Chapter 16

S-Domain Analysis

1. Calculate the driving point functions of the following network.

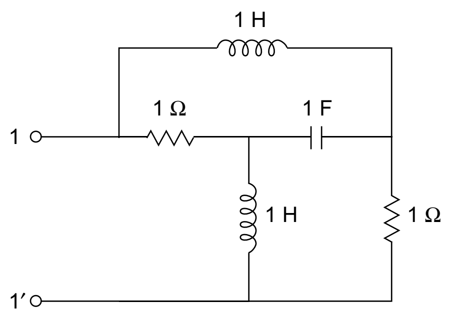

Fig. 16.1

Solution: The network is transferred in to the S-domain as follows.

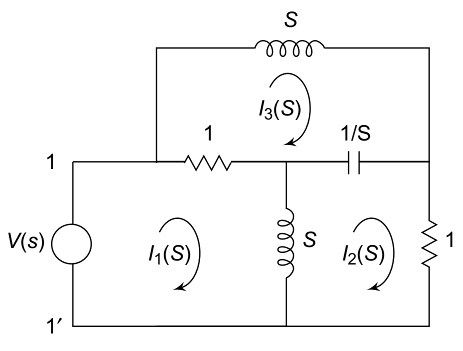

Fig. 16.2

Consider the $V(S)$ be applied at the terminals 1 1′ writing mesh equations in 3 loops in the figure.

$$I_1(S)\,(1 + S) - I_2(S)\cdot S - I_3(S) = V(S)$$

$$-I_1(S)S + I_2(S)\left(1 + S + \frac{1}{S}\right) - \frac{1}{S} I_3(S) = 0$$

$$-I_1(S) - I_2(S)\,\frac{1}{S} + I_3(S)\left(1 + S + \frac{1}{S}\right) = 0$$

$$I_1(S) = \frac{\begin{vmatrix} V(s) & -S & -1 \\ 0 & (1+S+1/S) & -1/S \\ 0 & -1/S & (1+S+1/S) \end{vmatrix}}{\begin{vmatrix} (1+S) & -S & -1 \\ -S & (1+S+1/S) & -1/S \\ -1 & -1/S & (1+S+1/S) \end{vmatrix}}$$

$$I_1(S) = \frac{V(S)\left[\left(S+1+\frac{1}{S}\right)^2 - \frac{1}{S^2}\right]}{(1+S)\left[\left(1+S+\frac{1}{S}\right)^2 - \frac{1}{S^2}\right] + S\left[-S\left(1+S+\frac{1}{S}\right) - \frac{1}{S}\right] - 1\left[\left(1+S+\frac{1}{S}\right)\right]}$$

$$I(S) = \frac{V(S)\left[S^3 + 2S^2 + 3S + 2\right]}{2S^3 + 3S^2 + 2S + 1}$$

The driving point impedance function

$$Z(S) = \frac{V(S)}{I_1(S)} = \frac{2S^3 + 3S^2 + 2S + 1}{S^3 + 2S^2 + 3S + 2}$$

The driving point admittance function

$$Y(S) = \frac{I_1(S)}{V(S)} = \frac{S^3 + 2S^2 + 3S + 2}{2S^3 + 3S^2 + 2S + 1}$$

2. Determine $Z_{11}(S)$, $Y_{11}(S)$ and $G_{21}(S)$ for the network shown in figure.

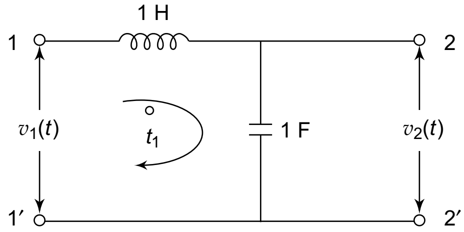

Fig. 16.3

Solution: The above network is transformed into S-domain

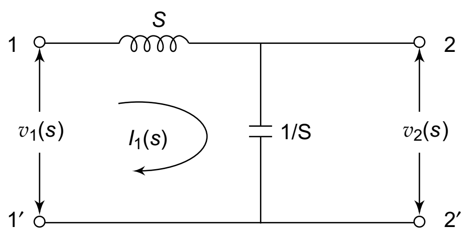

Fig. 16.4

The loop equation for the network is

$$V_1(S) = S\, I_1(S) + \frac{1}{S}\, I_1(S)$$

$$V_1(S) = \left(S + \frac{1}{S}\right) I_1(S)$$

Driving point impedance $Z_{11}(S) = \dfrac{V_1(S)}{I(S)} = \dfrac{S^2+1}{S}$

$$Y_{11}(S) = \frac{1}{Z_{11}(S)} = \frac{S}{S^2+1}$$

$$G_{21}(S) = \frac{V_2(S)}{V_1(S)}$$

From the figure, $V_2(S) = I_1(S)\,\dfrac{1}{S}$

where $$I_1(S) = \frac{S}{S^2+1} V_1(S)$$

$\therefore$ $$V_2(S) = \frac{V_1(S)}{S^2+1}$$

$$G_{21}(S) = \frac{V_2(S)}{V_1(S)} = \frac{1}{S^2+1}$$

3. Find the transform function $G_{21}(S)$ for the two-port network shown in the figure.

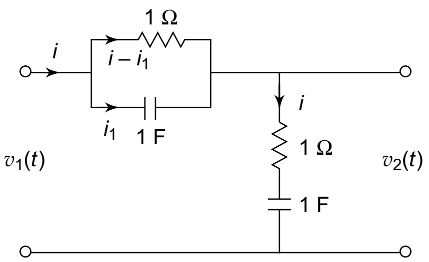

Fig 16.5

Solution: The loop equations for the network are

$$v_1(t\ ) = 1\ (i + i_1) + 1 \cdot i + \frac{1}{1}\int i\, dt$$

$$= i - i_1 + i + \int i\, dt$$

$$= 2i - i_1 + \int i\, dt \qquad (1)$$

$$v_1(t) = \frac{1}{1}\int i\, dt\ + 1 \cdot i + \frac{1}{1}\int i\, dt$$

$$= \int i_1\, dt + i \int i_1\, dt \qquad (2)$$

By taking Laplace transforms for the above equations

$$V_1(S) = 2I(S) - I_1(S) + \frac{I(S)}{S} \qquad (3)$$

$$V_1(S) = \frac{I_1(S)}{S} + I(S) + \frac{I(S)}{S} \qquad (4)$$

we have $$v_2(t) = 1 \cdot i + \frac{1}{1}\int i\, dt$$

$$= i + \int i\, dt \qquad (5)$$

Taking Laplace transforms

$$V_2(S) = I(S) + \frac{I(S)}{S} \qquad (6)$$

$$V_2(S) = I(S)\left(\frac{S+1}{S}\right) \qquad (7)$$

Subtracting (4) from (3), we get

$$0 = I(S) - I_1(S)\left[1 + \frac{1}{S}\right]$$

$$I_1(S) = \frac{I(s)}{(1+1/S)} \tag{8}$$

Substituting (7) in (4)

$$V_1(S) = I(S)\left[\frac{1}{S+1} + 1 + \frac{1}{S}\right]$$

$$= I(S)\left[\frac{S + S(S+1) + S + 1}{S(S+1)}\right] \tag{9}$$

Then $$G_{21}(S) = \frac{V_2(S)}{V_1(S)} = \frac{(S+1)^2}{S^2+3S+1}$$

$$G_{21}(S) = \frac{S^2+2S+1}{S^2+3S+1}$$

4. Determine the voltage transfer function of the following network.

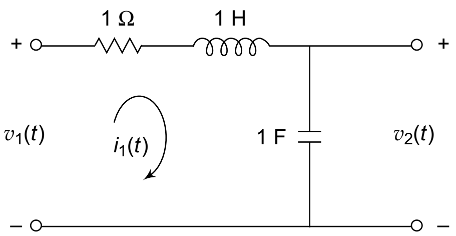

Fig. 16.6

Solution: The transform network is shown in the figure.

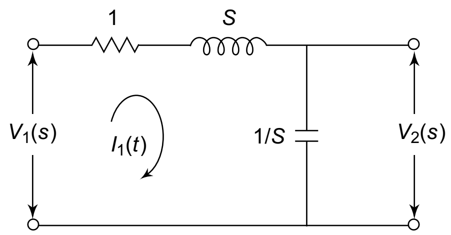

Fig. 16.7

Taking the loop equation

$$V_1(S) = I_1(S) + SI_1(S) + \frac{1}{S}\, I_1(S)$$

$$V_2(S) = I_1(S)\, \frac{1}{S}$$

The transfer function

$$G_{21}(S) = \frac{V_2(S)}{V_1(S)} = \frac{I_1(S)\,1/S}{I_1(S)\left[1+S+\dfrac{1}{S}\right]}$$

$$\therefore \qquad G_{21}(S) = \frac{1}{S^2+S+1}$$

5. Determine the current gain of the following network.

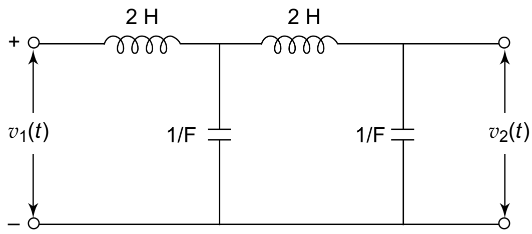

Fig. 16.8

Solution: Taking transform network, we get

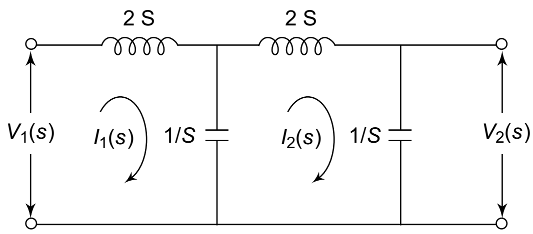

Fig. 16.9

Mesh equations are

$$V_1(S) = 2SI_1(S) + \frac{1}{S}\ I_1(S) - \frac{1}{S}\ I_2(S)$$

$$0 = -\frac{1}{S}\ I_1(S) + \left(\frac{2}{S} + 2S\right) I_2(S)$$

$$V_1(S) = \left(2S \frac{+1}{S}\right) I_1(S) - \frac{1}{S}\ I_2(S) \qquad (1)$$

$$0 = -\frac{1}{S}\ I_1(S) + \left(2S + \frac{2}{S}\right) I_2(S) \qquad (2)$$

Solving the mesh equations for $I_1(S)$

$$I_1(S) = \frac{\begin{vmatrix} V_1(S) & -1/S \\ 0 & 2S + \dfrac{2}{S} \end{vmatrix}}{\begin{vmatrix} 2S + \dfrac{1}{S} & -1/S \\ -1/S & 2S + \dfrac{2}{S} \end{vmatrix}}$$

$$I_1(S) = \frac{S\,(2S^2 + 2)\,V_1(S)}{4S^4 + 6S^2 + 1}$$

$$I_2(S) = \frac{\begin{vmatrix} 2S + \dfrac{1}{S} & V_1(S) \\ -1/S & 0 \end{vmatrix}}{\begin{vmatrix} 2S + \dfrac{1}{S} & -1/S \\ -1/S & 2S + \dfrac{2}{S} \end{vmatrix}} = \frac{S(V_1(S))}{4S^4 + 6S^2 + 1}$$

Current gain $\quad L(S) = \dfrac{I_2(S)}{I_1(S)} = \dfrac{1}{2S^2 + 2}$

6. Determine the driving point impedance z_d of the network given below.

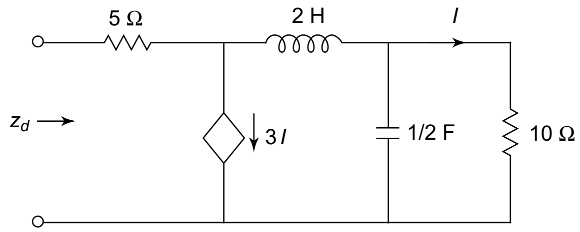

Fig. 16.10

Solution: Taking transform of the above network, we have

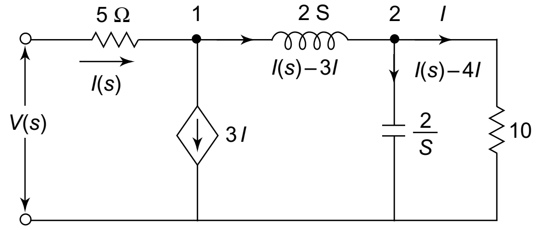

Fig. 16.11

At node 2,

$$V_2 = (I(S) - 4I)\ \frac{2}{S} = 10\ I$$

$$I(S)\ \frac{2}{S} = 10\ I + 4\ I\ \frac{2}{S}$$

$$I(S)\ \frac{2}{S} = \left(10 + \frac{8}{3}\right) I$$

$$I = \frac{I\ (S)}{5S + 4}$$

Applying Kirchhoff's voltage law at the outer loop, we get

$$V(S) = 5I(S) + 2S\ [I(S) - 3I] + 10\ I$$

Substituting I in the above equation

$$V(S) = 5I(S) + 2SI(S) - \frac{6\ S\ I(S)}{5S + 4} + \frac{10\ I(S)}{5S + 4}$$

Driving point impedance $Z_d = \dfrac{V(S)}{I(S)}$

$$Z_d = \frac{V(S)}{I(S)} = 5 + 2S - \frac{6S}{5S + 4} + \frac{10}{5S + 4}$$

$$Z_d = \frac{10S^2 + 27S + 30}{5S + 4}$$

7. Obtain the pole zero configuration of the impedance function of the network shown below.

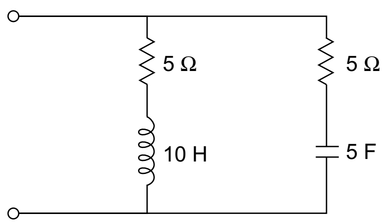

Fig. 16.12

Solution: Taking transform network, we have

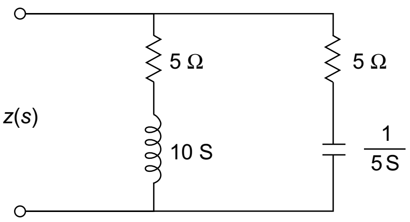

Fig. 16.13

$$Z(S) = \frac{(5+10S)\left(5+\frac{1}{5S}\right)}{5+10\,S+5+\frac{1}{5S}} = \frac{\frac{(5+10S)\,(25S+1)}{5S}}{\frac{25S+50S^2+25S+1}{5S}}$$

$$Z(S) = \frac{(10S+5)\,(25S+1)}{50S^2+50S+1}$$

$$= \frac{10\times 25\left(S+\frac{5}{10}\right)\left(S+\frac{1}{25}\right)}{50\left(S^2+S+\frac{1}{50}\right)}$$

$$= \frac{5\,(S+0.5)\,(S+0.04)}{\left(S^2+S+\frac{1}{50}\right)}$$

$$= \frac{5\,(S+0.5)\,(S+0.04)}{(S+0.98)\,(S+0.02)}$$

The zeros are lying at $S = -0.5$ and $S = -0.04$

The poles are lying at $S = -0.98$ and -0.02

9. Find the driving point impedance of the network shown in the figure.

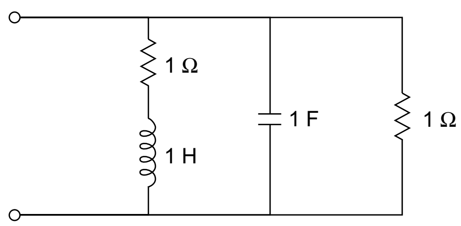

Fig. 16.14

Solution: The transform network is shown

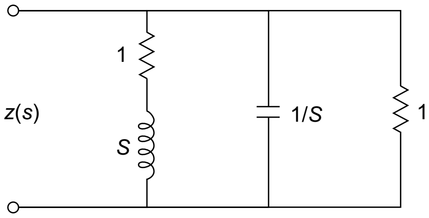

Fig. 16.15

The driving point impedance

$$Z(S) = (S+1) \left\| \left(\frac{1}{S}\right) \right\| (1)$$

$$= (S+1) \left\| \left[\frac{(1/S)^1}{1+1/S} \right] \right.$$

$$= (S+1) \left\| \left(\frac{1}{S+1} \right) \right.$$

$$= \frac{(S+1)\left(\frac{1}{S+1}\right)}{(S+1)+\frac{1}{S+1}} = \frac{1}{\frac{(S+1)^2+1}{S+1}}$$

$$Z(S) = \frac{S+1}{S^2+2S+2}$$

10. Draw the pole zero diagram for the given network function and hence obtain $v(t)$

$$V(S) = \frac{4S}{(S+2)\,(S+3)}$$

Solution: In the network function

$$P(S) = 4S$$

$$Q(S) = (S+2)\,(S+2)$$

By taking partial fractions, we have

$$V(S) = \frac{K_1}{S+2} + \frac{K_2}{S+3}$$

The time domain response can be obtained by taking inverse transform

$$v(t) = K_1\, e^{-2t} + K_2\, e^{-3t}$$

K_1 and K_2 may be determined by using pole zero plot.

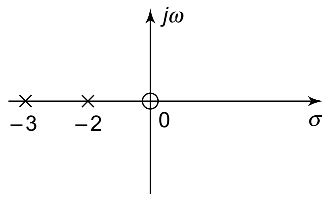

Fig. 16.16

The determine K_1, we have to find out the distances and phase angles from other zeros and poles to that particular pole.

Hence $$K_1 = H\,\frac{M_{02}\,e^{j\phi_{02}}}{M_{32}\,e^{j\phi_{32}}}$$

$$M_{02} = 2;\ M_{32} = 1, \text{ where } H = 4$$

$$\phi_{02} = 180°;\ \phi_{32} = 0°$$

$$K_1 = \frac{4 \times 2\,l}{1 \times e^{j0}} = -8$$

Similarly $$K_2 = H\,\frac{M_{03}\,e^{j\phi_{03}}}{M_{23}\,e^{j\phi_{23}}}$$

$$= \frac{4 \times 3 \times e^{j0}}{1 \times e^{j0}} = 12$$

Therefore $$v(t) = -\,8e^{-2t} + 12e^{-3t}$$

11. For the equation

$$P(S) = S^6 + 11S^5 + 42S^4 + 72S^3 + 71S^2 + 61S + 30 = 0$$

Determine the number of roots

(*i*) with positive real parts
(*ii*) with zero real parts
(*iii*) with negative real parts

Solution: Routh array is shown below:

$$\begin{array}{c|cccc}
S^6 & 1 & 42 & 71 & 30 \\
S^5 & 11 & 72 & 61 & \\
S^4 & \frac{390}{11} & \frac{720}{11} & 30 & \\
S^3 & \frac{672}{13} & \frac{672}{13} & & \\
S^2 & 30 & 30 & & \\
S^1 & 0 & 0 & & \\
S^0 & ? & & &
\end{array}$$

The elements in the sixth row become zero

Therefore $\qquad P_1(S) = 30S^2 + 30$

$\qquad\qquad = 30(S^2 + 1)$

Therefore, the polynomial $P(S)$ is

$$P(S) = (S^2 + 1)\,(S^4 + 11S^3 + 41S^2 + 61S + 30) = 0$$

The roots of equation $S^2 + 1 = 0$ are $S = \pm j$

The roots of equation $S^4 + 11S^3 + 41S^2 + 61S + 30 = 0$ may be determined by forming with array

$$\begin{array}{c|ccc} S^4 & 1 & 41 & 30 \\ S^3 & 11 & 61 & \\ S^2 & \dfrac{390}{11} & 30 & \\ S^1 & \dfrac{672}{13} & & \\ S^0 & 30 & & \end{array}$$

Here, there no roots with positive real parts. All the Four roots to have negative real parts. Therefore, we find that for the given polynomial, there are

(*i*) no roots with positive real parts
(*ii*) two roots with zero real parts
(*iii*) four roots with negative real parts

Objective-Type Questions

1. What is the driving point impedance at port 1 with port 2 open circuited in the figure.

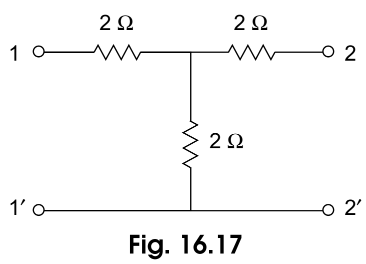

Fig. 16.17

(a) 4 Ω $\qquad$ (b) 5 Ω
(c) 6 Ω $\qquad$ (d) 7 Ω

2. What is the driving point admittance at port 1 with port 2 open circuited in the figure of questions 1.
   (a) 0.5 ℧ (b) 4 Ω
   (c) 2 Ω (d) 0.25 ℧
3. For voltage transfer ratio, the maximum degree of $P(S)$ must equal to the degree of $Q(S)$
   (a) True (b) False
4. If the poles lie on the imaginary axis
   (a) the system is oscillatory (b) the system is stable
   (c) the system is partially stable (d) none of the above
5. If there is no change in the sign of elements in the first column, then
   (a) all the roots have zero real parts
   (b) all the roots have negative real parts
   (c) all the roots have positive real parts
   (d) none of the above