Chapter 4

Introduction to Alternating Currents and Voltages

1. Determine the rms value of the sawtooth waveform shown below.

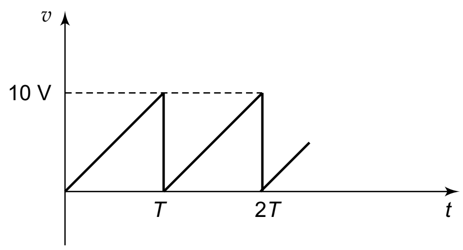

Fig. 4.1

Solution: From the figure shown, the period is T.
The equation for the figure is

$$v(t) = \frac{10t}{T},\ 0 < t < T$$

$$\text{rms value } V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \left(\frac{10t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{100}{T^3}\int_0^T t^2 dt}$$

$$= \sqrt{\frac{100}{T^3}\left(\frac{t^3}{3}\right)_0^T}$$

$$= \sqrt{\frac{100}{T^3} \times \frac{T^3}{3}} = \frac{10}{\sqrt{3}} = 5.77 \text{ V}$$

2. Find the rms values of
   (*i*) $v(t) = 25 \cos \omega t + 15 \sin \omega t$ and
   (*ii*) $i(t) = 100 \sin \omega t - 10 \cos 2 \omega t$

   Solution:

   (*i*) rms value of $v(t) = 25 \cos \omega t + 15 \sin \omega t$ is

$$V_{\text{rms}} = \sqrt{\frac{(25)^2}{2} + \frac{(15)^2}{2}} = 20.62 \text{ V}$$

   (*ii*) rms value of $i(t) = 100 \sin \omega t - 10 \cos 2 \omega t$ is

$$I_{\text{rms}} = \sqrt{\frac{(100)^2}{2} + \frac{(10)^2}{2}}$$

$$= \sqrt{\frac{10000}{2} + \frac{100}{2}} = 71.06 \text{ V}$$

3. Determine the rms value of a half-wave rectified sinusoidal voltage of peak value, $V_m = 10$ V peak.

   Solution: The wave of half-wave rectified sinewave is shown in the figure.

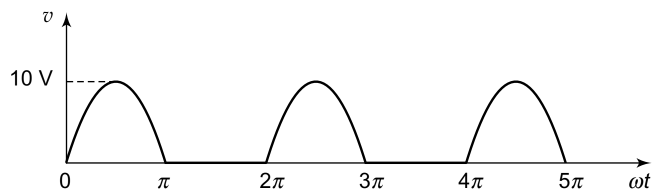

Fig. 4.2

The equation for the waveform is

$$v = 10 \sin \omega t \quad \text{for } 0 < \omega t < \pi$$
$$= 0 \quad \text{for } \pi < \omega t < 2\pi$$

the period is 2π

rms value of the waveform is

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} (10 \sin \omega t)^2 \, d(\omega t)}$$

$$= \sqrt{\frac{1}{4} V_m^2}$$

$$V_{\text{rms}} = \frac{V_m}{2}$$

4. A sawtooth voltage wave increases linearly from 0 to 200 V in the interval from 0 to 2 seconds. At t_1 = 2 sec, its value drops to zero suddenly. The wave repeats this patterns. Find the rms value of the voltage wave.

   Solution: The waveform representation for the above problem is shown in the figure.

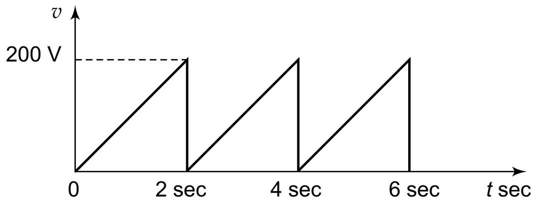

Fig. 4.3

The equation for the waveform is

$$v(t) = 100t \qquad 0 < t < 2 \text{ sec}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\int_0^2 (100t)^2\,dt}$$

$$= \sqrt{\frac{1}{2}10^4\left(\frac{t^3}{3}\right)_0^2}$$

$$V_{\text{rms}} = \sqrt{\frac{4\times 10^4}{3}} = 115.47 \text{ volts.}$$

5. Find the average value of signal shown in the figure.

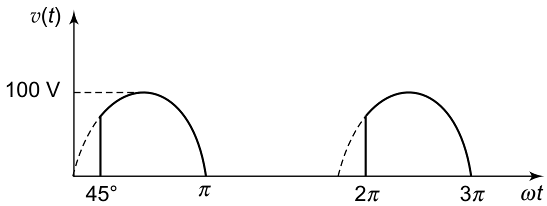

Fig.4.4

Solution: The equation for the signal is

$$v = 100 \sin \omega t \qquad \text{for } \frac{\pi}{4} < \omega t < \pi$$

$$= 0 \qquad \text{otherwise}$$

$$V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100 \sin \omega t \, d(\omega t)$$

$$= \frac{100}{2\pi} (-\cos \omega t)_{\pi/4}^{\pi}$$

$$V_{av} = \frac{-100}{2\pi} = \left(-1 - \frac{1}{\sqrt{2}}\right) = \frac{170.7}{2\pi} = 27.16 \text{ volts.}$$

6. Determine Y_{rms} of the waveform shown in figure.

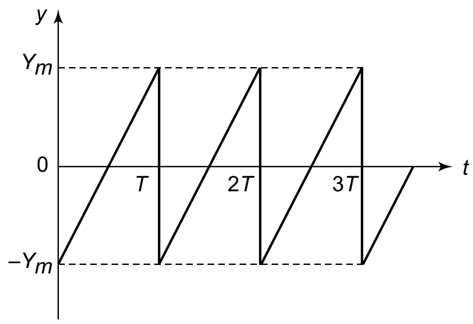

Fig. 4.5

Solution: The equation for the given waveform is

$$y = Y_m\left(\frac{2}{T}t - 1\right) \quad 0 < t < T$$

Effective value or rms value of the waveform $y_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T y^2 \, dt}$

$$= \sqrt{\frac{1}{T}\int_0^T y_m^2\left(\frac{2}{T}t - 1\right)^2 dt}$$

$$= \sqrt{\frac{y_m^2}{T}\int_0^T\left(\frac{4}{T^2}t^2 + 1 - \frac{4t}{T}\right)dt}$$

$$= \sqrt{\frac{y_m^2}{T}\left(\frac{4}{3T^2}T^3 + T - \frac{4}{2T}T^2\right)}$$

$$y_{\text{rms}} = \sqrt{y_m^2\left(\frac{4}{3} + 1 - 2\right)}$$

$$= \sqrt{\frac{y_m^2}{3}}$$

$$y_{\text{rms}} = \frac{y_m}{\sqrt{3}}$$

7. Determine k in the waveform shown below. Where k is some fraction of the period T such that the effective value is 2.

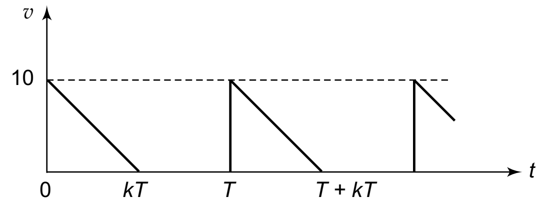

Fig. 4.6

Solution: The equation for the above waveform is

$$v(t) = \frac{-10}{kT}t \qquad \text{for } 0 < t < kT$$
$$= 0 \qquad \text{for } kT < t < T$$

Effective value

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{kT}\left(\frac{-10}{kT}t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{kT}\frac{100}{k^2T^2}(t^2)\,dt}$$

$$= \sqrt{\frac{1}{T}\left[\frac{100}{k^2T^2}\,\frac{T^3k^3}{3}\right]}$$

$$2 = 10\sqrt{\frac{k}{3}}$$

$$k = 0.12$$

8. Find the rms value of the function shown in the figure described as follows.

$$0 < t < 0.1; \quad y = 20\,(1 - e^{-100t})$$

$$0.1 < t < 0.2; \quad y = 20\,e^{-50}(t - 0.1)$$

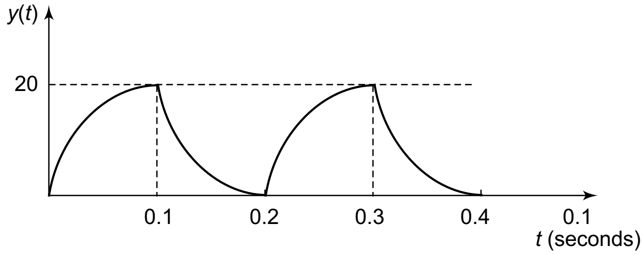

Fig. 4.7

Solution: The rms value of the waveform

$$y_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T y^2(t)\,dt}$$

$$= \sqrt{\frac{1}{0.2}\left[\int_0^{0.1}[20(1-e^{-100t})]^2\,dt + \int_{0.1}^{0.2}[20e^{-50(t-0.1)}]^2\,dt\right]}$$

$$= \sqrt{\frac{1}{0.2}\int_0^{0.1}400(1-2e^{-100t}+e^{-200t})\,dt + \int_{0.1}^{0.2}400e^{-100(t-0.1)}\,dt}$$

$$= \sqrt{2000[t+0.02e^{-100t}-0.005\,e^{-200t}]_0^{0.1}}$$

$$+[-0.01e^{-100(t-0.1)}]_{0.1}^{0.2}$$

$$y_{\text{rms}} = \sqrt{190}$$

$$y_{\text{rms}} = 13.78$$

9. Determine the average and rms values of the waveform shown in the figure, where in the first interval $v = 20\,e^{-200t}$.

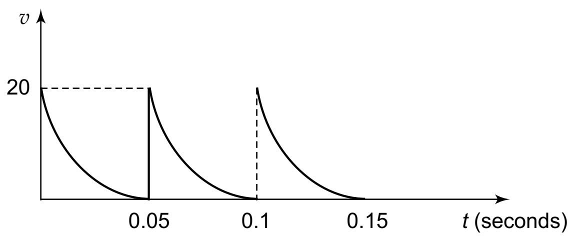

Fig. 4.8

Solution: Average value

$$v_{av} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{0.05}\int_0^{0.05} 20\,e^{-200t}\,dt$$

$$= \frac{20}{0.05\times(-200)}\left[e^{-200t}\right]_0^{0.05}$$

$$= -2[e^{-10} - e^0] = 2$$

$$v_{\text{rms}} = \left[\frac{1}{T}\int_0^T v^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{0.05}\int_0^{0.05} 400e^{-400t}\,dt\right]^{1/2}$$

$$v_{\text{rms}} = 4.47$$

Objective-type Questions

1. The rms value of the dc value $V = 4$ Voltage is
   (a) 2 (b) 4
   (c) $2\sqrt{2}$ (d) $\sqrt{2}$
2. The effective value of $a_1 \cos \omega t + b_1 \sin wt$ is
   (a) $\sqrt{\dfrac{a_1 + b_1}{2}}$ (b) $a_1 + b_1$
   (c) $\sqrt{\dfrac{a_1^2}{2} + \dfrac{b_1^2}{2}}$ (d) $\sqrt{\dfrac{a_1^2}{2}} + \sqrt{\dfrac{b_1^2}{2}}$
3. In waveforms with half-wave symmetry, the average value is
   (a) infinite (b) constant
   (c) maximum (d) zero
4. What is the average power in pure resistance of 10 Ω which carries a current $i(t) = 14.14 \cos \omega t$ amp?
   (a) 1000 W (b) 14.14 W
   (c) 10 W (d) 100 W
5. The average value of the half-wave rectified sinewave having maximum value of 10 V is
   (a) zero (b) 3.18
   (c) 0.318 (d) 0.0318
6. The effective value of the half-wave rectified sinewave having peak value of 20 V is
   (a) 20 (b) 0.318
   (c) 10 (d) 5 V
7. The effective value of the function $v = 50 + 30 \sin \omega t$ is
   (a) 54.3 (b) 50
   (c) 30 (d) 80

8. A 25 ohm resistor has an average power of 400 watts. Determine the maximum value of the current function if it is triangular.
   (a) 5.66
   (b) 6.93
   (c) 25
   (d) 12.5