Chapter 3

Useful Theorems in Circuit Analysis

1. Find the equivalent resistance across the terminals *A* and *B* of the network shown in the figure below using star-delta transformation. All halves are in ohms.

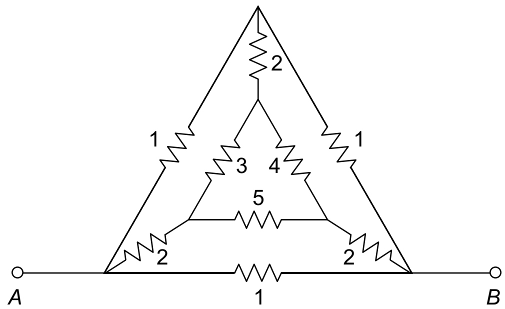

Fig. 3.1

Solution: In the above figure, by converting Δ to *Y* of internal Δ connection, we have

$$\frac{3\times 4}{3+4+5} = \frac{12}{12} = 1$$

$$\frac{4\times 5}{3+4+5} = \frac{20}{12} = 1.66$$

$$\frac{5\times 3}{3+4+5} = \frac{15}{12} = 1.25$$

The circuit can be drawn as

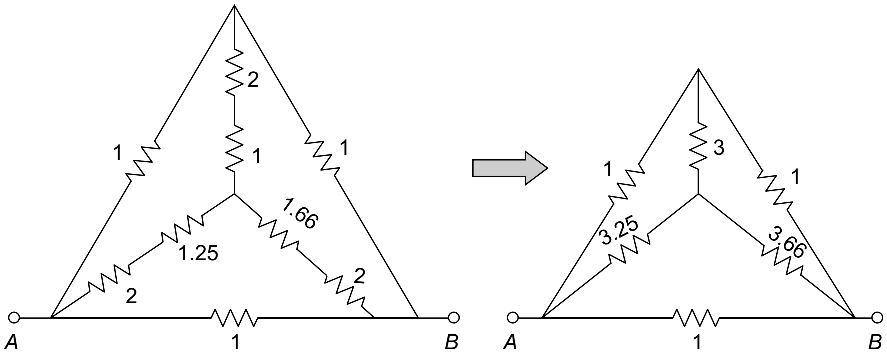

Fig. 3.2

By converting Y to Δ again

$$\frac{3(3.66) + (3.66)(3.25) + (3.25)3}{3.66} = 8.91\ \Omega$$

$$\frac{3(3.66) + (3.66)(3.25) + (3.25)3}{3.25} = 10.03\ \Omega$$

$$\frac{3(3.66) + (3.66)(3.25) + (3.25)3}{3} = 10.875\ \Omega$$

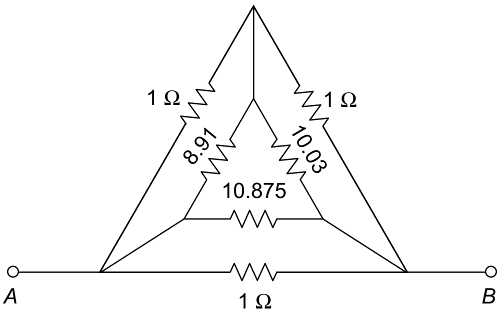

Fig. 3.3

Parallel combination of resistances gives

$$\frac{8.91 \times 1}{1 + 8.91} = 0.899\ \Omega$$

$$\frac{10.03 \times 1}{10.03 + 1} = 0.909\ \Omega$$

$$\frac{10.875 \times 1}{10.875 + 1} = 0.9157\ \Omega$$

The equivalent circuit is shown below.

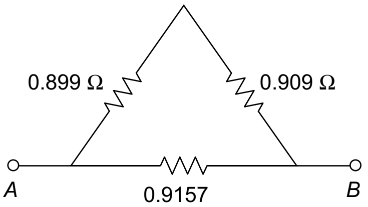

Fig. 3.4

The equivalent resistance is 0.899 Ω and 0.909 Ω. The resistances are in series and equivalent is in parallel with 0.9157 Ω.

$$R_{eq} = (0.899 + 0.909)//(0.9157)$$

$$R_{eq} = 0.6078\ \Omega$$

2. Determine the voltages and currents of the resistances in the following, using source transformation

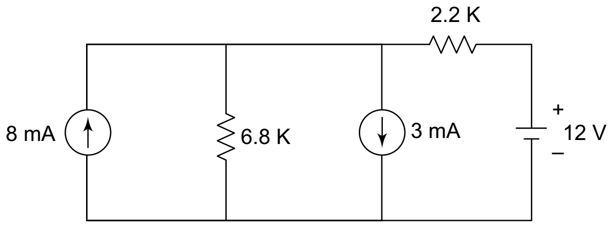

Fig. 3.5

Solution: By converting 12 V voltage source into current source, we have

$$I = \frac{V}{R} = \frac{12}{2.2\text{k}} = 5.45 \text{ mA}$$

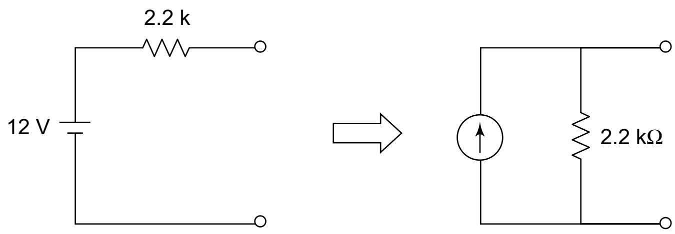

Fig. 3.6

The circuit is shown in the figure below.

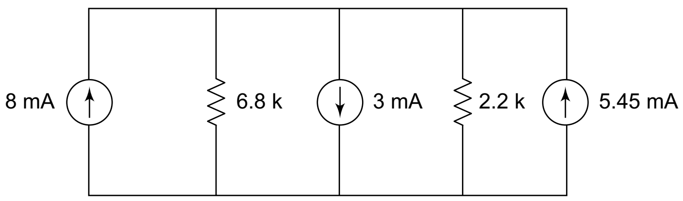

Fig. 3.7

Total current in the circuit

$$I = (8 + 5.45 - 3)\text{ mA} = 10.45\text{ mA}$$

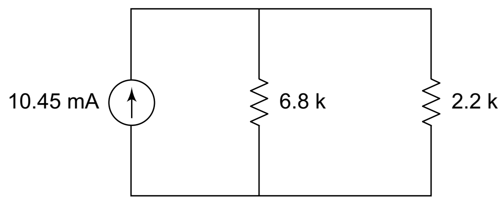

Fig. 3.8

The current in 6.8 k Ω is

$$I_{6.8\text{k}} = 10.45 \times 10^{-3} \times \frac{2.2\text{ k}}{9\text{k}}$$

$$= 2.55\text{ mA}$$

$$I_{2.2\text{k}} = 10.45 \times 10^{-3} \times \frac{6.8\text{ k}}{9\text{k}} = 7.9\text{ mA}$$

$$V_{6.8\text{k}\Omega} = (2.55)\ (6.8) \times 10^{-3} \times 10^{3} = 17.3\text{ volts}$$

$$V_{2.2\text{k}\Omega} = 7.9 \times 10^{-3} \times 2.2 \times 10^{3} = 17.3\text{ volts.}$$

3. Use superposition theorem, determine the current through $R_L = 10\ \Omega$ in the network shown below.

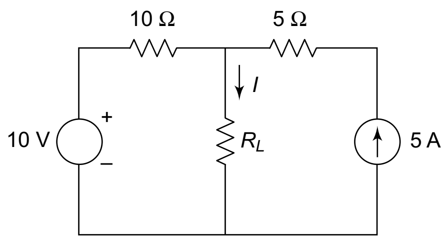

Fig. 3.9

Solution: The current I' due to 10 V source when 5 A source is zero.

$$I' = \frac{10}{10+10} = 0.5\text{ A}$$

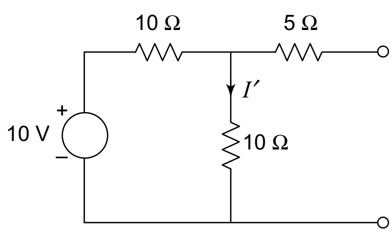

Fig. 3.10

Now, the current I' due to 5A source when 10 V source is zero

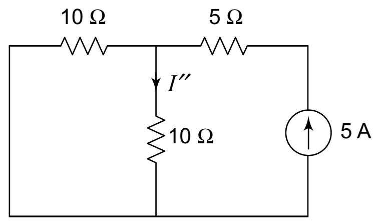

Fig. 3.11

$$I' = 5 \times \frac{10}{10+10} = \frac{50}{20} = 2.5 \text{ A}$$

When both the sources acts simultaneously

$$I = I' + I'' = 0.5 + 2.5 = 3 \text{ A}$$

4. Find the Thevenin's equivalent circuit at terminals of AB of the following network.

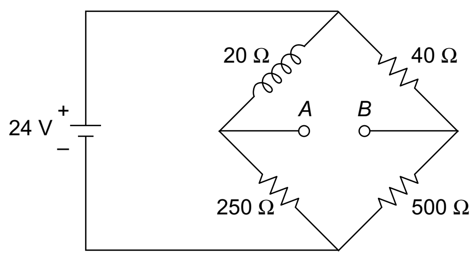

Fig. 3.12

Solution: The circuit can be redrawn as shown below

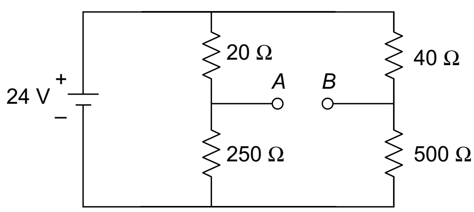

Fig. 3.13

The voltage across 250 Ω is

$$V_A = 24 \times \frac{250}{250+20}$$

$$V_A = 22.22$$

The voltage across 500 Ω is

$$V_B = 24 \times \frac{500}{540} = 22.22$$

$$V_{AB} = V_A - V_B = 22.22 - 22.22 = 0 \text{ V}$$

The Thevenin's resistance

$$R_{Th} = \frac{20 \times 250}{270} + \frac{40 \times 500}{540}$$

$$R_{Th} = 18.5 + 37.03 = 53.5\ \Omega$$

The Thevenin's equivalent circuit is

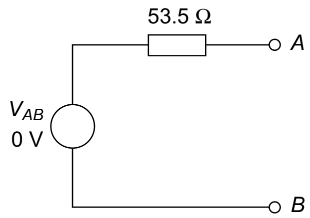

Fig. 3.14

5. Convert the branches in the figure to single voltage source in series with a single resistor at the terminals *AB*.

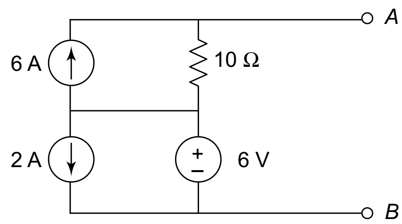

Fig. 3.15

Solution: The superposition theorem is applied here.

(*a*) Voltage across 10 Ω due to 6 V source is zero.

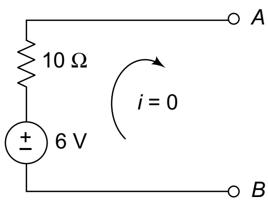

Fig. 3.16

(*b*) Voltage across 10 Ω due to 6 A source is 60 V.

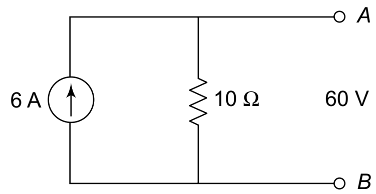

Fig. 3.17

(c) Voltage across 10 Ω due to 2 A source is zero

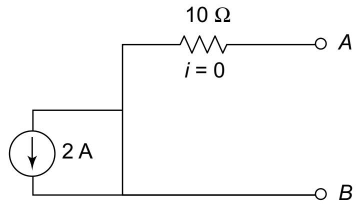

Fig. 3.18

Therefore, the voltage across AB terminals.

= the voltage across 10 Ω + voltage across 6 V source

= 60 + 6 = 66 V

The resistance seen into the terminals R_{AB} = 10 Ω
The circuit is

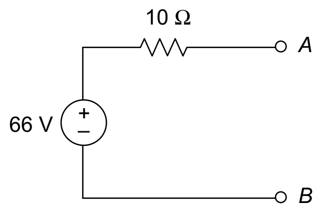

Fig. 3.19

6. Find the current in the 9 Ω resistor of the circuit given by Thevenin's theorem.

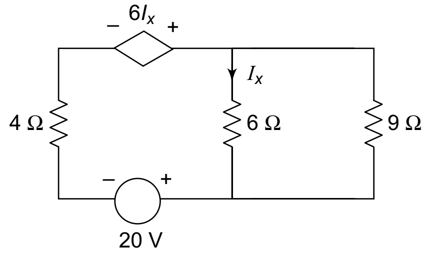

Fig. 3.20

Solution: To find the Thevenin's voltage

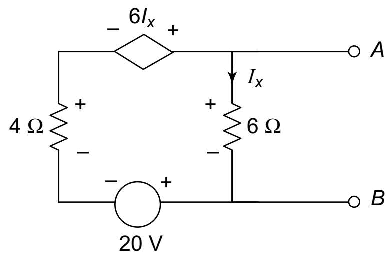

Fig. 21

$$6I_x + 20 + 4I_x = 6I_x$$
$$4I_x = 20$$
$$I_x = 5 \text{ amp}$$
$$V_{OC} = V_{Th} = 5 \times 6 = 30 \text{ V}$$

Since the circuit contains a dependent voltage source

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

I_{SC} can be obtained from the figure below.

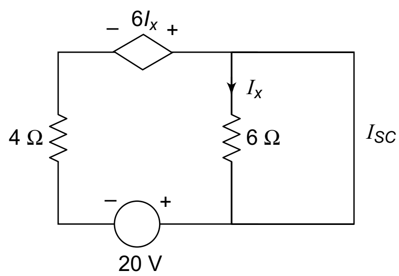

Fig. 3.22

Since no current passes through 6 Ω, then $I_x = 0$ then dependent source is shorted.

$$\therefore \qquad I_{SC} = \frac{20}{4} = 5 \text{ A}$$

$$\therefore \qquad R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{30}{5} = 6\ \Omega$$

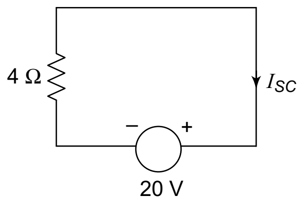

Fig. 3.23

The Thevenin's equivalent circuit is

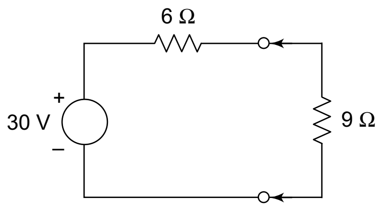

Fig. 3.24

The current in the 9 Ω resistor is

$$I_9 = \frac{30}{6+9} = 2\text{ A}.$$

7. Find the current 4 Ω resistor of the circuit shown below using superposition theorem.

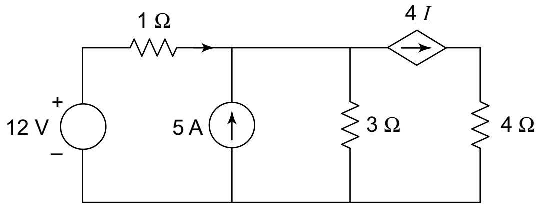

Fig. 3.25

Solution: When 12 V voltage source alone acting the circuit becomes

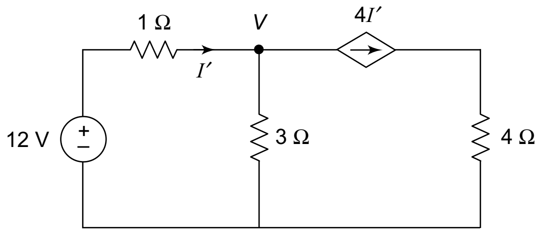

Fig. 3.26

Using nodal analysis

$$\frac{12-V}{1} = \frac{V}{3} + 4\,(12 - \text{V})$$

or $$V = 13.5\text{ V}$$

$$I' = \frac{12-13.5}{1} = -\,1.5\text{ amp.}$$

when current source 5 A is acting alone the circuit is

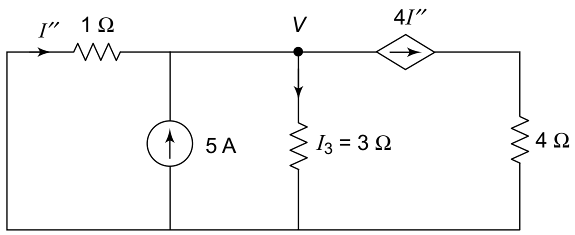

Fig. 3.27

$$I'' + 5 = 4\,I'' + \frac{V}{3}$$

and $$I' = \frac{-V}{1}$$

$$I'' + 5 = 4\,I'' - \frac{I''}{3}$$

$$3\,I'' - \frac{I''}{3} = 5$$

$$\frac{8}{3} I'' = 5$$

$$I'' = \frac{15}{8} \text{ amp.}$$

The current in 4-Ω resistor

$$I = 4\,(I' + I'') = 4(-\,1.5 + 1.877)$$

$$I_{4\,\Omega} = 1.5 \text{ A}$$

8. Find the Thevenin's equivalent circuit for given network across terminals *a*, *b*.

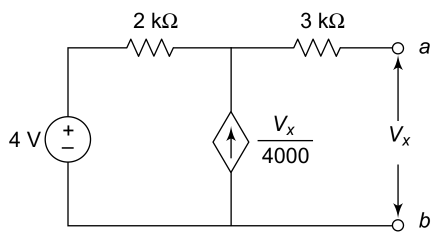

Fig. 3. 28

Solution: Applying KVL in the loop, we have

$$-\,4 - 2 \times 10^3 \times \frac{V_x}{4000} + V_{ab} = 0$$

$$-4 - 2000\ \frac{V_x}{4000} + V_x = 0$$

$$V_x = 8\text{V} = V_{OC}$$

To find the Thevenin's resistance, we should obtain the short circuit current I_{SC}.

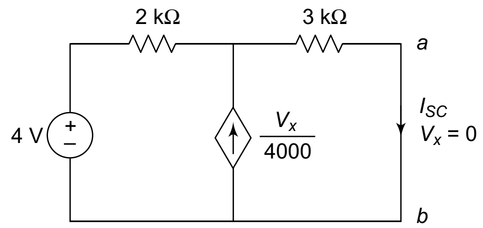

Fig. 3.29

$$I_{SC} = \frac{4}{(2+3)\,k} = 0.8 \text{ mA}$$

$$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{8}{0.8\text{ mA}} = 10 \times 10^3\ \Omega$$

The Thevenin's equivalent circuit is

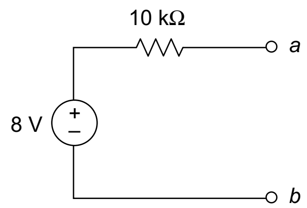

Fig. 3.30

9. Calculate the current through the 10-Ω resistor using Millman's theorem.

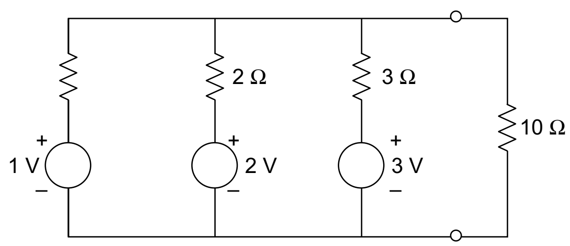

Fig. 3.31

Solution: By Millman's theorem, the equivalent voltage source

$$E = \frac{E_1 Y_1 + E_2 Y_2 + E_3 Y_3}{Y_1 + Y_2 + Y_3}$$

$$= \frac{1 + 2 \times \frac{1}{2} + 3 \times \frac{1}{3}}{1 + \frac{1}{2} + \frac{1}{3}} = \frac{3}{\frac{6+3+2}{6}} = \frac{18}{11} V$$

The equivalent resistance

$$Z = \frac{1}{1 + \frac{1}{2} + \frac{1}{3}} = \frac{6}{11}\,\Omega$$

The equivalent circuit is

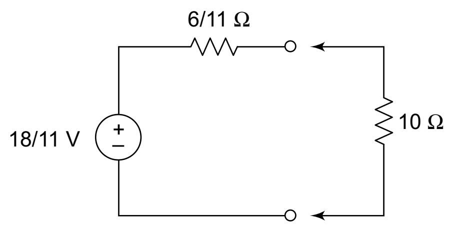

Fig. 3.32

The current through 10 Ω resistor is

$$I = \frac{18/11}{\frac{6}{11} + 10} = \frac{9}{58} \text{ amp.}$$

10. Draw the dual of the network shown below.

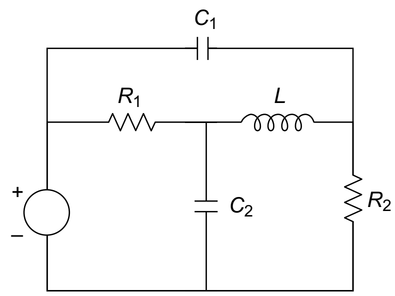

Fig. 3.33

Solution: Place nodes in each loop and on the reference node outside the circuit.
Joining the nodes through each element and placing the dual of each element in the line, we get the dual circuit.

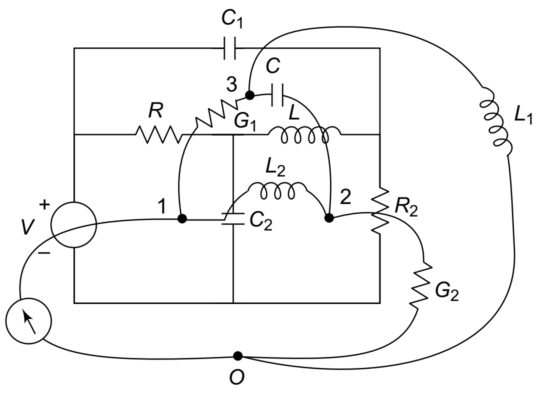

Fig. 3.34

The dual circuit is redrawn as shown below.

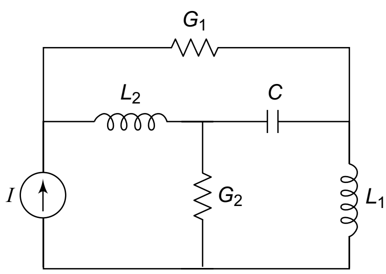

Fig. 3.35

Objective-Type Questions

1. When three equal resistances of 3 Ω each are connected in star, what is the resistance in one of the arms in an equivalent delta circuit?
   (a) 10 Ω (b) 3 Ω
   (c) 9 Ω (d) 27 Ω
2. Reciprocity theorem applies to only one of the following networks.
   (a) Linear networks only
   (b) Bilateral networks only
   (c) Linear/bilateral networks
   (d) All types of networks
3. Compensation theorem is applicable to
   (a) linear networks only
   (b) non-linear network only
   (c) linear and non-linear networks only
   (d) neither of the two
4. The ratios of source and response in a circuit are required to be interchanged for application of

(a) compensation theorem
(b) superposition theorem
(c) reciprocity theorem
(d) Millman's theorem

5. Millman's theorem is applicable during determination of
   (a) load condition for max power transfer
   (b) dual of a network
   (c) load current in a network with more than one voltage source
   (d) load current in a network of generators and impedance with two output terminals.