Chapter 7

Steady State AC Analysis

1. Using nodal analysis, determine the node voltage V_2 in the network given below.

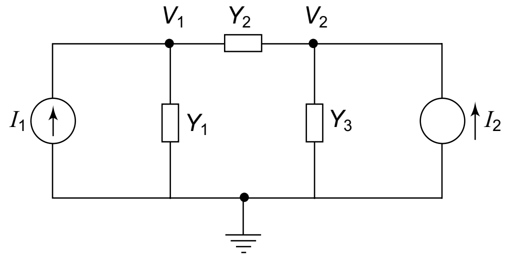

Fig. 7.1

$$I_1 = 10\angle 0^\circ \text{ A};\ I_2 = 10\angle 60^\circ \text{ A};\ Y_1 = (0.5 - j1.0) \text{ S}$$
$$Y_2 = (0.2 - j0.6)S;\ Y_3 = (0.8 - j0.6)\ S$$

Solution: To obtain the voltage V_1, applying KCL at node 1 considering the branch currents

$$I_1 = V_1\, Y_1 + (V_1 - V_2)\, Y_2$$
$$V_1\,(Y_1 + Y_2) - V_2\, Y_2 = I_1 \tag{1}$$

To obtain the voltage V_2, applying KCL at node 2 considering the branch currents

$$I_2 = (V_2 - V_1)\, Y_2 + V_2\, Y_3$$
$$-V_1\, Y_2 + V_2\,(Y_2 + Y_3) = I_2 \tag{2}$$

From the given data, the equations (1) and (2) becomes

$$V_1\,(0.5 - j1.0) + 0.2 - j0.6) - V_2\,(0.2 - j0.6) = 10\angle 0^\circ$$
$$-V_1\,(0.2 - j0.6) + V_2\,(0.2 - j0.6 + 0.8 - j0.6) = 10\angle 60^\circ$$

By simplifying, we get

$$(0.7 - j1.6)\, V_1 - (0.2 - j0.6)\, V_2 = 10\angle 0^\circ$$

$$-(0.2 - j0.6)\,V_1 + (1 - j1.2)\,V_2 = 10\,\underline{|60^\circ}$$

By using Cramers' rule

$$\begin{bmatrix} (0.7 - j1.6) & -(0.2 - j0.6) \\ -(0.2 - j0.6) & (1 - j1.2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\underline{|0^\circ} \\ 10\underline{|60^\circ} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} (0.7 - j1.6) & -(0.2 - j0.6) \\ -(0.2 - j0.6) & (1 - j1.2) \end{vmatrix} = -0.9 - j2.2$$

$$\Delta_2 = \begin{vmatrix} (0.7 - j1.6) & 10\underline{|0^\circ} \\ -(0.2 - j0.6) & 10\underline{|60^\circ} \end{vmatrix} = 19.356 - j7.938$$

$$V_2 = \frac{19.356 - j7.938}{-(0.9 + j2.2)} = 8.8\,\underline{|89.951^\circ}$$

2. For the given network, find current through $(2 + j3)\ \Omega$ impedance using mesh analysis.

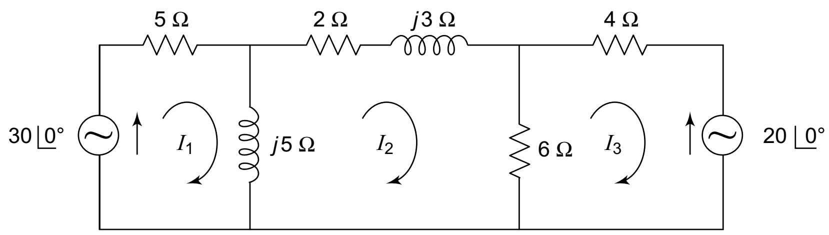

Fig. 7.2

Solution: Applying KVL to loop 1, 2 and 3

$$(5 + j5)I_1 - j5I_2 = 30\,\underline{|0^\circ}$$

$$-j5I_1 + (8 + j8)I_2 - 6I_3 = 0$$

$$-6I_2 + 10I_3 = -20\,\underline{|0^\circ}\ \text{V}$$

$$\begin{bmatrix} 5 + j5 & -j5 & 0 \\ -j5 & 8 + j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\underline{|0^\circ} \\ 0 \\ -20\underline{|0^\circ} \end{bmatrix}$$

To obtain current in $(2 + j3)\ \Omega$ impedance

$$I_2 = \frac{\Delta_2}{\Delta}$$

where
$$\Delta_2 = \begin{bmatrix} 5+j5 & 30\angle 0^\circ & 0 \\ -j5 & 0 & -6 \\ 0 & -22\angle 0^\circ & 10 \end{bmatrix}$$

and
$$\Delta = \begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} = 70 + j60$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{(5+j5)(-120\angle 0^\circ) - 30\angle 0^\circ(-j50)}{70+j60}$$

$\therefore$ $I_2 = 1.74 \angle 40.1^\circ$ A

3. For the given network, determine the voltage of nodes 1 and 2 with respect to the selected reference.

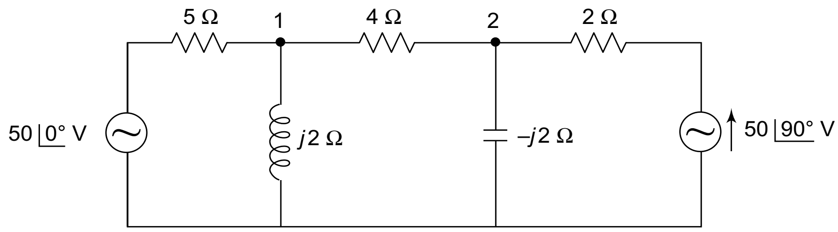

Fig. 7.3

Solution: Applying KCL at nodes 1 and 2 to determine voltage V_1 and V_2.

$$\frac{V_1 - 50\angle 0^\circ}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} = 0$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{-j2} + \frac{V_2 - 50\angle 90^\circ}{2} = 0$$

$$V_1\left[\frac{1}{5} + \frac{1}{j2} + \frac{1}{4}\right] - \frac{V_2}{4} = \frac{50\angle 0^\circ}{5} \qquad (1)$$

$$\frac{-V_1}{4} + V_2\left[\frac{1}{4} - \frac{1}{j2} + \frac{1}{2}\right] = \frac{50\angle 90^\circ}{2} \qquad (2)$$

The above equations are written in matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} \left(\dfrac{1}{5}+\dfrac{1}{j2}+\dfrac{1}{4}\right) & \left(\dfrac{-1}{4}\right) \\ \left(\dfrac{-1}{4}\right) & \left(\dfrac{1}{4}-\dfrac{1}{j2}+\dfrac{1}{2}\right) \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 25\angle 90^\circ \end{bmatrix}$$

$$V_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & -\dfrac{1}{4} \\ 25\angle 90^\circ & \left(\dfrac{1}{4}-\dfrac{1}{j2}+\dfrac{1}{2}\right) \end{vmatrix}}{\begin{vmatrix} \left(\dfrac{1}{5}+\dfrac{1}{j2}+\dfrac{1}{4}\right) & \left(-\dfrac{1}{4}\right) \\ \left(-\dfrac{1}{4}\right) & \left(\dfrac{1}{4}-\dfrac{1}{j2}+\dfrac{1}{2}\right) \end{vmatrix}} = 24.7 \angle 72.25^\circ\ V$$

$$V_2 = \frac{\begin{vmatrix} \left(\dfrac{1}{5}+\dfrac{1}{j2}+\dfrac{1}{4}\right) & 10\angle 0^\circ \\ \left(\dfrac{-1}{4}\right) & 25\angle 90^\circ \end{vmatrix}}{\begin{vmatrix} \left(\dfrac{1}{5}+\dfrac{1}{j2}+\dfrac{1}{4}\right) & \left(\dfrac{-1}{4}\right) \\ \left(\dfrac{-1}{4}\right) & \left(\dfrac{1}{4}-\dfrac{1}{j2}+\dfrac{1}{2}\right) \end{vmatrix}} = 33.6 \angle 53.75^\circ\ V$$

4. Apply the superposition theorem to the network given below and find current I.

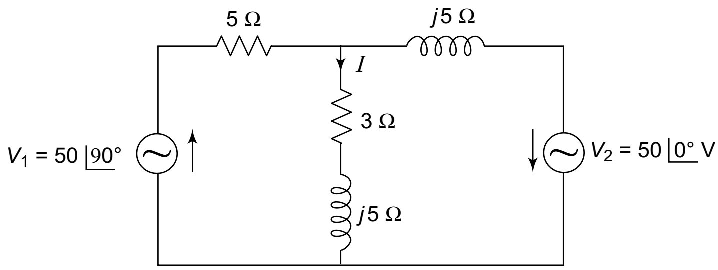

Fig. 4

Solution: According to the superposition theorem, the current due to the $50 \angle 90^\circ$ voltage source is I', with voltage source $50 \angle 0^\circ$ V short circuited.

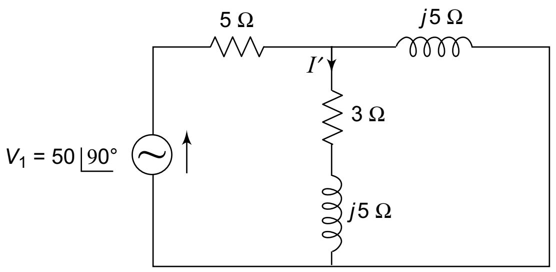

Fig. 5

Total impedance of the circuit is

$$Z_T = \frac{(3+j5)(j5)}{3+j10} = 6.3 \angle 25.4^\circ\ \Omega$$

Total current in the circuit

$$I_T = \frac{V_1}{Z_1} = \frac{50\angle 90^\circ}{6.3\angle 25.4^\circ} = 7.93 \angle 64.6^\circ \text{A}$$

The current in $(3 + j5)\ \Omega$ due to V_1 is

$$I' = \frac{(7.93\angle 64.6^\circ)(j5)}{3+j10} = 3.8 \angle 81.3^\circ\text{A}$$

The current due to $50 \angle 0^\circ$ V voltage source is I'', with voltage source $50 \angle 90^\circ$ V being shorted.

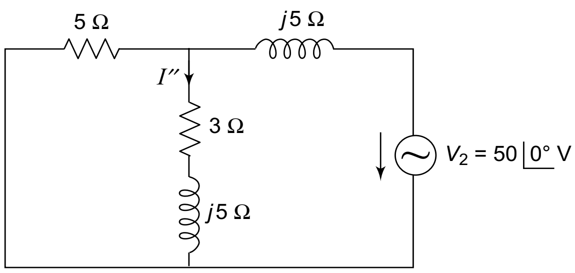

Fig. 6

The total impedance of the circuit at terminals V_2 is

$$Z_T = j5 + \frac{5(3+j5)}{8+j5} = \frac{-25+j40+15+j25}{8+j5}$$

$$Z_T = 6.97 \angle 66.74^\circ\ \Omega$$

The total current delivered by V_2 is

$$I_T = \frac{V_2}{Z_T} \times \frac{50\angle 0^\circ}{6.97\angle 66.74^\circ} = 7.17\angle -66.74^\circ\text{ A}$$

Therefore, the current

$$I'' = -I_T \times \frac{5}{8 + j5} = -7.17\angle -66.74^\circ \times \frac{5}{8 + j5}$$

$$I'' = 3.8\angle -98.74^\circ\text{ A}$$

The current in $(3 + j5)\ \Omega$ impedance due to two sources $I = I' + I''$

$$= 3.8\angle 81.3^\circ + 3.8\angle -98.74^\circ$$

$$I = 0$$

5. In the circuit shown in the given figure. Find the value of R_L which results in maximum power transfer. Calculate the value of the maximum power.

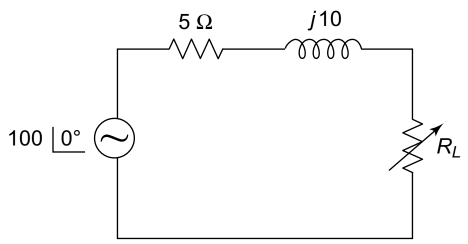

Fig. 7.7

Solution: The value of R_L for which the maximum power transfer

$$R_L = |\,5 + j10\,| = \sqrt{5^2 + 10^2} = 11.18\ \Omega$$

Then the current in the circuit is

$$I = \frac{100\angle 0^\circ}{11.18 + 5 + j10} = \frac{100\angle 0^\circ}{19.02\angle 31.718^\circ}$$

$$I = 5.26\angle -31.718^\circ$$

The maximum power across RL is

$$P_{max} = I^2 R = (5.26)^2\ 11.18 = 309\text{ watts.}$$

6. Obtain the Thevenin equivalent circuit at terminals AB shown in the figure.

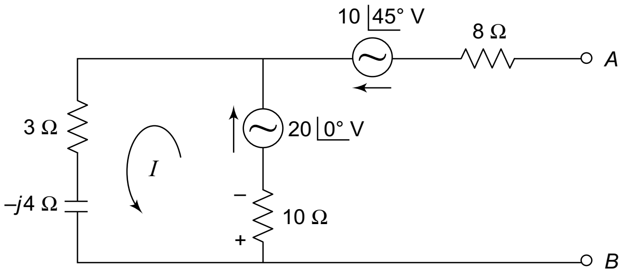

Fig. 7.8

Solution: When terminals AB are open circuited, the current is

$$I = \frac{20\angle 0^\circ}{10+3-j4} = 1.47 \angle 17.1^\circ \text{ A}$$

The voltage drop across the 10 Ω resistor

$$V_{10} = I(10) = 14.7 \angle 17.1^\circ\ A$$

$$V_{Th} = V_{AB} = 20 \angle 0^\circ - 10 \angle 45^\circ - 14.7 \angle 17.1^\circ$$

$$V_{Th} = 11.39 \angle 264.4^\circ \text{ V}$$

The impedance $$Z' = 8 + \frac{10(3-j4)}{10+3-j4} = 10.97 - j2.16\ \Omega$$

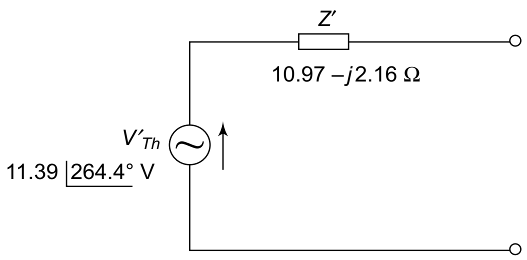

Fig. 7.9

7. Find the Norton equivalent circuit for the circuit shown in the figure.

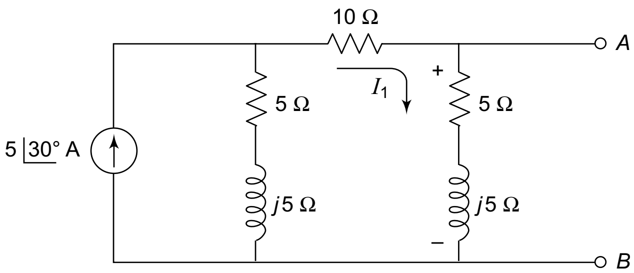

Fig. 7.10

Solution: The equivalent impedance Z_{eq} at terminals AB

$$Z_N = Z_{eq} = \frac{(5 + j5)\,(15 + j5)}{(5 + j5 + 15 + j5)} = 4 + j3\ \Omega$$

$$= 5\ \underline{|36.9^\circ}\ \Omega$$

when AB terminals are short circuited.

The current through short circuit is

$$I_N = 5\underline{|30} \times \frac{5 + j5}{15 + j5} = 2.24\underline{|56.6^\circ}\ \text{A}$$

The Norton equivalent circuit is

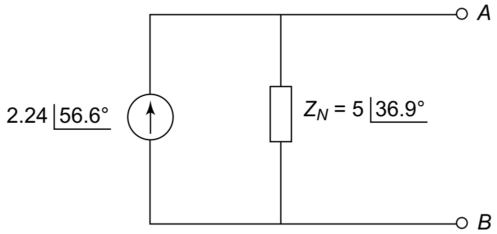

Fig. 11

8. By application of the Thevenin's theorem, find the current I in the circuit shown in the figure. The voltage source shown is sinusoidal having a 10 V rms value and the frequency is such that the inductor has an impedance of magnitude 20 Ω and each capacitor has an impedance of magnitude 10 Ω.

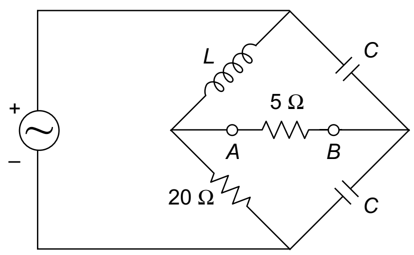

Fig. 12

Solution: The impedance of the circuit at terminals AB when voltage source is short circuited.

$$Z_{AB} = \frac{20 \times 20}{20 + 20} + \frac{10 \times 10}{10 + 10} = 15\ \Omega$$

The Thevenins equivalent voltage

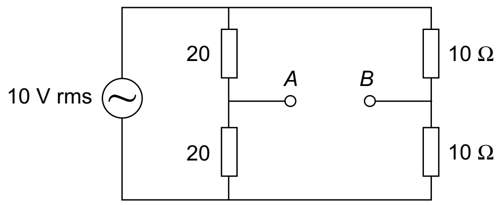

Fig. 7.13

$$V_{AB} = 10 \times \frac{20}{40} = 5 \text{ V}$$

The Thevenin's equivalent circuit is

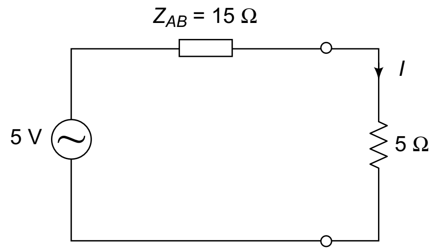

Fig. 7.14

The current I flowing through 5 Ω resistor will be

$$I = \frac{5}{15+5} = \frac{5}{20} = 0.25 \text{ A}$$

9. In the circuit shown in the figure, the resistance R_g is variable between 2 and 55 ohms. What value of R_g results in maximum power transfer across terminals AB?

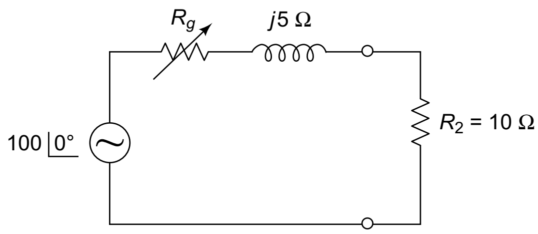

Fig. 7.15

Solution: In the given circuit load R_2 is fixed. Thus the maximum power transfer do not apply. However, the current result when R_g is minimum. Minimum given value of R_g is 2 ohm.

Then $$Z_T = 2 + j5 + 10 = (12 + j5)\ \Omega$$

Therefore $$I_{max} = \frac{V}{Z_T} = \frac{100\angle 0^\circ}{12 + j5} = 7.7\angle -22.6^\circ$$

$$\text{Maximum power} = I^2_{max}\, R$$
$$= (7.7)^2 \times 10 = 593 \text{ watts.}$$

Objective-Type Questions

1. One of the following theorems makes use of equivalent voltage generator in circuit analysis.
   (a) Norton's theorem
   (b) Maximum power transfer theorem
   (c) Thevenin's theorem
   (d) Superposition theorem
2. Conditions of maximum power transfer apply to
   (a) current source alone
   (b) voltage source alone
   (c) both current and voltage sources
   (d) none of the above
3. Superposition theorem applies to
   (a) d.c. circuits only
   (b) a.c. circuits only
   (c) a.c. and d.c. with voltage sources
   (d) a.c. and d.c. with voltage and current sources or mixed-type sources
4. During analysis of Thevenin's theorem, we have to consider
   (a) Power dissipated only in the load
   (b) Power dissipated only in the generator