PAPER 6

1. (a) Explain KCL and KVL.

Solution

Refer Sections 1.12 and 1.9 (Chapter 1)

1. (b) A capacitor is charged to 1 volt at $t = 0$. A resistor of 1 ohm is connected across its terminals. The current is known to be of the form $i(t) = e^{-t}$ amperes for $t > 0$. At a particular time the current drops to 0.37A at that instant determine.

(i) At what rate is the voltage across the capacitor changing?

(ii) What is the value of the charge on the capacitor?

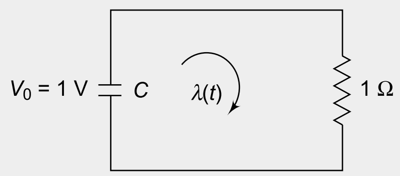
(iii) What is the voltage across the capacitor?

(iv) How much energy is stored in the electric field of the capacitor?

(v) What is the voltage across the resistor?

Solution

Refer Problem 11.3 (Chapter 11).



The current equation is

given as $i(t) = i(0^+)\, e^{-t/RC}$; given $i(t) = e^{-t/RC}$

$$i(0^+) = 1\text{A};\ RC = 1;\ C = 1\text{F}$$

When $i(t) = 0.37$ amperes

$$i(t) = 0.37 = e^{-t/1}$$

$$-t \log_e e = \log_e 0.37$$

$$t = 0.9942 \text{ sec}$$

$$i(t) = C\frac{dV(t)}{dt} \Rightarrow \frac{dV(t)}{dt} = \frac{i(t)}{C} = \frac{0.37}{1} = 0.37 \text{ V/sec}$$

or $$V_i(t) = \frac{1}{C}\int_0^t i(t)\,dt + V_0$$

$$= -\frac{1}{C}\int_0^t e^{-t}\,dt + V_0 \qquad [\therefore\ i(t) = -\,i(t)]$$

$$= \frac{-1}{1}\,\frac{e^{-t}}{(-1)} + 1 = e^{-t}$$

$$V_c(t) = e^{-t} \text{ for } t > 0$$

$$\therefore \quad \frac{dV_C(t)}{dt} = -e^{-t} = -e^{-0.9942} = -0.37 \text{ V/sec}$$

(ii) Charge on the capacitor

$$Q = C\,V_c = 1.e^{-t} = 0.37 \text{ coulombs}$$

(iii) Voltage across the capacitor

$$V_C(t) = e^{-t} = 0.37 \text{ volts}$$

(iv) Energy stored in the capacitor

$$W_C = \frac{1}{2}\,C\,V_c^2 = \frac{1}{2}1(e^{-t})^2 = \frac{e^{-2t}}{2} = 0.06845 \text{ joules}$$

(v) Voltage across the resistor at $t = 0.9942$ sec

$$V_R = i(t).R = e^{-t} = 0.37 \text{ V}$$

2. (a) Define Magneto Motive Force (MMF); reluctance, and flux density in a magnetic circuit. Specify the units of each of the above quantities.

Solution

Refer Section 9.11 (Chapter 9).

2. (b) Explain "dot convention" for a set of magnetically coupled coils. A cast steel electromagnet has an air gap of length 2 mm and an iron path of length 30 cms. Find the MMF needed to produce a flux density of 0.8T in the air gap. The relative permeability of the steel core at this flux density is 1000. Neglect leakage and fringing.

Solution

For "dot convention" refer Section 9.4 (Chapter 9).

Refer Example 10.2 (Chapter 10).

Air-gap length $l_g = 2 \text{ mm} = 2 \times 10^{-3}$ m

Iron path length $l_i = 30 \text{ cm} = 30 \times 10^{-2}$ m

Flux density $B = 0.8\text{T} = 0.8 \text{ Wb/m}^2$

$$\mu_r = 1000$$

$$\text{Total A.T} = \text{mmf} = H_i l_i + H_g l_g$$

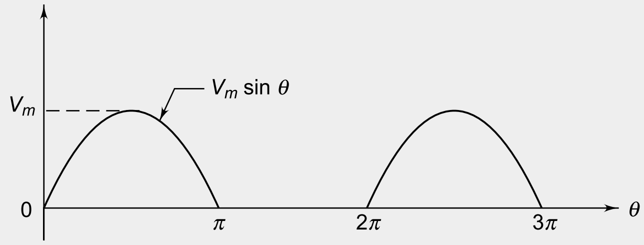
$$\frac{B \times l_i}{\mu_0\,\mu_g} + \frac{B}{\mu_0}\,l_g$$

$$= \frac{0.8 \times 30 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000} + \frac{0.8 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}}$$

$$= 1464 \text{ A.T.}$$

Hence, total MMF required to produce a flux density of 0.8T = 1464 AT.

3. (a) Find R.M.S. and average value of the following waveform.



Solution

Refer problem 4.13 (Chapter 4)

R.M.S. value, $$V_{\text{r.ms.}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\frac{(1-\cos 2\theta)}{2}\, d\theta}$$

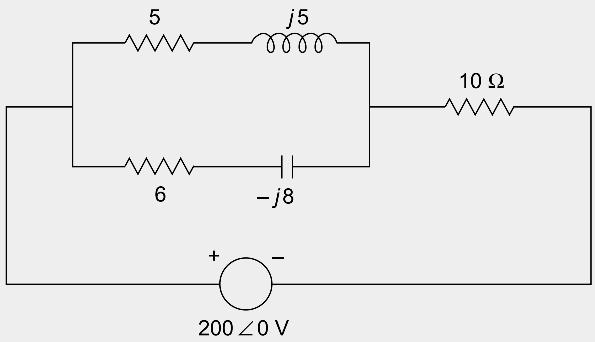
$$= \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{V_m}{2}$$

Average value, $$V_{\text{ave}} = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta\, d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{2\pi}$$

$$= \frac{V_m}{\pi}$$

3. (b) Find the total current and the power consumed by the circuit.



Solution

Total impedance of the circuit,

$$Z_{\text{T}} = (5 + j5) \,||\, (t - j8) + 10$$
$$Z_T = 16.15 + j0.769$$

$$I = \frac{V}{Z} = \frac{200\angle 0}{16+5+j0.769} = 12.35 - j0.588\text{A}$$

$$= 12.36\ \angle -2.72°$$

Power consumed $= I^2R$

$$= (12.36)^2 \times 16.15 = 2467\text{W}$$

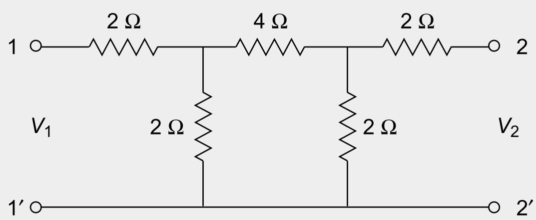
or $VI\cos\theta = 200 \times 12.36 \times \cos(-2.72) = 2467$ W.

4. (a) For a series RL circuit obtain the locus of current as inductance is changed from 0 to ∞ when the applied voltage is constant.

Solution

Refer Section 8.1(b) (Chapter 8)

4. (b) Obtain the z-parameters of the following two-parts network.
Two-port network



Solution

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{6\times 2}{6+2} + 2 = 3.5\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{6\times 2}{6+2} + 2 = 3.5\ \Omega$$

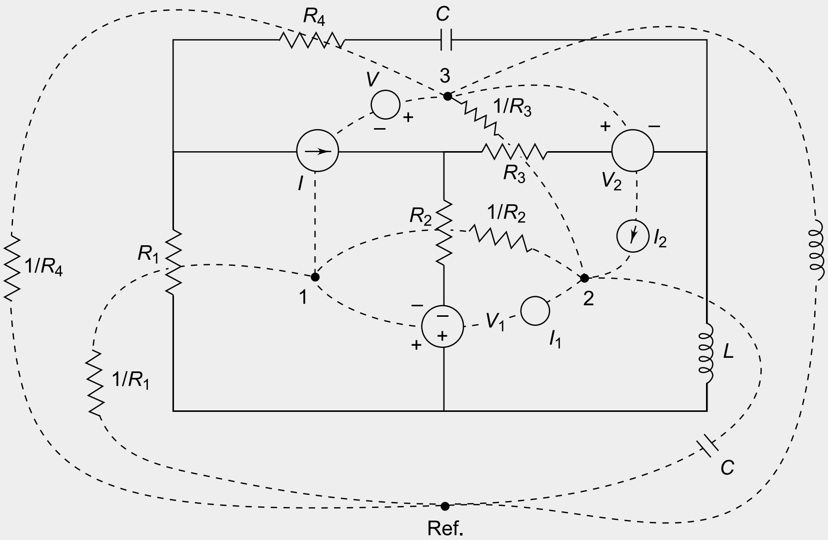
$$z_{12} = \left.\frac{V_1}{I_1}\right|_{I_1=0} = \frac{\frac{I_2\times 2}{6+2}\times 2}{I_2} = 0.5\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{\frac{I_1\times 2}{6+2}\times 2}{I_2} = 0.5\ \Omega$$

The parameters of the network are

$$z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3.5 & 0.5 \\ 0.5 & 3.5 \end{bmatrix}$$

5. What is duality? Explain the procedure for obtaining the dual of the given planar network shown below.

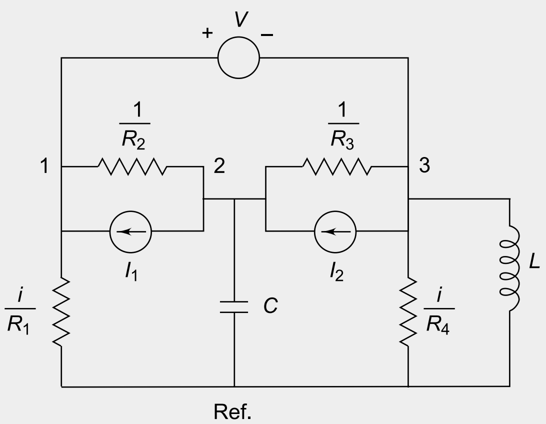


Solution

<u>Refer Section 3.8 (Chapter 3)</u>

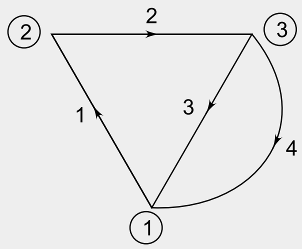
Rule 1 If a voltage source in the original network produces a c.w current in the mesh, the corresponding dual element is a current source whose direction is towards node representing the corresponding mesh.

Rule 2 If a current source in the original network produces a current in clockwise direction in the mesh, the voltage source in the dual network will have a polarity such that the node representing the corresponding mesh is positive.



Dual of the planar circuit given in 6(a).

6. (a) Construct the incident matrix for the graph show in figure.



Solution

Refer Section 2.4 (Chapter 2)

The dimensions of incidence matrix 'A' is $n \times b$ where n is number of nodes and b is number of branches, hence the dimensions of the incidence matrix for the above graph is 3×4.

Incidence matrix

n — nodes

b — branches

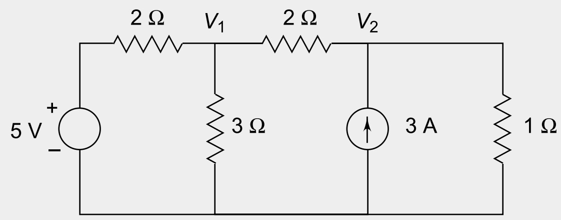
$A =$

$n \backslash p$	1	2	3	4
1	1	0	−1	−1
2	−1	1	0	0
3	0	−1	1	1

The incidence matrix is given by

$$A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

6. (b) Use nodal analysis, to determine the voltage V_1 and V_2 in the circuit shown.



Solution

Refer Section 2.12 (Chapter 2).

The nodal equation for the two nodes are

$$\frac{V_1 - 5}{2} + \frac{V_1}{3} + \frac{V_1 - V_2}{2} = 0 \qquad ...1$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} = 3 \quad \text{...2}$$

From 1 $\quad 1.333\ V_1 - 0.5\ V_2 = 2.5$

From 2 $\quad -0.5\ V_1 + 1.5\ V_2 = 3$

Solving the above equations for V_1 and V_2 yields

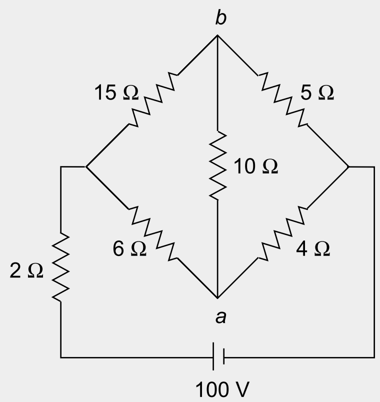
$V_1 = 3$ V and $V_2 = 3$ V.

7. (a) State and explain the Thevenin's theorem? State for what type problems this theorem is useful.

Solution

Refer Section 3.3 (Chapter 3).

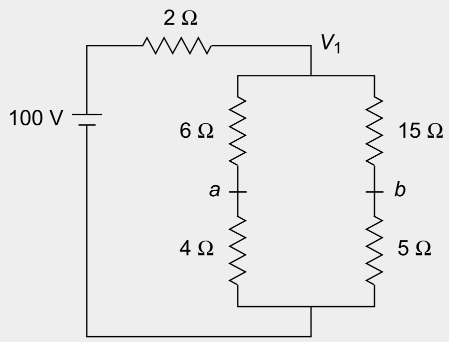
7. (b) Find the current through 10 Ω resistor using Thevenin's theorem.



Solution

Refer Problem 3.6 (Chapter 3).

Let us redraw the circuit by removing 10Ω.



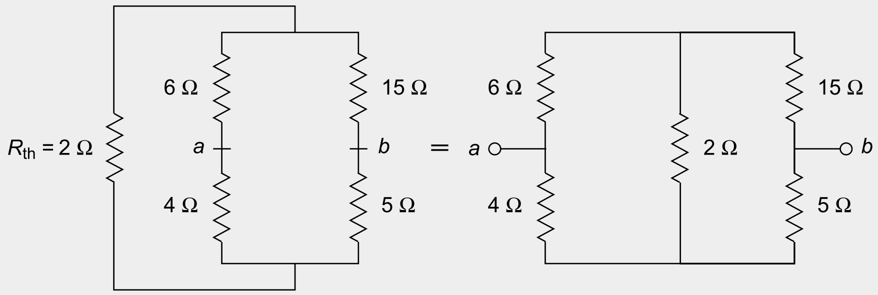
Applying KCL at V_1

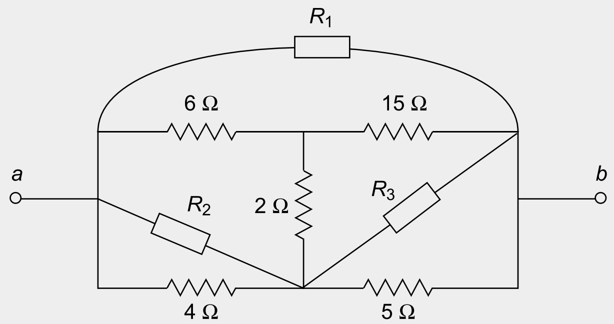
$$\frac{V_1 - 100}{2} + \frac{V_1}{10} + \frac{V_1}{20} = 0$$

from which $\qquad V_1 = 76.92 \text{ V}$

$$V_{\text{th}} = V_a - V_b$$

$$= \frac{V_1}{6+4} \times 4 - \frac{V_1}{15+5} \times 5 = 11.538 \text{ V}$$

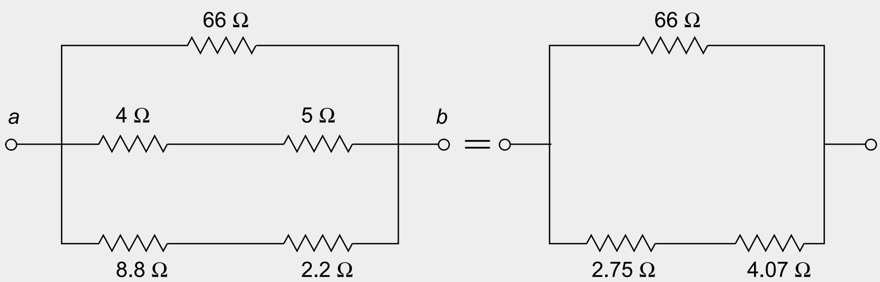




$$R_1 = \frac{6 \times 15 + 15 \times 2 + 2 \times 6}{2} = 66$$

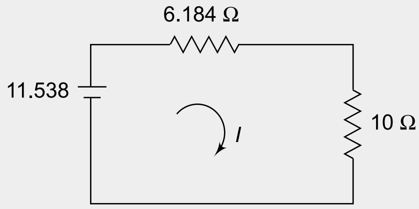
$$R_2 = \frac{132}{15} = 8.8$$

$$R_3 = \frac{132}{6} = 22$$



$$R_{ab} = R_{th} = \frac{66 \times 6.82}{72.82} = 6.184\ \Omega$$

Thevenin's equivalent circuit is given by



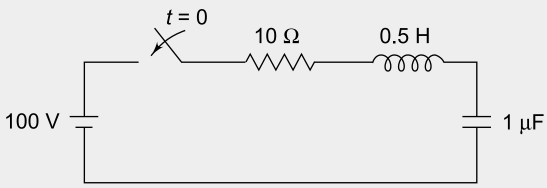
where $I = \dfrac{11.538}{16.184} = 0.7129\text{A}$

8. (a) For R-L-C series circuit with d.c. excitation discuss the under-damped, over-damped and critically damped cases.

Solution

Refer Section 11.4 (Chapter 11).

8. (b) Obtain the current $i(t)$ for $t \geq 0$ using tune domain approach.



Solution

Refer Example 11.3 (Chapter 11).

Writing KVL for the above circuit.

$$100 = 10i + 0.5\frac{di}{dt} + \frac{1}{1 \times 10^{-6}}\int i\, dt$$

Differentiating w.r.t. t

$$0 = 10\frac{di}{dt} + 0.5\frac{d^2 i}{dt^2} + 10^6 i$$

$$\frac{d^2 i}{dt^2} + 20\frac{di}{dt} + 2 \times 10^6 i = 0$$

$$(D^2 + 20D + 2 \times 10^{-6})i = 0 \text{ where } D = \frac{di}{dt}$$

$$D_1, D_2 = \frac{-20 \pm \sqrt{400 - 4 \times 2 \times 10^6}}{2}$$

$$D_1 = -10 + j1414;\ D_2 = -10 - j1414$$

The roots are in the form of $-K_1 \pm jK_2$

Therefore the solution for the current is given by

$$i(t) = e^{-k1t}\,[C_1 \cos k_2 t + C_2 \sin k_2\, t]$$

$$i(t) = e^{-10t}\,[C_1 \cos 1414\, t + C_2 \sin 1414\, t]$$

Substitute the initial conditions to find C_1 and C_2

At $t = 0$; the current following through the circuit is zero.

$$i = 0 = 1\,[C_1 \cos 0 + C_2 \sin 0]$$

$$C_1 = 0$$

$$i(t) = e^{-10t}\, C_2 \sin 1414t.$$

$$\frac{d\,i(t)}{dt} = C_2\,[e^{-10t}\, 1414 \cos 1414\, t + e^{-10t}\,(-10) \sin 1414t]$$

At $t = 0$, the voltage across the inductor

$$L\,\frac{d\,i(t)}{dt} = 100$$

$$\frac{d\,i(t)}{dt} = 200$$

$$200 = C_2\, e^{-(10 \times 0)}\, 1414$$

$$C_2 = 0.1414$$

The equation for current is given by

$$i(t) = e^{-10t}\,(0.1414 \sin 1414\, t)$$

$$i(t) = 0.1414\, e^{-10t} \sin 1414t$$