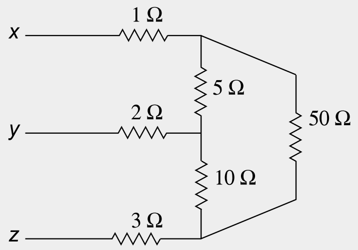
PAPER 5

1. (a) Obtain the expressions for star-delta equivalence of resistive networks.

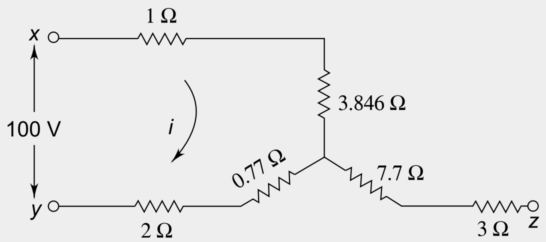
Solution

Refer Section 3.1 (Chapter 3).

(b) Determine the voltage appearing across terminals *y–z*, if a d.c. voltage of 100V is applied across *x–y* terminals in the figure below.



Solution Converting delta network to star network



Current, $$i = \frac{100}{1 + 3.846 + 0.77 + 2} = \frac{100}{7.616} = 13.13\text{A}$$

Voltage across $y_z{}^N, V_z = -13.13 \times (2 + 0.77)$

$= -36.37$ V

2. (a) State and explain Faraday's law of electromagnetic induction. What are statically and dynamically induced emfs.

Solution

Refer Section 1.6 (Chapter 1).

First law : It states that whenever the magnetic flux linked with a circuit changes an emf is always induced in it.

Second law : It states that the magnitude of the induced emf is equal to the rate of change of flux linkage.

Explanation : Suppose a coil with 100 turns undergoes a change of flux from zero refers to 2 mwb in one millisecond.

Initial flux linkages = 0
Final flux linkages = $100 \times 2 \times 10^{-3}$ wb.T

$$\text{Induced emf} = \frac{100 \times 2 \times 10^{-3} - 0}{1 \times 10^{-3}} = 2000 \text{ V}$$

Induced emf can be expressed as $e = \frac{d}{dt}(NQ) = N\frac{dQ}{dt}\ v$

Generally, a minus sign is associated with the $N\frac{dQ}{dt}$ to signify the fact that the induced emf sets up current in a such a direction that the magnetic effect produced by it opposes the very cause producing it. It is called Lenz's law

$$\therefore \qquad e = -N\frac{dQ}{dt}$$

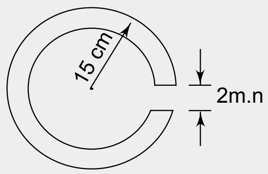
Statically induced EMF
EMF induced in a coil due to the change of its own flux linked with it or emf induced in one coil by the influence of the other coil is known as statically induced emf.
Dynamically Induced EMF:
When a coil with certain number of turns or a conductor is rotated in a magnetic filed (as in d.c. generator's), an emf is induced in it which is known as dynamically induced emf.

2. (b) An iron ring 15 cms in diameter and 10 cm^2 in area of cross section is wound with a coil of 200 turns. Determine the current in the coil to establish a flux density of 1 wb/m^2 if the relative permeability of iron is 500. In case if an air gap of 2 mm is cut in the ring, what is the current in the coil to establish the same flux density.

Solution

<u>Refer Example 10.12, Chapter 10: Refer Problem 10.13 Chapter 10</u>

(i) *Without air gap*
Diameter of Iron ring = 15 (cm) = 15×10^{-2} m
Area of Iron ring = 10 cm^2 = 10×10^{-4} m^2
Number of turns (N) = 200



Reluctance of Iron ring ($\Re_i$) $= \frac{l_i}{\mu_0\, \mu_r . A}$

Length of Iron path (l_i) $= \pi . d$
$= \pi \times 15 \times 10^{-2}$ m

$$\Re_i = \frac{15\pi \times 10^{-2}}{4\pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}} = 7.5 \times 105 \text{ AT/Wb}$$

mmf = Flux × reluctance
$I \times 200 = B.A.\ \Re i$

$$I = \frac{1 \times 10 \times 10^{-4} \times 7.5 \times 10^5}{200} = 3.75 \text{ A}$$

(ii) With 2 mm air gap cut in the iron ring reluctance of air gap

$$(\Re_g) = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 15.915 \times 10^5 \text{ AT/Wb}$$

With 2 mm air gap the length of the Iron path is reduced by 2 mm.

$\therefore$ $l_i = 15\pi \times 10^{-2} - 2 \times 10^{-3}$

But this is negligibly small.

$\therefore$ Total reluctance = $\Re_i + \Re_g = 23.415 \times 10^5$ AT/Wb

$$\therefore \quad I = \frac{\phi.\Re}{N} = \frac{B.A.\Re}{N}$$

$$= \frac{1 \times 10 \times 10^{-4} \times 23.415 \times 10^5}{200}$$

Required current (I) = 11.707A

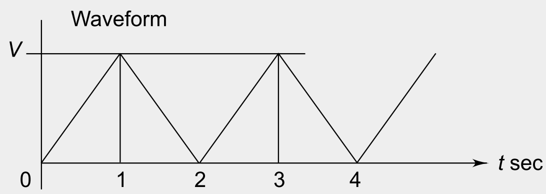
If the gap length is taken into consideration:

$$\text{Total emf} = \frac{B_i l_i}{\mu_0 \mu_r} + \frac{B_i l_g}{\mu_0}$$

$$= \frac{1(\pi \times 15 \times 10^{-2} - 2 \times 10^{-3})}{4\pi \times 10^{-7} \times 500} + \frac{1 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} \quad 2338.35 \text{ AT}$$

$$\therefore \quad I = \frac{2338.35}{200} = 11.691\text{A}$$

3. (a) Find the form factor for the following waveform.



Solution

Refer Section 4.4.7 (Chapter 4)

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}}$$

Average value of the triangular waveform 0 to 2 sec

$$\text{Vav} = \frac{1}{2}\left[\int_0^1 V \,.\, t\, dt + \int_1^2 -V\,(t-2)\, dt\right]$$

$$= \frac{1}{2}\left[V\frac{t^2}{2}\Big|_0^1 + -V\frac{t^2}{2}\Big|_1^2 + 2V.t|_1^2\right]$$

$$= \frac{1}{2}\left[\frac{V}{2} - \frac{3}{2}V + 2V\right] = V/2$$

$$\text{R.M.S. value, } (V_{\text{r.m.s.}}) = \left[\frac{1}{2}\left\{\int_0^1 V^2 t^2\, dt + \int_1^2 V^2 (t-2)^2\, dt\right]^{1/2}$$

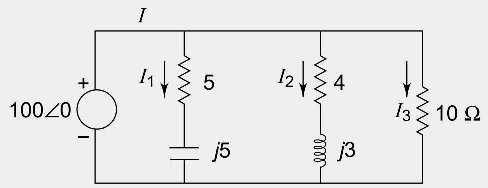
$$= \left[\frac{1}{2}\left\{V^2 \frac{t^3}{3}\Big|_0^1 + V^2 \frac{t^3}{3}\Big|_1^2 + 4V^2\, t|_1^2 - 4V^2 \frac{t^2}{2}\Big|_1^2\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2}\left\{\frac{V^2}{3} + \frac{7V^2}{3} - 2V^2\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2}\left\{\frac{8V^2 - 6V^2}{3}\right\}\right]^{1/2} = \frac{V}{\sqrt{3}}$$

$$\text{Form factor} = V/\sqrt{3}\,/V/2 = \frac{2}{\sqrt{3}} = 1.155$$

3. (b) Find the branch currents, total current and the total power in the circuit shown below:



Solution

$$\text{Branch currents } I_1 = \frac{100 + j0}{5 - j5} = 10 + j10$$

$$I_2 = \frac{100 + j0}{4 - j3} = 16 - j12$$

$$I_3 = \frac{100 + j0}{10} = 10 + j0$$

$$\text{Total current } (I) = I_1 + I_2 + I_3$$
$$= 36 - J2$$
$$= 36.055 \angle -3.179°$$
$$\text{Total power} = VI \times \cos Q$$
$$= 100 \times 36.055 \times \cos 3.179°$$
$$= 3599.95 \text{ watts.}$$

4. (a) Obtain the expression for the frequency at which maximum voltage occurs across the capacitance in series resonance circuit in terms of the Q-factor and resonance frequency.

Solution

<u>Refer Section 8.3 (Chapter 8)</u>

From Section 8.3 we know that

The frequency at which V_c is maximum is given by

$$f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_c = \frac{1}{2\pi}\left[\sqrt{\frac{1}{LC}\left[1 - \frac{R^2C}{2L}\right]}\right]$$

$$= \frac{1}{2\pi}\left[\sqrt{\frac{R^2}{LC}\left(\frac{1}{R^2} - \frac{C}{2L}\right)}\right]$$

$$= \frac{1}{2\pi}\frac{R}{\sqrt{LC}}\left[\sqrt{\frac{1}{R^2} - \frac{C}{2L}}\right]$$

$$= \frac{1}{2\pi}\frac{R}{\sqrt{LC}}\left[\sqrt{\frac{C}{L}\left[\frac{L}{CR^2} - \frac{1}{2}\right]}\right]$$

$$= \frac{1}{2\pi\sqrt{LC}} \cdot R\sqrt{\frac{C}{L}}\left[\frac{L}{CR^2} - \frac{1}{2}\right]^{1/2}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}\,;\; Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{Q} = R\sqrt{\frac{C}{L}}$$

$$\therefore \qquad f_c = \frac{f_o}{Q}\left[\frac{L}{CR^2} - \frac{1}{2}\right]^{1/2}$$

4. (b) In a series RLC circuit if the applied voltage is 10V, and resonance frequency is 1 kHz, and Q factor is 10, what is the maximum voltage across the inductance.

Solution

$$\text{Resonance freq } (fr) = \frac{1}{2\pi\sqrt{LC}} = 1000 \qquad (1)$$

$$\text{Quality factor } (Q) = \frac{1}{R}\sqrt{\frac{L}{C}} = 10 \qquad (2)$$

$$\sqrt{LC} = \frac{1}{2\pi \times 1000} = 6283.18$$

$$LC = 39.47 \times 10^6$$

From 1,
$$\frac{1}{2\pi} = \sqrt{LC}\ 1000 \qquad (3)$$

From 2,
$$\frac{1}{R} = \sqrt{\frac{C}{L}}\ 10 \qquad (4)$$

From 3 and 4

$$\frac{1}{2\pi R} = 10^4\ \sqrt{LC}\ \sqrt{\frac{C}{L}}$$

$$\frac{1}{2\pi\ RC} = 10000$$

$$RC = 1.59154 \times 10^{-5} \simeq 1.6 \times 10^{-5}.$$

The maximum voltage across the inductance occurs at frequency greater than the resonance frequency which is given by

$$f_L = \frac{1}{2\pi\sqrt{LC - \frac{(RC)^2}{2}}}$$

$$f_L = \frac{1}{2\pi\sqrt{39.47 \times 10^6 - \frac{(1.6 \times 10^{-5})^2}{2}}} = 1002.5$$

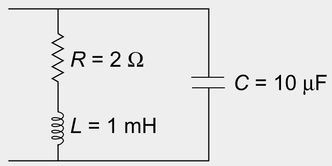
It can be observed that, the above frequency is approximately equal to resonance frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.47 \times 10^6}}$$

Hence we can take the voltage across the inductor

$$= Q \times V$$
$$= 10 \times 10$$
$$= 100 \text{ volts}$$

(c) In a parallel resonance circuit shown in figure find the resonance frequency, dynamic resistance and bandwidth.



Solution

The circuit shown in the above figure is the most common form of parallel resonant circuit in practical use and is also called the tank circuit.

The admittance of the circuit is

$$Y = \frac{1}{z} = \frac{1}{z_C} + \frac{1}{Z_L}$$

$$Y = \frac{1}{-jX_C} + \frac{1}{R + jX_L}$$

$$= j\omega C + \frac{1}{R + j\omega L}$$

$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)$$

At resonance the susceptance part is zero.

Hence at $\omega = \omega_r,\ C = \dfrac{L}{R^2 + \omega_r^2 L^2} = 0$

$$R^2 + \omega_r^2 L^2 = \frac{L}{C}$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2 \Rightarrow \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad (1)$$

Resonance frequency, $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$ (2)

$\therefore$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-3}}\sqrt{\frac{1 \times 10^{-3}}{10 \times 10^{-6}} - 4}$$

$$= 1559.4 \text{ Hz}$$

Dynamic impedance:
The input admittance at resonance is given by

$$Y_r = \frac{R}{R^2 + \omega_r^2 L^2}$$

The impedance at resonance is

$$Z_r = \frac{1}{y_r} = \frac{R^2 + \omega_r^2 L^2}{R} = R + \frac{\omega_r^2 L^2}{R}$$

Substituting $\omega r^2 L^2$ from Eq. 1 gives,

$$Z_r = R + \frac{\frac{L}{C} - R^2}{R} = R + \frac{L}{CR} - R$$

$Z_r = \frac{L}{CR}$ which is called dynamic impedance.

This is a pure resistance because it is independent of the frequency.

$$\text{Here, dynamic resistance} = \frac{1 \times 10^{-3}}{10 \times 10^{-6} \times 2}$$

$$= 50\,\Omega$$

Bandwidth of the parallel resonance circuit $= \frac{\omega_r}{Q}$

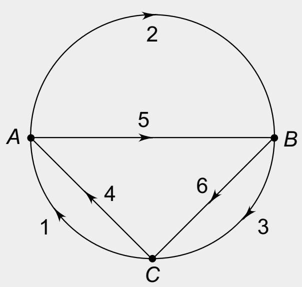
$$\omega_r = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$= 9797.95$$

$$Q_o = \frac{\omega o L}{R} = \frac{9797.95 \times 1 \times 10^{-3}}{2} = 4.898$$

$$\text{Bandwidth} = \frac{1559.4}{4.898} = 318.311\text{+Z}$$

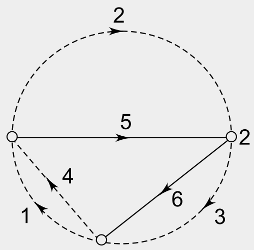
5. For the topological graph shown in figure, obtain the fundamental Tie set matrix choosing the tree containing two elements 5 and 6.



Solution

Refer Section 2.7 (Chapter 2)

The tree of the graph is shown with solid lines (5 and 6) and the links are shown with dashed lines (1, 2, 3, 4).



For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop. In a loop there exists a closed path and a circulating current, which is called the link current.

The fundamental loop formed by one link at a time, has a unique path in the tree rolling the two nodes of the link. This loop is also called *f*-loop or a tie-set. Every link defines a fundamental loop of the network.

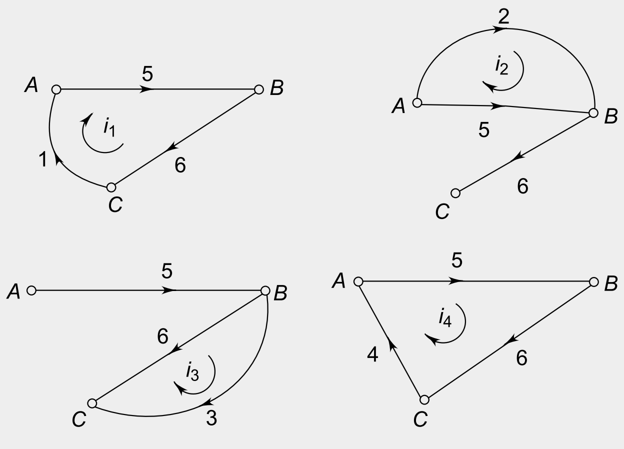
No. of nodes in the graph $n = 3 = (A, B, C)$

No. of branches, $b = 6 = (1, 2, 3, 4, 5, 6)$

No. of tree branches or twigs $= n - 1 = 2 = (5, 6)$

No. of link branches, $l = b - (n - 1) = 4$ (1, 2, 3, 4)

The following are the figures of the Tie-sets.



Tie set Matrix can be formed by considering the four fundamental loops. Corresponding to the link branches 1, 2, 3, 4.

If V_1, V_2, V_3, V_4, V_5 and V_6 are the respective branch voltages.

The KVL equations for the three *f*-loops can be written as

$$V_1 + V_5 + V_6 = 0$$
$$V_2 - V_5 = 0$$
$$V_3 - V_6 = 0$$
$$V_4 + V_5 + V_6 = 0$$

In order to apply KVL to each loop, we take the reference direction of the loop which coincides with the reference direction of the link defining the loop.

The above equations can be written in matrix form as

$[B][V_b] = 0$, where B is a 4×6 Tie-set matrix.

Loops Branches $\longrightarrow$

$$\begin{array}{c} \downarrow \\ l_1 \\ l_2 \\ l_3 \\ l_4 \end{array}
\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, Tie-set Matrix, $B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

6. (a) State and explain the superposition theorem.

Solution

<u>Refer Section 3.2 (Chapter 3)</u>

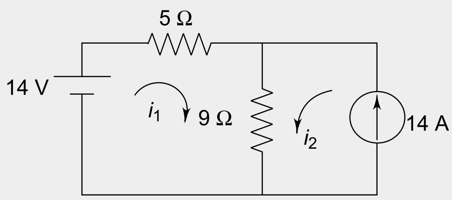
6. (b) Is superposition valid for power? Explain.

Solution

Superposition theorem is valid only for linear systems.

Superposition cannot be applied for power because the equation for power is non linear.

Let us consider a network with a voltage source and current source as shown below and find the power consumed in 9Ω resistor by super position.



When 14V source is acting the current in 9Ω is 1A

The power = $i^2 \times 9 = 9$ watts

When 14A source is acting the current in 9Ω is 5A

The power $= i^2 \times 9 = 225$ watts

Total power = 225 + 9 = 234 Watts

When both are acting the KVL for loop 1 and 2

are $14 = 5i_1 + 9(i_1 + i_2)$

$14i_1 = -112$

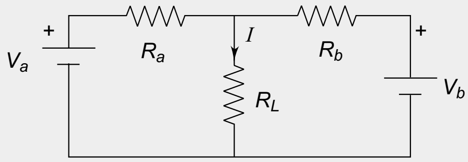
$i_1 = -8\text{A}; i_2 = 14\text{A}$

Current in 9Ω resistor is $i_1 + i_2 = 6\text{A}$

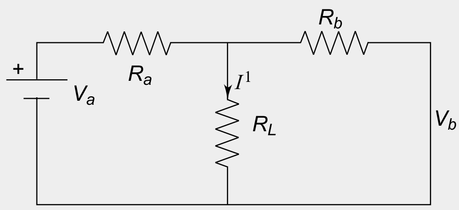
Power $= (6)^2 \times 9 = 324$ watts

Since power is not the same in both the cases, the superposition theorem does not hold true.

Consider the circuit shown below.

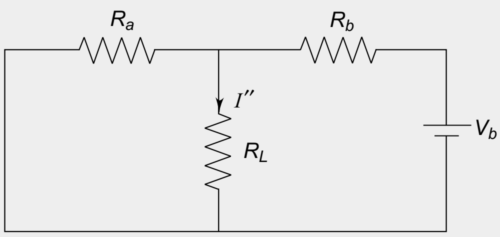


When V_a is acting.



I^1 be the current through R_L; and Power $= (I)^2 R_L$

When V_b is acting I'' be the current



through B_L and Power $= (I'')^2 R_L$

Total current through R_L by superposition

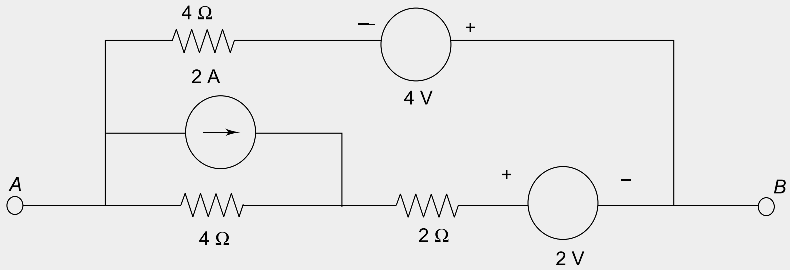
$I = I' + I''$, and power $= I^2 R_L$

$(I')^2 R_L + (I'')^2 R_L \neq I^2 R_L$

because $I^2 = (I' + I'')^2 = (I')^2 + (I'')^2 + 2I' I''$

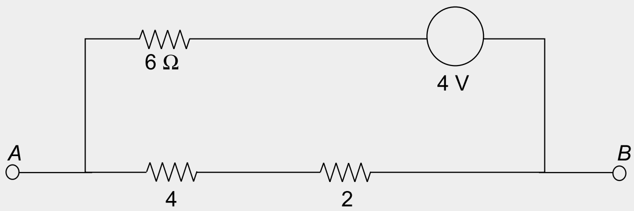
Hence, $(I')^2 + (I'')^2 \neq I^2$ and therefore superposition theorem is not valid for power.

7. Using superposition theorem, find V_{AB}.



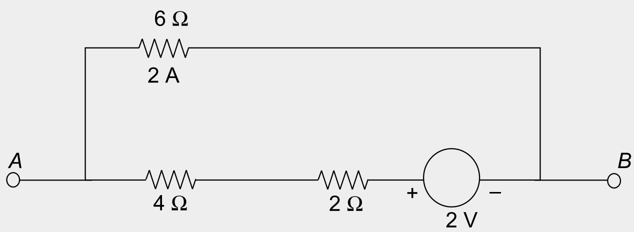
Solution

When 4V source is acting



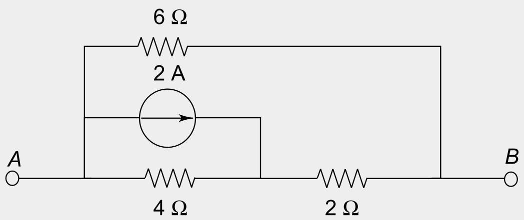
$V_{AB1} = -2$ V

When 2 V source is acting.



$V_{AB2} = 1$ V

When 2A source is acting



$V_{AB3} = -4$ V

Voltage across $AB = V_{AB1} + V_{AB2} + V_{AB2}$

$= -2 + 1 - 4$

$= -5$ volts

8. (a) Explain why the voltage across capacitor cannot change instantaneously?

Solution

Refer Section 1.7 (Chapter 1)

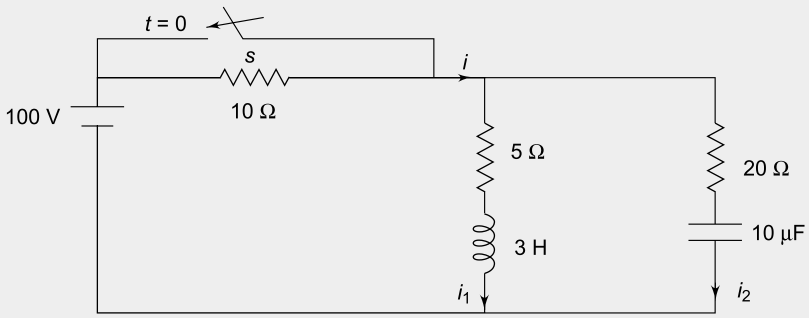
8. (b) What is the significance of time constant for R-L circuit? What are the difficult ways of defining time constant?

Solution

Refer Section 11.2 (Chapter 11)

8. (c) Switch S is closed at $t = 0$. Find initial conditions for voltage across capacitor.

$$i,\ i_2,\ \frac{di_1}{dt} \text{ and } \frac{di_2}{dt}$$



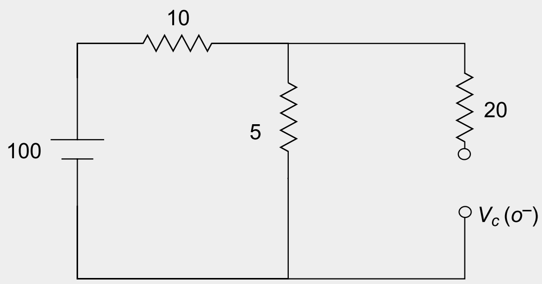
Solution

At $\qquad t = \bar{0}\,;\ i = i_1 + i_2$

Since $\qquad i_2 = 0,\ i = i_1\,(0^+) = \dfrac{V}{R_1 + R_2} = \dfrac{100}{15} = 6.67\text{A}$

$$i_1(0^+) = i_L(0^-) = i_L(0^+) = 6.667\text{A}$$

$$V_C(0^+) = V_C(0^+) = \frac{100}{10+5} \times 5 = 33.33\text{V}$$

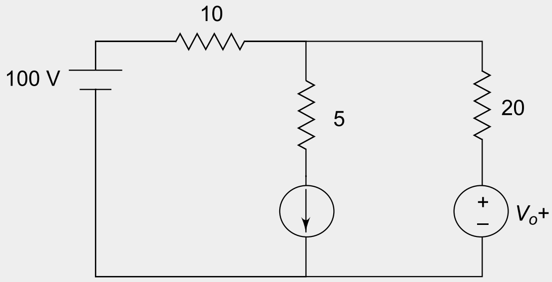


At $\qquad t = 0^+;$

$$20i_2 + V_c(0^+) = 100$$

$$i_2\,(0^+) = \frac{100 - 33.33}{20}$$

$$i_2(0^+) = 3.33\text{A}$$



Applying KVL for the loops at $t = 0^+$

$$5i_1 + 3\frac{di_1}{dt} = 100$$

$$3\frac{di_1}{dt} = 100 - 5i_1$$

$$\left.\frac{di_1}{dt}\right|_{t=0^+} = \frac{100 - 5i_1\,(0^+)}{3} = \frac{100 - 5 \times 6.667}{3}$$

$$\frac{di_1}{dt}(0^+) = 22.21 \text{ A/sec.}$$

$$20i_2 + \frac{1}{C}\int i_2\,dt = 100$$

$$20\frac{di_2}{dt} + \frac{i_2}{C} = 0$$

$$\left.\frac{di_2}{dt}\right|_{t=0^+} = \frac{-i_2\,(0^+)}{20 \times 10 \times 10^{-6}} = \frac{-3.33}{2 \times 10^{-4}}$$

$$\frac{di_2}{dt}\,(0^+) = -16.65 \times 10^3 \text{ A/sec.}$$