Chapter 15

Two-Port Networks

1. Determine the Z-parameters for the circuit shown in the figure.

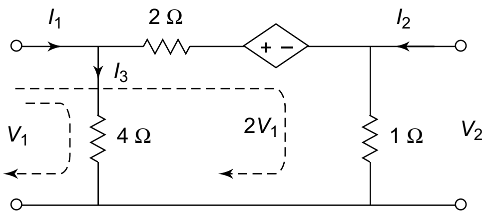

Fig. 15.1

Solution: Let I_3 be the current in 4 Ω

$$V_1 = 4\,I_3 \Rightarrow I_3 = \frac{V_1}{4}$$

$$V_2 = 2(I_1 - I_3) + I_2 \Rightarrow V_2 = 2\,I_1 - \frac{V_1}{2} + I_2$$

Applying KVL to outer loop

$$uI_3 - (I_1 - I_3 + I_2) - 2V_1 - 2(I_1 - I_3) = 0$$

$$2V_1 = -\,3I_1 - I_2 + 6I_3$$

$$2V_1 = -\,3I_1 - I_2 + \frac{V_1}{4}$$

$$\frac{V_1}{2} = -\,3I_1 - I_2$$

$$V_1 = -\,6I_1 - 2I_2 \qquad (1)$$

Substituting in V_2

$$V_2 = 2I_1 + I_2 - (-3I_1 - I_2)$$

$$V_2 = 5I_1 - 2I_2 \qquad (2)$$

From equations (1) and (2)

$$Z_{11} = -6; \qquad Z_{12} = -2$$

$$Z_{21} = 5; \qquad Z_{22} = -2$$

2. For the current shown in problem 1, find Y-parameters.

Solution: Subtracting Equation (1) from (2)

$$V_1 - V_2 = -6I_1 - 2I_2 - 5I_1 + 2I_2$$

$$I_1 = \frac{-V_1}{11} + \frac{V_2}{11} \qquad (3)$$

In order to eliminate I_1; multiply equation 2 with $\frac{6}{5}$ and add it to equation 1

$$V_2 \times \frac{6}{5} = 6I_1 - \frac{12}{5} I_2$$

$$V_1 = -6I_1 - 2I_2$$

$$V_1 + \frac{6}{5} V_2 = \frac{-22}{5} I_2$$

$$I_2 = -\frac{5}{22} V_1 - \frac{3}{11} V_2 \qquad (4)$$

From equations (3) and (4)

$$Y_{11} = \frac{-1}{11}; \qquad Y_{12} = \frac{1}{11}$$

$$Y_{21} = \frac{-5}{22}; \qquad Y_{22} = \frac{-3}{11}$$

3. For the Two-port network shown in the figure, the input impedance with cd open = 125 + J50. Input impedance with cd short = 200 + J150; output impedance with ab open = 100 Ω. Find Z_A; Z_B and Z_C.

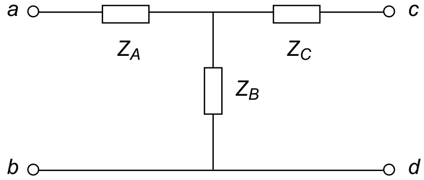

Fig. 15.2

Solution: From the given data

$$Z_{11} = 125 + J50$$

$$Z_{22} = 100$$

$$\frac{1}{Y_{11}} = 200 + J150$$

The following relations hold good for a T network

$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{21} - Z_{12}$$

$$Z_C = Z_{22} - Z_{12}$$

Also $$Y_{11} = \frac{Z_{22}}{4Z} = \frac{Z_{22}}{Z_{11}\, Z_{22} - Z_{12}\, Z_{21}}$$

$\therefore$ $$Z_{11}\, Z_{22} - Z_{12}\, Z_{21} = (100)\,(200 + J150)$$

$$Z_{12} = Z_{21}$$

$\therefore$ $$Z_{12}^2 = -\,100\,(200 + J150) + Z_{11}\, Z_{22}$$

$$= -\,100\,(200 + J150) + 100\,(125 + J50)$$

$$Z_{12}^2 = 12500 \angle 233.13^\circ$$

$$Z_{12} = 111.8 \angle 116.565^\circ = -\,50 + J100$$

$\therefore$ $$Z_A = 125 + J100 + 50 - J100 = 175 - J50$$

$$Z_B = Z_{12} = -\,50 + J100$$

$$Z_C = 100 + 50 - J100 = 150 - J100.$$

4. Find the transmission parameters for the network shown in the figure.

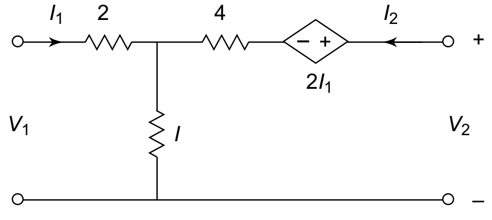

Fig. 15.3

Solution: $V_1 = AV_2 - BI_2$; $A = \dfrac{V_1}{V_2}\Big|_{I_2=0}$; $-B = \dfrac{V_1}{I_2}\Big|_{V_2=0}$

$I_1 = CV_2 - DI_2$, $C = \dfrac{I_1}{V_2}\Big|_{I_2=0}$; $-D = \dfrac{I_1}{I_2}\Big|_{V_2=0}$

when $I_2 = 0$; $V_1 = 3I_1$

and $-V_2 + 2I_1 + I1 = 0$

$4\ V_2 + 3I_1 = 0$

$$V_2 = 3I_1 \Rightarrow \frac{I_1}{V_2} = \frac{1}{3} = \text{C}$$

$$V_1 = 3I_1$$

$$V_1 = 3\ \frac{V_2}{3} \Rightarrow \frac{V_1}{V_2} = 1 = \text{A}$$

When $V_2 = 0$; the network is as shown in the figure.

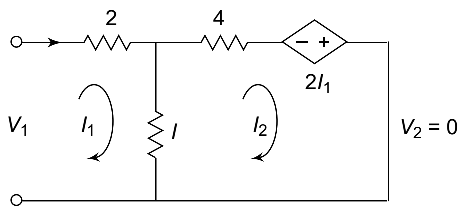

Fig. 15.4

Loop equation for the 1st loop

$$V_1 = 3I_1 - I_2$$

Loop equation for the 2nd loop

$$-2I_1 + I_2 - I_1 + 4I_2 = 0$$

$$3I_1 = 5I_2 \Rightarrow \frac{I_1}{I_2} = \frac{5}{3} = -\text{D}$$

$$V_1 = 3 \times \frac{5}{3}\ I_2 - I_2 = 4I_2$$

$$\frac{V_1}{I_2} = 4 = -\text{B}$$

5. For the parallel networks shown in the figure obtain the Y-parameters of the combination.

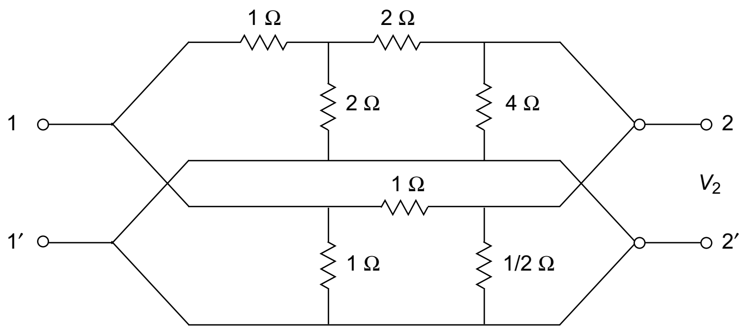

Fig. 15.5

Solution: The Y-parameters of the upper network are given

$$Y_{11} = \frac{1}{2}\mho; \qquad Y_{21} = \frac{-1}{4}\mho;$$

$$Y_{22} = \frac{5}{8}\mho \text{ and } \qquad Y_{12} = \frac{-1}{4}\mho$$

The Y-parameters of the lower network are given by

$$Y_{11} = 2; \qquad Y_{12} = -1;$$
$$Y_{22} = 3 \text{ and } \qquad Y_{21} = -1$$

Y-parameters of the combination are given by

$$Y_{11} = \frac{1}{2} + 2; \qquad Y_{12} = \frac{-1}{4} - 1$$

$$Y_{21} = \frac{-1}{4} - 1; \qquad Y_{22} = \frac{5}{8} + 3$$

6. If the Y-parameters of a network are given by $Y_{11} = \frac{1}{2}$; $Y_{12} = Y_{21} = 1$ and $Y_{22} = \frac{1}{2}$. Find inverse transmission parameters

Solution:

$$\Delta Y = \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{vmatrix} = \frac{-3}{4}$$

$$A^1 = \frac{-Y_{11}}{Y_{12}} = \frac{1}{2}; \quad B^1 = \frac{-1}{Y_{12}} = -1$$

$$C^1 = \frac{-\Delta Y}{Y_{12}} = \frac{3}{4}; \quad D^1 = \frac{-Y_{22}}{Y_{12}} = \frac{-1}{2}$$

7. Obtain the Z-parameters for the two-port unsymmetrical lattice network shown in the figure.

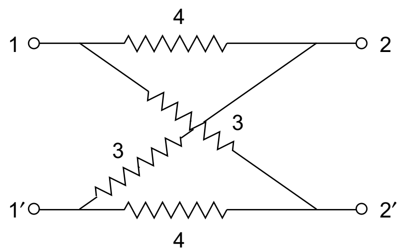

Fig. 15.6

Solution:
$$Z_{11} = \frac{(Z_a + Z_b)(Z_d + Z_c)}{Z_a + Z_b + Z_c + Z_d}$$

where Z_a, Z_d are called series colours and Z_b, Z_c are called the diagonal of the network.

Since the network is symmetrical

$$Z_{11} = \frac{Z_a + Z_b}{2} = \frac{7}{2} = Z_{22}$$

$$Z_{12} = Z_{21} = \frac{Z_b - Za}{2} = \frac{3-4}{2} = \frac{-1}{2}$$

8. For the network shown in the figure, obtain Z-parameters, h-parameters and inverse h-parameters.

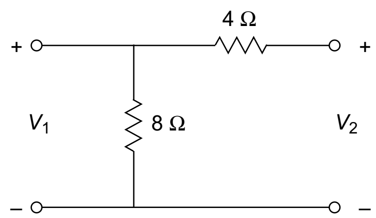

Fig. 15.7

Solution: The *Z*-parameters can be directly written from the figure.

$$Z_{11} = 8\ \Omega; \qquad Z_{12} = Z_{21} = 8\ \Omega; \qquad Z_{22} = 12\ \Omega$$

h-parameters can be expressed in terms of *Z*-parameters as

$$h_{11} = \frac{\Delta Z}{Z_{22}} \qquad \text{where } \Delta Z = \begin{vmatrix} 8 & 8 \\ 8 & 12 \end{vmatrix} = 32$$

$$\therefore\ h_{11} = \frac{32}{12} = \frac{8}{3}; \qquad h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{8}{12} = \frac{2}{3}$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} = \frac{-8}{12} = \frac{-2}{3}; \qquad h_{22} = \frac{1}{Z_{22}} = \frac{1}{12}$$

$$g_{11} = \frac{1}{Z_{11}} = \frac{1}{8}\mho; \qquad g_{12} = \frac{-Z_{12}}{Z_{11}} = -1$$

$$g_{21} = \frac{Z_{21}}{Z_{11}} = 1; \qquad g_{22} = \frac{\Delta Z}{Z_{11}} = 8$$

Objective-Type Questions

1. A Two-port network
   (a) is always an active network
   (b) is always a passive network
   (c) both active or passive
2. A Two-port network without sources are called
   (a) active parts (b) passive parts (c) linear parts
3. A Two-port network with sources in their branches are called
   (a) active parts (b) passive parts (c) non-linear parts
4. The Two-port network is reciprocal of
   (a) the determinant of the Transmission matrix = 1
   (b) $Z_{12} = Z_{21}$
   (c) both a and b
5. For a Two-port bilateral network
   (a) AD – BC = 0 (b) AD – BC = – 1 (c) AD – BC = 1
6. The *Z*-parameters of a symmetrical *T* network are $Z_{11} = 3\ \Omega$; $Z_{12} = Z_{21} = 2$; $Z_{22} = 3\ \Omega$, then the series and diagonal elements of an equivalent lattice are
   (a) 3 Ω; 2 Ω (b) 2 Ω; 3 Ω (c) 1 Ω : 5 Ω

7. The h-parameters for the current shown in the figure are given by

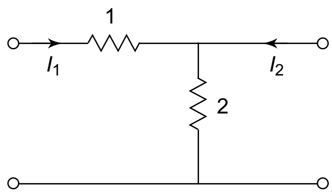

Fig. 15.8

(a) $\begin{matrix} 1 & 1 \\ -1 & \frac{1}{2} \end{matrix}$ (b) $\begin{matrix} 1 & -1 \\ 1 & \frac{1}{2} \end{matrix}$ (c) $\begin{matrix} 1 & \frac{1}{2} \\ -1 & 1 \end{matrix}$

8. The Z-parameters of the network given in question number 7 are

(a) $\begin{matrix} 2 & 2 \\ 2 & 3 \end{matrix}$ (b) $\begin{matrix} 2 & 3 \\ 2 & 2 \end{matrix}$ (c) $\begin{matrix} 3 & 2 \\ 2 & 2 \end{matrix}$

9. Dimensionally the units of h_{11} represents

(a) admittance (b) impedance (c) voltage gain

10. Dimensionally the units of h_{12} represent

(a) open circuit output admittance
(b) short circuit input impedance
(c) open circuit reverse voltage gain