Chapter 11

Transients

1. The switch in the circuit shown below is closed at $t = 0$. Find $v_2(t)$ for all $t \geq 0$ by time domain method. Assume zero initial current in the inductance.

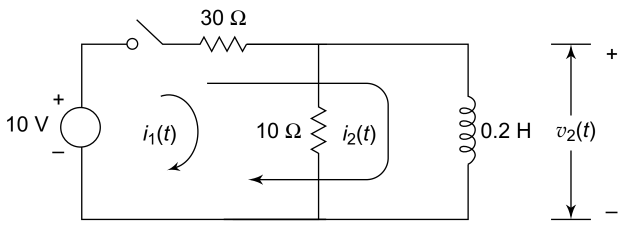

Fig. 11.1

Solution: By applying KVL to loop 1, we get

$$10 = 30\,[i_1(t) + i_2(t)] + 10i_1 \tag{1}$$

$$40i_1(t) + 30i_2(t) = 10 \tag{2}$$

By applying KVL to loop 2, we get

$$30\,[i_1(t) + i_2(t)] + 0.2\,\frac{di_2(t)}{dt} = 10 \tag{3}$$

From equation (1)

$$i_1(t) = 0.25 - 0.75\,i_2(t) \tag{4}$$

Substituting (4) into (3), we get

$$30\,[0.25 - 0.75\,i_2(t) + i_2(t)] + 0.2\,\frac{di_2(t)}{dt} = 10$$

$$\frac{di_2(t)}{dt} - 37.5\,i_2(t) = 2.5 \tag{5}$$

$$(D - 37.5)\,i_2(t) = 2.5$$

$$i_2(t) = e^{-37.5t} \int 2.5\, e^{37.5t}\, dt + Ce^{-37.5\, t}$$

$$i_2(t) = 0.066 + Ke^{-37.5t}$$

At $\quad t = 0;\ i_2(t) = 0$

$$K = -\,0.066$$

Substituting the value of K, we get

$$i_2(t) = 0.066\ [1 - e^{-37.5t}] \text{ amps}$$

$$v_2(t) = L\,\frac{di_2(t)}{dt}$$

$$= L\,\frac{d}{dt}\ [0.066(1 - e^{-37.5t})]$$

$$= 0.2\ (0.066)\ [0 - e^{-37.5t}\ (-\,37.5)]$$

$$v_2(t) = 0.495\ e^{-37.5t} \text{ volts.}$$

2. A series R-L circuit is excited by 100 volts d.c. source by closing the switch at $t = 0$. Find the values of i, $\frac{di}{dt}$, and $\frac{d^2 i}{dt^2}$ at $t = 0+$ for $R = 10\Omega$, $L = 1$ H. The current at $t = 0$ in the inductor was zero.

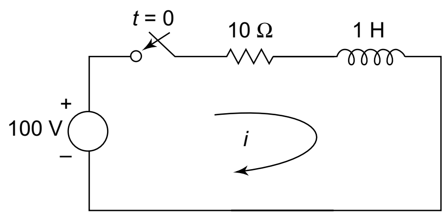

Fig. 11.2

Solution: Applying KVL in the circuit

$$10i + 1\frac{di}{dt} = 100$$

$$(D + 10)i = 100$$

The complete solution is $i = i_C + i_P$

$$i = e^{-Pt} \int Ke^{Pt}\, dt + Ce^{-Pt}$$

$$P = 10 \text{ and } K = 100$$

$$i = e^{-10t} \int 100e^{10t}\, dt + Ce^{-10t}$$

$$i = 10 + Ce^{-10t}$$

At $t = 0; i = 0$

$e = -10$

$\therefore \quad i = 10 - 10e^{-10t}$

$$\frac{di}{dt} = 0 + 100e^{-10t}; \ \frac{di}{dt}\ (0+) = 100$$

$$\frac{d^2i}{dt^2} = -1000e^{-10t}; \ \frac{d^2i}{dt^2}\ (0+) = -1000$$

3. In the circuit given below, steady state conditions are reached with the switch K in the position 1. At $t = 0$, the switch is changed over to the position 2. Using time domain methods, determine the current through the inductor $i(t)$ for all $t \geq 0+$.

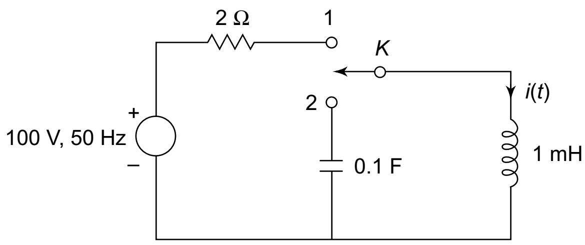

Fig. 11.3

Solution: The steady-state current through R_L circuit is

$$i(t) = \frac{10}{2} = 5 \text{ amp}$$

For $t \geq 0$, the KVL is applied in the circuit, we get

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$10^{-3}\frac{di}{dt} + \frac{1}{0.1}\int idt = 0$$

Differentiating once, we have

$$10^{-3}\frac{d^2i}{dt^2} + 10i = 0$$

$$(D^2 + 10^4)i = 0$$

The current equation is

$$i(t) = C_1 \cos 100t + C_2 \sin 100t$$

At $t = 0$ $i(t) = 5$ amp and $C_1 = 5$

By differentiating the above equation, we get

$$\frac{di}{dt} = -\,100C_1 \sin 100t + C_2\, 100 \cos 100t$$

At $\quad t = 0;\ \frac{di}{dt} = 0$

4. A series circuit shown in the figure at $t = 0$, the switch is closed. Find
   (i) the equations for i, V_R and V_L
   (ii) the current at $t = 0.4$ sec
   (iii) the time at which $V_R = V_L$

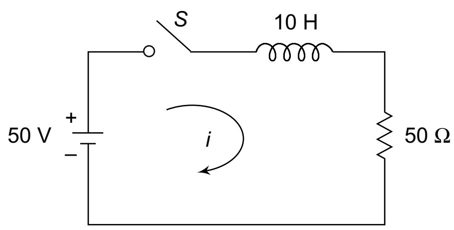

Fig. 11.4

Solution:

(i) When the switch is closed for $t > 0$, we have

$$50i + 10\frac{di}{dt} = 50$$

$$(\mathrm{D} + 5)i = 5$$

The complete solution for the above equation is

$$i = \mathrm{C}e^{-5t} + 1$$

At $\quad t = 0,\ i_0 = 0$ and $C = -\,1$

Therefore $\quad i = (1 - e^{-5t})$ amp

The corresponding voltages across the circuit elements

$$V_R = Ri = 50\,(1 - e^{-5t}) \text{ volts}$$

$$V_2 = L\frac{di}{dt} = 10 \times 5e^{-5t} = 50e^{-5t}$$

(ii) At $t = 0.4$ sec

$$i = (1 - e^{-5(0.4)}) = (1 - 0.082) = 0.918 \text{ amp}$$

(iii) When $V_R = V_L$, the voltage across each element is 25 V

$$v_L = 50e^{-5t} = 25$$

$$e^{-5t} = 0.5$$

$$5t = 0.693$$

$$t = 0.1386 \text{ sec.}$$

5. In the given circuit, switch K is closed at time $t = 0$, the steady-state condition having reached previously. Obtain an expression for the current in the circuit at any time t. If $R_1 = R_2 = 100$ ohms, $V = 10$ volts, and $L = 1$ henry. Calculate at time $t = 5$ sec, (i) current i (ii) voltage drop across R_2 and (iii) voltage across L.

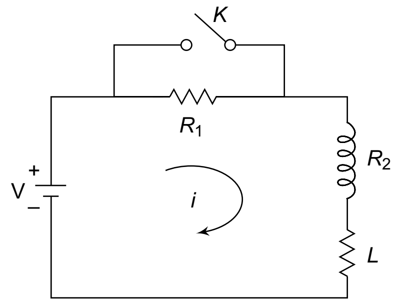

Fig. 11.5

Solution: When switch K is closed, by applying KVL, we get

$$L\frac{di}{dt} + R_2\, i = V$$

$$\left(D + \frac{R_2}{L}\right) i = \frac{V}{L}$$

The complete current is

$$i = Ce^{-\frac{R_2}{L}t} + e^{\frac{-R_2}{L}t}\int \frac{V}{L} e^{\frac{R_2}{L}t}\, dt$$

$$i = Ce^{-\frac{R_2}{L}t} + \frac{V}{R_2}$$

At $t = 0$, the current $i = \dfrac{V}{R_1 + R_2}$

$\therefore$ $$\frac{V}{R_1 + R_2} = C + \frac{V}{R_2}$$

$$C = \frac{-VR_1}{R_2(R_1 + R_2)}$$

Then the current equation becomes

$$i = \frac{V}{R_2}\left[1 - \frac{R_1}{R_1 + R_2} e^{-\frac{R_2}{L}t}\right]$$

At $$t = 5 \times 10^{-3}$$

$$i = \frac{10}{100}\left[1 - \frac{100}{200}\, e^{\frac{-100}{1} \times 10^{-3}}\right] \text{amp}$$

$$i = 69.7 \text{ mA}$$

Voltage across $R_2 = 69.7 \times 10^{-3} \times 100 = 6.97$ volts

Voltage across $L = 10 - 6.97 = 3.03$ volts

6. In the given circuit, capacitor C has an initial voltage $v_C(-0) = 10$ volts and at the same instant, the current in the inductor L is zero. Switch K is closed at time $t = 0$. Obtain the expression for the voltage $v(t)$ across the inductor L.

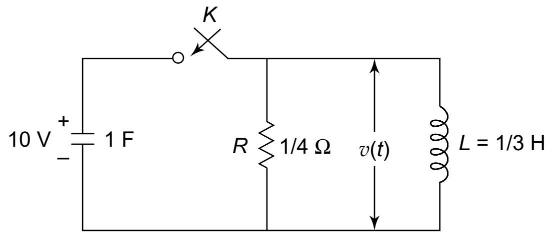

Fig. 11.6

Solution: Applying KCL to the circuit

$$C\frac{dv}{dt} + Gv + \frac{1}{L}\int v dt = 0$$

Differentiating the above equation

$$C\frac{d^2v}{dt^2} + G\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\left(CD^2 + GD + \frac{1}{L}\right)v = 0$$

Substituting the element values

$$(D^2 + 4D + 3)v = 0$$

The characteristic equation is

$$(D + 1)(D + 3) = 0$$

The solution is $v(t) = C_1 e^{-t} + C_2 e^{-3t}$

At $$t = 0+;\ V(0+) = 10 \text{ V}$$

and $$\frac{1}{L}\int v\, dt = 0$$

and $$C_1 + C_2 = 10$$

By differentiating the above equation

$$\frac{d\,v(t)}{dt} = -C_1\, e^{-t} - 3C_2\, e^{-3t}$$

At $\quad t = 0 \; \frac{d\,v(t)}{dt} = -40$

$\therefore \quad -40 = -C_1 - 3C_2$

Solving the equations, we get

$$C_1 = -5 \text{ and } C_2 = 15$$

Therefore, $\quad v(t) = -5e^{-t} + 15e^{-3t}$

7. In the network shown in the figure, the values of R, L and C are $\frac{1}{4}$ ohm, $\frac{1}{4}$ H and 1 F respectively. If $I = 5$ amps and switch K is opened at $t = 0$. Obtain an expression for voltage of the node a for $t \geq 0$.

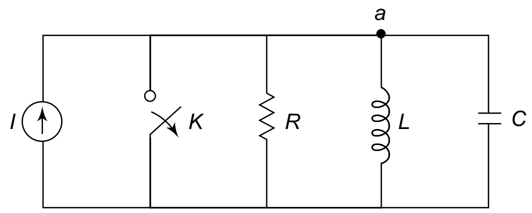

Fig. 11.7

Solution: By applying Kirchhoff's current law

$$C\,\frac{dv}{dt} + Gv + \frac{1}{2}\int v\,dt = I$$

$$1\,\frac{dv}{dt} + 4v + 4\int v\,dt = I$$

Differentiating the above equation, we get

$$\frac{d^2v}{dt^2} + 4\,\frac{dv}{dt} + 4v = 0$$

Characteristic equation is

$$(D^2 + 4D + 4) = 0$$

The roots are repeated at -2, therefore the solution is

$$v(t) = C_1\, e^{-2t} + C_2\, te^{-2t}$$

At $t = 0$, $v(t) = 0$ since the voltage across the capacitor $v(0-) = v(0+) = 0$

Substituting in the above equation

$$C_1 = 0$$

On differentiating the above equation, we get

$$\frac{dv}{dt} = -2C_1\, e^{-2t} - 2C_2\, te^{-2t} + C_2\, e^{-2t}$$

At $\quad t = 0+,\; C\dfrac{dv}{dt}(0+) = I$

$$\frac{dv(0+)}{dt} = \frac{5}{1} = 5 \text{ volts/sec}$$

Substituting in the above equation,

$$5 = -2C_1 - C_2$$

$\therefore \quad C_2 = 5$

Hence the solution becomes

$$v(t) = 5t\, e^{-2t}$$

Objective-Type Questions

1. A circuit consist of a series resistance $R = 10\ \Omega$ and inductance $L = 2$ H and is connected to a voltage $V = 100$ volts at $t = 0$. The current passing through inductor L at $t = 0+$ is
   (a) 10 (b) infinite (c) zero (d) 5
2. The time constant of a series circuit, $R = 5\ \Omega$ and $L = 2$ H, is
   (a) 10 (b) $\frac{2}{5}$ (c) $\frac{5}{2}$ (d) $e^{\frac{-5}{2}}$
3. When a series circuit, $R = 10\ \Omega$ and $C = 2$ F, is connected to a constant voltage of 50 V at $t = 0$, the current passing through the circuit at $t = 0+$ is
   (a) infinite (b) zero (c) 5 (d) $\frac{50}{2\omega}$
4. The time constant of a series circuit consisting of $R = 100\ \Omega$ and C = 1 F is
   (a) $\frac{1}{100}$ (b) 100 (c) 20 (d) e^{-100}
5. A series RL circuit with $R = 30\ \Omega$ and $L = 20$ H has a constant voltage $V = 100$ V applied at $t = 0$. Determine the complementary function of the current passing through it.
   (a) $Ke^{1.5t}$ (b) 0 (c) Ke^{2t} (d) Ke^{30t}