Chapter 9

Coupled Circuits and Magnetic Circuits

1. The self-inductances of two mutually compled coils are L_1 = 20 mH and L_2 = 100 mH. The coefficient of coupling is 0.6. (a) Find the value of mutual inductance. (b) What is the value of maximum mutual inductance?

Solution: (a) $M = k\sqrt{L_1 L_2}$

$$= 0.6\ \sqrt{20 \times 100 \times 10^{-6}} = 0.6\sqrt{0.2} \times 10^{-2} = 2.68 \text{ mH}$$

(b) $M = \sqrt{L_1 L_2} = \sqrt{20 \times 100 \times 10^{-6}} = 4.47$ mH

2. For the circuit shown in Fig. 1, write the equations for v_1, v_2 and v_3.

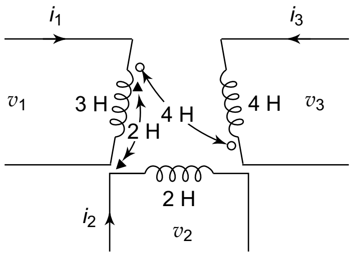

Fig. 9.1

Solution:

$$v_1 = \frac{3di_1}{dt} + \frac{2di_2}{dt} - \frac{4di_3}{dt}$$

$$v_2 = \frac{2di_2}{dt} + \frac{2di_1}{dt}$$

$$v_3 = \frac{4di_3}{dt} + \frac{4di_1}{dt}$$

3. Two identical coils connected in series have an equivalent. Inductance of 0.9 H when connected in aiding, and an equivalent inductance of 0.6 H. When connected in opposite direction. Find the mutual inductance and co-efficient of coupling.

Solution: $L_1 + L_2 + 2\,M = 0.9$
$L_1 + L_2 - 2\,M = 0.6$

Solving the above equations $4M = 0.3$

$$M = 75 \text{ mH}$$

Since the coils are identical $L_1 = L_2 = L$

$$0.9 = 2\,L + 2\,M$$
$$L = 375 \text{ mH}$$
$$L_1 = L_2 = 375 \text{ mH}$$

Coefficients of coupling, $K = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{75}{\sqrt{375 \times 375}} = 0.2$

4. For the circuit shown in Fig. 2 find the values of v_1 and v_2, $L_1 = 4$ H; $L_2 = 5$ H; $K = 0.6$. $i_1 = 3 \sin (30t - 30°)$ A; $i_2 = 2 \sin (30t - 30°)$.

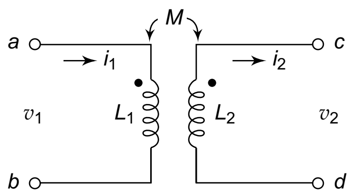

Fig. 9.2

Solution: $v_1 = L_1 \dfrac{di_1}{dt} - \dfrac{Mdi_2}{dt}$; $v_2 = L_2 \dfrac{di_2}{dt} - \dfrac{Mdi_1}{dt}$

$$M = K\sqrt{L_1 L_2} = 0.6\sqrt{4 \times 5} = 2.683$$

$$\frac{di_1}{dt} = 90 \cos (30t - 30°)$$

$$\frac{di_2}{dt} = 60 \cos (30t - 30°)$$

$$v_1 = 4 \times 90 \cos (30t - 30°) - 2.683 \times 60 \cos (30t - 30°)$$
$$v_1 = 199.02 \cos (30t - 30°)$$
$$v_2 = 5 \times 60 \cos (30t - 30°) - 2.683 \times 90 \cos (30t - 30°)$$
$$v_2 = 58.53 \cos (30t - 30°)$$

5. Write the mesh equations for the circuit shown in Fig. 3.

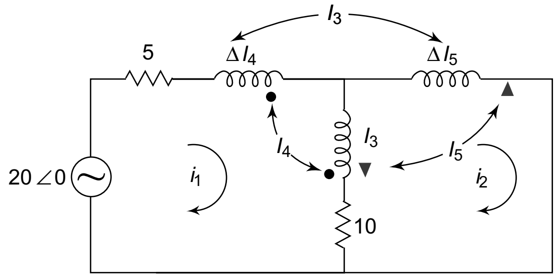

Fig. 9.3

Solution: For the 1st mesh

$$5i_1 + J4i_1 + J_3(i_1 - i_2) + 10(i_1 - i_2) + T_3 i_2$$
$$-J_4(i_2 - i_1) - J_5 i_2 = 20\angle 0.$$
$$(15 + J_{11})i_1 - (10 + j_9) = 20\angle 0 \quad (1)$$

For the 2nd mesh.

$$J_5 i_2 + 10(i_2 - i_1) + J_3(i_2 - i_1) - J_4 i_1 \; J_5 i_2 - (i_2 - i_1) \; J_{51} + J_3 i_1$$
$$(-10 + J)i_1 + (10 - J_2)i_2 = 0 \quad (2)$$

6. Write the mesh equations for the current shown in Fig. 4.

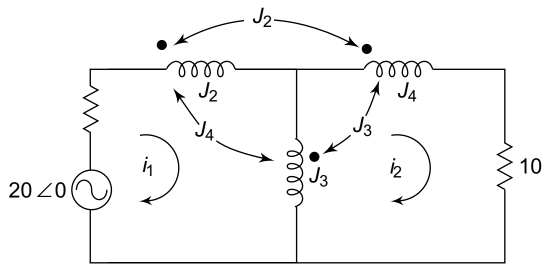

Fig. 9.4

Solution: For the 1st mesh

$$5i_1 + J_2 i_1 + J_3(i_1 - i_2) + J_2 i_2 + (i_1 - i_2)J_4$$
$$+ J_3 i_2 + J_4 i_1 = 25\angle 0.$$

First three terms are self-induced voltages and later four are mutually induced voltages

$$(5 + 13\ \text{J})\ i_1 - 2\text{J}\ i_2 = 25\angle 0 \quad (1)$$

For the 2nd mesh

$$J_4 i_2 + 10i_2 + (i_2 - i_1)J_3 + J_2 i_1 + J_3(i_1 - i_2)$$
$$-J_3 i_2 - J_4 i_1 = 0$$
$$-J_2 i_1 + (10 + J)\ i_2 = 0 \quad (2)$$

7. Calculate the effective inductance of the current shown in Fig. 5.

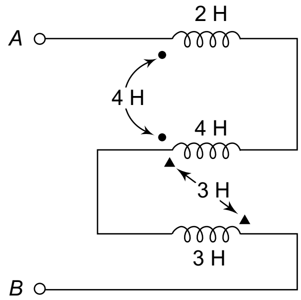

Fig. 9.5

Solution: Let i be the current from A to B and v be the voltage across AB

$$v = \frac{di}{dt}\,[2 + 4 + 3 - 4 - 4 + 3 + 3]$$

The first three terms are self-inducted terms and the later four terms are mutual terms.

$$\therefore \qquad v = 7\,\frac{di}{dt}$$

$$L = 7\text{ H}$$

8. For the circuit shown in Fig. 6 find the ratio of output voltage to the input voltage.

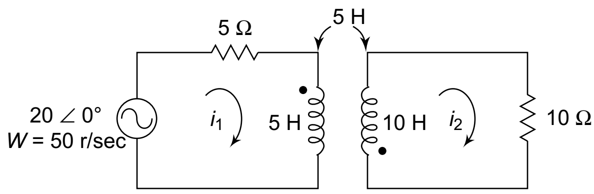

Fig. 9.6

Solution: The primary mesh equation is

$$20\underline{|0} = (5 + J250)\,i_1 + J250\,i_2$$

Secondary mesh equation is

$$O = J250\,i_1 + (10 + J500)\,i_2$$

$$i_2 = \frac{\begin{vmatrix} (5 + J\,250) & 20\underline{|0} \\ J\,250 & 0 \end{vmatrix}}{\begin{vmatrix} (5 + J\,250) & J\,250 \\ J\,250 & (10 + J\,500) \end{vmatrix}} = \frac{500\underline{|90}}{125050\underline{|177.7^\circ}}$$

$$i_2 = 0.04 \underline{|-87.7^\circ}$$

$$V_2 = 10 \times i_2 = 0.4\underline{|-87.7^\circ}$$

$$\frac{V_L}{V_1} = \frac{0.4\underline{|-87.7}}{20\underline{|0}} = 0.02\underline{|-87.7^\circ}$$

9. Calculate the effective inductance for the circuit shown in Fig. 7.

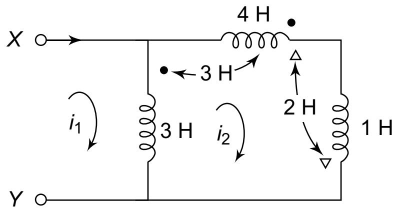

Fig. 9.7

Solution: If i_1 and i_2 are the currents in the two meshes and v is the voltage air is the terminals XY.

The mesh equation are

$$v = 3\ \frac{d}{dt}\ (i_1 - i_2) - 3\ \frac{di_2}{dt} \text{ for the first mesh}$$

$$\Rightarrow \quad \frac{3di_1}{dt} - 6\frac{di_2}{dt} = v \tag{1}$$

$$O = \frac{4di_2}{dt} - 3\frac{d}{dt}\ (i_1 - i_2) + \frac{2di_2}{dt} + \frac{di_2}{dt}$$

$$+ 2\ \frac{di_2}{dt} + 3\frac{d}{dt}\ (i_2 - i_1) + 3\ \frac{di_2}{dt}$$

$$\Rightarrow \quad -6\ \frac{di_1}{dt} + 18\frac{di_2}{dt} \tag{2}$$

Multiplying Eq. (1) with 3 yields

$$1 \times 3 = 9\frac{di_1}{dt} - 18\frac{di_2}{dt} = 3\ v$$

$$-6\frac{di_1}{dt} + 18\frac{di_2}{dt} = 0$$

$$3\ \frac{di_1}{dt} = -3\ v$$

$$v = \frac{di_1}{dt} \quad \therefore L_{eq} = 1 \text{ H}$$

10. Find the source voltage to the voltage across the 50 Ω is 100 V for the network shown in Fig. 8.

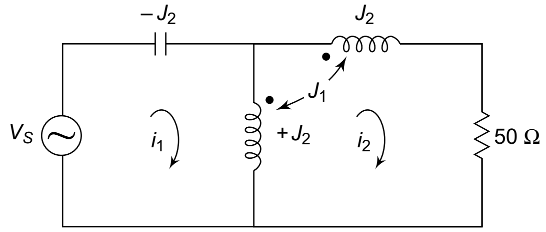

Fig. 9.8

Solution: Mesh equations

$$V_S = (-J_2 + J_2)\,i_1 - J_2 i_2 + J i_2$$
$$i_2 = 2\text{A}$$
$$V_S = -J i_2$$
$$= 2\angle{-90°}$$

11. The inductance matrix for the circuit of three series connected in coupled coils is given below. Draw the circuit and indicate the dots.

$$L = \begin{bmatrix} 8 & -2 & 1 \\ -2 & 4 & -6 \\ 1 & -6 & 6 \end{bmatrix}$$

Solution:

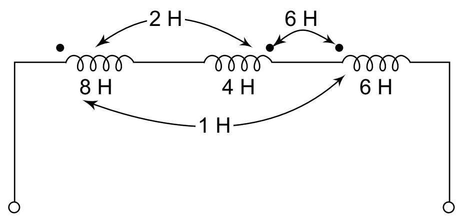

Fig. 9.9

12. Calculate the required mmf to produce a flux of 10 mwB across an air gap of 1.5 mm of length having an effective area of 100 cm^2 of a cost Iron ring of mean Iron path of 0.25 m and cross sectional area of 100 cm^2 as shown in Fig. 10. The relative permeability of the cast iron is 170.

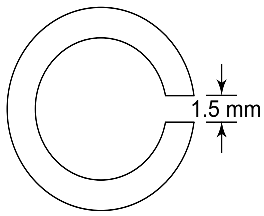

Fig. 9.10

Solution:

Air gap: Area of the gap = 100×10^{-4} m^2

$$\text{Flux density } (B_g) = \frac{10 \times 10^{-3}}{100 \times 10^{-4}} = 1 \text{ Tesla}$$

$$\text{H of the gap} = \frac{B_g}{\mu_0} = \frac{1}{4\pi \times 10^{-7}} \text{ AT/m}$$

$$\text{mmf required for the gap} = Hg \times lg = \frac{1}{4\pi \times 10^{-7}} \times 1.5 \times 10^{-3}$$

= 1194 AT

Cast Iron: Flux density (B_c) = 1 Tesla

$$H_c = \frac{B_c}{\mu_0 \mu_r} = \frac{1}{4\pi \times 10^{-7} \times 170} = 4681 \text{ AT/m}$$

mmf required for the Iron path = Hc lc

= 4681×0.25 = 1171 AT

Total AT required = 1194 + 1171 = 2365 AT

13. If the above Iron ring is wound with 1000 Turns calculate the current required to produce an air gap flux of 10 mwB.

Solution: Total AT = 2365

No of Turns = 1000

$$\therefore \quad I = \frac{2365}{1000} = 2.365 \text{ Amperes.}$$

14. A circular ring consists of semicircular sections of cat iron and cast steel with an air gap of 0.25 mm as shown in Fig. 11. The mean path of the Iron and steel are 0.35 m and 0.25 m respectively. Find the ampere–turns required to produce a flux of 5 mwB. The relative permeabilities of Iron and steel are 170 and 800 respectively. The cross-sectional area of the ring is 5 cm^2.

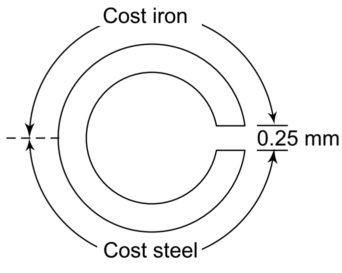

Fig. 9.11

Solution:
$$B = \frac{Q}{A} = \frac{5 \times 10^{-3}}{10 \times 10^{-4}} = 5 \text{ wb/m}^2$$

Air gap: $H = \dfrac{B}{\mu_0} = \dfrac{5}{4\pi \times 10^{-7}} = \text{AT/m}$

$$\text{AT}_g = \frac{5}{4\pi \times 10^{-7}} \times 0.25 \times 10^{-3} \text{ AT}$$

Cast Iron path: $H = \dfrac{B}{\mu_0 \mu_r} = \dfrac{5}{4\pi \times 10^{-7} \times 170}$

$$\text{Ampere-turns for iron path} = \frac{5}{4\pi \times 10^{-7} \times 170} \times 0.35 \text{ AT}$$

Cast Steel path: $H = \dfrac{5}{4\pi \times 10^{-7} \times 800}\ 0.25 \text{ A.T}$

$$\text{Total ampere-turns} = \frac{5}{4\pi \times 10^{-7}}\left[0.25 \times 10^{-3} + \frac{0.35}{170} + \frac{0.25}{800}\right]$$
$$= 1044 \text{ AT}$$

Objective-Type Questions

1. Permeance in analogous to
   (a) conductance (b) resistance (c) reluctance
2. Two coils have inductances of 32 mH and 2 mH and a coefficient of compling of 0.6. The mutual inductance between the coils is
   (a) 4.8 mH (b) 6.8 mH (c) 51.2 mH
3. Two coils have inductances of 2 mH and 4 mH. The coefficient of coupling is 0.5. If the two coils are connected in series opposition, the total inductance will be
   (a) 3 mH (b) 4 mH (c) 3.7715 mH
4. The relative permeability of a ferromagnetic material is usually
   (a) 1 (b) more than 1000 (c) less than 1
5. Tesla is a unit of
   (a) field strength (b) flux density (c) mmf
6. Fringing is the phenomenon associated with the
   (a) attraction of lines of force when passing through air
   (b) repulsion of lines of force when passing through air
   (c) decrease of lines of force when passing through air
7. Leakage factor is defined as
   (a) $\dfrac{\text{useful flux}}{\text{total flux}}$ (b) $\dfrac{\text{total flux}}{\text{useful flux}}$ (c) $\dfrac{\text{leakage flux}}{\text{useful flux}}$