t)}{d\,t} + L_2 \frac{d\,i(t)}{d\,t} + M_A \frac{d\,i(t)}{d\,t} + M_A \frac{d\,i(t)}{d\,t} + L_3 \frac{d\,i(t)}{d\,t} - M_B \frac{d\,i(t)}{d\,t} - M_B \frac{d\,i(t)}{d\,t} - M_C \frac{d\,i(t)}{d\,t} - M_C \frac{d\,i(t)}{d\,t}$$

$$\therefore \quad V(t) = [L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C] \frac{d\,i(t)}{d\,t}$$

is the required voltage equation.

We have $V(t) = L \dfrac{d\,i(t)}{d\,t}$

$$L \frac{d\,i(t)}{d\,t} = [L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C] \frac{d\,i(t)}{d\,t}$$

$\therefore$ $\boxed{L = L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C}$ is the equivalent inductance.

2. (c) Two identical coils connected in series gave an inductance of 800 mH and when one of the coils is reversed gave an inductance of 400 mH. Determine self-inductance, mutual inductance between the coils and the co-efficient of coupling.

Solution: Let 'L' be the self inductance of the coils and M be the Mutual inductance between the coils.

<u>Given Data</u>

Two identical coils connected in series gave an inductance of 800 mH

i.e. $L + L + 2M = 800$ [$\because$ identical coils $L_1 = L_2 = L$]

$2L + 2M = 800$ (1)

When one of the coils is reversed gave an inductance of 400 mH

i.e. $L + L - 2M = 400$

$2L - 2M = 400$ (2)

Add (1) and (2) we get $4L = 1200$

$$\boxed{L = 300 \text{ mH}}$$

Subtracting (2) from (1) we get $4M = 400$ mH

$$\boxed{M = 100 \text{ mH}}$$

$\therefore$ Self inductance of each coil = L = 300 mH

Mutual inductance between the coils = M = 100 mH

Co-efficient of coupling = $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$\therefore \quad K = \dfrac{M}{\sqrt{LL}}$ $\quad [\because L_1 = L_2 = L]$

$\therefore \quad K = \dfrac{M}{\sqrt{L^2}} = \dfrac{M}{L}$

$\therefore \quad K = \dfrac{100\,\text{mH}}{300\,\text{mH}}$

$\therefore \quad \boxed{K = 1/3}$

$\therefore$ co-efficient of coupling = 1/3.

3. (a) Derive the expression for $i(t)$ when the switch is moved from position 1 to position 2 at $t = 0$ in the circuit (Fig. Set 1.6) shown. The switch was in position 1 for a long time. Sketch the variation of $i(t)$. Also determine $V_C(t)$.

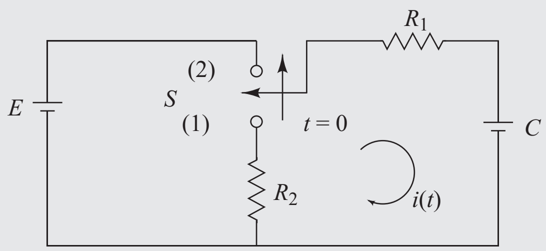


Fig. Set 1.6

Solution: When switch is in position 1 for a *long time* (steady state) capacitor is *not charged* to any voltage.

i.e., $V_C = 0$ when switch is at position (1)

Switch of position (2): Capacitor doesn't allow sudden change in voltage

$\therefore \quad V_C(t = 0^+) = 0$ (Here acting as short circuit initially)

$\therefore \quad i(t = 0^+) = \dfrac{\text{E}}{\text{R}_1}$ (Initial conditions)

Applying KVL, $E = i(t)\, R_1 + \dfrac{1}{C}\displaystyle\int_0^t i(t)\, dt$

Differentiating once, $O = R_1 \dfrac{d\,i(t)}{d\,t} + \dfrac{i(t)}{C}$ (1)

$$\Rightarrow \left(D + \frac{1}{R_1 C}\right) i = 0 \qquad \left(\text{where } D = \frac{d}{dt}\right)$$

$\therefore$ $i(t) = Ce^{-t/R_1C}$ is the solution of eqn. (1)

By initial condition $i(0) = \dfrac{E}{R_1}$

$$\therefore \quad \boxed{i(t) = \frac{E}{R_1} e^{-t/R_1C}}$$

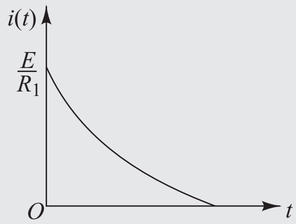


Fig. Set 1.7

$$V_C(t) = \frac{1}{C}\int_0^t i(t)\,dt = \frac{E}{R_1C}\int_0^t e^{-t/R_1C}\,dt$$

$$= \frac{E}{R_1C}(-R_1C)(e^{-t/RC} - 1)$$

$$\therefore \quad \boxed{V_C(t) = E(1 - e^{-t/RC})}$$

3. (b) In the circuit (Fig. Set 1.8) shown, determine the voltage V_{AB} to be applied to the circuit if a current of 2.5 A is required to flow in the capacitor. Determine also total power factor and total active and reactive powers. Draw the phasor diagram.

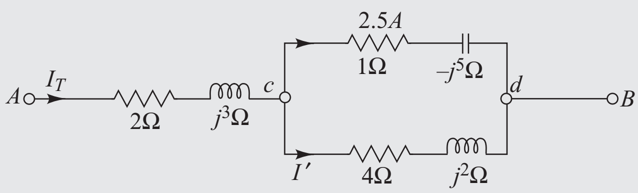


Fig. Set 1.8

Solution: $V_{cd} = 2.5\,(1 - j5) = I'(4 + j2)$

(Assuming "I'" is the current through '$4 + j2$' Ω)

$$\therefore \quad I' = \frac{2.5\,(1-5j)}{4+2j} = 2.85 \angle{-105.25^\circ} = -0.75 - 2.75j$$

$$\therefore \quad I_T = 2.5 - 0.75 - 2.75j \qquad (\because I_T = 2.5 + I')$$

$$= 1.75 - 2.75j = 3.26 \angle{-57.53^\circ}$$

$$\therefore \quad V_{AB} = I_T(2 + j3) + 2.5(1 - j5)$$

$$= (1.75 - 2.75j)(2 + j3) + 2.5(1 - j5)$$

$$= 14.25 - 12.75j = 19.12 \angle{-41.82^\circ}$$

$$Z_{AB} = \frac{V_{AB}}{I_T} = \frac{19.12\angle{-41.82^\circ}}{3.26\angle{-57.53^\circ}} = 5.865 \angle 15.71^\circ$$

$$\theta = 15.71^\circ$$

Total power factor = $\cos\theta = \cos 15.71° = 0.962$

Total active power $= V_{AB}\, I_T \cos\theta$
$(\mathrm{P_{arg}})$

$$= 19.12 \times 3.26 \times 0.962 = 59.96 \text{ W}$$

Total reactive power $= V_{AB}\, I_T \sin\theta$
(P_o)

$$= 19.12 \times 3.26 \times \sin 15.71° = 16.87 \text{ VAR}$$

Apparent power $= V_{AB}\, I_T\, (P_o) = 19.12 \times 3.26 = 62.3312\ V_A$

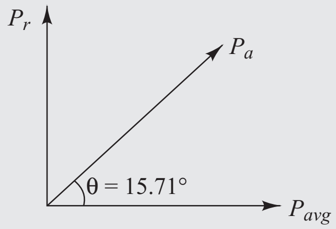


Fig. Set 1.9

4. (a) Obtain the response of *R-L-C* series circuit for impulse excitation. Use Laplace transform method

Solution:

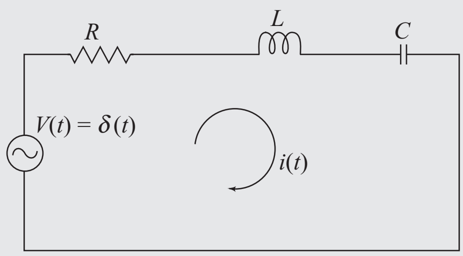


Fig. Set 1.10

Apply KVL to the *R-L-C* series circuit

$$\delta(t) = i(t)\, R + L\, \frac{di(t)}{dt} + \frac{1}{C}\int i(t)\, dt$$

Apply Laplace transform

$$L\,\{\delta(t)\} = L\left\{i(t)\, R + L\,\frac{d\,i(t)}{dt} + \frac{1}{C}\int_0^t i(t)\, dt\right\}$$

$$1 = I(S)\, R + L\,[SI(S) - I(0)] + \frac{1}{C}\left[\left[\frac{I(S)}{S}\right] - \frac{I(0^-)}{S}\right]$$

$$1 = I(S)\left[R + LS + \frac{1}{CS}\right] \qquad [\because \text{ Initial conditions are zero}]$$

$$I(S) = \frac{1}{R + LS + \dfrac{1}{CS}}$$

$$I(S) = \frac{1}{\dfrac{L}{S}\left[S^2 + S\dfrac{R}{L} + \dfrac{1}{LC}\right]}$$

$$I(S) = \frac{S}{L\left[S^2 + S\dfrac{R}{L} + \dfrac{1}{LC}\right]} \qquad (1)$$

Applying partial fraction

$$\frac{(S/L)}{S^2 + S\dfrac{R}{L} + \dfrac{1}{LC}} = \frac{1}{L}\left[\frac{S}{S^2 + \dfrac{R}{L}S + \dfrac{1}{LC}}\right] = \frac{1}{L}\left[\frac{S}{\left(S + \dfrac{R}{2L}\right)^2 - \left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}}\right]$$

$$= \frac{1}{L}\left[\frac{S + \dfrac{R}{2L} - \dfrac{R}{2L}}{\left(S + \dfrac{R}{2L}\right)^2 - \left(\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}\right)}\right]$$

$$= \frac{1}{L}\left[\frac{S + \dfrac{R}{2L}}{\left(S + \dfrac{R}{2L}\right)^2 - \left[\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}\right]} - \frac{R}{2L}\,\frac{1}{\left(S + \dfrac{R}{2L}\right)^2 - \left[\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}\right]}\right]$$

Let $\quad K_1 = -\dfrac{R}{2L}$ and $K_2 = \sqrt{\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}}$

But $L\left(\dfrac{S+k}{(S+k)^2 + \omega^2}\right) = e^{-kt}\cos \omega t$ and $L\left(\dfrac{\theta}{S^2 + \theta^2}\right) = \sin \omega t]$

$$= \frac{1}{L}\left[\frac{S - K_1}{(S - k_1)^2 - K_2^2} + \frac{K_{1/k_2} . k_2}{(S - k_1)^2 - K_2^2}\right]$$

If $\left(\dfrac{R}{2L}\right)^2 > \dfrac{1}{LC}$, $\qquad k_2 > 0$

Applying inverse Laplace transform of (1)

$$i(t) = \frac{1}{L}\left[e^{k_1 t}\cosh k_2 t - \frac{k_1}{k_2}e^{k_1 t}\sinh k_2 t\right]$$

If $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, k_2 imaginary $k_2^2 \to -k_2^2$

$$i(t) = \frac{1}{L}\,[e^{k_1 t}\cos k_2 t + \frac{k_1}{k_2}\,e^{k_1 t}\sin k_2 t]$$

If $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, $k_2 = 0$

$$i(t) = \frac{1}{L}\,[e^{k_1 t} + k_1\,e^{k_1 t}\,t] = \frac{k_1}{L}\,e^{k_1 t}\,(1 + k_1 t)$$

4. (b) Obtain the S domain equivalent for the following network elements.

Solution:

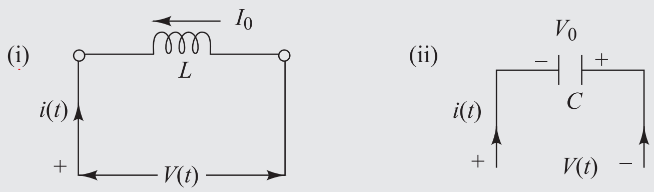


Fig. Set 1.11

(i) $$V(t) = L\,\frac{d\,i(t)}{dt} \tag{1}$$

$$i(t) = \frac{1}{L}\int V(t)\,dt \tag{2}$$

$V(S) = L[SI(S) - I(0^-)]$ Applying Laplace Transform to (1)

$I(S) = \frac{1}{SL}\,V(S) + \frac{i(0^-)}{S}$ Applying Laplace Transform to (2)

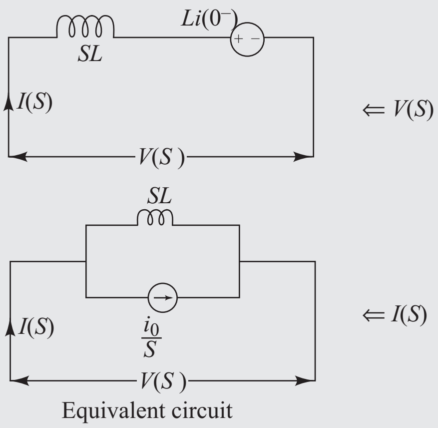


Fig. Set 1.12

(ii) We have

$$i(t) = C\frac{dV(t)}{dt}$$

Applying Laplace Transform

$$L\{i(t)\} = C\left\{\frac{dV(t)}{dt}\right\}$$

$$I(S) = C\,[SV(S) - V(0)] \qquad (1)$$

$$dV(t) = \frac{1}{C}\int i(t)\,dt$$

Integrating

$$V(t) = \frac{1}{C}\int i(t)\,dt$$

Apply laplace transform

$$L\{V(t)\} = \frac{1}{C}L\{i(t)\,dt\}$$

$$V(S) = \frac{1}{Cs}\;I(S) + \frac{V(o)}{S} \qquad (2)$$

For Eq. (2) circuit is

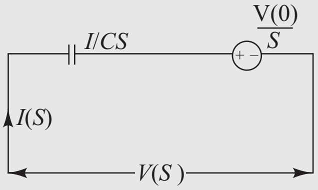


Fig. Set 1.13

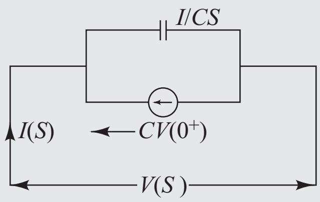
For Eq. (1) circuit is 

Fig. Set 1.14

4. (c) Define RMS Value, Average Value, form factor of a periodic quantity.

Solution: *Average Value*

For any alternating quantity $f(t)$ with time period 'T' average value f_{avg} is given by

$$f_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

Waveforms with half-wave symmetry i.e.,

$f(t) = -f\left(t + \frac{T}{2}\right)$ have zero average values. For these waveforms f_{avg} is computed over the positive half of the period which is called as half-cycle average.

Average value is that value of direct current which gives same amount of the charge to the network in same amount of time as given by the alternating current to the same electrical network.

RMS Value

For any alternating quantity $f(t)$ with time period 'T', r.m.s. value $f_{r.m.s}$ is given by,

$$f_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

If $f(t) = a_0 + (a_1 \cos \omega t + a_2 \cos 2\omega t \ldots) + (b_1 \sin \omega t + b_2 \sin 2\omega t \ldots)$

then $f_{r.m.s.} = \left[a_0^2 + \frac{1}{2}\left(a_1^2 + a_2^2 \cdots\right) + \frac{1}{2}\left(b_1^2 + b_2^2 \cdots\right)\right]^{1/2}$

R.M.S. value is equal to that direct current which when allowed to flow through a given circuit for a given time, produces same amount of heat as produced by the alternating current when allowed to flow through the same circuit for the same time.

From Factor [K_f]

The ratio of r.m.s value of an alternating quantity to its average value is called form factor.

$$K_f = \frac{\mathrm{f}_{\mathrm{r.m.s}}}{\mathrm{f}_{\mathrm{avg}}} = \frac{\sqrt{\frac{1}{T}\int_0^T t^2(t)\,dt}}{\frac{1}{T}\int_0^T f(t)\,dt}$$

5. (a) Draw the dual network for the following circuit. Shown in Fig. Set 1.15.

Solution:

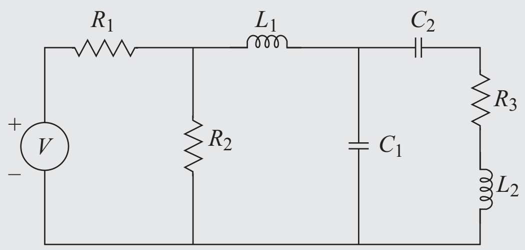


Fig. Set 1.15

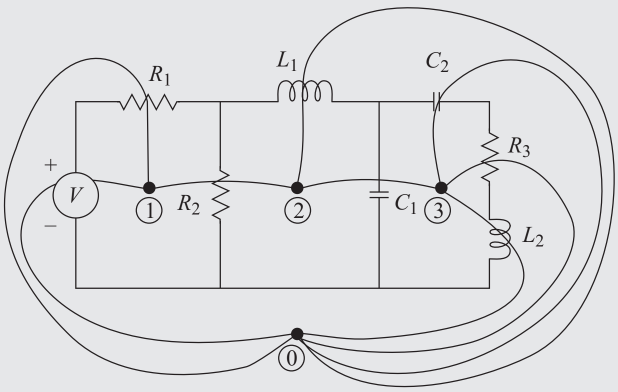


Fig. Set 1.16

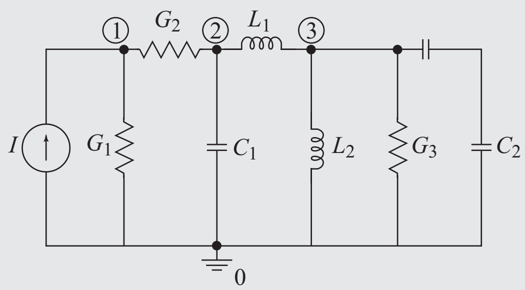


Fig. Set 1.17

5. (b) Explain, what are the dual quantities?

Solution:

Refer to Set No. 2 Questions 1 (c)

5. (c) Draw the phasor diagram of *R*, *L*, *C* elements connected parallel across a sinusoidal voltage source?

Solution

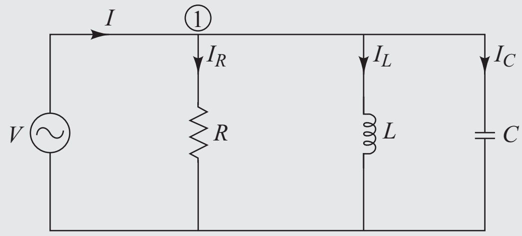


Fig. Set 1.18

Let Z be the equivalent impedance

Apply KCL at Node (1)

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{\bar{V}}{Z} = \frac{\bar{V}}{R} + \frac{\bar{V}}{jx_L} + \frac{\bar{V}}{-jx_C}$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jx_L} - \frac{1}{jx_C}$$

$$\left|\frac{1}{Z}\right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_L} - \frac{1}{x_C}\right)^2}$$

$$|Y| = \left|\frac{1}{Z}\right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{x_L} - \frac{1}{x_C}\right)^2}$$

The term Z is known as complex impedance the term Y is known as complex admittance of the parallel RLC circuit and we have three cases.

Case (i)

if $x_C > x_L$ (low freq.) or $I_L > I_C$

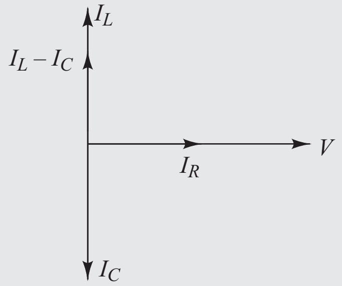


Fig. Set 1.19

Current triangle

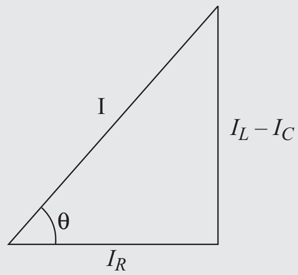


Fig. Set 1.20

$$\theta = \tan^{-1}\left(R\left(\frac{1}{x_L} - \frac{1}{x_C}\right)\right)$$

Impedence triangle

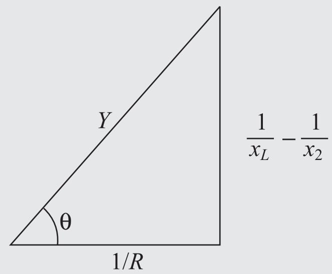


Fig. Set 1.21

Case (ii)

if $x_L > x_C$ (high freq.) of $I_C > I_L$

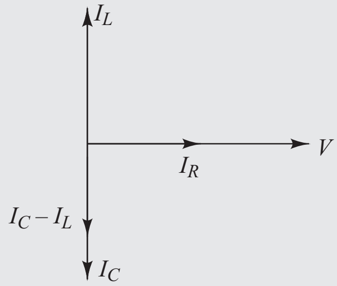


Fig. Set 1.22

Impedence triangle

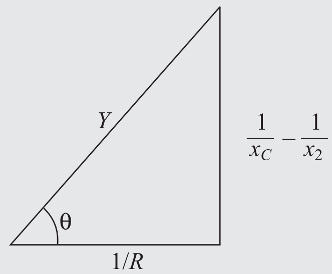


Fig. Set 1.23

$$\theta = \tan^{-1}\left(R\left(\frac{1}{x_L} - \frac{1}{x_C}\right)\right)$$

Case (iii)

if $x_L = x_C$

then $|Z| = |R|$

The circuit is Purely Real circuit i.e. it contains only Resistive element.

6. (a) Write the standard Y-parameter equations. Obtain the Y-parameters in terms of Z-parameters.

Solution: Y-parameter

Y-parameters are also known as short circuit parameters.
The standard Y-parameter equations are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad (2)$$

Multiply equation (1) with Y_{21} and equation (2) with Y_{11}

$$Y_{21} I_1 = Y_{11} Y_{21} V_1 + Y_{21} Y_{12} V_2$$

$$Y_{11} I_2 = Y_{11} Y_{21} V_1 + Y_{11} Y_{22} V_2$$

Subtracting we get

$$(Y_{21} Y_{12} - Y_{11} Y_{22})V_2 = Y_{21} I_1 - Y_{11} I_2 - \Delta Y V_2 = Y_{21} I_1 - Y_{11} I_2$$

$$V_2 = \frac{Y_{21}}{-\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2$$

We know

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{-Y_{21}}{\Delta Y} \qquad [\text{where } \Delta Y = Y_{11}Y_{22} - Y_{21}Y_{12}]$$

$$Z_{22} = \left.\frac{V_2}{I_1}\right|_{I_1 = 0} = \frac{Y_{11}}{\Delta Y}$$

Multiply equation (1) with Y_{22} and equation (2) with Y_{12}.

$$Y_{22} I_1 = Y_{22} Y_{11} V_1 + Y_{22} Y_{12} V_2$$

$$Y_{12} I_2 = Y_{12} Y_{21} V_1 + Y_{12} Y_{22} V_2$$

Subtracting

$$V_1 (Y_{11} Y_{22} - Y_{12} Y_{21}) = Y_{22} I_1 - Y_{12} I_2$$

$$V_1 = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2$$

and
We know

$$Z_{11} = \left.\frac{V_1}{I_2}\right|_{I_2 = 0} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = \frac{-Y_{12}}{\Delta Y}$$

$\therefore$ Z-parameters interms of Y-parameters

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Similarly we can get Y-parameters in terms of Z.

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

and $Z = Y^{-1}$ (or) $Y = Z^{-1}$.

6. (b) Obtain Z-parameters for the circuit shown in Figure 7 and there by obtain ABCD parameters.

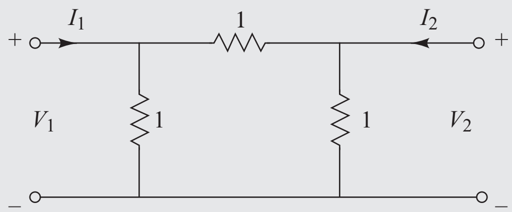


Fig. Set 1.24

Solution:

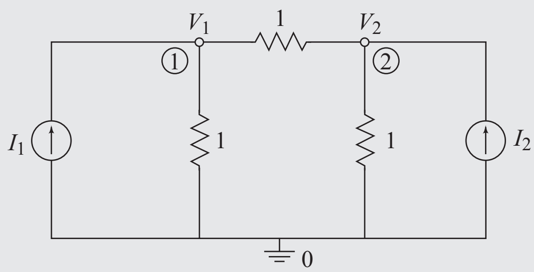


Fig. Set 1.25

Apply Nodal Analysis

At Node (1)

$$I_1 = \frac{V_1}{1} + \frac{V_1 + V_2}{1}$$

$$I_1 = 2V_1 - V_2 \tag{1}$$

At Node (2)

$$I_2 = \frac{V_2}{1} + \frac{V_2 - V_1}{1}$$

$$I_2 = 2V_2 - V_1$$

$$\text{We get } Y = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$Z = Y^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

We have

$$A = \frac{Z_{11}}{Z_{21}} = \frac{2/3}{1/3} = 2$$

$$B = \frac{\Delta Z}{Z21} = \frac{1/3}{1/3} = 1$$

$$C = \frac{1}{Z_{21}} = \frac{1}{1/3} = 3$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{2/3}{1/3} = 2$$

$$\therefore \qquad \boxed{A = 2 \quad B = 1 \quad C = 3 \quad D = 2}$$

7. (a) Figure Set 1.26 shows a resistive T network and a resistive P network connected in parallel. Find the overall Y parameters of the combination.

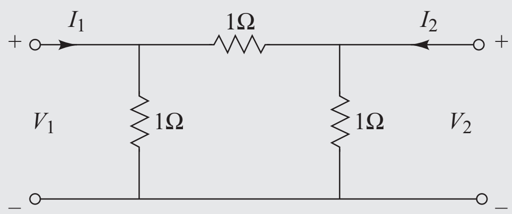


Fig. Set 1.26

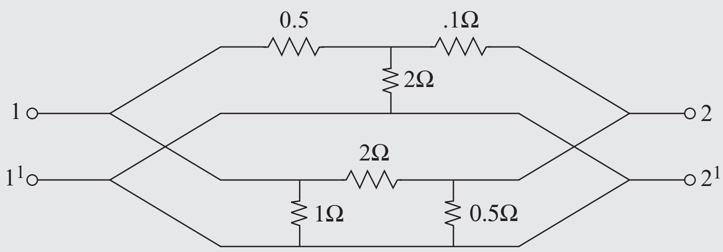


Fig. Set 1.27

Solution: For '*T*' network

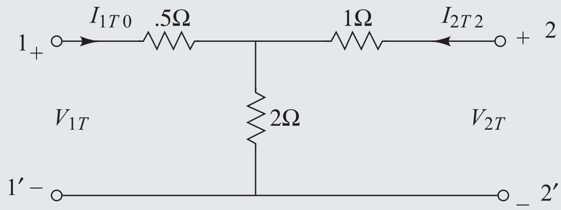


Fig. Set 1.28

$$I_{1T} = Y_{11T} V_{1T} + Y_{12T} V_{2T} \quad \therefore \; Y_{11T} = \left.\frac{I_{1T}}{V_{1T}}\right|_{V_{2T}=0} = 0.854$$

$$I_{2T} = Y_{21T} V_{1T} + Y_{22T} V_{2T}.$$

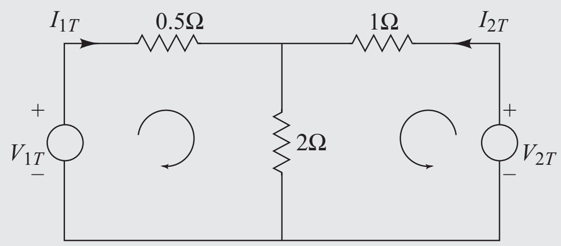


Fig. Set 1.29

$$V_{1T} = \frac{I_{1T}}{2} + 2(I_{1T} + I_{2T}) \quad \Rightarrow \quad V_{1T} = \frac{5}{2}\, I_{1T} + 2I_{2T}$$

$$V_{2T} = I_{2T} + 2(I_{1T} + I_{2T}) \quad \Rightarrow \quad V_{2T} = 2I_{1T} + 3I_{2T}$$

$$Z\text{-parameters} = \begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 3 \end{pmatrix} \quad Y\text{-parameters} = (Z)^{-1}$$

$$Y_T = \frac{1}{3.5}\begin{pmatrix} 3 & -2 \\ -2 & 5/2 \end{pmatrix} = \begin{pmatrix} 0.857 & -0.571 \\ -0.571 & 0.714 \end{pmatrix}$$

For π network

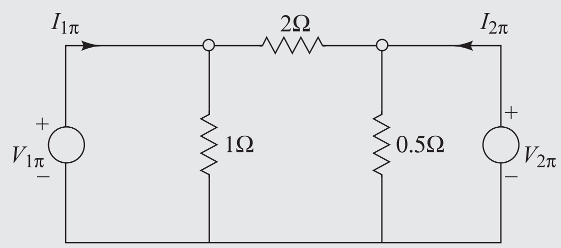


Fig. Set 1.30

$$I_{1\pi} = \frac{V_{1\pi}}{1} + \frac{V_{1\pi} - V_{2\pi}}{2}$$

$$I_{2\pi} = \frac{V_{2\pi}}{0.5} + \frac{V_{2\pi} - V_{1\pi}}{2}$$

$$\Rightarrow \qquad I_{1\pi} = \frac{3V_{1\pi}}{2} - \frac{V_{2\pi}}{2}$$

$$I_{2\pi} = \frac{\sqrt{V_{2\pi}}}{2} - \frac{V_{1\pi}}{2}$$

$$Y_{T1} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{pmatrix} = \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 2.5 \end{pmatrix}$$

The overall Y-parameters of the combination is

$$Y_{OV} = Y_T + Y_\pi = \begin{pmatrix} \frac{3}{3.5} + \frac{3}{2} & \frac{-2}{3.5} - \frac{1}{2} \\ \frac{-2}{3.5} - \frac{1}{2} & \frac{5}{7} + \frac{5}{2} \end{pmatrix}$$

$$\therefore \qquad Y_{0V} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} 2.357 & -1.071 \\ -1.071 & 3.214 \end{pmatrix}$$

7. (b) Find the characteristic impedance of a symmetric T network.

Solution: <u>T-Network</u>

Consider a symmetrical T-network as shown in Fig. 1.

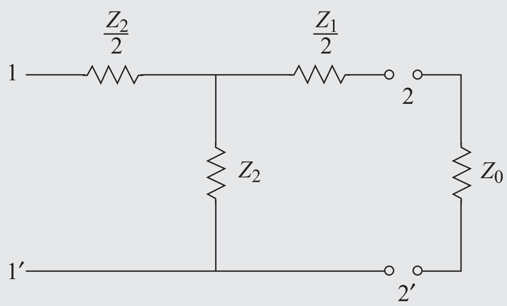


Fig. Set 1.31

If the image impedances at port 1–1′ and port 2–2′ are equal to each other, the image impedance is then called the characteristic or the iterative impedance, Z_0. Thus, if the network in figure is terminated in Z_0, its input impedance will be Z_0. The value of input impedance for T-network when it is terminated in Z_0 is given by

$$Z_{\text{in}} = \frac{Z_1}{2} + \frac{Z_2\left[\frac{Z_1}{2} + Z_0\right]}{\frac{Z_1}{2} + Z_2 + Z_0}$$

Also $\quad Z_{\text{in}} = Z_0$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left[\frac{Z_1}{2} + Z_0\right]}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_2Z_0)}{Z_1 + 2Z_2 + 2Z_0}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_0Z_2}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

$$\therefore \quad Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

$\therefore$ The characteristic impedance of a symmetrical T-section is

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}\ .$$

8. (a) What is constant k-filter? What is the difference between constant k-filter and m-derived filter? What are the limitations of constant k-filter?

Solution: Refer to Text Book (Chapter 6)

8. (b) Find the circuit parameters of a constant k-band pass filter having a pass band from 500 Hz and a characteristic resistance of 100 Ω.

Solution: k = 100 Ω, f_1 = 500 Hz, F_2 = ? (Not given)

Assume f_2 = 10000 Hz

for band pass filter.

$$L_1 = \frac{k}{\pi(f_2 - f_1)} = \frac{100}{\pi(9500)} = 3.35 \text{ mH}$$

$$C_1 = \frac{f_2 - f_1}{4\pi k\, f_1 f_2} = \frac{9500}{4 \times \pi \times 100 \times 500 \times 10000} = 1.512\ \mu\text{F}$$

$$L_2 = C_1\, k^2 = 15.12 \text{ mH}$$

$$C_2 = \frac{L_1}{k^2} = 0.335\ \mu\text{F}$$

Each of the two series erms of the constant *h*, *T*-section filter is given by

$$\frac{L_1}{2} = \frac{3.35}{2} = 1.675 \text{ mH}$$

$$2C_1 = 2 \times 1.512\ \mu\text{F} = 3.024\ \mu\text{F}$$

And the shunt erm elements of the network are given by

$$C_2 = 0.335\ \mu\text{F and } L_2 = 15.12 \text{ mH}$$

Constant-k T-section bendpass filter

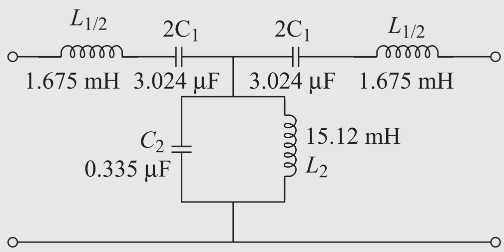


Fig. Set 1.32

Constant-k π-section bandpass filter

$$\frac{C_2}{2} = 0.1675\ \mu\text{F}$$

$$2L_2 = 30.24 \text{ mH}$$

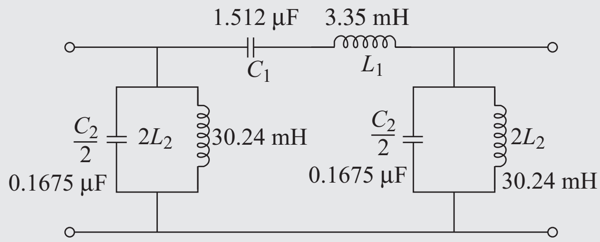


Fig. Set 1.33