Chapter 5

Complex Impedance

1. A 250 V, 50 Hz voltage is applied to a coil of inductance 5 H and resistance of 2 Ω in series with a capacitance C. What value must 'C' have in order that the voltage across the coil shall be 280 V. Draw the phasor diagram.

Solution:

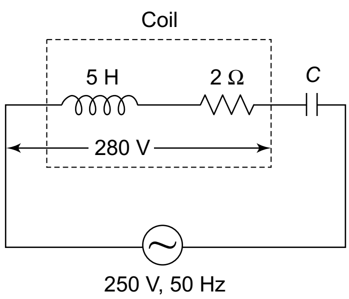

Fig. 5.1

Impedance of the coil $Z_{\text{coil}} = R + j\omega L$

$$Z_{\text{coil}} = 2 + j_{2\pi} \times 50 \times 5 = 2 + j1570$$

$$Z_{\text{coil}} = 2 + j1570 \approx 1570 \angle 90°$$

Voltage across the coil is

$$V_{\text{coil}} = 280 \text{ V}$$

$$I = \frac{280}{1570 \angle 90°} = 0.178 \angle -90°$$

Total impedance $$Z_T = \frac{V}{I} = \frac{250}{0.178 \angle -90°} = j1404.5 \ \Omega$$

$$Z_{\text{capacitance}} = Z_T - Z_{\text{coil}}$$
$$= j1404.5 - j1570$$
$$= -j165.5\ \Omega$$

$$\therefore \qquad X_C = -j165.5$$

$$\frac{1}{\omega C} = -j165.5 \text{ and } C = 192\ \mu\text{F}$$

$$\frac{1}{2\pi \times 50 \times C} = -j165.5$$

Phasor diagram

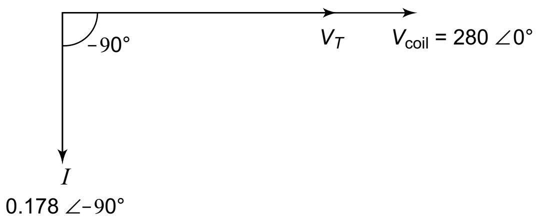

Fig. 5.2

2. Find the values of R and C in the circuit shown so that $V_b = HV_a$ and V_a and V_b are in phase quadrature.

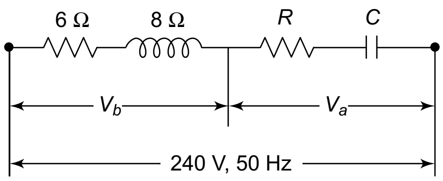

Fig. 5.3

Solution: Consider applied voltage to the circuit is 240 $\angle 0°$ volts

Total voltage applied

$$V_a + V_b = 240\ \angle 0°$$
$$V_a + 4V_a = 240$$
$$V_a = \frac{240}{5} = 48 \text{ volts}$$

Since the current blowing through all the elements is the same, we have

$$\frac{V_b \angle 0^\circ}{6 + j8} = \frac{V_a \angle -90^\circ}{R - j \times C}$$

$$R - jX_C = \frac{(6 + j8)\, V_a \angle -90^\circ}{V \angle 0^\circ}$$

$$= \frac{(6 + j8)\, 48 \angle -90^\circ}{4 \times 48 \angle 0^\circ}$$

$$= 2.5 \angle -36.9^\circ$$

$$R - jX_C = (2 - j1.5)\ \Omega$$

$\therefore\ R = 2\ \Omega$ $\qquad \dfrac{1}{\omega C} = 1.5$

$X_C = 1.5\ \Omega$ $\qquad \dfrac{1}{2\pi \times 50 \times C} = 1.5$

$\therefore$ $\qquad C = 2.12 \times 10^{-3}$ F

3. The applied voltage to a series circuit is $v = 50 \sin (2000t - 25^\circ)$ volts and the resultant current through the circuit is $i = 8 \sin (2000\, t + 5^\circ)$ amps. Find the circuit elements.

Solution: The effective applied voltage

$$V_{\text{rms}} = \frac{50}{\sqrt{2}} \angle -25^\circ \text{ Volts}$$

The effective current in the circuit

$$I_{\text{rms}} = \frac{8}{\sqrt{2}} \angle 5^\circ \text{ ampers.}$$

The phasor diagram is

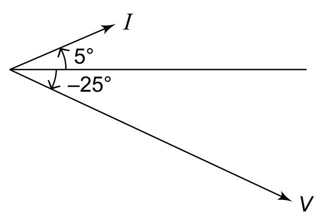

Fig. 5.4

Phase angle between voltage and current is 30° leading.

$$\text{The impedance } Z = \frac{V}{I} = \frac{(50/\sqrt{2})\,\angle{-25°}}{(8/\sqrt{2})\,\angle{5°}} = 6.25\,\angle{-30°}\ \Omega$$

$$Z = 6.25\,\angle{-30°} = 5.412 - j3.125$$

$$= R - jX_C$$

$$\therefore \quad R = 5.412$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times C} = 3.125$$

$$C = \frac{1}{2000 \times 3.125} = 160 \times 10^{-6}\ \text{F}$$

4. For the parallel circuit shown below, find the branch currents and the total current and construct the phasor diagram.

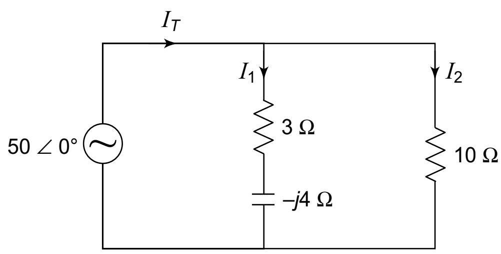

Fig. 5.5

Solution: The current through (3 – j4) Ω is

$$I_1 = \frac{50\,\angle{0°}}{3 - j4} = \frac{50\,\angle{0°}}{5\,\angle{-53.1°}}$$

$$I_1 = 10\,\angle{53.1°}\ \text{amp} = (6 + j8)\ \text{amp}$$

The current through 10 Ω is

$$I_2 = \frac{50\,\angle{0°}}{10} = 5\,\angle{0°}\ \text{amp.} = 5\ \text{amp.}$$

Total current $I_T = I_1 + I_2$

$$= 6 + j8 + 5 = 11 + j8$$

$$I_T = 13.6\,\angle{36°}\ \text{amp.}$$

The phasor diagram

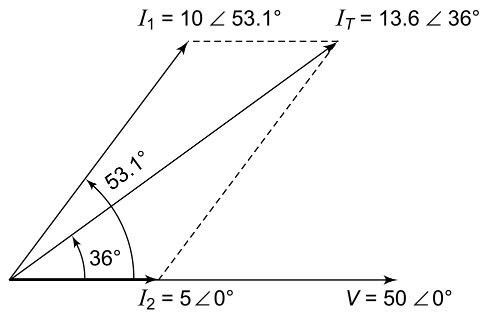

Fig. 5.6

5. A three-element series circuit contains one inductance $L = 0.02$. The applied voltage and resulting currents are shown on the phasor diagram. If $\omega = 500$ rad/sec, what are the two circuit elements?

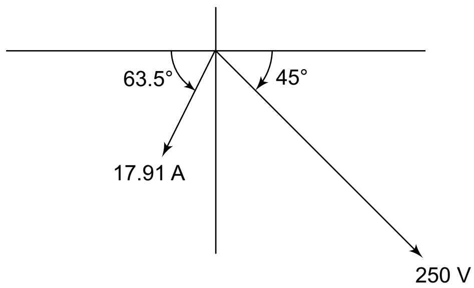

Fig. 5.7

Solution: The voltage applied to the circuit

$$V = 250 \angle -45° \text{ volts}$$

The current through the circuit $I = 7.91 \angle -116.5°$

The total impedance $$Z = \frac{V}{I} = \frac{250\angle -45°}{7.91\angle -116.5°}$$

$$Z = 31.60 \angle 71.5° \ \Omega$$

$$Z = (10 + j30)\ \Omega$$

Since the angle of the impedance is positive. The circuit is an *RL* circuit

$$\therefore \qquad Z = R + j\,(X_{L1} + X_{L2})$$

$$= 10 + j\omega\,(L_1 + L_2)$$

$$= 100 + j500\ (L_1 + L_2)$$

Equating reactive parts, we get

$$500\ (L_1 + L_2) = 30$$

$$L_1 + L_2 = \frac{30}{500} = 0.06 \text{ H}$$

$$L_1 = 0.02 \text{ H}$$

$$L_2 = 0.04 \text{ H}$$

6. The total current entering the parallel circuit shown in the figure is given by $I_T = 18\ \underline{|45°}$ A. Determine the potential difference between points A and B.

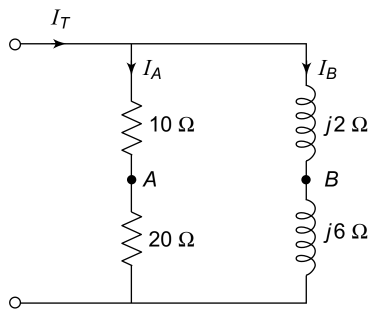

Fig. 5.8

Solution: Total current in the circuit $I_T = 18\ \underline{|45°}$ A

Current in 30 Ω branch $I_A = I_T \dfrac{Z_B}{Z_A + Z_B}$

$$I_A = \frac{18\ \underline{|45°} \times j8}{30 + j8} = 4.66\ \underline{|120°} \text{ A}$$

and current in $j8$ Ω branch $I_B = I_T \dfrac{Z_A}{Z_A + Z_B}$

$$I_B = \frac{18\ \underline{|45°} \times 30}{30 + j8} = 17.5\ \underline{|30°} \text{ A}$$

The voltage across 20 Ω resistance is

$$V_{20} = V_A\ (20) = 4.66 \times 20\ \underline{|120°}$$

$$= 93.2 \angle 120° \text{ V}$$

The voltage across $j6\ \Omega$ is

$$V_{j6} = I_B\,(j6) = 17.5 \angle 30° \times 6 \angle 90°$$

$$= 105 \angle 120° \text{ V}$$

∴ Voltage across terminals AB

$$V_{AB} = V_{20} - V_{j6} = 93.2 \angle 120° - 105 \angle 120°$$

$$V_{AB} = 11.8 \angle -60° \text{ V}$$

7. A series circuit has $R = 8\ \Omega$ and C = 30 μF. At what frequency will the current lead the voltage by 30°?

Solution: The impedance of the circuit

$$Z = R - jX_C = \sqrt{R^2 + X_C^2} \angle \tan^{-1}\left(-\frac{X_C}{R}\right)$$

$$-30° = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$\frac{-X_C}{R} = \tan(-30°) = -0.577$$

$$\frac{X_C}{R} = 0.577$$

$$\frac{X_C}{8} = 0.577$$

$$X_C = 4.618\ \Omega$$

$$\frac{1}{\omega C} = 4.618$$

$$\omega = \frac{1}{4.618 \times 30 \times 10^{-6}} = 7218.13$$

$$2\pi f = 7218.13$$

$$f = \frac{7218.13}{2\pi} = 1149 \text{ Hz}$$

8. An RC series circuit with $R = 10\ \Omega$ an impedance with an angle – 45° at a frequency $f_1 = 500$ Hz. Find the frequency for which the absolute value of the impedance is twice that of f_1.

Solution: Since the impedance angle is negative, the series circuit is RC and phase angle

$$\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$-45° = \tan^{-1}\left(\frac{-X_C}{10}\right)$$

$$\frac{-X_C}{10} = \tan(-45°)$$

$$X_C = 10$$

$$C = \frac{1}{2\pi\ f_C \times 10} = 31.83\ \mu\text{F}$$

The impedance value is

$$Z = R - jX_C = 10 - j10 = 14.14\ \underline{|-45°}$$

If the absolute value of impedance is doubled, then

$$\cos(-\phi) = \frac{R}{Z} = \frac{10}{28.28} = 0.3536$$

$$\phi = -69.29°$$

$$-X_C = R\tan\phi = -26.46$$

$$\frac{1}{2\pi f_2 C} = 26.46$$

$$f_2 = \frac{1}{2 \times \pi \times 26.46 \times 31.83 \times 10^{-6}} = 198\ \text{Hz}$$

9. A voltmeter place across the 3 Ω resistor shown in figure reads 45. What is the ammeter reading.

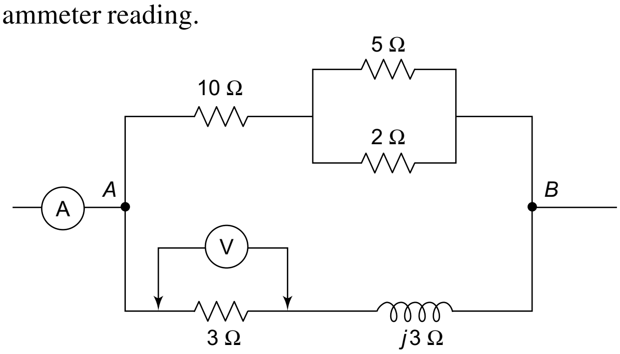

Fig. 5.9

Solution: The current through 3-Ω resistor

$$I = (3) = 45 \text{ V}$$

$$I_1 = 15 \text{ amp.}$$

The voltage across each branch $V_{AB} = 15\ (3 + j3)$

$$= 63.63 \angle 45°$$

Since same voltage is applied to another branch, the current I_2 is

$$I_2 = \frac{V_{AB}}{Z_{eq}} = \frac{63.63 \angle 45°}{[10 + (5 // 2)]}$$

$$= \frac{63.63 \angle 45°}{11.428} = 5.567 \angle 45° \text{ amp}$$

Total current $I_T = I_1 + I_2$

$$= 15 + 5.567 \angle 45° = 19.34 \angle 11.74° \text{ amp}$$

The ammeter reading is 19.34 amp.

10. In the circuit shown in the figure at a frequency $\omega = 400$ rad/s the current leads the voltage by 63.4°. Find R and the voltage across each circuit element. Draw the voltage phasor diagram.

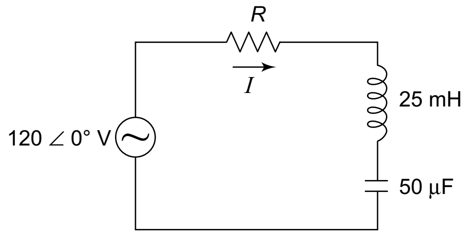

Fig. 5.10

Solution: Inductive reactance $X_L = \omega_L = 400 \times 25 \times 10^{-3} = 10\ \Omega$

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{400 \times 50 \times 10^{-6}} = 50\ \Omega$

Total impedance $Z = R + j(X_L - X_C)$

$$= R + j(10 - 50)$$

$$= (R - j40)\ \Omega$$

Since the current leads the voltage by 63.4°

$$Z\angle\theta = \sqrt{R^2 + (40)^2}\ \angle\tan^{-1}\left(\frac{-40}{R}\right)$$

$$Z\angle -63.4^\circ = \sqrt{R^2 + (40)^2}\ \angle\tan^{-1}\left(\frac{-40}{R}\right)$$

$$\tan^{-1}\left(\frac{-40}{R}\right) = -\,63.4^\circ$$

$$R = \frac{-40}{\tan(-63.4^\circ)} = 20\ \Omega$$

Total impedance is

$$Z = 20 - j40 = 44.7\ \angle -63.4^\circ\ \Omega$$

Current in the circuit is

$$I = \frac{V}{Z} = \frac{120\angle 0^\circ}{44.7\angle -63.40} = 2.68\ \angle 63.4^\circ \text{ amp.}$$

Voltage across resistor $V_R = IR = 20 \times 2.68\ \angle 63.4^\circ$

$$= 53.6\ \angle 63.4^\circ$$

Voltage across inductive reactance

$$V_L = IX_L = 2.68\ \angle 63.4^\circ \times 10\ \angle +90^\circ$$

$$= 26.8\ \angle 153.4^\circ$$

Voltage across capacitive reactance

$$V_C = IX_C = 2.68\ \angle 63.4^\circ \text{ x } 50\ \angle -90^\circ$$

$$= 134\ \angle -26.6^\circ \text{ V}$$

The phasor diagram is

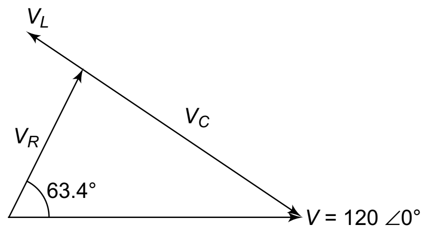

Fig. 5.11

Objective-Type Questions

1. A voltage 5 V, 10 kHz applied to a capacitance of 0.005 μF, then the value of current is
   (a) 1.57 mA (b) 3.18 A
   (c) 1.57 A (d) 5 A
2. Capacitive reactance is ________ proportional to frequency and capacitance.
   (a) directly (b) inversely
   (c) equal to (d) none of the above
3. A voltage 5 V, 10 kHz applied to an inductance of 100 mH, then the current flowing through it is
   (a) 5 amp (b) 10 amp
   (c) 795.8 μA (d) 795.8 μA
4. At what frequency is the reactance of 50 μH inductor equal to 800 Ω?
   (a) 255 kHz (b) 2.55 kHz
   (c) 25.5 MHz (d) 2.55 MHz
5. What is the phase angle between the capacitor current and applied voltage in a parallel RC circuit?
   (a) 90° (b) 0°
   (c) 45° (d) 25°
6. In a certain parallel RC circuit, the resistor current is 10 mA, and the capacitor current is 15 mA. Determine the phase angle and total current.
   (a) 50°, 8 mA (b) 56.3°, 18 mA
   (c) 90°, 18 mA (d) 45°, 8 mA
7. When the frequency of the applied voltage in a series RL circuit is increased, what happens to the phase angle?
   (a) Increases (b) Decreases
   (c) Remains same (d) Zero