PAPER 2

1. (a) Discuss Kirchhoff's Laws.

Solution Refer Sections 1.9 and 1.12.

(b) Derive the expression for self, mutual inductance and coefficient of coupling.

Solution Refer Sections 10.3. and 10.5.

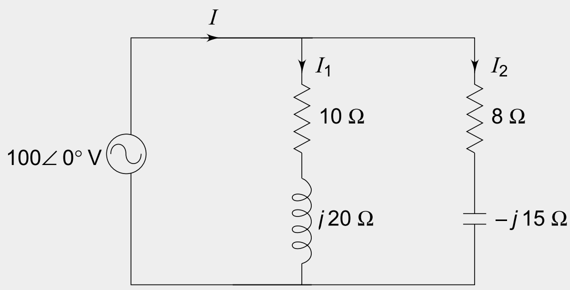
(c) Explain source transformation with example.

Solution Refer Section 2.15.

2. (a) What is the use of operator j?

Solution Refer Appendix C.

(b) For the circuit shown in figure, find the current I drawn from the source.



The impedance as seen by the source is

$$Z = (10 + j20) \,//\, (8 - j15)$$

$$= \frac{380 + j10}{18 + 5j} = 19.742 - j4.928$$

$$\therefore \text{ Current drawn from source } I = \frac{V}{Z} = \frac{100}{19.742 - j4.928}$$

$$= 4.768 + j1.19$$

$$= 4.914 \angle 14.01°$$

or

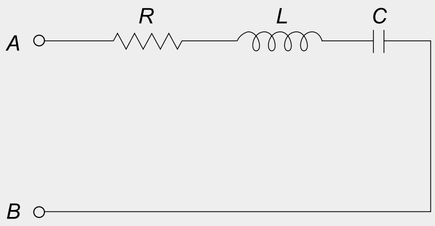
$$I_1 = \frac{100}{10 + 20j} = 2 - 4j$$

$$I_2 = \frac{100}{8 - 15j} = 2.768 + 5.1903j$$

$$I = I_1 + I_2 = 4.768 + j1.19$$

$$= 4.914 \angle 14.01°$$

3. (a) A series RLC circuit with $Q = 250$ is resonant at 1.5 MHZ. Find the frequencies at half power points and also bandwidth.



Solution Given $Q = 250$

$$Q = \frac{\omega_o L}{R}$$

$$250 = \frac{2\pi \times f_o \times L}{R} \Rightarrow \frac{R}{L} = \frac{2\pi \times 1.5 \times 10^6}{250} = 37.7 \times 10^3$$

Lower half power frequency $f_1 = f_r - \dfrac{R}{4\pi L}$

$$= 1.5 \times 10^6 - \frac{37.7 \times 10^3}{\Delta\pi}$$

$$= 1.5 \times 10^6 - 3 \times 10^3$$

$$= 1.496 \text{ MHz}$$

Upper half power frequency $f_2 = f_r + \dfrac{R}{4\pi L}$

$$= 1.5 \times 10^6 + \frac{37.7 \times 10^3}{4\pi}$$

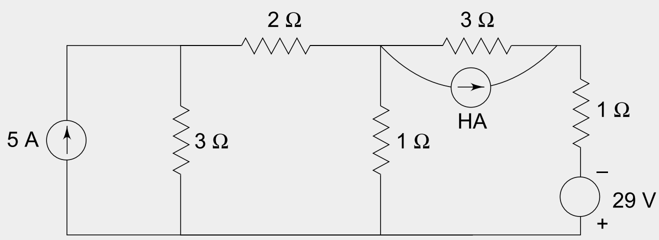
$$= 1.5\text{M} + 3k = 1.53 \text{ MHz}$$

Bandwidth $= f_2 - f_1 = 1.53$ M – 1.496 M = 6 kHz

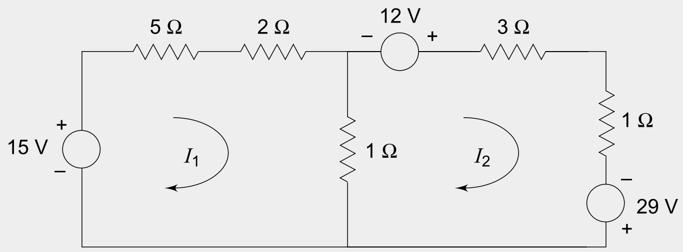
(b) Distinguish between the average value and rms value of an alternating current.

Solution Refer Section 4.4.

4. Write and solve the equation for Mesh Current in the network shown.



Solution By source transformation technique transform 5A and 4A current sources into voltage sources.
5A current source in parallel with 3 Ω can be transformed to 15V in series with 3 Ω and 4A current source in parallel with 3 Ω can be transformed to 12 volts in series with 3 Ω. The equivalent circuit is as shown below:



The mesh equations are

$$5I_1 + 1(I_1 - I_2) = 15$$

$$1(I_2 - I_1) + 4I_2 = 41$$

$$\Rightarrow \quad -I_1 + 5I_2 = 41 \tag{1}$$

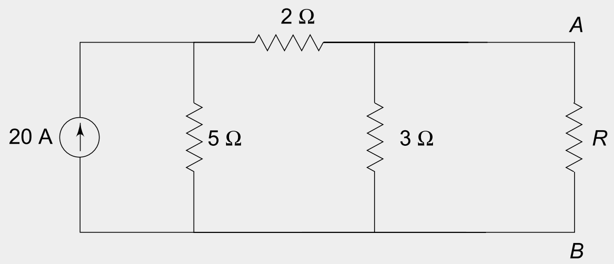
$$6I_1 - I_2 = 15 \tag{2}$$

on solving equations (1) and (2) we get

$$I_1 = 4 \text{ Amps}$$

$$I_2 = 9 \text{ Amps}$$

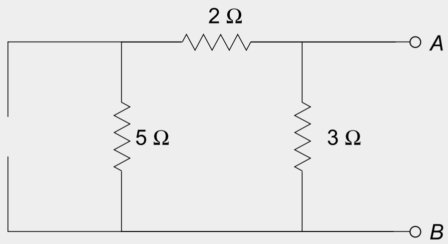
5. The circuit shown in the figure below has resistance R which absorbs maximum power. Compute the value of R and maximum power.



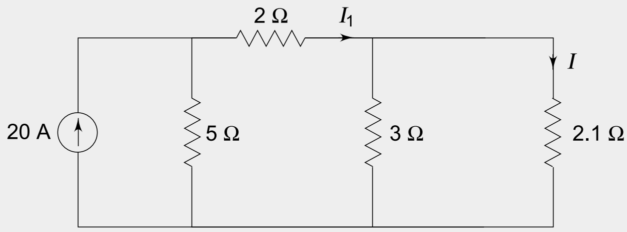
Solution According to maximum power transfer theorem, maximum power can be transferred when load resistance is equal to the interval resistance of the source which can be calculated as the resistance seen from AB with source open.

$$\therefore R_{\text{th}} = 3\,/\,/\,7$$

$$= \frac{21}{10} = 2.1\ \Omega$$



Now the circuit can be drawn as



According to current dividing rule

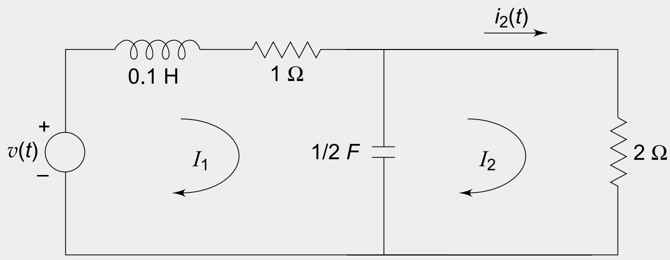
$$I_1 = \frac{20 \times 5}{(5 + 3.235)} = 12.14 \text{ A}$$

$$I_2 = \frac{I_1 \times 3}{5.1} = \frac{12.14 \times 3}{5} = 7.14 \text{ A}$$

So the maximum power that can be delivered to resistor R is

$$I^2R = (7.14)^2 \times 2.1 = 107 \text{ watts.}$$

6. In the figure shown below $v(t) = 10$ V, find $i_2(t)$. Assume all initial conditions to be zero. Use Laplace transform technique.



Solution Writing mesh equation

$$\frac{10}{S} = 0.1S\ I_1(s) + I_1(s) + \frac{2}{s}(I_1(s) - I_2(s))$$

$$0 = \frac{2}{S}(I_2(s) - I_1(s)) + 2\,I_2(s) = 0$$

$$\left(\frac{2}{s} + 0.1s + 1\right) I_1(s) - \frac{2}{s} I_2(s) = \frac{10}{s} \quad (1)$$

$$-\frac{2}{s} I_1(s) + \left(\frac{2}{s} + 2\right) I_2(s) = 0 \quad (2)$$

on solving equations (1) and (2) we get

$$I_2(s) = \frac{100}{s(s^2+11s+60)}$$

$$\frac{1}{S(S^2+11S+60)} = \frac{AS+B}{S^2+11S+60} + \frac{C}{S}$$

$$A = \frac{-1}{60};\ \ B = \frac{-11}{60};\ \ C = \frac{1}{60}$$

$$\therefore L^{-1}\left[\frac{1}{S\left(S^2+11S+60\right)}\right] = L^{-1}\left[\frac{1}{60}\frac{1}{S} - \frac{1}{60}\frac{S}{S^2+11S+60} - \frac{11}{60}\cdot\frac{1}{S^2+11S+60}\right]$$

$$= \frac{1}{60}L^{-1}\left[\frac{1}{S} - \frac{\left(S+\frac{11}{2}\right)}{\left(S+\frac{11}{2}\right)^2 + \left(\frac{\sqrt{199}}{2}\right)^2} + \frac{319}{\left(S+\frac{11}{2}\right)^2 + \left(\frac{\sqrt{119}}{2}\right)^2}\right]$$

$$\therefore i_2(t) = \frac{100}{60}\left[1 - e^{\frac{11}{2}t}\cos\frac{\sqrt{119}}{2}t - 319\times\frac{2}{\sqrt{119}}e^{\frac{11}{2}t}.\sin\frac{\sqrt{119}}{2}t\right]$$

$$\therefore i_2(t) = \left[1.667 - 1.667e^{-5.5t}\cos 5.45t - 97.47e^{5.5t}\sin 5.45t\right]$$

7. (a) In a two-port bilateral network show that $AD - BC = 1$.

Solution Refer Section 15.8.2.

(b) Derive an expression for DC response in an RC circuit.

Solution Refer Section 11.3.