Chapter 2

Method of Analysing Circuits

1. Find the value of R_1 and R_2 in the network shown in Fig. 2.1 using mesh analysis.

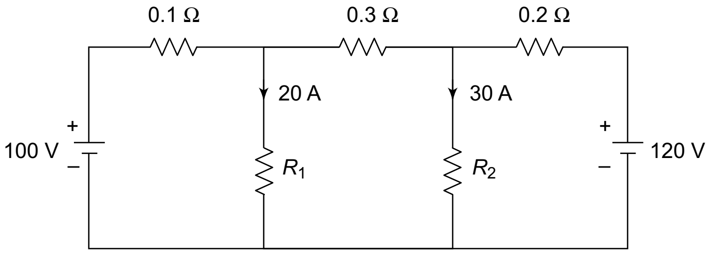

Fig. 2.1

Solution:

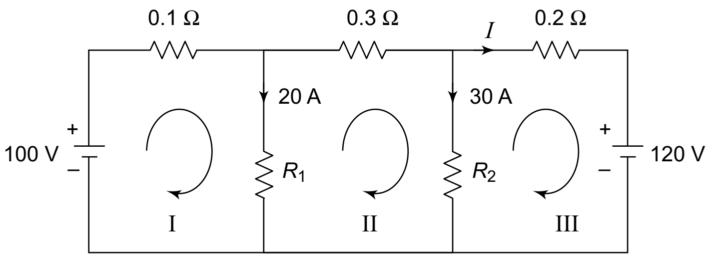

Fig. 2.2

Assume the current flowing in a 0.2-Ω resistor is I
Total current becomes $I_T = 50 + I$
Mesh I

$$0.1(50 + I) + 20R_1 = 100$$
$$5 + 0.1I + 20R_1 = 100$$
$$20R_1 + 0.1I = 95$$

Mesh II

$$-20R_1 + 0.3(30 + I) + 30R_2 = 0$$

$$20R_1 - 30R_2 - 0.3I = -9$$

Mesh III

$$-30R_2 + 0.2I = -120$$

$$30R_2 - 0.2I = 120$$

$$\begin{bmatrix} 20 & 0 & 0.1 \\ 20 & -30 & -0.3 \\ 0 & 30 & -0.2 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ I \end{bmatrix} = \begin{bmatrix} 95 \\ -9 \\ 120 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 20 & 0 & 0.1 \\ 20 & -30 & -0.3 \\ 0 & 30 & -0.2 \end{vmatrix} = 20(6 + 9) + 0.1(600)$$

$$\Delta = 360$$

$$\Delta_1 = \begin{vmatrix} 95 & 0 & 0.1 \\ -9 & -30 & -0.3 \\ 120 & 30 & -0.2 \end{vmatrix} = 1758$$

$$\Delta_2 = \begin{bmatrix} 20 & 95 & 0.1 \\ 20 & -9 & -0.3 \\ 0 & 120 & -0.2 \end{bmatrix} = 1376$$

$$R_1 = \frac{\Delta_1}{\Delta} = \frac{1758}{360} = 4.88\Omega$$

$$R_2 = \frac{\Delta_2}{\Delta} = \frac{1376}{360} = 3.82\Omega$$

2. For the circuit shown below, find the currents in all the branches of the circuit using node voltage method.

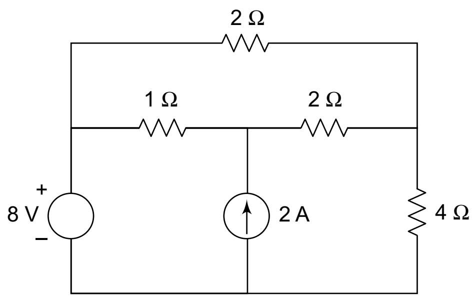

Fig. 2.3

Solution: Consider two nodes, *A* and *B*, in the circuit

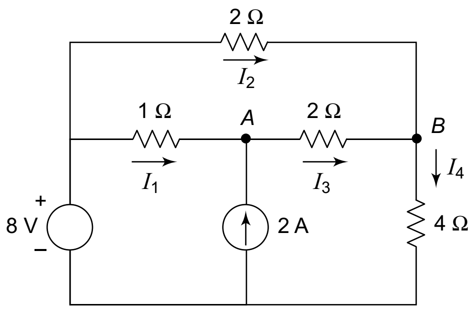

Fig. 2.4

Applying KCL at node *A*, consider all currents are entering the node.

$$\frac{8-V_A}{1} + 2 + \frac{V_B - V_A}{2} = 0$$

$$\frac{-3V_A}{2} + \frac{V_B}{2} = -10$$

$$3V_A - V_B = 20 \qquad (1)$$

Applying KCL at node *B* by considering all currents entering the node *B*

$$\frac{V_A - V_B}{2} + \frac{-V_B}{4} + \frac{8-V_B}{2} = 0$$

$$2V_A - 5V_B = -16 \qquad (2)$$

By solving the above equations, we get

$$V_A = 8.92 \text{ volts, } V_B = 6.77 \text{ volts}$$

current in a $k\Omega$ resistor

$$I_1 = \frac{8-V_B}{1} = 8 - 8.92 = -0.92 \text{ amp}$$

Current in 2-Ω resistor

$$I_2 = \frac{8-V_B}{2} = \frac{8-6.77}{2} = 0.615 \text{ amp}$$

Current in the 2-Ω resistor

$$I_3 = \frac{V_A - V_B}{2} = \frac{8.92 - 6.77}{2}$$

$$I_3 = 1.075 \text{ amp}$$

Current in the 4-Ω resistor

$$I_4 = \frac{6.77}{4} = 1.69 \text{ amp.}$$

3. Determine the value of I in the following circuit shown in Fig. 2.5.

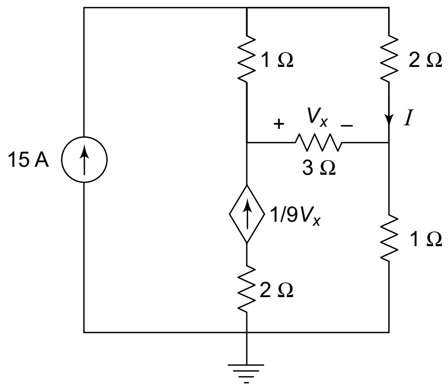

Fig. 2.5

Solution: Consider loop currents as shown in Fig. 6.6.

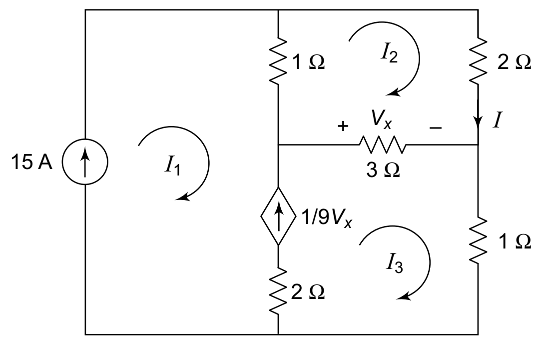

Fig. 2.6

From the figure

$$I_1 = 15 \text{ A}$$

$$I_3 - I_1 = \frac{1}{9} V_x$$

$$(I_3 - I_2)3 = V_x$$

$$\therefore \quad I_3 - I_1 = \frac{1}{9} \times 3(I_3 - I_2)$$

$$I_3 - I_1 - \frac{I_3}{3} + \frac{I_2}{3} = 0$$

$$\frac{2 I_3}{3} - I_1 + \frac{I_2}{3} = 0$$

$$\frac{2I_3}{3} + \frac{I_2}{3} = 15$$

$$I_2 + 2I_3 = 45 \qquad (2)$$

The second loop equation

$$(I_2 - I_3)\,3 + (I_2 - I_1)1 + 2I_2 = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0$$

$$2I_2 - I_3 = 5 \qquad (3)$$

From (2) and (3), we get

$$I_2 = 11\,\text{A}$$

$$I = I_2 = 11\,\text{A}$$

4. Find V_1 for the network given below.

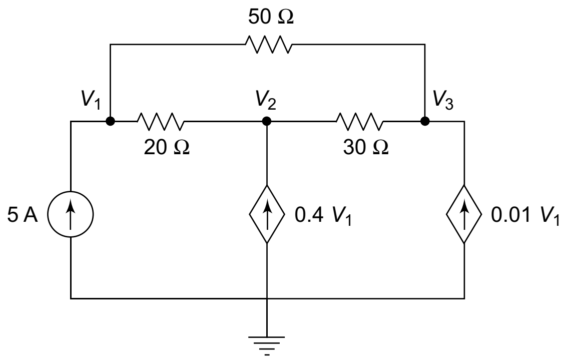

Fig. 2.7

Solution: Applying KCL at node V_1 we get

$$\frac{V_1 - V_2}{20} + \frac{V_1 - V_3}{50} = 5 \qquad (1)$$

Applying KCL at node 3, we have

$$\frac{V_3 - V_2}{30} + \frac{V_3 - V_1}{50} = 0.01\ V_1 \qquad (2)$$

From the circuit, we have

$$V_2 = 0.4\ V_1 \qquad (3)$$

By solving the above equations, we get

$$V_1 = 148.1 \text{ Volts}$$

5. Find V_x for the circuit given below.

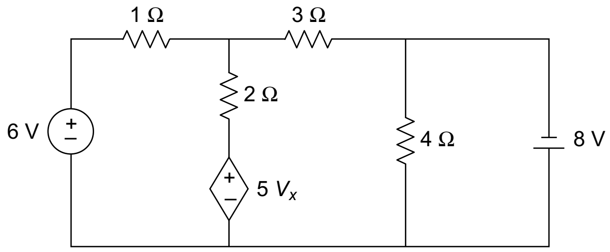

Fig. 2.8

Solution: Considering nodes 1, 2 and 3 as shown in Fig. 2.9 to find out the values of V_1, V_2 and V_3.

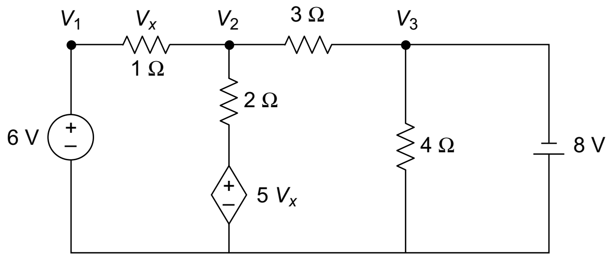

Fig. 2.9

From the circuit,

$$V_1 = 6\text{V}$$

$$V_1 - V_2 = V_x$$

$$\therefore \quad V_x = 6 - V_2 \tag{1}$$

Applying KCL at node 3 is

$$\frac{V_3 - V_2}{3} + \frac{V_3}{4} = 0$$

$$\frac{V_3}{3} + \frac{V_3}{4} = \frac{V_2}{3}$$

$$7V_3 = 4V_2 \tag{2}$$

Applying KCL at node 2

$$\frac{V_2 - 5V_x}{2} + \frac{V_2 - V_3}{3} + \frac{V_2 - V_1}{1} = 0 \tag{3}$$

where $\quad V_x = 6 - V_2$

$$V_2 = \frac{7}{4} V_3$$

$$V_1 = 6 \text{ V}$$

Substituting these values in equation (3), we get

$$V_2 = 5.727 \text{ Volts}$$

$$\therefore \quad V_x = 6 - 5.727 = 0.272 \text{ Volts}$$

6. Use mesh analysis to find the current supplied by 31 V source and the current in 4 Ω resistor of the circuit given below.

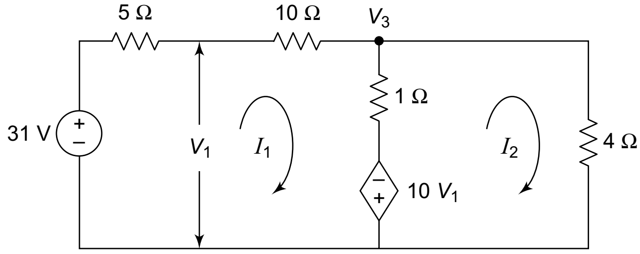

Fig. 2.10

Solution: The two loop equations are

$$31 - 5I_1 - 10I_1 - (I_1 - I_2) + 10V_1 = 0$$

$$16I_1 - I_2 = 31 + 10V_1 \quad (1)$$

$$10V_1 - (I_2 - I_1)\,1 - 4I_2 = 0$$

$$I_1 - 5I_2 = 10V_1 \quad (2)$$

From the circuit, we have

$$I_1 = \frac{31 - V_1}{5}$$

$$\therefore \quad V_1 = 31 - 5I_1 \quad (3)$$

Substituting eq. (3) in eq. (1) and eq. (2) we get

$$66I_1 - I_2 = 341 \quad (4)$$

$$-51I_1 + 5I_2 = -310 \quad (5)$$

From the above equations, we get

$$I_1 = 5\text{A};\ I_2 = -11\text{A}$$

7. The figure shows a network having a voltage controlled voltage source. Determine the potential difference between points *B* and *C*. Use nodal analysis.

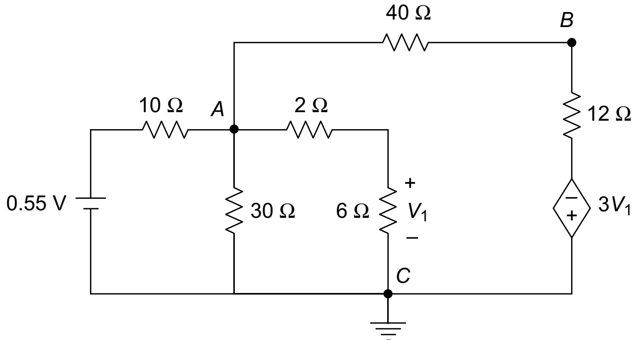

Fig. 2.11

Solution: Consider nodes A, B shown in the figure.
At node A, by applying KCL we get

$$V_A\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{8}+\frac{1}{40}\right)-\frac{0.55}{10}-\frac{V_B}{40}=0 \qquad (1)$$

At node B, by applying KCL we get

$$V_B\left(\frac{1}{12}+\frac{1}{40}\right)-\frac{V_A}{40}+\frac{3V_1}{12}=0 \qquad (2)$$

From the circuit we have

$$V_1 = \frac{V_A\,(b)}{8} \qquad (3)$$

Substituting the value of V_1 in eq. (i) and eq. (2) then

$$34V_A - 3V_B = 6.6 \qquad (4)$$

$$78V_A + 52V_B = 0 \qquad (5)$$

Solving the above equation for V_B, we get

$$V_B = V_{BC} = -\,0.257\,\text{V}$$

8. Determine the current supplied by a battery in the network shown in the figure.

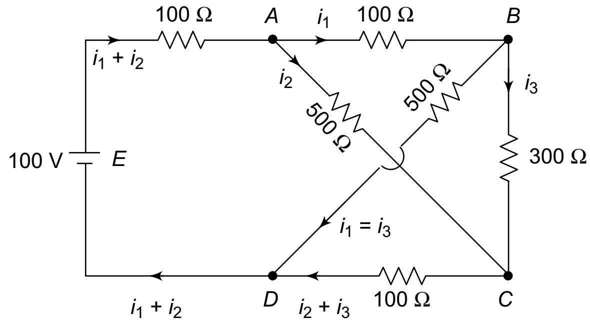

Fig. 2.12

Solution: Let i_1, i_2 and i_3 be the currents flowing as shown in the figure. The current supplied by the battery will be $(i_1 + i_2)$.

Applying KVL to the loop $ABCA$, we get

$$-\,100i_1 - 300i_3 + 500i_2 = 0$$

$$i_1 - 5i_2 + 3i_3 = 0 \qquad (1)$$

Applying KVL to the loop BCDB, we get

$$-\,300i_3 - 100(i_2 + i_3) + 500(i_2 - i_3) = 0$$

$$5i_1 - i_2 - 9i_3 = 0 \qquad (2)$$

Applying KVL to the loop $ABDEA$, we get

$$-100i_1 - 500(i_1 - i_3) - 100(i_1 - i_2) + 100 = 0$$

$$7i_1 + i_2 - 5i_3 = -1$$

$$\Delta = \begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ -1 & 1 & -5 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{vmatrix}$$

$$i_1 = \frac{\Delta_1}{\Delta} = 0.2 \text{ A}, \ i_2 = \frac{\Delta_2}{\Delta} = 0.1 \text{ A}$$

The current supplied by the battery = 0.2 + 0.1

$$i = 0.3 \text{ A}$$

9. Find out the various branch currents in the elements of the network shown in the figure.

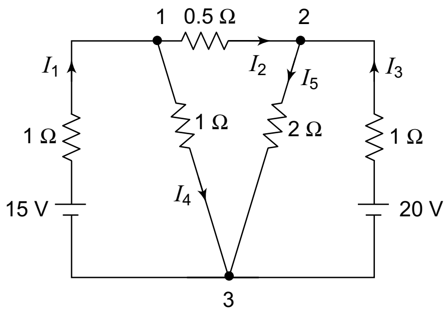

Fig. 2.13

Solution: Applying nodal analysis to the circuit, the nodal equation for node 1

$$I_1 = I_2 + I_4$$

$$\frac{15 - V_1}{1} = \frac{V_1 - V_2}{0.5} + \frac{V_1}{1}$$

$$4V_1 - 2V_2 = 15 \quad (1)$$

At node 2, we have

$$I_5 = I_2 + I_3$$

$$\frac{V_2}{2} = \frac{V_1 - V_2}{0.5} + \frac{20 - V_2}{1}$$

$$4V_1 - 7V_2 = -40 \quad (2)$$

Solving equations (1) and (2), we get

$V_2 = 11$ volts; $V_1 = 9.25$ volts

$$I_1 = \frac{15 - V_1}{1} = \frac{15 - 9.25}{1} = 5.75\,\text{A}$$

$$I_2 = \frac{V_2 - V_1}{0.5} = \frac{11 - 9.25}{0.5} = 3.5\,\text{A}$$

$$I_3 = \frac{20 - V_2}{1} = 20 - 11 = 9\,\text{A}$$

$$I_4 = \frac{V_1}{1} = 9.25\,\text{A}$$

$$I_5 = I_2 + I_3 = 3.5 + 9 = 12.5\,\text{A}$$

Objective-type Questions

1. In the network, there are seven branches and four junctions. The required number of currents would be
   (a) 4 (b) 5
   (c) 7 (d) 6
2. The circuit shown in figure is equivalent to a load of

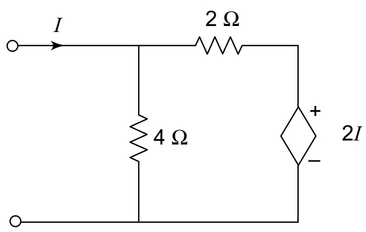

Fig. 2.14

   (a) $\frac{4}{3}$ ohms (b) $\frac{8}{3}\,\Omega$
   (c) 4Ω (d) 2Ω
3. The number of independent loops for a network with 3 nodes and 6 branches is
   (a) 2 (b) 1
   (c) 4 (d) 6
4. A circuit consists of two resistances, 4Ω and 4Ω in parallel. The total current passing through the circuit is 10 A. The current passing through R_1 is
   (a) 5A (b) 10A
   (c) 4A (d) 2A
5. A network has eight nodes and five independent loops. The number of branches in the network is
   (a) 13 (b) 11
   (c) 12 (d) 5