Chapter 1

Circuit Elements and Kirchhoff's Laws

1. How many coulombs do 93.75×10^{16} electrons represent?

Solution: $$Q = \frac{\text{number of electrons}}{\text{number of electrons in one coulomb}}$$

$$= \frac{93.75 \times 10^{6}}{6.25 \times 10^{18}} = 0.15 \text{ C}$$

2. If 50 joules of energy are available for every 10 coulombs of charge, what is the voltage?

Solution: $$V = \frac{W}{Q} = \frac{50\,\text{J}}{10\,\text{C}} = 5 \text{ V}$$

3. In the circuit shown in Fig. 1.1, how much voltage is needed to produce 5 A of current?

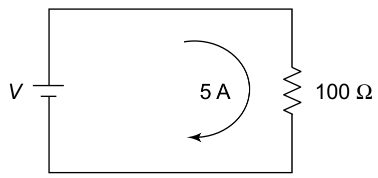

Fig. 1.1

Solution: $V = IR$

$= (5 \text{ A})\,(100\ \Omega) = 500 \text{ V}$

Thus, 500 V are required to produce 5 A of current through a 100 Ω resistor.

4. Calculate the power in each of the three circuits of Fig. 1.2.

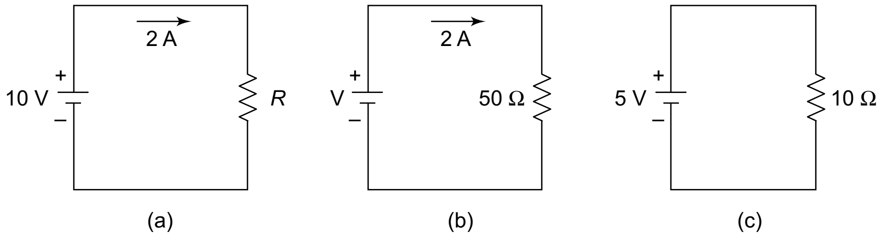

Fig. 1.2

Solution: In circuit (a), V and I are known. The power is determined as follows:

$$P = VI = (10\text{ V})\ (2\text{ A}) = 20\text{ W}$$

In circuit (b), I and R are known. The power is determined as follows:

$$P = I^2 R = (2\text{ A})^2\ (50\ \Omega) = 200\text{ W}$$

In circuit (c), V and R are known. The power is determined as follows:

$$P = \frac{V^2}{R} = \frac{(5\text{ V})^2}{10\Omega} = 2.5\text{ W}$$

5. Find the value of R_H in Fig. 1.3 shown below.

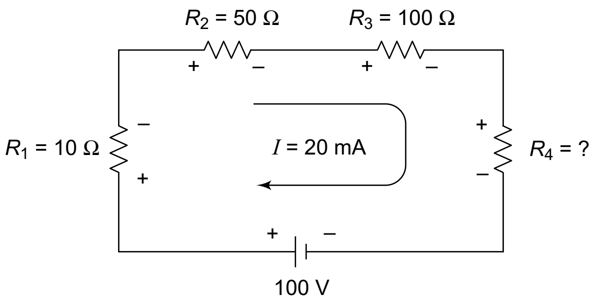

Fig. 1.3

Solution: Let us find the voltage drop across each of the resistors:

$$V_1 = IR_1 = (200)\ (10) = 2\text{ V}$$
$$V_2 = IR_2 = (200)\ (50) = 10\text{ V}$$
$$V_3 = IR_3 = (200)\ (100) = 20\text{ V}$$

By Kirchhoff's voltage law we can find V_4, the voltage drop across the unknown resistor:

$$V_S - V_1 - V_2 - V_3 - V_4 = 0$$
$$100 - 2 - 10 - 20 - V_4 = 0$$
$$V_4 = 68\text{ V}$$

By Ohm's law, we can calculate R_4:

$$R_4 = \frac{V_4}{I} = \frac{68\,\text{V}}{200\,\text{mA}} = 340\ \Omega$$

6. Determine the voltage across R_1 and the voltage across R_2 in the voltage divider shown in Fig. 1.4.

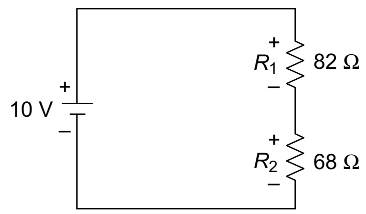

Fig. 1.4

Solution: Total resistance $R_T = R_1 + R_2 = 82 + 68 = 150\ \Omega$

By using voltage divider formula,

$$V_1 = \left(\frac{R_1}{R_T}\right) V_S = \left(\frac{82}{150}\right) 10 = 5.47\ \text{V}$$

Using Kirchhoff's voltage low,

$$V_S = V_1 + V_2$$
$$V_2 = V_S - V_1 = 10 - 5.47 = 4.53\ \text{V}$$

7. Determine the total amount of power in the series circuit shown in Fig. 1.5? Also find the power in each resistor.

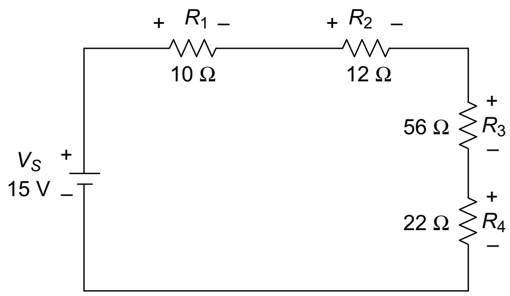

Fig. 1.5

Solution: Total resistance $R_T = 10 + 12 + 56 + 22 = 100\ \Omega$

Total power $P_T = V_S^2/R_T = \dfrac{(15)^2}{100} = 2.25$ W

Current in the circuit $I = \dfrac{V_s}{R_T} = \dfrac{15}{100} = 0.15$ A

Power in each resistor can be calculated using $P = I^2 R$

$$P_1 = (0.15)^2\ (10) = 0.225\ \text{W}$$
$$P_2 = (0.15)^2\ (12) = 0.270\ \text{W}$$
$$P_3 = (0.15)^2\ (56\ \Omega) = 1.260\ \text{W}$$
$$P_4 = (0.15)^2\ (22\ \Omega) = 0.495\ \text{W}$$

The total power

$$P_T = 0.225 + 0.270 + 1.26 + 0.495 = 2.25\ \text{W}$$

8. Use Kirchhoff's current law to find the current measured by ammeters A_1 and A_2 shown in Fig. 1.6.

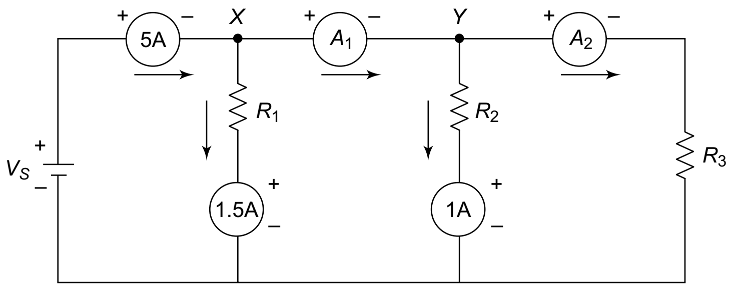

Fig. 1.6

Solution: The total current into junction X is 5 A. Two currents flow out of junction X. 1.5 A through resistor R_1 and the current through A_1. Kirchhoff's current law at junction X is

$$5\ \text{A} = 1.5\ \text{A} + I_{A1}$$
$$\therefore \quad I_{A1} = 3.5\ \text{A}$$

The total current into junction Y is $I_{A1} = 3.5$ A
Two currents flow out of junction: 1 A through resistor R_2 and the current through A_2 and R_3. Kirchhoff's current law at junction Y is

$$3.5\ \text{A} = 1\ \text{A} + I_{A2}$$
$$\therefore \quad I_{A2} = 2.5\ \text{A}$$

9. Determine the current through each resistor in the circuit of Fig. 1.7.

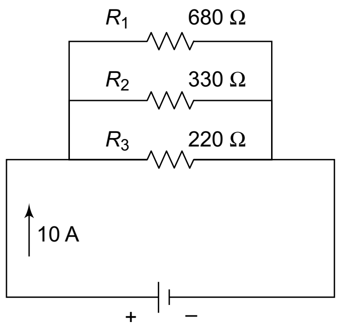

Fig. 1.7

Solution: First calculate the total parallel resistance

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\left(\dfrac{1}{680\,\Omega}\right) + \left(\dfrac{1}{330\,\Omega}\right) + \left(\dfrac{1}{220\,\Omega}\right)} = 110.5\ \Omega$$

Total current is 10 A. Each branch current can be calculated.

$$I_1 = \left(\frac{R_T}{R_1}\right) I_T = \left(\frac{110.5}{680}\right) 10 = 1.63 \text{ A}$$

$$I_2 = \left(\frac{R_T}{R_2}\right) I_T = \left(\frac{110.5}{330}\right) 10 = 3.35 \text{ A}$$

$$I_3 = \left(\frac{R_T}{R_3}\right) I_T = \left(\frac{110.5}{220}\right) 10 = 5.02 \text{ A}$$

10. The amplifier in the stereo system of Fig. 1.8 drives four speakers as shown. If the maximum voltage to the speakers is 15 V, how much power must the amplifier be able to deliver to the speakers?

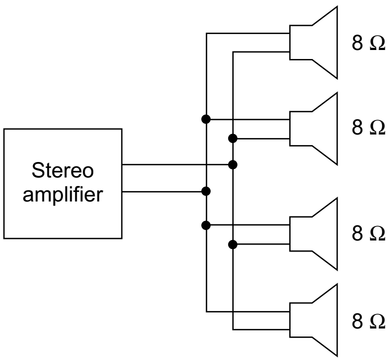

Fig. 1.8

Solution: The speakers are connected in parallel to the amplifier output, so the voltage across each is the same. The maximum power to each speaker is

$$P = \frac{V^2}{R} = \frac{(15)^2}{8} = 28.125 \text{ W}$$

The total power that the amplifier must be capable of delivering to the speaker system is four times the power in the individual speaker because

the total power is the sum of individual powers.

$$P_T = P + P + P + P = 4P = 112.5 \text{ W}$$

11. Find the voltage across a 50-Ω resistor shown in Fig. 1.9

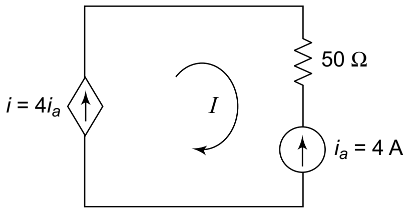

Fig. 1.9

Solution: The current I in the circuit

$$I = 4i_a - i_a = 3i_a = 12 \text{ A}$$

The voltage across the 50-Ω resistance

$$V_{50} = I \times 50 = 12 \times 50 = 600 \text{ V}$$

12. In the circuit shown in the Fig. 1.10, find the current i and voltage across the 10-Ω resistance.

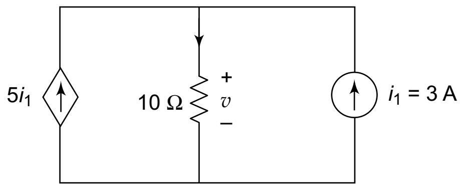

Fig. 1.10

Solution: The current in the 10-Ω resistance

$$i = i_1 + 5i_1 = 6i_1 = 6 \times 3 = 18 \text{ A}$$

The voltage across the 10-Ω resistance

$$v = 10i = 10 \times 18 = 180 \text{ V}$$

13. Find the resistance value R such that the power in the 5-Ω resistance is 50 W. Also find the maximum current and power delivered by the source as R is adjusted.

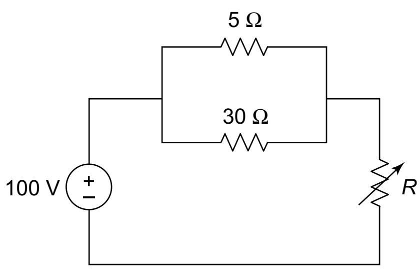

Fig. 1.11

Solution: Power in the 5 Ω resistor $P = I^2 R = 50$ W

$$P = I^2 \times 5 = 50$$

$$I_5 = \sqrt{\frac{50}{5}} = 3.16 \text{ A}$$

Voltage across 5 Ω, $V_5 = 3.16 \times 5 = 15.8$ V

$$V_5 = V_{30} = 15.8 \text{ V}$$

The current is 30 Ω = $\frac{V_{30}}{30} = \frac{15.8}{30} = 0.53$ A

Total current, $I_T = I_R = I_5 + I_{30}$

$$I_R = 3.16 + 0.53 = 3.69 \text{ A}$$

The voltage across resistance R is

$$V_R = V_{100} - V_{30} = 100 - 15.8$$

$$V_R = 84.2 \text{ V}$$

The resistance $R = \frac{V_R}{I_R} = \frac{84.2}{3.69} = 22.8\ \Omega$

Maximum current is delivered when $R = 0$

The maximum current

$$I_{\max} = \frac{100}{R_{eq}} = \frac{100}{(30)/(5)} = \frac{100}{4.29}$$

$$I_{\max} = 23.31 \text{ A}$$

The maximum power delivered by the source

$$= VI_{\max} = 100 \times 23.31 = 233.1 \text{ W}$$

Objective-type Questions

1. What is the basic particle of a negative charge?
   (a) Electron (b) Atom (c) Neutron (d) Proton
2. How much charge, in coulomb, is there in 10×10^{12} electrons?
   (a) 1×10^{-6} C (b) 6×10^{-6} C
   (c) 1.6×10^{-6} C (d) 10^{-6} C
3. What voltage do you need to produce 3 mA of current in a 3-Ω resistance?
   (a) 3 V (b) 9 V (c) 6 V (d) 0 V
4. You have a resistor across which you measure 25 V, and your ammeter indicates 50 mA of current. What is the resistors value in kilohms? In ohms?
   (a) 0.5 kΩ, 500 Ω (b) 500 kΩ, 0.5 Ω
   (c) 50 kΩ, 50 Ω (d) 0.5 kΩ, 0.5 Ω

5. What is the resistance of a 75-W bulb that takes 0.5 A?
   (a) 300 Ω (b) 300 kΩ (c) 0.3 Ω (d) 30 Ω
6. What is the R_T for twelve 47-Ω resistors in series?
   (a) 654 Ω (b) 456 Ω (c) 664 Ω (d) 564 Ω
7. Four equal-value resistors are connected in series with a 5 V source. Five milliamperes of current are measured. What is the value of each resistor?
   (a) 250 Ω (b) 500 Ω (c) 50 Ω (d) 25 Ω
8. If two series resistors of equal value are connected across a 20 V source, how much voltage is there across each resistor?
   (a) 20 V (b) 5 V (c) 10 V (d) 25 V
9. A circuit has a 100 Ω, a 330 Ω and a 680 Ω resistor in series. A current of 1 A flows through the circuit. What is the total power?
   (a) 100 W (b) 1110 W (c) 110 W (d) 111 W
10. A total current of 2.5 A flows into the junction of three parallel branches. What is the sum of all three branch currents?
   (a) 2.5 A (b) 7.5 A (c) 5 A (d) 10 A
11. A parallel circuit has the following resistors in parallel: 220 Ω, 100 Ω, 68 Ω, 56 Ω and 22 Ω. Which resistor has the most current through it? The least current?
   (a) The 22-Ω resistor has the most current, the 220-Ω resistor has least current.
   (b) The 220-Ω resistor has the most current, the 22-Ω resistor has the least current.
   (c) The 100-Ω resistor has the most current, the 56-Ω resistor has the least current.
   (d) The 56-Ω resistor has the most current, the 100-Ω has the least current.
12. A load resistor is connected to an output on a voltage divider. What effect does the load resistor have on the output voltage?
   (a) It remains the same. (b) It increases the output voltage.
   (c) It is zero. (d) It decreases the output voltage.