Chapter 17

Filters and Attenuators

1. A *t*-section low pass filter has series inductance 80 mH and shunt capacitance 0.022 μF. Determine the cut-off frequency and nominal design impedance. Obtain the equivalent π-section.

Solution: The cut-off frequency f_c is given by

$$f_c = \frac{1}{\pi\sqrt{2c}}$$

$$= \frac{1}{\pi\sqrt{(80\times10^{-3})(0.022\times10^{-6})}} \text{ Hz}$$

$$= 7.587 \text{ kHz}$$

The design impedance (R_0) is given by

$$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{80\times10^{-3}}{0.022\times10^{-6}}} = 1.907 \text{ k}\Omega$$

The low-pass filters for *T*-section and π-section are shown below.

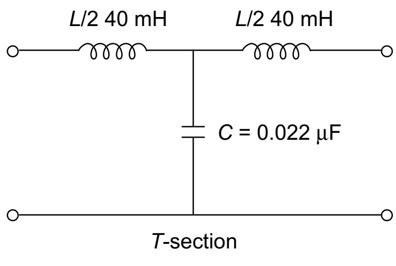


Fig. 17.1

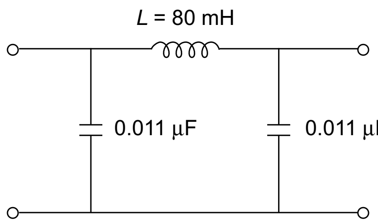


Fig. 17.2

2. Design a high-pass filter to have a design impedance of 500 Ω and a cut-off frequency of 1 KHz.

Solution: Design impedance = 500 Ω

Cut off frequency = 1000 Hz

$$L = \frac{K}{4\pi f_c} = \frac{500}{4\pi \times 1000} = \frac{7}{176} \text{ H}$$

$$C = \frac{1}{4\pi K f_c} = \frac{1}{4\pi \times 500 \times 1000} = \frac{7}{44} \ \mu\text{F}$$

The T and π sections are shown below.

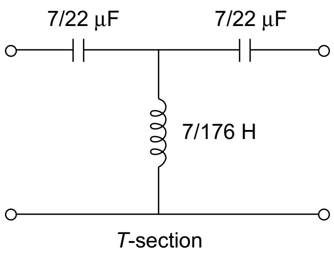


Fig. 17.3

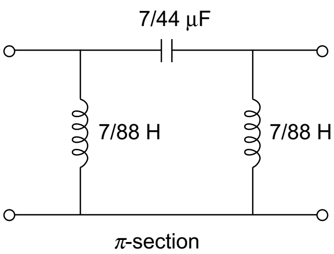


Fig. 17.4

3. Design are m-derived low-pass filter, both T and π sections having design impedance $R_0 = 500\ \Omega$, cut-off frequency $f_c = 1500$ Hz and infinite alternation frequency, $f_\infty = 2000$ Hz.
Solution: From the theory of m-derived filter,

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{1500}{2000}\right)^2} = 0.661$$

For the prototype low-pass filter, for $f_c = 1500$ Hz and $R_0 = 500\ \Omega$, series arm inductance is given by

$$L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 1500} \text{ H} = 106.103 \text{ mH}$$

and shunt-arm capacitance is given by

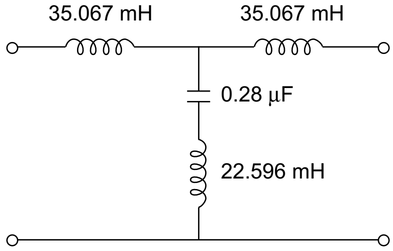
$$C = \frac{1}{\pi R_0 f_c} = \frac{1}{\pi \times 500 \times 1500} \text{ F} = 0.424\ \mu\text{F}$$

T-section: The elements are

$$\frac{mL}{2} = \frac{0.661 \times 106.103}{2} = 35.067 \text{ mH}$$

$$mC = 0.661 \times 0.424 = 0.280\ \mu\text{F}$$

$$\frac{1-m^2}{4m}\ L = \frac{1-(0.661)^2}{4 \times 0.661} \times 106.103 = 22.596 \text{ mH}$$



T-section

Fig. 17.5

In the π-section of m-derived low-pass filter, the values of the elements are

$$\frac{mC}{2} = \frac{0.661 \times 0.424}{2} = 0.14\ \mu\text{F}$$

$$mL = 0.661 \times 106.103 = 70.134 \text{ mH}$$

$$\frac{1-m^2}{4m}\ C = \frac{1-(0.661)^2}{4 \times 0.661} \times 0.424 = 0.090\ \mu\text{F}$$

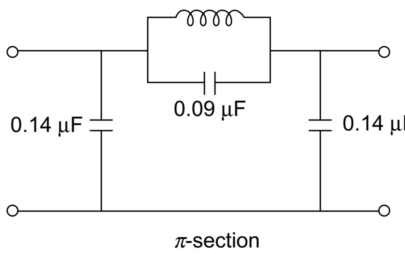


Fig. 17.6

4. Design an m-derived high-pass filter having a design impedance of 500 Ω and a cut-off frequency of 1 KHz. Take $m = 0.2$.
Solution: The constant K-filter has the values (Problem 2)

$$L = \frac{7}{176} \text{ H and } C = \frac{7}{44} \ \mu\text{F}$$

$$\frac{C}{m} = \frac{7}{44 \times 0.2} = \frac{35}{44} \ \mu\text{F}$$

$$\frac{L}{m} = \frac{7}{176 \times 0.2} = \frac{35}{176} \ \text{H}$$

$$\left(\frac{4m}{1-m^2}\right) C = \frac{0.8}{0.96} \times \frac{7}{44} \times 10^{-6} = \frac{35}{264} \ \mu\text{F}$$

$$\left(\frac{4m}{1-m^2}\right) L = \frac{0.8}{0.96} \times \frac{7}{176} = \frac{35}{1056} \ \text{H}$$

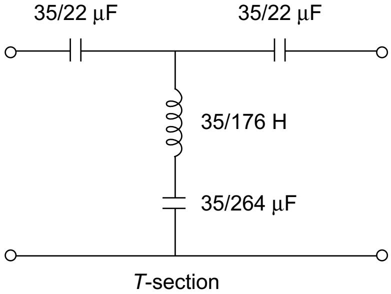


Fig. 17.7

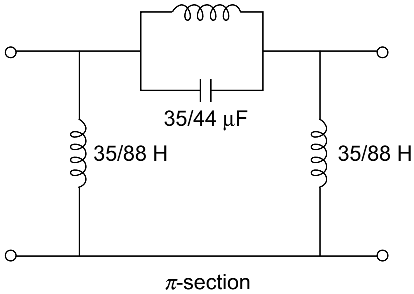


Fig. 17.8

5. Design a prototype band pass filter, both T and π sections having cut-off frequencies of 3000 Hz and 6000 Hz and nominal characteristics impedance of 600 Ω. Also find the resonant frequency of shunt arm of series arm.
Solution: The series arm impedance of a band pass filter is given by

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{600}{\pi(6000 - 3000)} = 63.622 \text{ H}$$

$$\frac{L_1}{2} = 31.831 \text{ mH}$$

The series arm capacitance C_1 is given by

$$C_1 = \frac{f_1 - f_2}{4\,\pi\, R_0 f_1 f_2}$$

$$= \frac{6000 - 3000}{4\pi \times 600 \times 3000 \times 6000} = 0.022\ \mu\text{F}$$

$$2C_1 = 0.044\ \mu\text{F}$$

The shunt-arm inductance L_2 and capacitance C_2 are given by

$$L_2 = \frac{R_0(f_1 - f_2)}{4\pi\, f_1 f_2} = \frac{600\,(6000 - 3000)}{4 \times \pi \times 6000 \times 3000}$$

$$L_2 = 7.958 \text{ mH}$$

$$2L_2 = 15.916 \text{ mH}$$

$$C_2 = \frac{1}{\pi R_0\,(f_1 - f_2)} = \frac{1}{\pi \times 600 \times 3000}$$

$$C_2 = 0.177\ \mu\text{F}$$

Hence $$\frac{C_2}{2} = 0.0885\ \mu\text{F}$$

Resonant frequency is $f_0 = \sqrt{f_1\, f_2} = \sqrt{3 \times 6 \times 10^6} = 4242.7$ Hz

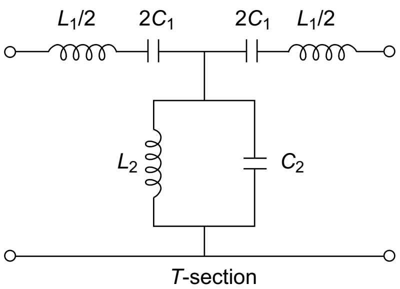


Fig. 17.9

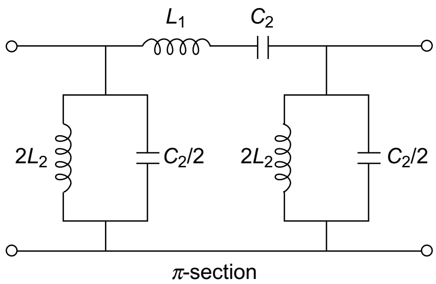


Fig. 17.10

6. Design prototype band stop filter section having cut-off frequency of 2000 Hz and 5000 Hz and a design resistance of 600 Ω.
Solution: The series arm inductance L_1 and capacitance C_1 are given by

$$L_1 = \frac{R_0(f_2 - f_1)}{\pi\, f_1\, f_2} = \frac{600(5000 - 2000)}{\pi \times 2000 \times 5000} = 57.296 \text{ mH}$$

$$\frac{L_1}{2} = 28.648 \text{ mH}$$

$$C_1 = \frac{1}{4\pi\, R_0\,(f_2 - f_1)}$$

$$= \frac{1}{4\,\pi \times 600(5000 - 2000)} = 0.044\ \mu\text{F}$$

$$2C_1 = 2 \times 0.044 = 0.088\ \mu\text{F}$$

The shunt inductance L_2 and capacitance C_2 are given by

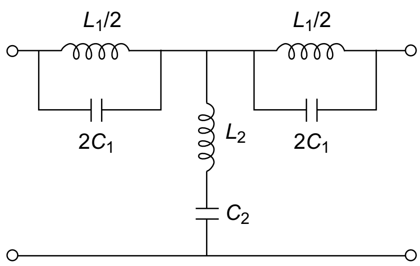
$$L_2 = \frac{R_0}{4\pi(f_2 - f_1)} = \frac{600}{4\pi \times (5000 - 2000)}$$

$$L_2 = 15.915 \text{ mH}$$

and
$$C_2 = \frac{f_2 - f_1}{4\pi\, R_0\,(f_1\, f_2)} = \frac{5000 - 2000}{\pi \times 600 \times 2000 \times 5000}$$

$$C_2 = 0.159\ \mu\text{F}$$

The band stop T-section filter is



Band stop T-section filter

Fig. 17.11

7. An attenuator is composed of symmetrical π-section having series arm of 275 Ω and shunt arm each of 450 Ω. Find
   (*i*) the characteristic impedance of the network, and
   (*ii*) attenuation per section

Solution: On the symmetrical π alternator

$$R_1 = R_0\left(\frac{N^2 - 1}{2N}\right)$$

$$R_1 = 275\ \Omega \text{ and } R_2 = 450\ \Omega$$

$$\therefore \qquad R_0\left(\frac{N^2-1}{2N}\right) = 275 \tag{1}$$

$$R_2 = R_0\left(\frac{N+1}{N-1}\right)$$

$$R_0\left(\frac{N+1}{N-1}\right) = 450 \tag{2}$$

Dividing Equation (1) by (2), we get N

$$N = 2.874 \text{ or } 0.348$$

If $N = 0.348$, it provides gain than attenuation

Thus $\quad N = 2.874$

(*i*) $\quad R_1 = R_0\left(\dfrac{N^2-1}{2N}\right)$

$$R_0 = R_1 \times \left(\frac{2N}{N^2-1}\right) = \frac{275\times 2\times 2.874}{(2.874)^2-1}$$

$$R_0 = 217.731\ \Omega$$

(*ii*) attenuation per section

$$D = 20\log_{10}(2.874) = 9.17\text{ dB}.$$

8. Design a symmetrical lattice attenuator to have attenuation of 20 dB and characteristic impedance of 500 Ω. What will be the equivalent *T*-configuration?

Solution: $\quad R_0 = 500\ \Omega$ and $D = 20$ dB

$$N = \text{antilog}_{10}\left(\frac{20}{20}\right) = 10$$

In symmetrical lattice attenuator

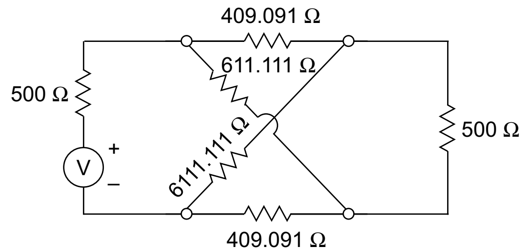
$$R_1 = R_0\left(\frac{N-1}{N+1}\right) = 500\left(\frac{10-1}{10+1}\right) = 409.091\ \Omega$$

$$R_2 = R_0\left(\frac{N+1}{N+1}\right) = 500\left(\frac{10+1}{10+1}\right) = 611.111\ \Omega$$

The equivalent design elements of *T*-sections are

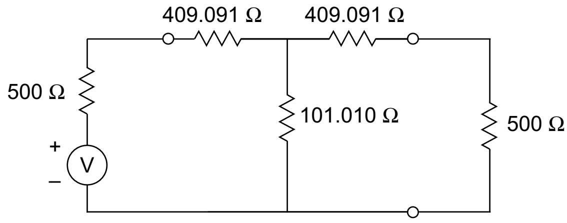
$$R_1 = R_0\left(\frac{N-1}{N+1}\right) = 500\left(\frac{10-1}{10+1}\right) = 409.091\ \Omega$$

$$R_2 = R_0\left(\frac{2N}{N^2-1}\right) = 500\left(\frac{2\times 10}{10^2-1}\right) = 101.010\ \Omega$$



Symmetrical lattice section

Fig. 17.12



Symmetrical *T*-network

Fig. 17.13

9. The series arm of a lattice equalizer used in conjunction with a telephone line of characteristic impedance $Z_0 = 600\ \Omega$ consists of a resistor of 400 Ω in series with an inductor of 40 mH. Find the shunt of the lattice.

Solution: The lattice equalizer used with telephone line will be a constant resistance equalizer. The typical constant resistance lattice attenuation equivalizer is shown below.

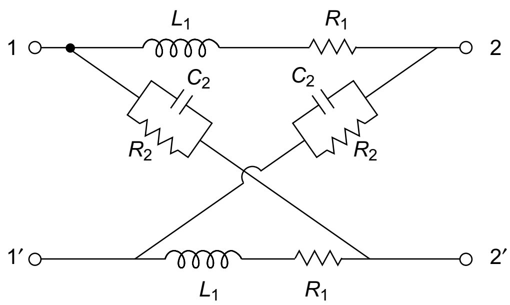


Fig. 17.14

From the given data

$$R_0 = 600\ \Omega,\ R_1 = 400\ \Omega,\ L_1 = 40\ \text{mH}$$

$$R_1\ R_2 = \frac{L}{C} = R_0^{\ 2}$$

Thus $$R_2 = \frac{R_0^2}{R_1} = \frac{(600)^2}{400} = 900\ \Omega$$

$$C_2 = \frac{L}{R_0^2} = \frac{0.04}{(600)^2} = 0.111\ \mu\text{F}$$

10. In a constant resistance lattice attenuation equalizer, having characteristic impedance of 1 Ω, the diagonal arm consists of a resistance L_2 = 1 Ω in series with a capacitor C_2 = 0.05 F. Find the series arm.
Solution: The constant resistance lattice attenuation equalizer is shown in the figure below.

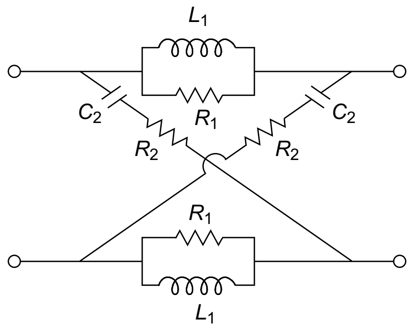


Fig. 17.15

From the given data R_2 = 1 Ω, C_2 = 0.05 F and R_0 = 1 Ω
For a constant resistance equalizer

$$R_1 R_2 = \frac{L_1}{C_2} = R_0^2$$

Thus $$R_1 = \frac{R_0^2}{R_2} = \frac{(1)^2}{1} = 1\ \Omega \text{ and } L_1 = C_2\, R_0^2$$
$$= (0.05)\,(1)^2 = 0.05\ \text{H}$$

Objective-Type Questions

1. In a broadcasting receiver, a filter circuit is used in *IF* circuit to limit the receiver bandwidth. The filter is a
   (a) band-stop filler (b) low-pass filter
   (c) band-pass filter (d) high-pass filter
2. In the *m*-derived low-pass filter, the resonant frequency is to be chosen so that it is
   (a) above the cut-off frequency
   (b) below the cut-off frequency

(c) same as that of the cut-off frequency
(d) none of the above

3. A *m*-derived low-pass filter has f_∞ position
   (a) above f_c (b) below f_c
   (c) at zero (d) at infinity
4. A line works as
   (a) attenuator (b) LPF
   (c) HPF (d) neither of the above
5. Attenuation is expressed in
   (a) decibels (b) nepers
   (c) both (d) none
6. The two networks having impedances z_1 and z_2 are said to be inverse if
   (a) $Z_1 Z_2 = R_0^{\,2}$ (b) $\dfrac{Z_1}{Z_2} = R_0^{\,2}$
   (c) $Z_1 R_0 = Z_2$ (d) none
7. Attenuation distortion in equalizers occurs due to
   (a) non uniform attenuation against frequency
   (b) uniform attenuation against frequency
   (c) non-uniform attenuation against time
   (d) uniform attenuation against time