Chapter 8

Frequency Domain Analysis

1. Determine the quality factor, bandwidth and half power frequencies of a series resonant current with $R = 5\ \Omega$, $L = 0.05$ H and $C = 5\ \mu$F

Solution: Resonance frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{0.05 \times 5 \times 10^{-6}}}$$

$$= 318.3 \text{ Hz}$$

Quality factor, $Q = \dfrac{\omega L}{R} = \dfrac{2\pi(318.3)\,(0.05)}{5}$

$$= 20$$

$$\text{B.W.} = \frac{f_r}{Q} = \frac{318.3}{20} = 15.915 \text{ Hz}$$

$$f_r = \sqrt{f_1 f_2}$$

$$f_r^2 = f_1 f_2 \quad \Rightarrow \quad f_1 = \frac{f_r^2}{f_2}$$

Also $f_2 - f_1 = 15.915$ Hz

$$f_2 - \frac{f_r^2}{f_2} = 15.915$$

$$f_2^2 - f_r^2 - 15.915 \times f_2 = 0$$

$$f_2^2 - 15.915\, f_2 - 10.13 \times 10^4 = 0$$

$$f_2 = 326 \text{ Hz}$$
$$f_1 = 310 \text{ Hz}$$

Half-power points can be also calculated using

$$f_1 = f_r - \frac{R}{4\pi L} = 318.3 - \frac{5}{4\pi \times 0.05} = 310 \text{ Hz}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 318.3 + \frac{5}{4\pi \times 0.05} = 326 \text{ Hz}$$

2. A coil with $R = 15\ \Omega$ and $L = 50$ mH is connected in series with a capacitor across a 240 V source resonates at 350 Hz. Find the (a) value of capacitance, (b) power dissipated in the coil, (c) Q factor, (d) voltage across the capacitor and coil.

Solution:

(a) At $f = f_r$, $x_L = x_C$

$$\therefore \qquad C = \frac{1}{4\pi^2 L f_r^2} = \frac{1}{4\pi^2 \cdot 40 \times 10^{-3} \cdot (350)^2}$$
$$= 5.17\ \mu\text{F}$$

(b) At resonance $I = \dfrac{V}{R} = \dfrac{240}{15} = 16$ A

Power dissipated $= I^2R = (16)^2 \times 15 = 3.84$ kW

(c) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{15}\sqrt{\dfrac{50 \times 10^{-3}}{5.17 \times 10^{-6}}} = 6.556$

(d) $V_C = -JQV = 6.556 \times 240 = 15.73 \cdot 47\ \underline{|-90^\circ}$ V

Let the voltage across the inductance of the coil then

$V_L = V_C$ in magnitude $\quad \therefore\ V_L = 1573.47\ \underline{|90^\circ}$

Let V_R is the voltage across the resistance of the coil then

$$V_R = V = 240\ \underline{|0}$$

The voltage across the coil $V_{\text{coil}} = V_L + V_R$

$$= 1573.47\ \underline{|90^\circ} + 240\ \underline{|0}$$
$$= 240 + j1573.47$$
$$= 1591.67\ \underline{|81.32^\circ}$$

3. For the parallel circuit shown in the figure.
   (a) Find resonance frequency.
   (b) Find the currents in all the branches at resonance
   (c) Quality factor

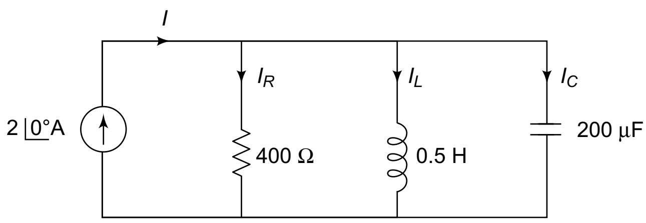

Fig. 8.1

Solution: (a) $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.5 \times 200 \times 10^{-6}}}$ = 15.91 Hz

(b) At resonance the current through resistance is the same as the current from source

$\therefore \qquad I_R = I = 2$ A

The voltage across the parallel branch = $I_R \cdot R$

$$V(t) = 2 \times 400 = 800 \underline{|0}$$

$$\therefore \qquad I_L(t) = \frac{800\underline{|0}}{J\omega L} = \frac{800\underline{|0}}{100 \times 0.5\underline{|90}} = 16\underline{|-90^\circ}$$

$$I_C(t) = \frac{800\underline{|0}}{-J/\omega C}\ \frac{800\underline{|0}}{100 \times 0.5\underline{|90}} = 16\underline{|-90^\circ}$$

(c) The quality factor = $\dfrac{I_L}{I}$ or $\dfrac{I_C}{I}$

$$= \frac{16}{2} = 8$$

4. For the tank circuit shown in the figure calculate the resonance frequency and quality factor.

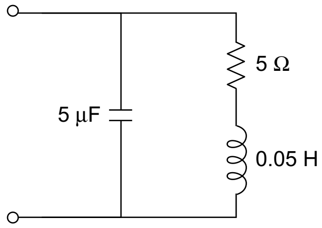

Fig. 8.2

Solution: $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \left(\dfrac{R_L}{L}\right)^2}$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.05 \times 5 \times 10^{-6}} - \left(\frac{5}{0.05}\right)^2}$$

$$= 317.91 \text{ Hz}$$

$$Q = \frac{\omega L}{R} = \frac{1997.5 \times 0.05}{5} = 19.97$$

5. For the parallel circuit shown in the figure. (a) Find the resonance frequency, (b) find the resonance frequency if $R_L = R_C$.

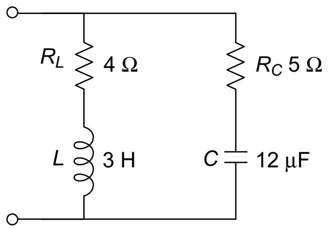

Fig. 8.3

Solution: (a) $f_r = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{\dfrac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$

$$= \frac{1}{2\pi\sqrt{3 \times 12 \times 10^{-6}}}\sqrt{\frac{(4)^2 - (3/12 \times 10^{-6})}{(5)^2 - \left(\dfrac{3}{12 \times 10^{-6}}\right)}}$$

$$= 26\sqrt{\frac{249984}{249975}} = 26 \text{ Hz}$$

(b) If $R_L = R_C$, $fr = 26$ Hz

6. For the circuit shown in the figure, find the value of capacitance which results in resonance when $f_r = 2000/\pi$.

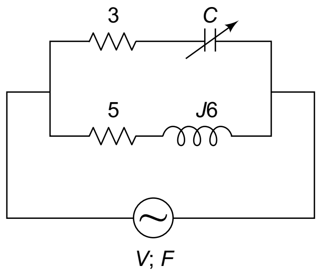

Fig. 8.4

Solution: At resonance, the imaginary part of the admittance is zero. Hence, the complex admittance is a real number.

$$Y = \frac{1}{5+J6} + \frac{1}{3-JX_C}$$

$$= \frac{5-J6}{61} + \frac{3+JX_C}{(3-JX_C)(3+JX_C)}$$

$$= \frac{5-J6}{61} + \frac{3+JX_C}{9+X_C^2}$$

Separating real and imaginary part

$$Y = \left(\frac{5}{61} + \frac{3}{9 \times X_C^2}\right) + J\left[\frac{X_C}{9+X_C^2} - \frac{6}{61}\right]$$

Equating J term to zero

$$\frac{X_C}{9+X_C^2} = \frac{6}{61}$$

$$6X_C^2 - 61X_C + 54 = 0$$

From which $X_C = 9.18$ or $0.979\ \Omega$

$\therefore$ $\frac{1}{\omega C} = 9.18$ or $\frac{1}{\omega C} = 0.979$

From the given data, $\omega = 4000$

$\therefore$ $$C = \frac{1}{4000 \times 9.18} = 0.272\ \mu\text{F}$$

or $$C = \frac{1}{4000 \times 0.979} = 255\ \mu\text{F}.$$

7. For the circuit shown in the figure. If the quality factor is 8. Determine the value of capacitance and coil resistance at a resonant frequency of 350 rad/sec.

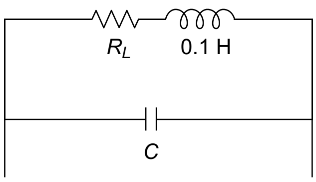

Fig. 8.5

Solution: $$Q = 8 = \frac{\omega_r L}{R} = \frac{350 \times 0.1}{R_L}$$

$$R_L = 4.375\ \Omega$$

$$\omega_r^2 = \frac{1}{LC}$$

$$C = \frac{1}{\omega_r^2 L} = \frac{1}{(350)^2 \times 0.1} = 81.63\ \mu\text{F}.$$

8. For the series parallel circuit shown in the figure, find the value of X which will produce resonance across at the terminals ab.

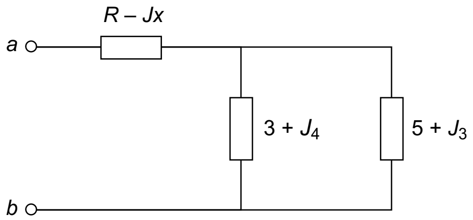

Fig. 8.6

Solution: Impedance of the equivalent parallel branch

$$\frac{(3+J4)\,(5-J3)}{(8+J)} = \frac{(27+J11)}{(8+J)}$$

$$= \frac{29.15\underline{|22.16}}{8.06\underline{|7.125}} = 3.616\ \underline{|15.035}$$

$$= 3.49 + J\,0.938$$

The value of X for the resonance is 0.938 Ω.

9. For the circuit shown in the figure, determine the resonance frequency, bandwidth, lower and upper frequencies of the bandwidth and voltage across the parallel branch, when the value of the current source is $i(t) = 5 \sin 900\,t$.

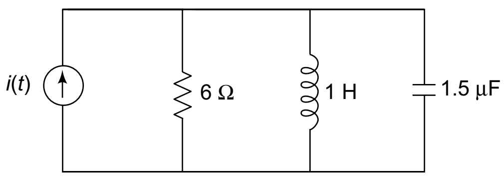

Fig. 8.7

Solution: $$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.5 \times 10^{-6}}} = 816.5 \text{ rad/sec}$$

$$V = \frac{5}{\sqrt{2}} \times 6 = 21.21$$

$$\omega_2 - \omega_1 = \frac{1}{RC} = \frac{1}{6 \times 1.5 \times 10^{-6}}$$

$$f_2 - f_1 = 17.684 \text{ kHZ}$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\frac{1}{2 \times 6 \times 1.5 \times 10^{-6}} + \sqrt{\left(\frac{1}{2 \times 6 \times 1.5 \times 10^{-6}}\right)^2 + \left(\frac{1}{1.5 \times 10^{-6}}\right)}$$

$$\omega_1 = -55555 + 55557$$

$$f_1 = 0.6366 \text{ Hz}$$

$$f_2 = 17684 + 0.6366$$

$$= 17684.6366 \text{ Hz.}$$

10. For the current shown in the figure, the $v(t) = 50 \sin 1500\, t$. Determine the resonant frequency, quality factor, band width, lower and upper limits of the bandwidth.

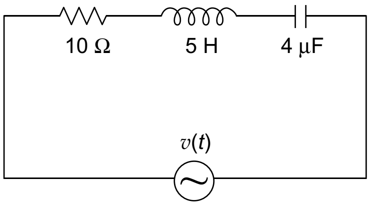

Fig. 8.8

Solution: $v(t) = 50 \sin 1500t$

$$V_{\text{r.m.s}} = \frac{50}{\sqrt{2}} = 35.355$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5 \times 4 \times 10^{-6}}} = 35.588 \text{ Hz}$$

$$Q = \frac{\omega L}{R} = \frac{223.605 \times 5}{10} = 111.8$$

$$f_1 = f_r - \frac{R}{4\pi L} = 35.588 - \frac{10}{4\pi 5} = 35.428$$

$$f_2 = f_r + \frac{R}{4\pi L} = 35.588 + \frac{10}{4\pi 5} = 35.747$$

Band width $= f_2 - f_1 = 0.319$ Hz.

11. For the circuit shown in the figure. The current is at its maximum when the value of $L = 2$ H and 0.707 times its maximum value when $L = 0.5$ H. Find the value of Q and other circuit constants when the supply frequency is constant at $\omega = 300$ rad/sec.

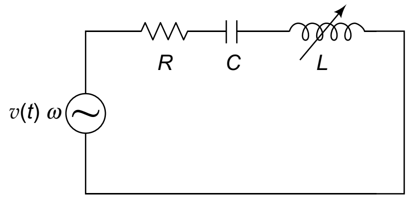

Fig. 8.9

Solution: Current is maximum at resonance, at resonance $\omega_r = \dfrac{1}{\sqrt{LC}}$

$$\therefore \qquad C = \frac{1}{\omega_r^2 L} = \frac{1}{(300)^2 \cdot 2} = 5.555\ \mu\text{F}$$

When the current is 0.707 times the maximum value its net reactance = resistance

$$\therefore \qquad \frac{1}{\omega C} - \omega L = R$$

$$R = \frac{1}{300 \times 5.555 \times 10^{-6}} - 300 \times 0.5 = 450.06\ \Omega.$$

12. For the circuit shown in the figure, draw the locus of the total current vector I.

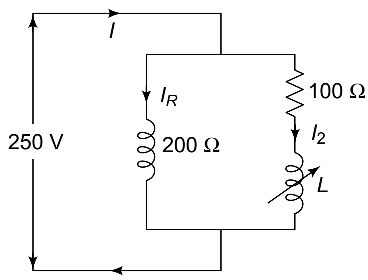

Fig. 8.10

Solution: Total current $I = I_R + I_L$

$I_R = \dfrac{250}{200} = 1.25$ A (fixed value) shown with vector OA.

I_L varies from minimum value when x_L is maximum to maximum value

when x_L is zero the locus diagram is shown in the figure. The maximum value of $I_L = \dfrac{250}{100} = 2.5$ A (shown with vector AB). The maximum current vector $OB = 3.75$ A in phase with V. For any intermediate value of X_L, vector AC represents I_L and the total current vector is OC ϕ represents the PF angle of the circuit.

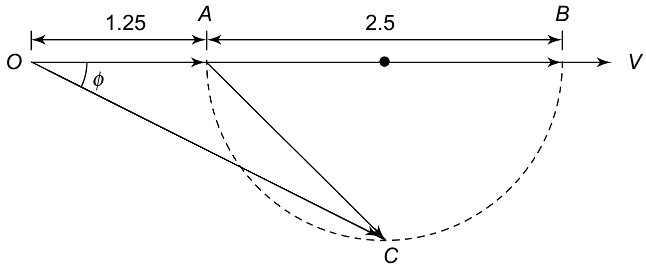

Fig. 8.11

13. For the parallel circuit shown in the figure draw the locus of currents.

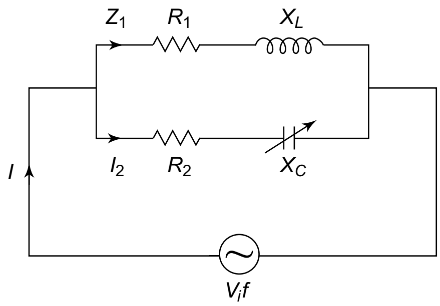

Fig. 8.12

Solution: Vector OA represents I_1; vector AB represents the maximum value of I_2 when $X_C = 0$. The total maximum current is given by $OB = OA + AB$. For any intermediate value of X_C; AC represents I_2 and the total current $I = OC = OA + AC$. The locus diagram is shown in the figure.

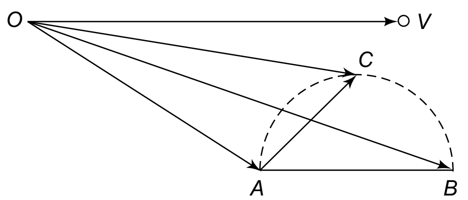

Fig. 8.13

14. Draw the locus of I_1 and I for the parallel circuit shown in the figure.

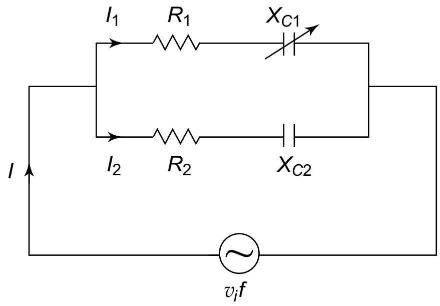

Fig. 8.14

Solution: Vector OA represents I_2, vector AB represents the maximum value of I_1. The total maximum current I is represented by OB.

For any intermediate value of X_{C1} vector AC represents I_2 and the total current is represented by OC. The locus diagram is shown in the figure.

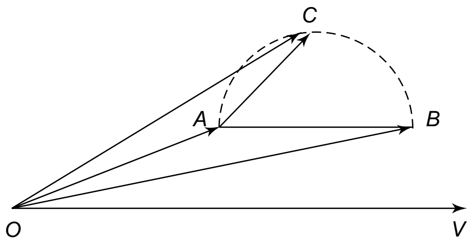

Fig. 8.15

Objective-Type Questions

1. In a parallel circuit, under the resonant condition
   (a) current is maximum and impedance is minimum
   (b) both current and impedance are minimum
   (c) current is minimum and impedance is maximum
2. If the Q factor of a series circuit is high
   (a) The pass band is narrow
   (b) Bandwidth is more
   (c) The passband is wide
3. The resonance frequency of a series RLC circuit is 350 Hz. Its band-width is 175 Hz; then the quality factor is given by
   (a) 350 Hz (b) 175 Hz (c) 2 Hz
4. Voltage magnification occurs in
   (a) series resonant circuit
   (b) parallel resonant circuit
   (c) both series and parallel circuits

5. Current magnification occurs in
   (a) series resonant circuit
   (b) parallel resonant circuit
   (c) both series and parallel circuits
6. The resonance frequency of a tank circuit is given by

   (a) $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ (b) $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R_L^2}{L^2}}$ (c) $f_r = \dfrac{1}{2\pi LC}$
7. For a parallel resonance circuit the net susceptance is
   (a) maximum (b) minimum (c) zero
8. The impedance of a parallel resonance circuit at frequencies above resonance is
   (a) predominantly inductive
   (b) predominantly capacitive
   (c) pure resistive
9. The net reactance at half power points of a series *RLC* circuit at resonance is
   (a) zero (b) resistance (c) impedance
10. What is the voltage across the resistance of a series *RLC* circuit at resonance. If the circuit is exerted by an a.c. voltage of 220 V.
   (a) 110 V (b) zero (c) 220 V