Chapter 6

Power and Power Factor

1. Determine the branch and total real and reactive powers in the parallel circuit shown. Use *j* notation.

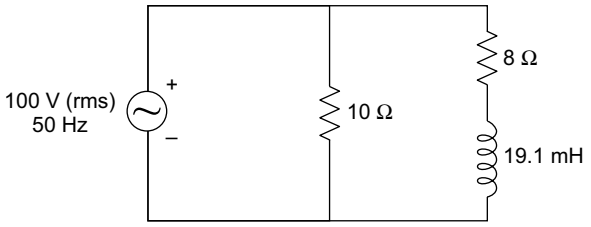

Fig. 6.1

Solution: The circuit in the *j* notation is shown in the figure.

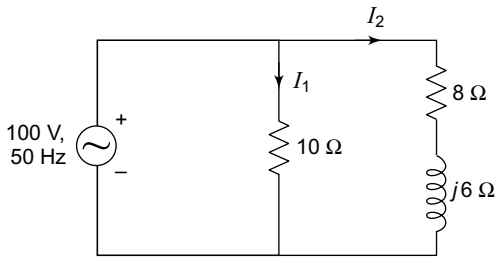

Fig. 6.2

$$X_L = WL$$
$$= 2\pi \times 50 \times 19.1 \times 10^{-3}$$
$$= 6\ \Omega$$

The current I_1 in the circuit $= \dfrac{100}{10} = 10$ A

Power in the 10 Ω resistor $= I_1^2 R = (10)^2 \times 10 = 1000$ W

The current I_2 in the circuit $= \dfrac{100}{8 + i6} = \dfrac{100}{10 \angle 36.86^\circ}$

$$I_2 = 10 \angle -36.86^\circ$$

Power in the $(8 + j6)\Omega$ branch $= I_2^2 R$

$= (10)^2 \times 8 = 800$ W

Total real power in the branch $= 1000 + 800 = 1800$ W

Reactive power in the $10 - \Omega$ branch $= I_1^2 (0) = 0$ VAR

Reactive power in the $(8 + j6) - \Omega$ branch $= I_2^2 \times 6 = 600$ VAR

Total reactive power $= 0 + 600 = 600$ VAR lagging

2. Two impedances $Z_1 = (4 + j5)\Omega$ and $Z_2 = (8 + j10)\ \Omega$ are connected in parallel across a 230 V, 50 Hz supply. Find (i) total admittance, (ii) Current drawn from supply and power factor, and (iii) Value of capacitance to be connected in parallel with the above admittances to raise the power factor unity.

Solution: The circuit is shown in the figure.

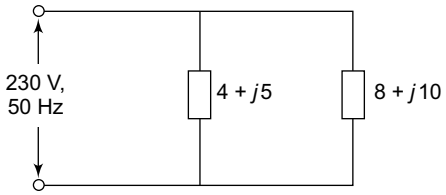

Fig. 6.3

Total admittance

$$Y_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{4 + j5} + \frac{1}{8 + j10}$$

$$Y_{eq} = \frac{1}{6.403 \angle 51.30^\circ} + \frac{1}{12.8 \angle 51.30^\circ}$$

$$Y_{eq} = 0.234 \angle -51.4^\circ \ v$$

Total current drawn from the supply

$$I_T = V \times Y_{eq} = 230 \times 0.234$$

$$I_T = 53.82 \text{ ampers}$$

Power factor $= \cos 51.4° = 0.6238$

$$Y_{eq} = 0.234 \angle -51.4^\circ = 0.145 - j0.1819$$

To raise the power factor unity the value of capacitive reactance to be

$$\frac{1}{X_C} = 0.1819$$

$$X_C = \frac{1}{0.1819} = 5.497$$

$$C = 5.79 \times 10^{-4} \text{ F}$$

3. The current in a circuit lags the voltage by 30°. If the input power be 400 W and the supply voltage be $v = 100 \sin (377t + 10°)$, find the complex power in voltamperes.

Solution: Voltage applied to the circuit

$$v = 100 \sin (377t + 10°) \text{ volts}$$

$$V_{max} = 100$$

$$V_{rms} = \frac{100}{\sqrt{2}}$$

Input power $P = VI \cos \theta$

$= 400$ watts

$$\frac{100}{\sqrt{2}} I \cos 30 = 400$$

$$I = \frac{400 \times 2\sqrt{2}}{100 \times \sqrt{3}} = 6.53 \text{ amp}$$

Reactive Power $P_{rea} = VI \sin 30°$

$$= 70.7\ (6.53)\ \frac{\sqrt{3}}{2}$$

$$P_{rea} = 230.8 \text{ VAR lagging}$$

Complex Power $S = P_{act} + j\, P_{rea}$

$$S = 400 + j230.8 = 461.8 \text{ VA}$$

4. In the circuit shown below, the total effective current is 30 ampers. Determine the three powers

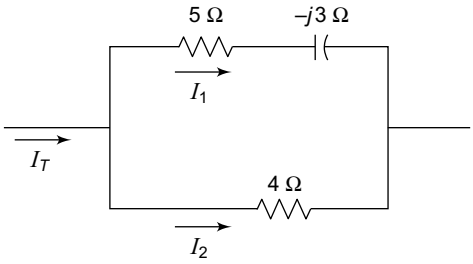

Fig. 6.4

Solution: Assume total current in the circuit

$$I_T = 30\angle 0^\circ$$

Current in the 4 Ω resistor is

$$I_2 = I_T \times \frac{5 - j3}{4 + 5 - j3}$$

$$I_2 = 30\angle 0^\circ \times \frac{5 - j3}{9 - j3} = 18.45\angle -12.55^\circ \text{ amp.}$$

Current in $(5 - j3)$ Ω is

$$I_1 = I_T \times \frac{4}{4 + 5 - j3}$$

$$I_1 = 30\angle 0^\circ \times \frac{4}{4 - j3} = 12.7\angle 18.45^\circ \text{ amp.}$$

(i) Total power (active power)

$$= I_1^2 \times 5 + I_2^2 \times 4$$
$$= (18.45)^2 \times 4 + (12.7)^2 \times 5$$
$$= 2160 \text{ watts}$$

(ii) Total Reactive power $= I_1^2 X$

$$= (12.7)^2 (3) = 483 \text{ VAR leading}$$

(iii) Complex Power

$$S = P_{act} - j P_{rea}$$

$$= 2160 - j483 = 2210\angle 12.6^\circ$$

Complex Power $S = 2210$ VA

5. An impedance carries an effective current of 18 amps. which results 3500 volt ampers at a power factor of 0.76 lagging. Find the impedance.

Solution: Effective current $I_{rms} = 18$ amp.

Power factor $\cos\theta = 0.76$

$$\theta = \cos^{-1}(0.76) = 40.53°$$

Complex Power $S = VI = 3500$

$$= I^2 Z = 3500$$

$$|Z| = \frac{3500}{18 \times 18} = 10.802$$

The impedance is $Z = 10.802 \angle 40.53^\circ$

$$Z = (8.21 + j7)\ \Omega$$

6. Determine the total power triangle for the parallel circuit given in the figure, if the power in the 2 Ω resistor is 20 watts.

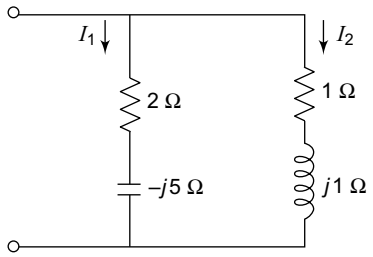

Fig. 6.5

Solution: The power in the 2 Ω resistor is 20 watts

$$I_1^2 (2) = 20$$

$$I_1 = 3.16 \text{ A}$$

Consider voltage $V \angle 0^\circ$ is applied to the circuit

$$I_1 = \frac{V \angle 0^\circ}{2 - j5} = \frac{V \angle 0^\circ}{5.38 \angle -68.2^\circ}$$

$$V \angle 0^\circ = 3.16 \times 5.38 = 17 \angle 0^\circ$$

Then $$I_1 = \frac{17 \angle 0^\circ}{5.38 \angle -68.2^\circ} = 3.16 \angle 68.2^\circ$$

$$I_2 = \frac{V}{Z_2} = \frac{17 \angle 0^\circ}{1 + j1} = \frac{17 \angle 0^\circ}{\sqrt{2} \angle 45^\circ}$$

$$I_2 = 12.02 \angle -45^\circ$$

Total current $I_T = I_1 + I_2$

$$= 3.16 \angle 68.2^\circ + 12.02 \angle -45^\circ$$

$$I_T = 11.16 \angle -29.9^\circ \text{ A}$$

Complex Power $= VI^* = 17 \angle 0^\circ \times 11.16 \angle +29.9^\circ$

$$= 18.9 \angle 29.9^\circ$$

Active Power $= VI \cos\theta = 17 \times 11.16 \times \cos 29.9°$

$= 164$ watts

Reactive Power P_{rac} $= VI \sin\theta$

$= 17 \times 11.16 \times \sin 29.9°$

$P_{rac} = 94$ VAR lagging

7. In the parallel circuit shown in figure, the total power is 1100 watts. Find the power in each resistor and the reading on the ammeter.

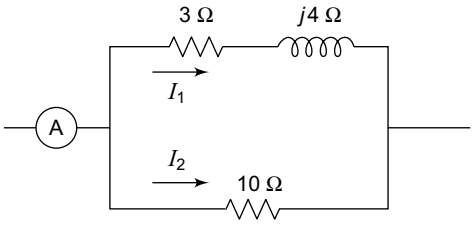

Fig. 6.6

Solution: Consider v is the voltage applied to parallel circuit.

$$I_1 = \frac{V}{Z_1} = \frac{V}{3 - j4} = \frac{V}{5 \angle \tan^{-1}\frac{4}{3}} = \frac{V}{5 \angle 53.1^\circ}$$

$$I_2 = \frac{V}{Z_2} = \frac{V}{10}$$

The ratio of powers in two branches

$$\frac{P_3}{P_{10}} = \frac{I_1^2 \times 3}{I_1^2 \times 10} = \left(\frac{I_1}{I_2}\right)^2 \times \frac{3}{10}$$

$$\frac{P_3}{P_{10}} = \left(\frac{2}{1}\right)^2 \times \frac{3}{10} = \frac{6}{5}$$

Total Power $P_T = P_3 + P_{10}$

$$\frac{P_T}{P_{10}} = \frac{P_T}{P_{10}} + 1$$

$$\frac{P_T}{P_{10}} = \frac{6}{5} + 1$$

Power in 10 Ω resistor $P_{10} = \left(\frac{5}{11}\right)P_T = \frac{5}{11} \times 1100 = 500$ W

Power in $(3 + j4)$ Ω is $P_3 = 1100 - 500 = 600$ W

$$I_1^2 R = 600$$

$$I_1^2 = \frac{600}{3} = 200$$

$$I_1 = 14.14 \text{ A}$$

Let $V = V \angle 0^\circ$

then, the current $I_1 = 14.14 \angle -53.1^\circ$

$$\frac{I_1}{I_2} = 2$$

$\therefore$ the current in 10 Ω resistor

$$I_2 = 7.07 \angle 0^\circ$$

Total current $I_1 = I_1 + I_2$

$$= 14.14 \angle -53.1^\circ + 7.07 \angle 0^\circ$$

Total current $I_T = 19.25 \angle -36^\circ$ A

8. A two element series circuit has a power of 940 watts and a power factor of 0.707 leading. If the applied voltage $v = 99 \sin (6000\, t + 30°)$ volts, determine the circuit constants.

Solution: The effective voltage applied to the circuit is

$$V_{rms} = \frac{99}{\sqrt{2}} \angle 30^\circ = 70 \angle 30^\circ$$

Active power $P = VI \cos \theta = 940$ watts

$$\cos \theta = 0.707$$

Phase angle $\theta = \cos^{-1} 0.707 = 45°$ leading

$$VI \cos \theta = 940$$

$$I = \frac{940}{70 \times 0.707} = 19 \text{ amp}$$

Current leads by 45° to voltage

$$I = 19 \angle 75^\circ$$

The impedance of the circuit $= \frac{V}{I} = \frac{70 \angle 30^\circ}{19 \angle 75^\circ}$

$$Z = 3.68 \angle -45^\circ$$

$$Z = 3.68 \angle -45^\circ = (2.6 - j2.6)\ \Omega$$

$$Z = R - jX_C = 2.6 - j2.6$$

The element are

$$R = 2.6\ \Omega$$

$$X_C = 2.6\ \Omega$$

$$\frac{1}{WC} = 2.6 \text{ and}$$

$$C = \frac{1}{6000 \times (2.6)} = 64.1\ \mu\text{F}$$

Objective-type Questions

1. Two alternating voltage quantities are represented by $e_1 = 60 \sin (\omega t - 30°)$ and $e_2 = 10 \cos \omega t$. Then
   (a) e_1 lags e_2 by 30° (b) e_2 leads by 60°
   (c) e_1 leads e_2 by 60° (d) e_2 leads e_1 by 120°
2. Active power is defined as
   (a) I^2R (b) $VI \cos \theta$
   (c) $VI \sin \theta$ (d) both (a) and (b)

3. Reactive power is defined as
   (a) VI (b) $VI \sin \theta$
   (c) $VI \cos \theta$ (d) I^2R
4. When the circuit has a leading power factor, the circuit is
   (a) capacitive circuit (b) inductive circuit
   (c) resistive circuit (e) none of the above
5. The circuit has an impedance of $Z = 3 + j4\ \Omega$ and applied phasor voltage $V = 100 \angle 30^\circ$ V, determine the true power.
   (a) 1200 W (b) 1600 VAR
   (c) 2000 VA (d) 0.6