Chapter **9**

Polyphase Circuits

9.1 POLYPHASE SYSTEM

In an ac system it is possible to connect two or more number of individual circuits to a common polyphase source. Though it is possible to have any number of sources in a polyphase system, the increase in the available power is not significant beyond the three-phase system. The power generated by the same machine increases 41.4 per cent from single phase to two-phase, and the increase in the power is 50 per cent from single phase to three-phase. Beyond three-phase, the maximum possible increase is only seven per cent, but the complications are many. So, an increase beyond three-phase does not justify the extra complications. In view of this, it is only in exceptional cases where more than three phases are used. Circuits supplied by six, twelve and more phases are used in high power radio transmitter stations. Two-phase systems are used to supply two-phase servo motors in feedback control systems.

In general, a three-phase system of voltages (currents) is merely a combination of three single phase systems of voltages (currents) of which the three voltages (currents) differ in phase by 120 electrical degrees from each other in a particular sequence. One such three-phase system of sinusoidal voltages is shown in Fig. 9.1.

9.2 ADVANTAGES OF THREE-PHASE SYSTEM

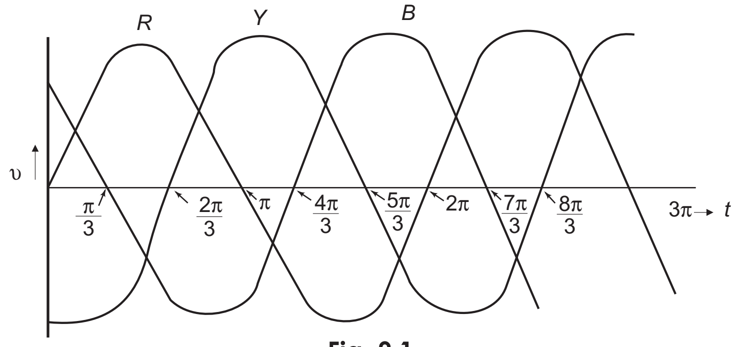


Fig. 9.1

It is observed that the polyphase, especially three-phase, system has many advantages over the single phase system, both from the utility point of view as well as from the consumer point of view. Some of the advantages are as under.

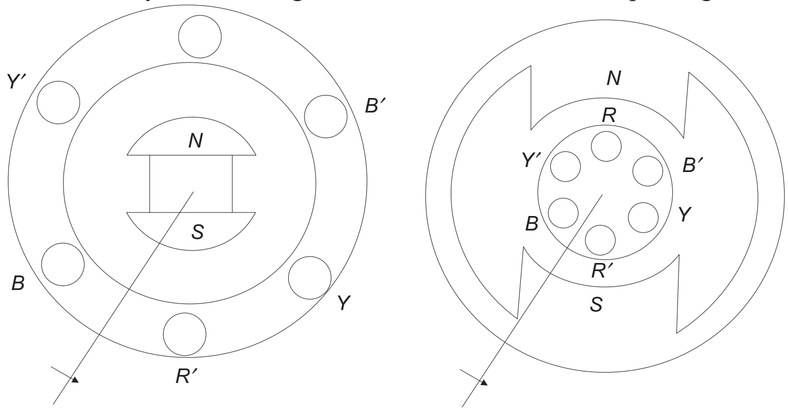
1. The power in a single phase circuit is pulsating. When the power factor of the circuit is unity, the power becomes zero 100 times in a second in a 50 Hz supply. Therefore, single phase motors have a pulsating torque. Although the power supplied by each phase is pulsating, the total three-phase power supplied to a balanced three-phase circuit is constant at every instant of time. Because of this, three-phase motors have an absolutely uniform torque.
2. To transmit a given amount of power over a given length, a three-phase transmission circuit requires less conductor material than a single-phase circuit.
3. In a given frame size, a three-phase motor or a three-phase generator produces more output than its single phase counterpart.
4. Three-phase motors are more easily started than single phase motors. Single phase motors are not self starting, whereas three-phase motors are.

In general, we can conclude that the operating characteristics of a three-phase apparatus are superior than those of a similar single phase apparatus. All three-phase machines are superior in performance. Their control equipments are smaller, cheaper, lighter in weight and more efficient. Therefore, the study of three phase circuits is of great importance.

9.3 GENERATION OF THREE-PHASE VOLTAGES

Three-phase voltages can be generated in a stationary armature with a rotating field structure, or in a rotating armature with a stationary field as shown in Fig. 9.2 (a and b).

Single phase voltages and currents are generated by single phase generators as shown in Fig. 9.3(a). The armature (here a stationary armature) of such a generator has only one winding, or one set of coils. In a two-phase generator the



(a) STATIONARY ARMATURE (b) STATIONARY FIELD

Fig. 9.2

armature has two distinct windings, or two sets of coils that are displaced 90° (electrical degrees) apart, so that the generated voltages in the two phases have 90 degrees phase displacement as shown in Fig. 9.3(b). Similarly, three-phase voltages are generated in three separate but identical sets of windings or coils that are displaced by 120 electrical degrees in the armature, so that the voltages generated in them are 120° apart in time phase. This arrangement is shown in Fig. 9.3(c). Here RR' constitutes one coil (R-phase); YY' another coil (Y-phase), and BB' constitutes the third phase (B-phase). The field magnets are assumed in clockwise rotation.

The voltages generated by a three-phase alternator is shown in Fig. 9.3(d). The three voltages are of the same magnitude and frequency, but are displaced from one another by 120°. Assuming the voltages to be sinusoidal, we can write

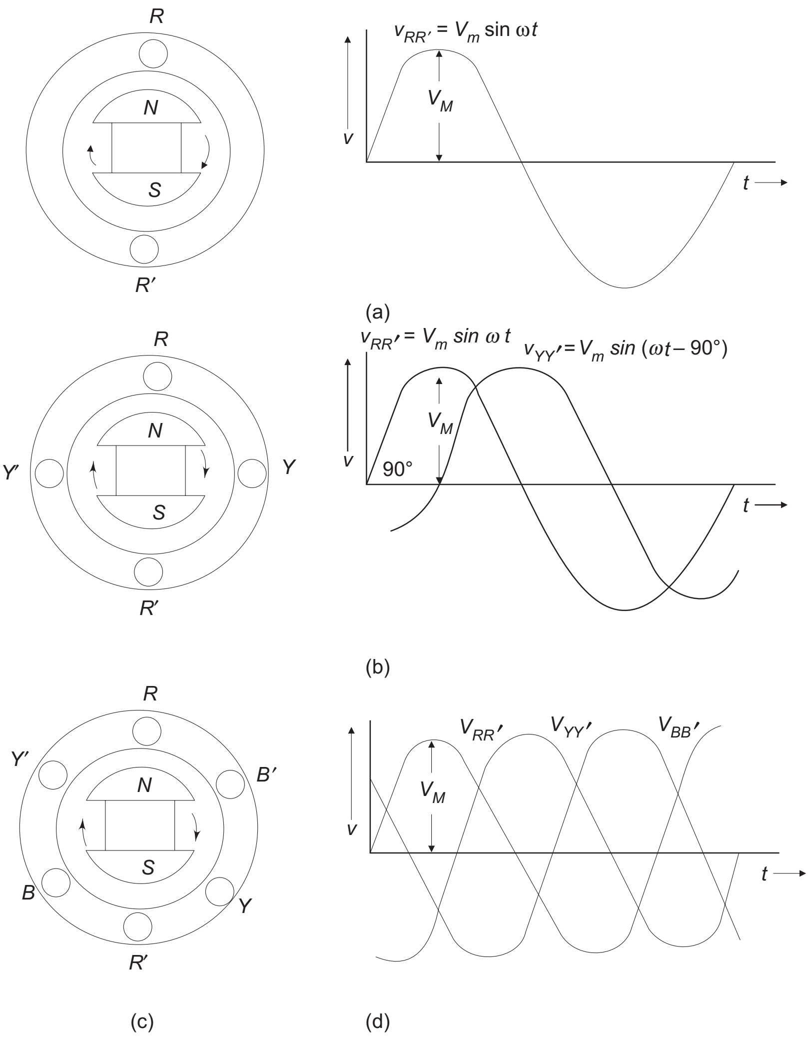


Fig. 9.3

the equations for the instantaneous values of the voltages of the three phases. Counting the time from the instant when the voltage in phase R is zero. The equations are

$$v_{RR'} = V_m \sin \omega t$$
$$v_{YY'} = V_m \sin (\omega t - 120°)$$
$$v_{BB'} = V_m \sin (\omega t - 240°)$$

At any given instant, the algebraic sum of the three voltages must be zero.

9.4 PHASE SEQUENCE

Here the sequence of voltages in the three phases are in the order $v_{RR'} - v_{YY'} - v_{BB'}$, and they undergo changes one after the other in the above mentioned order. This is called the *phase sequence*. It can be observed that this sequence depends on the rotation of the field. If the field system is rotated in the anticlockwise direction, then the sequence of the voltages in the three-phases are in the order $v_{RR'} - v_{BB'} - v_{YY'}$; briefly we say that the sequence is RBY. Now the equations can be written as

$$v_{RR'} = V_m \sin \omega t$$
$$v_{BB'} = V_m \sin (\omega t - 120)$$
$$v_{YY'} = V_m \sin (\omega t - 240)$$

Example 9.1 What is the phase sequence of the voltages induced in the three coils of an alternator shown in Fig. 9.4? Write the equations for the three voltages.

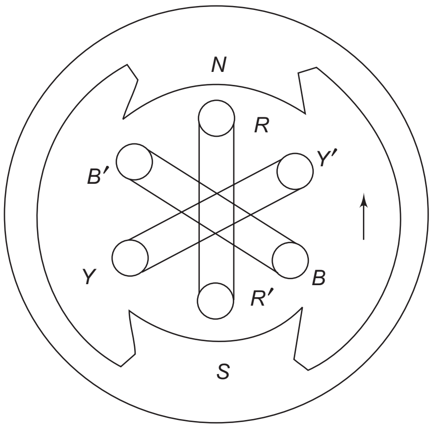


Fig. 9.4

Solution Here the field system is stationary and the three coils, *RR′*, *YY′* and *BB′*, are rotating in the anticlockwise direction, so the sequence of voltages is *RBY*, and the induced voltages are as shown in Fig. 9.4.

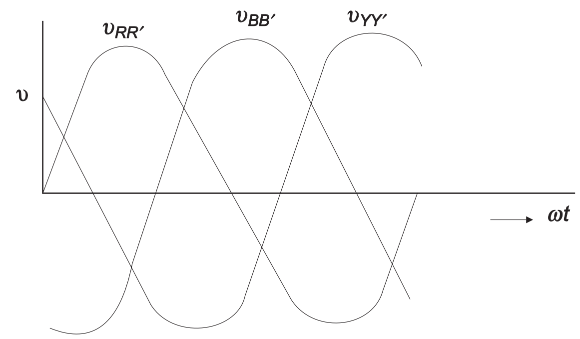


Fig. 9.5

$$v_{RR'} = V_m \sin \omega t$$
$$v_{BB'} = V_m \sin (\omega t - 120°)$$
$$v_{YY'} = V_m \sin (\omega t - 240°) \text{ or } V_m \sin (\omega t + 120°)$$

Example 9.2 What is the possible number of phase sequences in Fig. 9.4. What are they?

Solution There are only two possible phase sequences; they are *RBY*, and *RYB*.

9.5 INTER CONNECTION OF THREE-PHASE SOURCES AND LOADS

9.5.1 Inter Connection of Three-phase Sources

In a three-phase alternator, there are three independent phase windings or coils. Each phase or coil has two terminals, viz. *start* and *finish*. The end connections of the three sets of the coils may be brought out of the machine, to form three separate single phase sources to feed three individual circuits as shown in Fig. 9.6 (a and b).

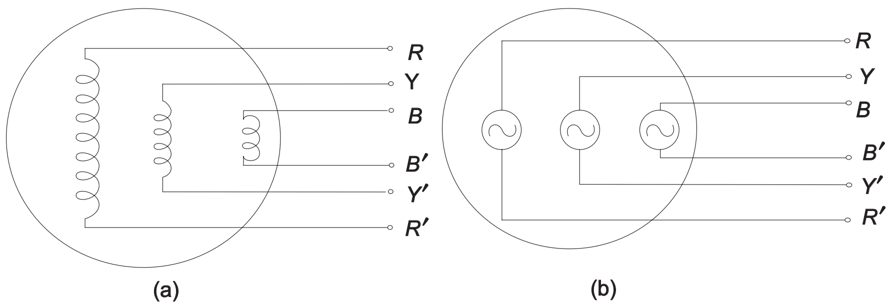


Fig. 9.6

The coils are inter-connected to form a wye (Y) or delta (Δ) connected three-phase system to achieve economy and to reduce the number of conductors, and

thereby, the complexity in the circuit. The three-phase sources so obtained serve all the functions of the three separate single phase sources.

9.5.2 Wye or Star-Connection

In this connection, similar ends (*start* or *finish*) of the three phases are joined together within the alternator as shown in Fig. 9.7. The common terminal so formed is referred to as the neutral point (N), or neutral terminal. Three lines are run from the other free ends (R, Y, B) to feed power to the three-phase load.

Figure 9.7 represents a three-phase, four-wire, star-connected system. The terminals R, Y and B are called the *line terminals* of the source. The voltage between any line and the neutral point is called the *phase voltage* (V_{RN}, V_{YN} and V_{BN}), while the voltage between any two lines is called the *line voltage* (V_{RY}, V_{YB} and V_{BR}). The currents flowing through the phases are called the phase currents, while those flowing in the lines are called the line currents. If the neutral wire is not available for external connection, the system is called a three-phase, three-wire, star-connected system. The system so formed will supply equal line voltages displaced 120° from one another and acting simultaneously in the circuit like three independent single phase sources in the same frame of a three-phase alternator.

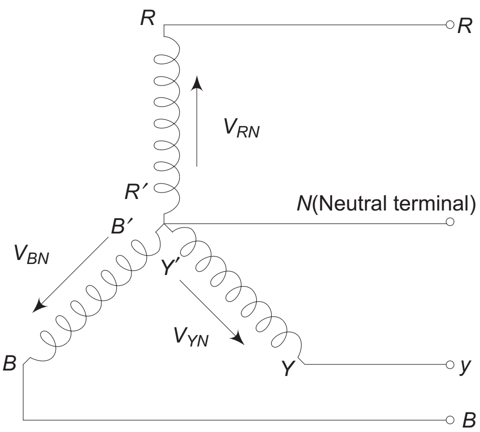


Fig. 9.7

Example 9.3 Figure 9.8 represents three phases of an alternator. Arrange the possible number of three-phase star connections and indicate phase voltages and line voltages in each case. ($V_{RR'} = V_{YY'} = V_{BB'}$)

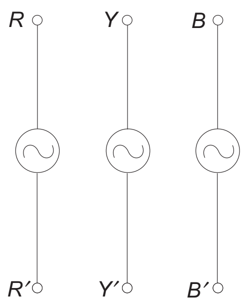


Fig. 9.8

Solution There are two possible star-connections and they can be arranged as shown in Fig. 9.9(a).

The phase voltages are

$$V_{RN}, V_{YN}, V_{BN} \text{ and } V_{R'N}, V_{Y'N}, V_{B'N}$$

The line voltages are

$$V_{RY}, V_{YB}, V_{BR} \text{ and } V_{R'Y'}, V_{Y'B'}, V_{B'R'}$$

Note The phases can also be arranged as shown in Fig. 9.9(b), in which case they do not look like a star; so the name star or wye-connection is only a convention.

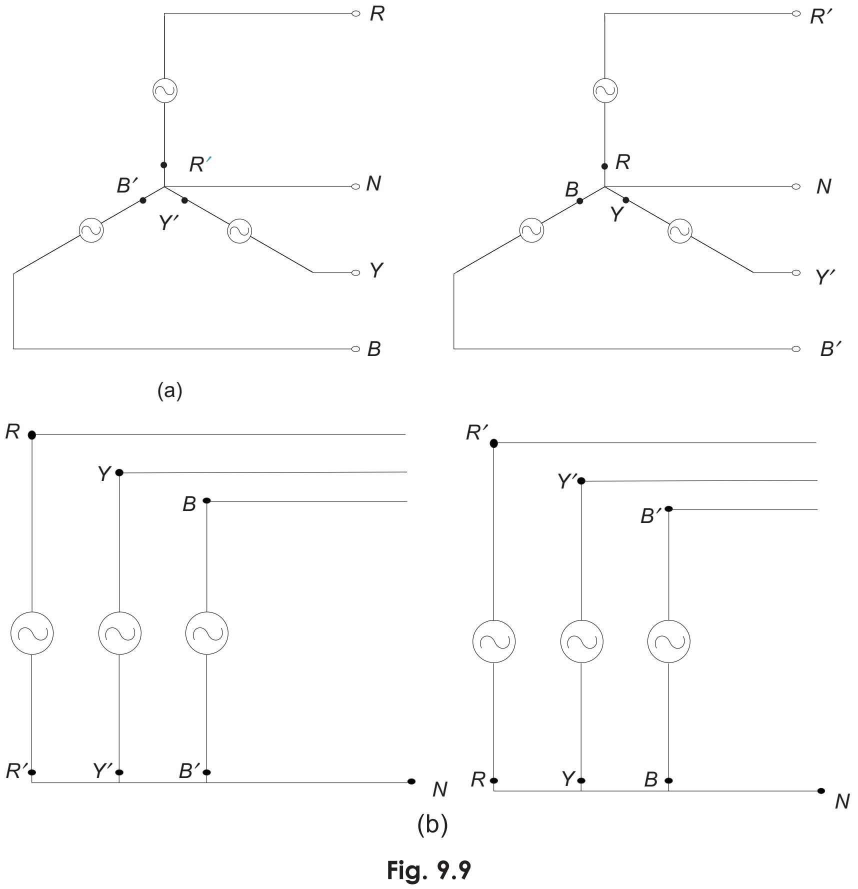


Fig. 9.9

9.5.3 Delta or Mesh-connection

In this method of connection the dissimilar ends of the windings are joined together, i.e. R' is connected to Y, Y' to B and B' to R as shown in Fig. 9.10.

The three line conductors are taken from the three junctions of the mesh or delta connection to feed the three-phase load. This constitutes a three-phase, three-wire, delta-connected system. Here there is no common terminal; only three line voltages V_{RY}, V_{YB} and V_{BR} are available.

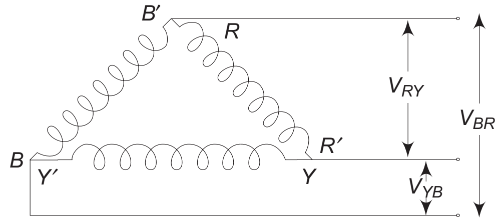


Fig. 9.10

These line voltages are also referred to as *phase voltages* in the delta-connected system. When the sources are connected in delta, loads can be connected only across the three line terminals, R, Y and B. In general, a three-phase source, star or delta, can be either balanced or unbalanced. A balanced three-phase source is one in which the three individual sources have equal magnitude, with 120° phase difference as shown in Fig. 9.3(d).

Example 9.4 Figure 9.11 represents three phases of an alternator. Arrange the possible number of three-phase, delta connections and indicate phase voltages and line voltages in each case (*Note* $V_{RR'} = V_{YY'} = V_{BB'}$).

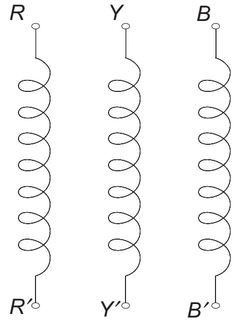


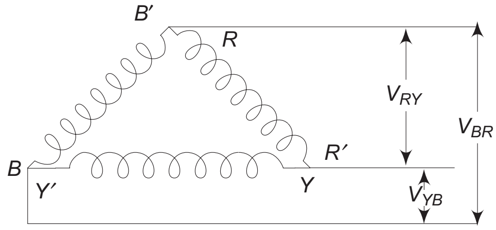
Fig. 9.11

Solution There are two possible delta connections which are shown as follows.

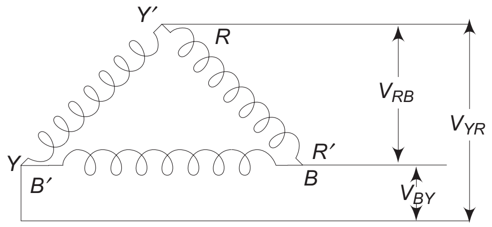
$$V_{\text{phase}} = V_{\text{line}}$$

The line voltages are

From Fig. 9.12(a) V_{RY}, V_{YB} and V_{BR} and V_{RB}, V_{BY} and V_{YR} from Fig. 9.12(b).



(a)

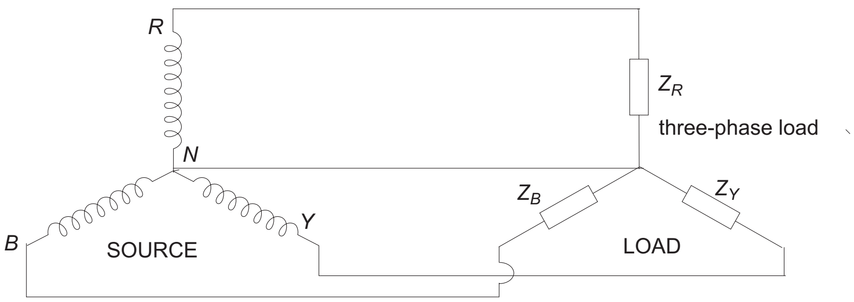


(b)

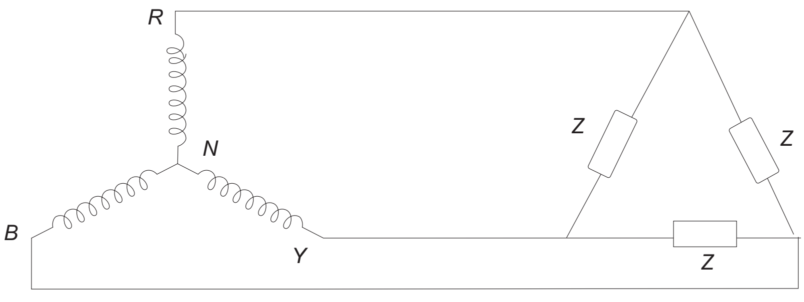
Fig. 9.12

9.5.4 Interconnection of Loads

The primary question in a star or delta-connected three-phase supply is how to apply the load to the three-phase supply. An impedance, or load, connected across any two terminals of an active network (source) will draw power from the source, and is called a single phase load. Like alternator phase windings, load can also be connected across any two terminals, or between line and neutral terminal (if the source is star-connected). Usually the three-phase load impedances are connected in star or delta formation, and then connected to the three-phase source as shown in Fig. 9.13.



(a)



(d)

Fig. 9.13

Figure 9.13(a) represents the typical inter-connections of loads and sources in a three-phase star system, and is of a three-phase four wire system. A three-

phase star connected load is connected to a three-phase star-connected source, terminal to terminal, and both the neutrals are joined with a fourth wire. Figure 9.13(b) is a three-phase, three-wire system. A three-phase, delta-connected load is connected to a three-phase star-connected source, terminal to terminal, as shown in Fig. 9.13(b). When either source or load, or both are connected in delta, only three wires will suffice to connect the load to source.

Just as in the case of a three-phase source, a three-phase load can be either balanced or unbalanced. A balanced three-phase load is one in which all the branches have identical impedances, i.e. each impedance has the same magnitude and phase angle. The resistive and reactive components of each phase are equal. Any load which does not satisfy the above requirements is said to be an unbalanced load.

Example 9.5 Draw the inter-connection between a three-phase, delta-connected source and a star-connected load.

Solution When either source or load, or both are connected in delta, only three wires are required to connect, the load to source, and the system is said to be a three-phase, three-wire system. The connection diagram is shown in Fig. 9.14.

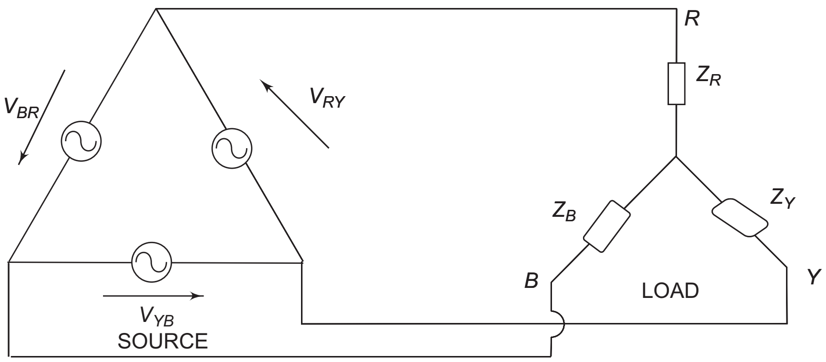


Fig. 9.14

The three line voltages are V_{RY}, V_{YB} and V_{BR}.

Example 9.6 Draw the inter-connection between a three-phase, delta-connected source and delta-connected load.

Solution Since the source and load are connected in delta, it is a three-wire system. The connection diagram is shown in Fig. 9.15.

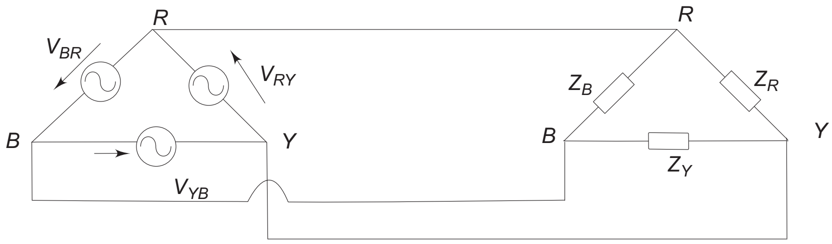


Fig. 9.15

9.6 STAR TO DELTA AND DELTA TO STAR TRANSFORMATION

While dealing with currents and voltages in loads, it is often necessary to convert a star load to delta load, and vice-versa. It has already been shown in Chapter 3 that delta (Δ) connection of resistances can be replaced by an equivalent star (Y) connection and vice-versa. Similar methods can be applied in the case of networks containing general impedances in complex form. So also with ac, where the same formulae hold good, except that resistances are replaced by the impedances. These formulae can be applied even if the loads are unbalanced. Thus, considering Fig. 9.16(a), star load can be replaced by an equivalent delta-load with branch impedances as shown.

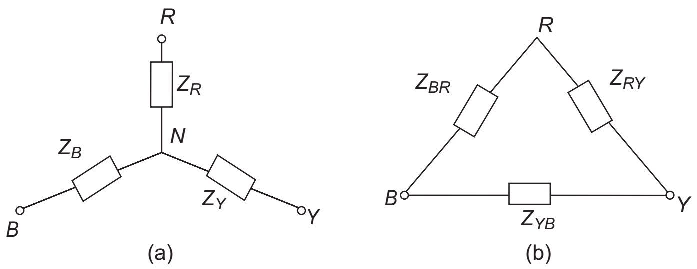


Fig. 9.16

Delta impedances, in terms of star impedances, are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

and $$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

The converted network is shown in Fig. 9.16(b). Similarly, we can replace the delta load of Fig. 9.16(b) by an equivalent star load with branch impedances as

$$Z_R = \frac{Z_{RY}\ Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_Y = \frac{Z_{RY}\ Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

and $$Z_B = \frac{Z_{BR}\ Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

It should be noted that all impedances are to be expressed in their complex form.

Example 9.7 A symmetrical three-phase, three-wire 440 V supply is connected to a star-connected load as shown in Fig. 9.17(a). The impedances in each branch are $Z_R = (2 + J3)\ \Omega$, $Z_Y = (1 - J2)\ \Omega$ and $Z_B = (3 + J4)\ \Omega$. Find its equivalent delta-connected load. The phase sequence is *RYB*.

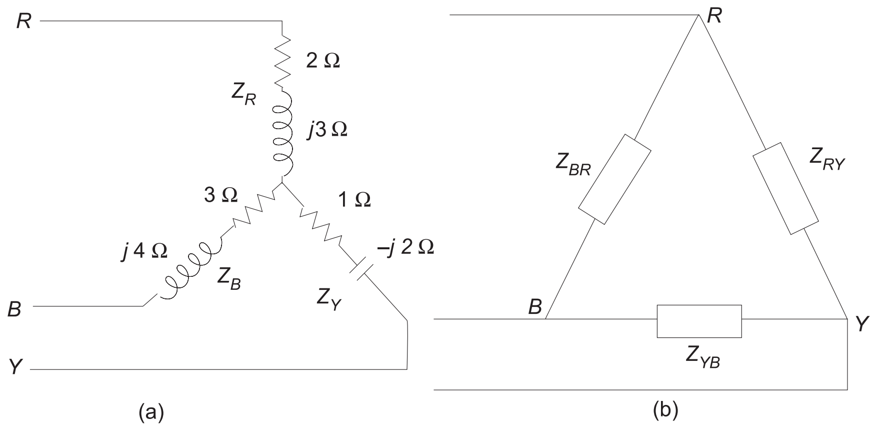


Fig. 9.17

Solution The equivalent delta network is shown in Fig. 9.17(b). From Section 9.6, we can write the equations to find Z_{RY}, Z_{YB} and Z_{BR}. We have

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_R = 2 + j3 = 3.61 \angle 56.3°$$

$$Z_Y = 1 - j2 = 2.23 \angle - 63.4°$$

$$Z_B = 3 + j4 = 5 \angle 53.13°$$

$$\begin{aligned} Z_R Z_Y + Z_Y Z_B + Z_B Z_R &= (3.61 \angle 56.3°)\,(2.23 \angle - 63.4°) \\ &\quad + (2.23\angle - 63.4°)\,(5\angle 53.13°) + (5\angle 53.13°)\,(3.61\angle 56.3°) \\ &= 8.05 \angle - 7.1° + 11.15 \angle - 10.27° + 18.05 \angle 109.43° \\ &= 12.95 + j14.04 = 19.10 \angle 47.3° \end{aligned}$$

$$Z_{RY} = \frac{19.10 \angle 47.3°}{5 \angle 53.13°} = 3.82 \angle - 5.83° = 3.8 - j0.38$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$= \frac{19.10 \angle 47.3°}{3.61 \angle 56.3°} = 5.29 \angle - 9° = 5.22 - j0.82$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

$$= \frac{19.10\ \angle 47.3^\circ}{2.23\ \angle -63.4^\circ} = 8.56\ \angle 110.7^\circ = -3.02 + j8$$

The equivalent delta impedances are

$$Z_{RY} = (3.8 - j0.38)\ \Omega$$
$$Z_{YB} = (5.22 - j0.82)\ \Omega$$
$$Z_{BR} = (-3.02 + j8)\ \Omega$$

Example 9.8 A symmetrical three-phase, three-wire 400 V, supply is connected to a delta-connected load as shown in Fig. 9.18(a). Impedances in each branch are $Z_{RY} = 10\ \angle 30^\circ\ \Omega$; $Z_{YB} = 10\ \angle -45^\circ\ \Omega$ and $Z_{BR} = 2.5\ \angle 60^\circ\ \Omega$. Find its equivalent star-connected load; the phase sequence is *RYB*.

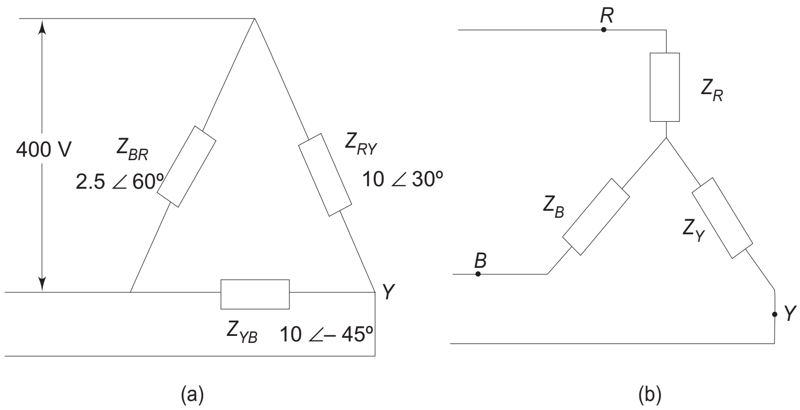


Fig. 9.18

Solution The equivalent star network is shown in Fig. 9.18(b). From Section 9.6, we can write the equations to find Z_R, Z_Y and Z_B as

$$Z_R = \frac{Z_{RY}\ Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_{RY} + Z_{YB} + Z_{BR} = 10\ \angle 30^\circ + 10\ \angle -45^\circ + 2.5\ \angle 60^\circ$$
$$= (8.66 + j5) + (7.07 - j7.07) + (1.25 + j2.17)$$
$$= 16.98 + j0.1 = 16.98\ \angle 0.33^\circ\ \Omega$$

$$Z_R = \frac{(10 \quad 30)(2.5 \quad 60)}{16.98 \quad 0.33} = 1.47\ \angle 89.67^\circ$$

$$= (0.008 + j1.47)\ \Omega$$

$$Z_Y = \frac{Z_{RY}\ Z_{YB}}{Z_{RY} \quad Z_{YB} \quad Z_{BR}}$$

$$= \frac{(10 \quad 30)(10 \quad 45)}{16.98 \quad 0.33} = 5.89\ \angle -15.33^\circ\ \Omega$$

$$Z_B = \frac{Z_{BR}\ Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

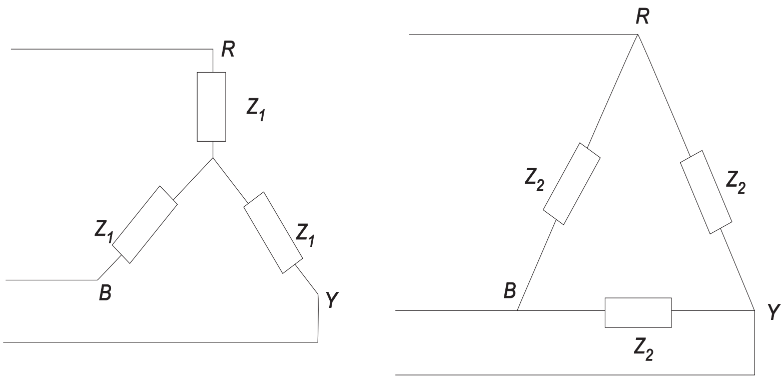
$$= \frac{(2.5 \quad 60\ )(10 \quad\quad 45\ )}{16.98 \quad 0.33} = 1.47\ \angle 14.67°\ \Omega$$

The equivalent star impedances are

$Z_R = 1.47\ \angle 89.67°\ \Omega$, $Z_Y = 5.89\ \angle -15.33°\ \Omega$ and $Z_B = 1.47\ \angle 14.67°\ \Omega$

9.6.1 Balanced Star-Delta and Delta-Star Conversion

If the three-phase load is balanced, then the conversion formulae in Section 9.6 get simplified. Consider a balanced star-connected load having an impedance Z_1 in each phase as shown in Fig. 9.19(a).



(a) (b)

Fig. 9.19

Let the equivalent delta-connected load have an impedance of Z_2 in each phase as shown in Fig. 9.19(b). Applying the conversion formulae from Section 9.6 for delta impedances in terms of star impedances, we have

$$Z_2 = 3Z_1$$

Similarly, we can express star impedances in terms of delta $Z_1 = Z_2/3$.

Example 9.9 Three identical impedances are connected in delta as shown in Fig. 9.20(a). Find an equivalent star network such that the line current is the same when connected to the same supply.

Solution The equivalent star network is shown in Fig. 9.20(b). From Section 9.6.1, we can write the equation to find $Z_1 = Z_2/3$

$$Z_2 = 3 + j4 = 5\ \angle 53.13°\ \Omega$$

$$\therefore \qquad Z_1 = \frac{5}{3}\ \angle 53.13° = 1.66\ \angle 53.15° = (1.0 + j1.33)\ \Omega$$

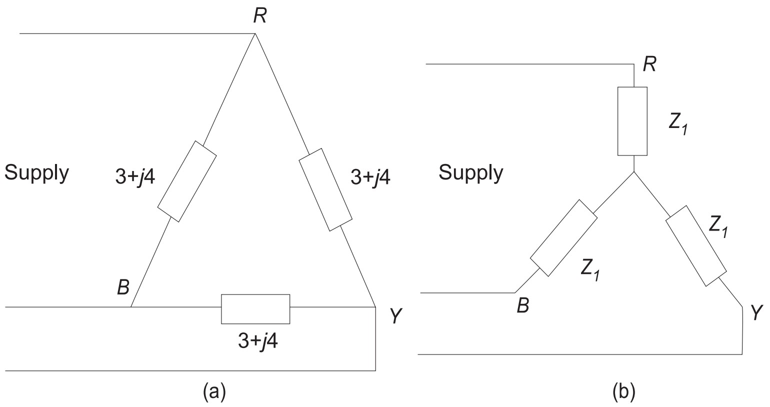


Fig. 9.20

9.7 VOLTAGE, CURRENT AND POWER IN A STAR CONNECTED SYSTEM

9.7.1 Star-Connected System

Figure 9.21 shows a balanced three-phase, Y-connected system. The voltage induced in each winding is called the phase voltage (V_{Ph}). Likewise V_{RN}, V_{YN} and V_{BN} represent the rms values of the induced voltages in each phase. The voltage available between any pair of terminals is called the *line voltage* (V_L). Likewise V_{RY}, V_{YB} and V_{BR} are known as *line voltages*. The double subscript notation is purposefully used to represent voltages and currents in polyphase circuits. Thus, V_{RY} indicates a voltage V between points R and Y, with R being positive with respect to point Y during its positive half cycle.

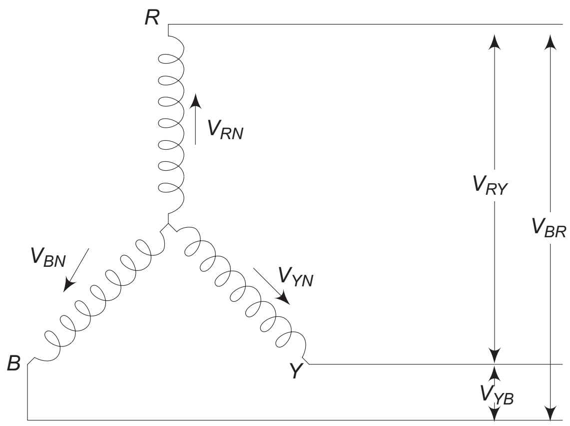


Fig. 9.21

Similarly, V_{YB} means that Y is positive with respect to point B during its positive half cycle; it also means that $V_{RY} = -V_{YR}$.

9.7.2 Voltage Relation

The phasors corresponding to the phase voltages constituting a three-phase system can be represented by a phasor diagram as shown in Fig. 9.22.

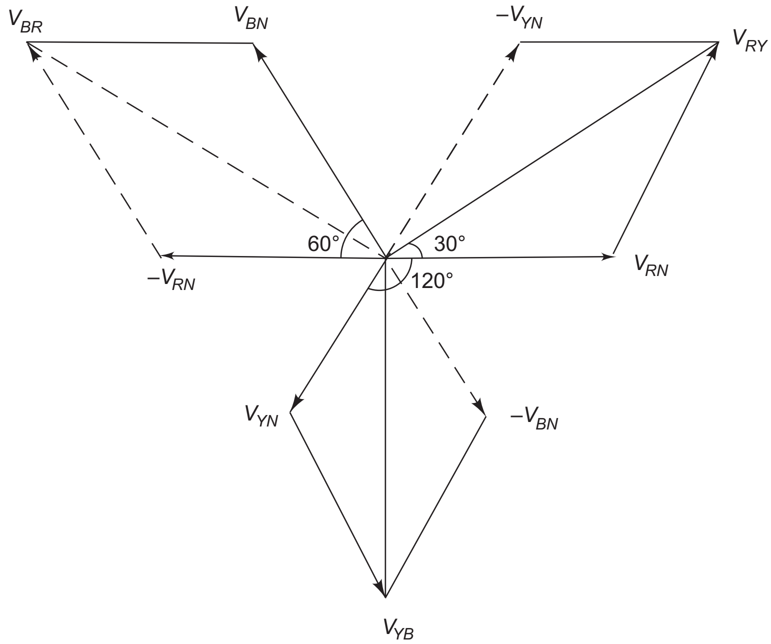


Fig. 9.22

From Fig. 9.22, considering the lines R, Y and B, the line voltage V_{RY} is equal to the phasor sum of V_{RN} and V_{NY} which is also equal to the phasor difference of V_{RN} and V_{YN} ($V_{NY} = -V_{YN}$). Hence, V_{RY} is found by compounding V_{RN} and V_{YN} reversed. To subtract V_{YN} from V_{RN}, we reverse the phasor V_{YN} and find its phasor sum with V_{RN} as shown in Fig. 9.22. The two phasors, V_{RN} and $-V_{YN}$, are equal in length and are 60° apart.

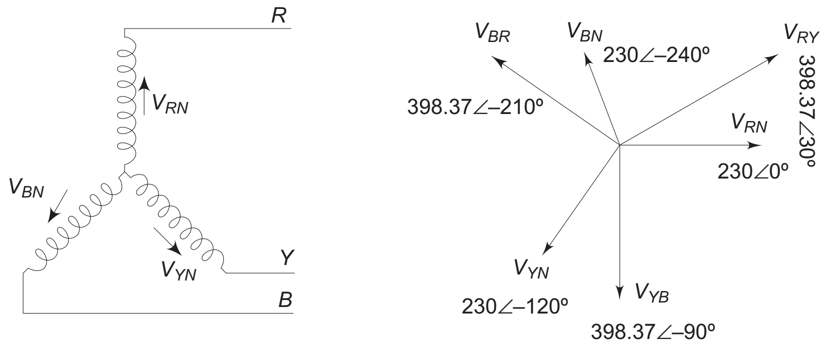
$$|V_{RN}| = -|V_{YN}| = V_{Ph}$$

$$\therefore \qquad V_{RY} = 2V_{Ph} \cos 60/2 = \sqrt{3}\ V_{Ph}$$

Similarly, the line voltage V_{YB} is equal to the phasor difference of V_{YN} and V_{BN}, and is equal to $\sqrt{3}\ V_{Ph}$. The line voltage V_{BR} is equal to the phasor difference of V_{BN} and V_{RN}, and is equal to $\sqrt{3}\ V_{Ph}$. Hence, in a balanced star-connected system

(i) Line voltage = $\sqrt{3}\ V_{Ph}$
(ii) All line voltages are equal in magnitude and are displaced by 120°, and
(iii) All line voltages are 30° ahead of their respective phase voltages (from Fig. 9.22).

Example 9.10 A symmetrical star-connected system is shown in Fig. 9.23(a). Calculate the three line voltages, given $V_{RN} = 230\ \angle 0°$. The phase sequence is *RYB*.



(a) (b)

Fig. 9.23

Solution Since the system is a balanced system, all the phase voltages are equal in magnitude, but displaced by 120° as shown in Fig. 9.23(b).

$$\therefore \quad V_{RN} = 230\ \angle 0°\ \text{V}$$

$$V_{YN} = 230\ \angle -120°\ \text{V}$$

$$V_{BN} = 230\ \angle -240°\ \text{V}$$

Corresponding line voltages are equal to $\sqrt{3}$ times the phase voltages, and are 30° ahead of the respective phase voltages.

$$\therefore \quad V_{RY} = \sqrt{3} \times 230\ \angle 0 + 30°\ \text{V} = 398.37\ \angle 30°\ \text{V}$$

$$V_{YB} = \sqrt{3} \times 230\ \angle -120° + 30°\ \text{V} = 398.37\ \angle -90°\ \text{V}$$

$$V_{BR} = \sqrt{3} \times 230\ \angle -240° + 30°\ \text{V} = 398.37\ \angle -210°\ \text{V}$$

9.7.3 Current Relations

Figure 9.24(a) shows a balanced three-phase, wye-connected system indicating phase currents and line currents. The arrows placed alongside the currents I_R, I_Y and I_B flowing in the three phases indicate the directions of currents when they are assumed to be positive and *not* the directions at that particular instant. The phasor diagram for phase currents with respect to their phase voltages is shown in Fig. 9.24(b). All the phase currents are displaced by 120° with respect to each other, 'ϕ' is the phase angle between phase voltage and phase current (lagging load is assumed). For a balanced load, all the phase currents are equal in magnitude. It can be observed from Fig. 9.24(a) that each line conductor is connected in series with its individual phase winding. Therefore, the current in a

line conductor is the same as that in the phase to which the line conductor is connected.

$\therefore \qquad I_L = I_{Ph} = I_R = I_Y = I_B$

It can be observed from Fig. 9.24(b) that the angle between the line (phase) current and the corresponding line voltage is $(30 + \phi)°$ for a lagging load. Consequently, if the load is leading, then the angle between the line (phase) current and corresponding line voltage will be $(30 - \phi)°$.

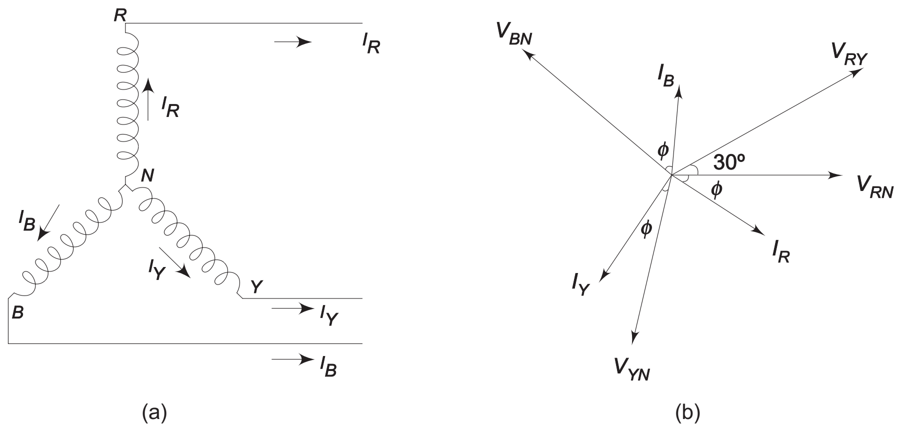


Fig. 9.24

Example 9.11 In Fig. 9.24(a), the value of the current in phase R is $I_R = 10 \angle 20°$ A. Calculate the values of the three line currents. Assume an RYB phase sequence.

Solution In a balanced star-connected system $I_L = I_{Ph}$, and is displaced by 120°. Therefore the three line currents are

$$I_R = 10 \angle 20° \text{ A}$$
$$I_Y = 10 \angle 20° - 120° \text{ A} = 10 \angle -100° \text{ A}$$
$$I_B = 10 \angle 20° - 240° \text{ A} = 10 \angle -220° \text{ A}$$

9.7.4 Power in the Star-Connected Network

The total active power or true power in the three-phase load is the sum of the powers in the three phases. For a balanced load, the power in each load is the same; hence total power = 3 × power in each phase

or $\qquad P = 3 \times V_{Ph} \times I_{Ph} \cos \phi$

It is the usual practice to express the three-phase power in terms of line quantities as follows.

$$V_L = \sqrt{3}\ V_{\text{Ph}}, I_L = I_{Ph}$$
$$P = \sqrt{3}\ V_L I_L \cos \phi\ W$$

or $\sqrt{3}\ V_L I_L \cos\phi$ is the active power in the circuit.

Total reactive power is given by

$$Q = \sqrt{3}\ V_L I_L \sin\phi \text{ VAR}$$

Total apparent power or volt-amperes

$$= \sqrt{3}\ V_L I_L\ VA$$

9.7.5 N-Phase Star System

It is to be noted that star and mesh are general terms applicable to any number of phases; but wye and delta are special cases of star and mesh when the system is a three-phase system. Consider an n-phase balanced star system with two adjacent phases as shown in Fig. 9.25(a). Its vector diagram is shown in Fig. 9.25(b).

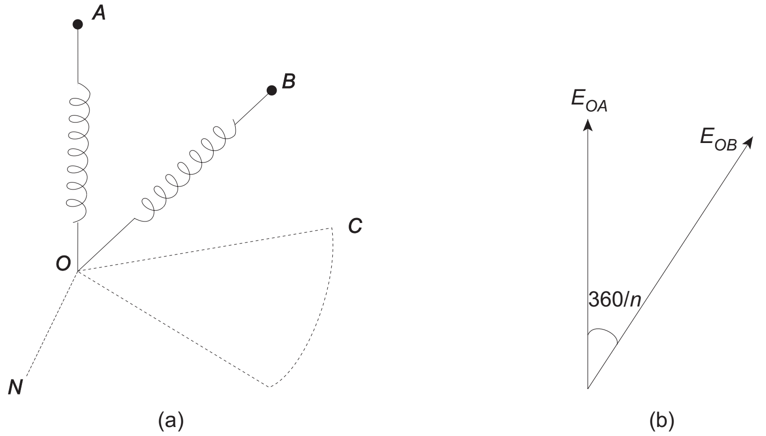


Fig. 9.25

The angle of phase difference between adjacent phase voltages is 360°/n. Let E_{Ph} be the voltage of each phase. The line voltage, i.e. the voltage between A and B is equal to $E_{AB} = E_L = E_{AO} + E_{OB}$. The vector addition is shown in Fig. 9.25 (c). It is evident that the line current and phase current are same.

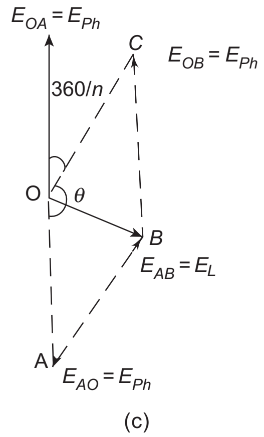


Fig. 9.25

$$E_{AB} = E_{AO} + E_{OB}$$

Consider the parallelogram $OABC$.

$$OB = \sqrt{OC^2 + OA^2 + 2 \times OA \times OC \times \cos\theta}$$

$$= \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph}^2 \cos\left(180° - \frac{360°}{n}\right)}$$

$$= \sqrt{2E_{ph}^2 \quad 2E_{ph}^2 \cos\frac{360}{n}}$$

$$= \sqrt{2}\ E_{ph} \sqrt{\left[1 - \cos 2\left(\frac{180°}{n}\right)\right]}$$

$$= \sqrt{2}\ E_{ph} \sqrt{2 \sin^2\left(\frac{180°}{n}\right)}$$

$$E_L = 2E_{ph} \sin\frac{180}{n}$$

The above equation is a general equation for line voltage, for example, for a three-phase system, $n = 3$; $E_L = 2\ E_{ph} \sin 60° = \sqrt{3}\ \ E_{ph}$.

Example 9.12 A balanced star-connected load of (4 + *J*3) Ω per phase is connected to a balanced 3-phase 400 V supply. The phase current is 12 A. Find (i) the total active power (ii) reactive power and (iii) total apparent power.

Solution The voltage given in the data is always the rms value of the line voltage unless otherwise specified.

$$\therefore \qquad Z_{Ph} = \sqrt{4^2 + 3^2} = 5\ \Omega$$

$$\text{PF} = \cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{4}{5} = 0.8$$

$$\sin\phi = 0.6$$

(i) Active power $= \sqrt{3}\ \ V_L I_L \cos\phi$ W

$= \sqrt{3} \times 400 \times 12 \times 0.8 = 6651$ W

(ii) Reactive power $= \sqrt{3}\ \ V_L\ I_L \sin\phi$ VAR

$= \sqrt{3} \times 400 \times 12 \times 0.6 = 4988.36$ VAR

(iii) Apparent power $= \sqrt{3}\ \ V_L I_L$

$= \sqrt{3} \times 400 \times 12 = 8313.84$ VA

9.8 VOLTAGE, CURRENT AND POWER IN A DELTA CONNECTED SYSTEM

9.8.1 Delta-Connected System

Figure 9.26 shows a balanced three-phase, three-wire, delta-connected system. This arrangement is referred to as mesh connection because it forms a closed circuit. It is also known as delta connection because the three branches in the circuit can also be arranged in the shape of delta (Δ).

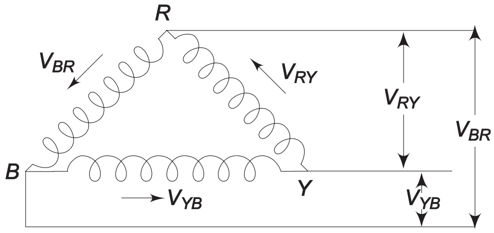


Fig. 9.26

From the manner of interconnection of the three phases in the circuit, it may appear that the three phases are short-circuited among themselves. However, this is not the case. Since the system is balanced, the sum of the three voltages round the closed mesh is zero; consequently, no current can flow around the mesh when the terminals are open.

The arrows placed alongside the voltages, V_{RY}, V_{YB} and V_{BR}, of the three phases indicate that the terminals R, Y and B are positive with respect to Y, B and R, respectively, during their respective positive half cycles.

9.8.2 Voltage Relation

From Fig. 9.27, we notice that only one phase is connected between any two lines. Hence, the voltage between any two lines (V_L) is equal to the phase voltage (V_{Ph}).

$\therefore$ $$V_{RY} = V_L = V_{\text{Ph}}$$

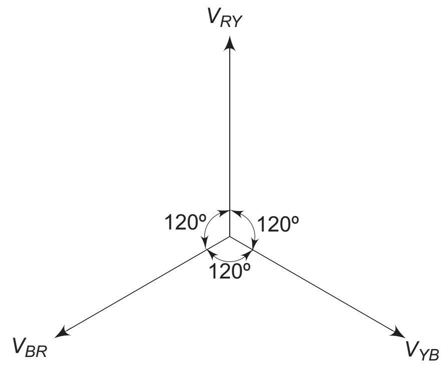


Fig. 9.27

Since the system is balanced, all the phase voltages are equal, but displaced by 120° from one another as shown in the phasor diagram in Fig. 9.27. The phase sequence *RYB* is assumed.

$$\therefore \qquad |V_{RY}| = |V_{YB}| = |V_{BR}| = V_L = V_{Ph}$$

Example 9.13 In Fig. 9.27, the voltage across the terminals R and Y is 400 $\angle 0°$. Calculate the values of the three line voltages. Assume *RYB* phase sequence.

Solution In a balanced delta-connected system, $|V_L| = |V_{\text{Ph}}|$, and is displaced by 120°; therefore the three line voltages are

$$V_{RY} = 400 \angle 0° \text{ V}$$

$$V_{YB} = 400 \angle -120° \text{ V}$$

$$V_{BR} = 400 \angle -240° \text{ V}$$

9.8.3 Current Relation

In Fig. 9.28 we notice that, since the system is balanced, the three phase currents (I_{Ph}), i.e. I_R, I_Y, I_B are equal in magnitude but displaced by 120° from one another as shown in Fig. 9.28(b). I_1, I_2 and I_3 are the line currents (I_L), i.e. I_1 is the line current in line 1 connected to the common point of R. Similarly, I_2 and I_3 are the line currents in lines 2 and 3, connected to common points Y and B, respectively. Though here all the line currents are directed outwards, at no instant will all the three line currents flow in the same direction, either outwards or inwards. Because the three line currents are displaced 120° from one another, when one is positive, the other two might both be negative, or one positive and one negative. Also it is to be noted that arrows placed alongside phase currents in Fig. 9.28(a), indicate the direction of currents when they are assumed to be positive and not their actual direction at a particular instant. We can easily determine the line currents in Fig. 9.28(a), I_1, I_2 and I_3 by applying KCL at the three terminals R, Y and B, respectively. Thus, the current in line 1, $I_1 = I_R - I_B$; i.e. the current in any line is equal to the phasor difference of the currents in the two phases attached to that line. Similarly, the current in line 2, $I_2 = I_Y - I_R$, and the current in line 3, $I_3 = I_B - I_Y$.

The phasor addition of these currents is shown in Fig. 9.28(b). From the figure,

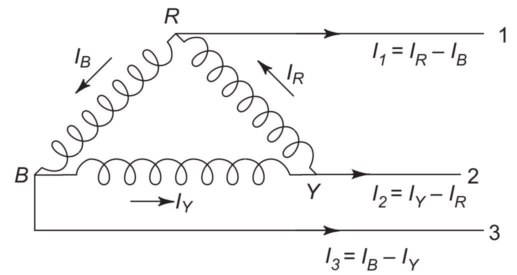
$$I_1 = I_R - I_B$$

$$I_1 = \sqrt{I_R^2 + I_B^2 + 2I_R I_B \cos 60°}$$

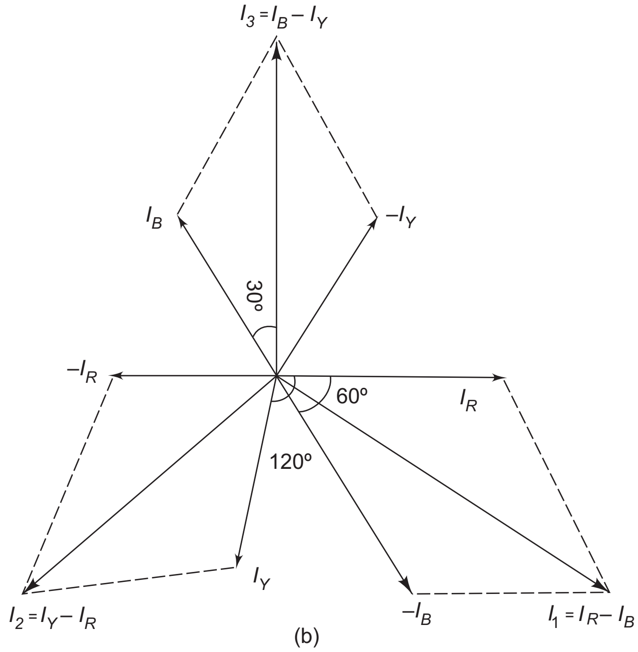
$$I_1 = \sqrt{3}\ I_{\text{Ph}}, \text{ since } I_R = I_B = I_{Ph}$$

Similarly, the remaining two line currents, I_2 and I_3, are also equal to $\sqrt{3}$ times the phase currents; i.e. $I_L = \sqrt{3}\ I_{Ph}$.

As can be seen from Fig. 9.28(b), all the line currents are equal in magnitude but displaced by 120° from one another; and the line currents are 30° behind the respective phase currents.



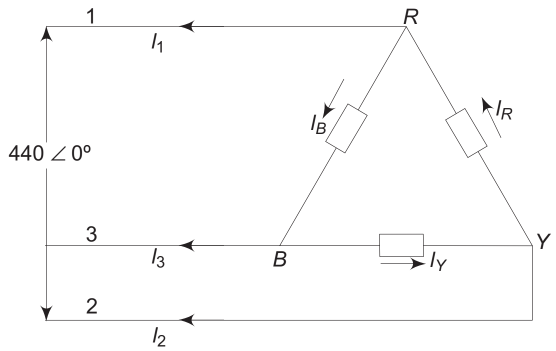
(a)



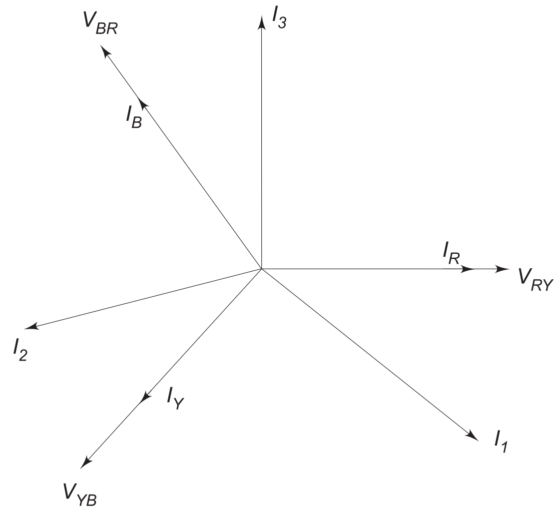
(b)

Fig. 9.28

Example 9.14 Three identical loads are connected in delta to a three-phase supply of 440 $\angle 0°$ V as shown in Fig. 9.29(a). If the phase current I_R is 15 $\angle 0°$ A, calculate the three line currents.



(a)



(b)

Fig. 9.29

Solution All the line currents are equal and 30° behind their respective phase currents, and $\sqrt{3}$ times their phase values, displaced by 120° from one another, assuming RYB phase sequence.

Let the line currents in line 1, 2 and 3 be I_1, I_2 and I_3, respectively.

$$I_1 = \sqrt{3} \times I_R \angle(\phi - 30°)$$
$$= \sqrt{3} \times 15 \angle -30° = 25.98 \angle -30° \text{ A}$$
$$I_2 = \sqrt{3} \times 15 \angle(-30 - 120)° = 25.98 \angle -150° \text{ A}$$
$$I_3 = \sqrt{3} \times 15 \angle(-30 - 240)° = 25.98 \angle -270° \text{ A}$$

The phasor diagram is shown in Fig. 9.29(b).

9.8.4 Power in the Delta-Connected System

Obviously the total power in the delta circuit is the sum of the powers in the three phases. Since the load is balanced, the power consumed in each phase is the same. Total power is equal to three times the power in each phase.

$$\text{Power per phase} = V_{Ph}\, I_{Ph} \cos\phi$$

where ϕ is the phase angle between phase voltage and phase current.

$$\text{Total power } P = 3 \times V_{Ph}\, I_{Ph} \cos\phi$$

In terms of line quantities

$$P = \sqrt{3}\; V_L I_L \cos\phi \text{ W}$$

Since $$V_{\text{Ph}} = V_L \text{ and } I_{\text{Ph}} = \frac{I_L}{\sqrt{3}}$$

for a balanced system, whether star or delta, the expression for the total power is the same.

Example 9.15 A balanced delta-connected load of $(2 + j3)\ \Omega$ per phase is connected to a balanced three-phase 440 V supply. The phase current is 10 A. Find the (i) total active power (ii) reactive power and (iii) apparent power in the circuit.

Solution $$Z_{\text{Ph}} = \sqrt{(2)^2 + (3)^2} = 3.6\ \angle 56.3^\circ\ \Omega$$

$$\cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{2}{3.6} = 0.55$$

So, $$\sin\phi = 0.83$$

$$I_L = \sqrt{3} \times I_{\text{Ph}} = \sqrt{3} \times 10 = 17.32 \text{ A}$$

(i) Active power $= \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3} \times 440 \times 17.32 \times 0.55 = 7259.78$ W

(ii) Reactive power $= \sqrt{3}\ V_L I_L \sin\phi$

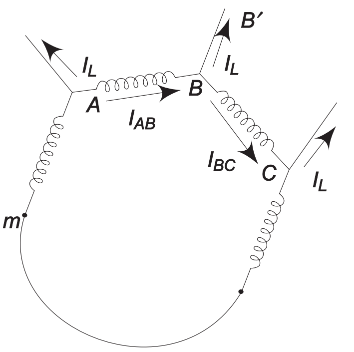
$= \sqrt{3} \times 440 \times 17.32 \times 0.83 = 10955.67$ VAR

(iii) Apparent power $= \sqrt{3}\ V_L I_L$

$= \sqrt{3} \times 440 \times 17.32 = 13199.61$ VA

9.8.5 N-Phase Mesh System

Figure 9.30(a) shows part of an n-phase balanced mesh system. Its vector diagram is shown in Fig. 9.30(b).



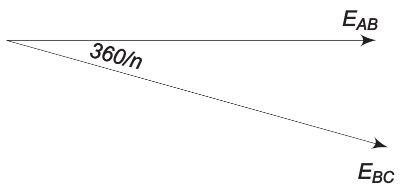


Fig. 9.30(a) **Fig. 9.30(b)**

Let the current in line BB' be I_L. This is same in all the remaining lines of the n-phase system. I_{AB}, I_{BC} are the phase currents in AB and BC phases respectively. The vector addition of the line current is shown in Fig. 9.30(c). It is evident from the Fig. 9.30(b) that the line and phase voltages are equal.

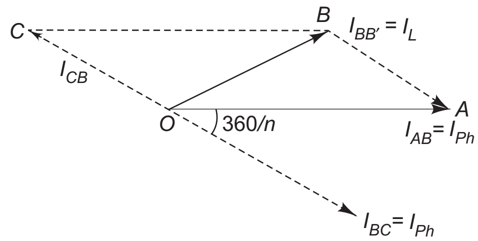


Fig. 9.30(c)

$$I_{BB} = I_L = I_{AB} + I_{CB}$$
$$= I_{AB} - I_{BC}$$

Consider the parallelogram OABC.

$$OB = \sqrt{OA^2 + OC^2 + 2 \times OA \times OC \times \cos\left(180 - \frac{360}{n}\right)}$$
$$= \sqrt{I_{Ph}^2 \quad I_{Ph}^2 \quad 2I_{Ph}^2 \cos\frac{360}{n}}$$
$$= \sqrt{2}\ I_{Ph}\sqrt{1 \quad \cos 2\left(\frac{180}{n}\right)}$$
$$= \sqrt{2}\ I_{Ph}\sqrt{2\sin^2\frac{180}{n}}$$
$$I_L = 2I_{ph}\sin\frac{180}{n}$$

The above equation is a general equation for the line current in a balanced n-phase mesh system.

9.9 THREE-PHASE BALANCED CIRCUITS

The analysis of three-phase balanced systems is presented in this section. It is no way different from the analysis of AC systems in general. The relation between voltages, currents and power in delta-connected and star-connected systems has already been discussed in the previous sections. We can make use of those relations and expressions while solving the circuits.

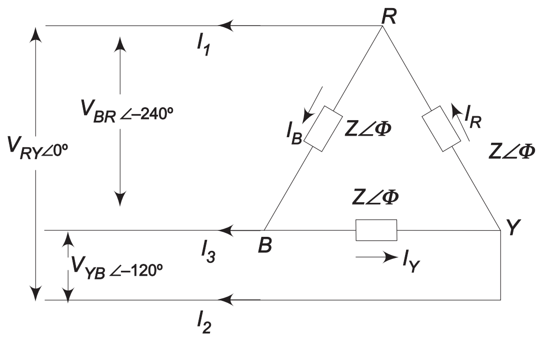
9.9.1 Balanced Three-Phase System-Delta Load

Figure 9.31(a) shows a three-phase, three-wire, balanced system supplying power to a balanced three-phase delta load. The phase sequence is RYB. We are required to find out the currents in all branches and lines.

Let us assume the line voltage $V_{RY} = V\angle 0°$ as the reference phasor. Then the three source voltages are given by

$$V_{RY} = V\angle 0° \text{ V}$$
$$V_{YB} = V\angle -120° \text{ V}$$
$$V_{BR} = V\angle -240° \text{ V}$$

These voltages are represented by phasors in Fig. 9.31(b). Since the load is delta-connected, the line voltage of the source is equal to the phase voltage of the load. The current in phase RY, I_R will lag (lead) behind (ahead of) the phase voltage V_{RY} by an angle ϕ as dictated by the nature of the load impedance. The angle of lag of I_Y with respect to V_{YB}, as well as the angle of lag of I_B with respect to V_{BR} will be ϕ as the load is balanced. All these quantities are represented in Fig. 9.31(b).



(a)

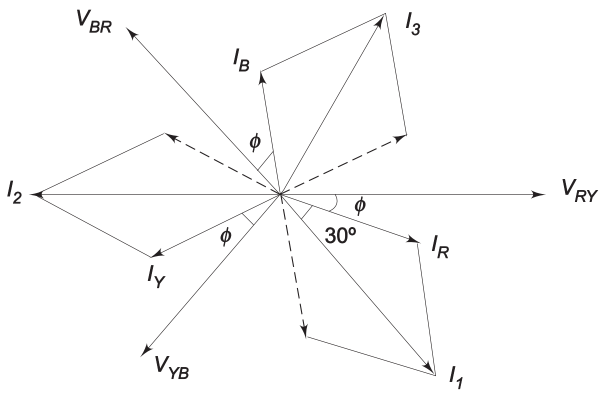


Fig. 9.31(b)

If the load impedance is $Z\angle\phi$, the current flowing in the three load impedances are then

$$I_R = \frac{V_{RY} \quad 0}{Z \quad \phi} = \frac{V}{Z} \angle -\phi$$

$$I_Y = \frac{V_{YB} \quad 120}{Z \quad \phi} = \frac{V}{Z} \angle -120° - \phi$$

$$I_B = \frac{V_{BR} \quad 240}{Z \quad \phi} = \frac{V}{Z} \angle -240° - \phi$$

The line currents are $\sqrt{3}$ times the phase currents, and are 30° behind their respective phase currents.

∴ Current in line 1 is given by

$$I_1 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-\phi - 30°), \text{ or } I_R - I_B \text{ (phasor difference)}$$

Similarly, the current in line 2

$$I_2 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-120 - \phi - 30°),$$

or $I_Y - I_R$ (phasor diference) = $\sqrt{3}\left|\frac{V}{Z}\right| \angle(-\phi - 150)°$, and

$$I_3 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-240 - \phi - 30)°, \text{ or } I_B - I_Y \text{ (phasor difference)}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right| \quad (\quad 270 \quad \phi)$$

To draw all these quantities vectorially, $V_{RY} = V\angle 0°$ is taken as the reference vector.

Example 9.16 A three-phase, balanced delta-connected load of (4 + *j*8) Ω is connected across a 400 V, $3-\phi$ balanced supply. Determine the phase currents and line currents. Assume the phase sequence to be *RYB*. Also calculate the power drawn by the load.

Solution Refering to Fig. 9.31(a), taking the line voltage V_{RY} = V∠0° as reference V_{RY} = 400 ∠0° *V*; V_{YB} = 400 ∠– 120° V, V_{BR} = 400 ∠– 240° *V*

Impedance per phase = (4 + *j*8) Ω = 8.94 ∠63.4° Ω

Phase currents are: $I_R = \dfrac{400 \angle 0°}{8.94 \angle 63.4°} = 44.74 \angle -63.4°$ A

$$I_Y = \frac{400 \quad 120}{8.94 \quad 63.4} = 44.74 \angle -183.4° \text{ A}$$

$$I_B = \frac{400 \quad 240}{8.94 \quad 63.4} = 44.74 \angle -303.4° \text{ A}$$

The three line currents are

$I_1 = I_R - I_B$ = (44.74 ∠– 63.4° – 44.74 ∠– 303.4°)

= (20.03 – *j*40) – (24.62 + *j*37.35) = (– 4.59 – *j*77.35) A

= 77.49 ∠266.6° A

Or the line current I_1 is equal to the $\sqrt{3}$ times the phase current and 30° behind its respective phase current

$$I_1 = \sqrt{3} \times 44.74 \angle -63.4° - 30° = 77.49 \angle -93.4°$$

or $= 77.49 \angle 266.6°$ A

Similarly, $I_2 = I_Y - I_R$

$$= \sqrt{3} \times 44.74 \angle -183.4° - 30° = 77.49 \angle -213.4° \text{ A} = 77.49 \angle 146.6° \text{ A}$$

$$I_3 = I_B - I_Y$$

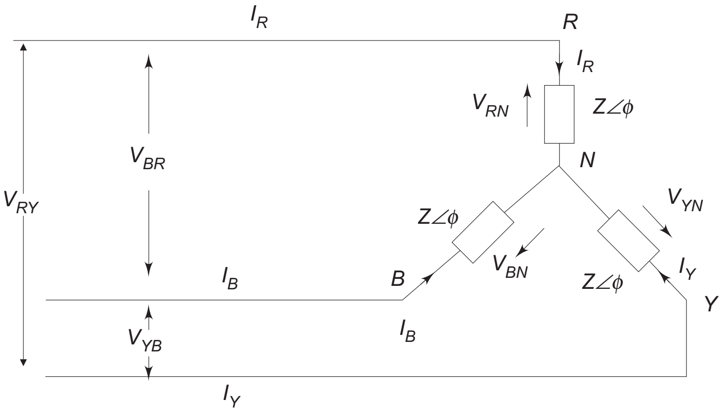
$$= \sqrt{3} \times 44.74 \angle -303.4° - 30° = 77.49 \angle -333.4° \text{ A} = 77.49 \angle 26.6° \text{ A}$$

Power drawn by the load is $P = 3V_{Ph} I_{Ph} \cos \phi$

or $\sqrt{3} \times V_L \times I_L \cos 63.4° = 24.039$ kW

9.9.2 Balanced Three Phase System-Star Connected Load

Figure 9.32(a) shows a three-phase, three wire system supplying power to a balanced three phase star connected load. The phase sequence RYB is assumed.



(a)

Fig. 9.32

In star connection, whatever current is flowing in the phase is also flowing in the line. The three line (phase) currents are I_R, I_Y and I_B.

V_{RN}, V_{YN} and V_{BN} represent three phase voltages of the network, i.e. the voltage between any line and neutral. Let us assume the voltage $V_{RN} = V\angle 0°$ as the reference phasor. Consequently, the phase voltage

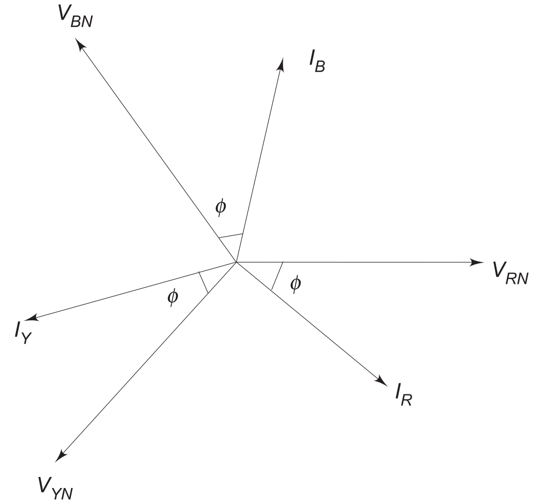
$$V_{RN} = V \angle 0°$$
$$V_{YN} = V \angle -120°$$
$$V_{BN} = V \angle -240°$$

Hence $$I_R = \frac{V_{RN}}{Z\angle\phi} = \frac{V\angle 0}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -\phi$$

$$I_Y = \frac{V_{YN}}{Z\angle\phi} = \frac{V\angle -120}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -120° - \phi$$

$$I_B = \frac{V_{BN}}{Z\angle\phi} = \frac{V\angle -240}{Z\angle\phi} = \left|\frac{V}{Z}\right| \angle -240° - \phi$$

As seen from the above expressions, the currents, I_R, I_Y and I_B, are equal in magnitude and have a 120° phase difference. The disposition of these vectors is shown in Fig. 9.32(b). Sometimes, a 4th wire, called neutral wire is run from the neutral point, if the source is also star-connected. This gives three-phase, four-wire star-connected system. However, if the three line currents are balanced, the current in the fourth wire is zero; removing this connecting wire between the source neutral and load neutral is, therefore, not going to make any change in the condition of the system. The availability of the neutral wire makes it possible to use all the three phase voltages, as well as the three line voltages. Usually, the neutral is grounded for safety and for the design of insulation.



(b)

Fig. 9.32

It makes no difference to the current flowing in the load phases, as well as to the line currents, whether the sources have been connected in star or in delta, provided the voltage across each phase of the delta connected source is $\sqrt{3}$ times the voltage across each phase of the star-connected source.

Example 9.17 A balanced star-connected load having an impedance (15 + j20) Ω per phase is connected to a three-phase, 440 V; 50 Hz supply. Find the line currents and the power absorbed by the load. Assume RYB phase sequence.

Solution Referring to Fig. 9.32 (a), taking V_{RN} as the reference voltage we have

$$V_{RN} = \frac{440\ \angle 0^\circ}{\sqrt{3}} = 254\ \angle 0^\circ$$

$$V_{YN} = 254\ \angle -120^\circ$$

$$V_{BN} = 254\ \angle -240^\circ$$

Impedance per phase, $Z_{Ph} = 15 + j20 = 25\ \angle 53.13^\circ\ \Omega$

The phase currents are $I_R = \dfrac{V_{RN}}{Z_{Ph}} = \dfrac{254 \quad 0}{25 \quad 53.13} = 10.16\ \angle -53.13^\circ$ A

$$I_Y = \frac{V_{YN}}{Z_{Ph}} = \frac{254 \quad 120}{25 \quad 53.13} = 10.16\ \angle -173.13^\circ \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_{Ph}} = \frac{254 \quad 240}{25 \quad 53.13} = 10.16\ \angle -293.13^\circ \text{ A}$$

The three phase currents are equal in magnitude and displaced by 120° from one another. Since the load is star-connected, these currents also represents line currents.

The power absorbed by the load (P)

$$= 3 \times V_{Ph} \times I_{Ph} \cos \phi$$

or

$$= \sqrt{3} \times V_L \times I_L \cos \phi$$

$$= \sqrt{3} \times 440 \times 10.16 \times \cos 53.13^\circ = 4645.78 \text{ W}$$

9.10 THREE-PHASE UNBALANCED CIRCUITS

9.10.1 Types of Unbalanced Loads

An unbalance exists in a circuit when the impedances in one or more phases differ from the impedances of the other phases. In such a case, line or phase currents are different and are displaced from one another by unequal angles. So far, we have considered balanced loads connected to balanced systems. It is enough to solve problems, considering one phase only on balanced loads; the conditions on other two phases being similar. Problems on unbalanced three-phase loads are difficult to handle because conditions in the three phases are different. However, the source voltages are assumed to be balanced. If the system is a three-wire system, the currents flowing towards the load in the three lines must add to zero at any given instant. If the system is a four-wire system, the sum of the three outgoing line currents is equal to the return current in the neutral wire. We will now consider different methods to handle unbalanced star-connected and delta-connected loads. In practice, we may come across the following unbalanced loads:

(i) Unbalanced delta-connected load

(ii) Unbalanced three-wire star-connected load, and
(iii) Unbalanced four-wire star-connected load.

9.10.2 Unbalanced Delta-connected Load

Figure 9.33 shows an unbalanced delta-load connected to a balanced three-phase supply.

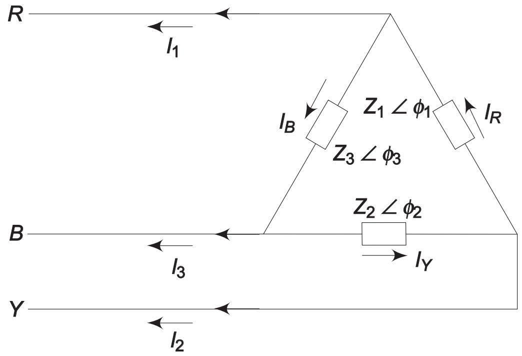


Fig. 9.33

The unbalanced delta-connected load supplied from a balanced three-phase supply does not present any new problems because the voltage across the load phase is fixed. It is independent of the nature of the load and is equal to the line voltage of the supply. The current in each load phase is equal to the line voltage divided by the impedance of that phase. The line current will be the phasor difference of the corresponding phase currents, taking V_{RY} as the reference phasor.

Assuming RYB phase sequence, we have

$$V_{RY} = V \angle 0° \text{ V}, \; V_{YB} = V \angle -120° \text{ V}, \; V_{BR} = V \angle -240° \text{ V}$$

Phase currents are

$$I_R = \frac{V_{RY}}{Z_1 \angle \phi} = \frac{V \quad 0}{Z_1 \quad \phi_1} = \left| \frac{V}{Z_1} \right| \angle - \phi_1 \text{ A}$$

$$I_Y = \frac{V_{YB}}{Z_2 \quad \phi_2} = \frac{V \quad 120}{Z_2 \quad \phi_2} = \left| \frac{V}{Z_2} \right| \angle - 120° - \phi_2 \text{ A}$$

$$I_B = \frac{V_{BR}}{Z_3 \quad \phi_3} = \frac{V \quad 240}{Z_3 \quad \phi_3} = \frac{V}{Z_3} \angle - 240° - \phi_3 \text{ A}$$

The three line currents are

$I_1 = I_R - I_B$ phasor difference
$I_2 = I_Y - I_R$ phasor difference
$I_3 = I_B - I_Y$ phasor difference

Example 9.18 Three impedances $Z_1 = 20\ \angle 30°\ \Omega$, $Z_2 = 40\ \angle 60°\ \Omega$ and $Z_3 = 10\ \angle{-90°}\ \Omega$ are delta-connected to a 400 V, $3-\phi$ system as shown in Fig. 9.34. Determine the (i) phase currents (ii) line currents, and (iii) total power consumed by the load.

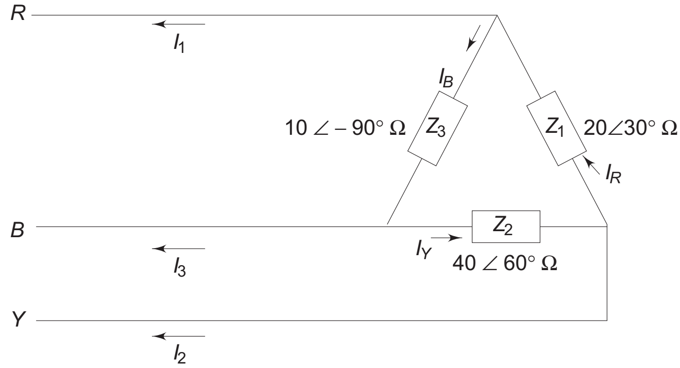


Fig. 9.34

Solution The three phase currents are I_R, I_Y and I_B, and the three line currents are I_1, I_2 and I_3. Taking $V_{RY} = V\ \angle 0°$ V as reference phasor, and assuming *RYB* phase sequence, we have

$$V_{RY} = 400\ \angle 0°\ \text{V},\ V_{YB} = 400\ \angle{-\ 120°}\ \text{V},$$

$$V_{BR} = 400\ \angle{-\ 240°}\ \text{V}$$

$$Z_1 = 20\ \angle 30°\ \Omega = (17.32 + j10)\ \Omega;$$

$$Z_2 = 40\ \angle 60°\ \ \Omega = (20 + j34.64)\ \Omega;$$

$$Z_3 = 10\ \angle{-\ 90°}\ \Omega = (0 - j10)\ \Omega$$

$$I_R = \frac{V_{RY}}{Z_1\ \angle\phi_1} = \frac{400 \quad 0}{20 \quad 30}\ \text{A} = 20\ \angle{-\ 30°}\ \text{A}$$

$$= (17.32 - j10)\ \text{A}$$

$$I_Y = \frac{V_{YB}}{Z_2 \quad \phi_2} = \frac{400\ \angle{-120°}}{40\ \angle 60°}\ \text{A} = 10\ \angle{-\ 180°}\ \text{A}$$

$$= (-\ 10 + j0)\ \text{A}$$

$$I_B = \frac{V_{BR}}{Z_3\ \angle\phi_3} = \frac{400\ \angle{-240°}}{10\ \angle{-90°}}\ \text{A} = 40\ \angle{-\ 150°}\ \text{A}$$

$$= (-\ 34.64 - j\,20)\ \text{A}$$

Now the three line currents are

$$I_1 = I_R - I_B = [(17.32 - j\,10) - (-\ 34.64 - j\,20)]$$

$$= (51.96 + j\,10)\ A = 52.91\ \angle 10.89°\ \text{A}$$

$$I_2 = I_Y - I_R = [(-\ 10 + j0) - (17.32 - j10)]$$

$$= (-\ 27.32 + j10)\ \text{A} = 29.09\ \angle 159.89°\ \text{A}$$

$$I_3 = I_B - I_Y = [(-\ 34.64 - j20) - (-\ 10 + j\,0)]$$

$$= (-\ 24.64 - j\,20)\ \text{A} = 31.73\ \angle{-\ 140.94°}\ \text{A}$$

(iii) To calculate the total power, first the powers in the individual phases are to be calculated, then they are added up to get the total power in the unbalanced load.

$$\text{Power in } R \text{ phase} = I_R^2 \times R_R = (20)^2 \times 17.32 = 6928 \text{ W}$$

$$\text{Power in } Y \text{ phase} = I_Y^2 \times R_Y = (10)^2 \times 20 = 2000 \text{ W}$$

$$\text{Power in } B \text{ phase} = I_B^2 \times R_B = (40)^2 \times 0 = 0$$

$\therefore$ Total power in the load = 6928 + 2000 = 8928 W

9.10.3 Unbalanced Four Wire Star-Connected Load

Figure 9.35 shows an unbalanced star load connected to a balanced 3-phase, 4-wire supply.

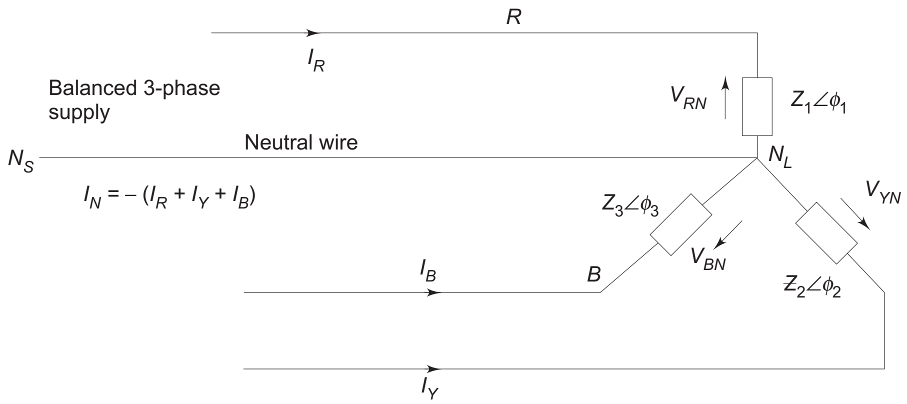


Fig. 9.35

The star point, N_L, of the load is connected to the star point, N_S of the supply. It is the simplest case of an unbalanced load because of the presence of the neutral wire; the star points of the supply N_S (generator) and the load N_L are at the same potential. It means that the voltage across each load impedance is equal to the phase voltage of the supply (generator), i.e. the voltages across the three load impedances are equalised even though load impedances are unequal. However, the current in each phase (or line) will be different. Obviously, the vector sum of the currents in the three lines is not zero, but is equal to neutral current. Phase currents can be calculated in similar way as that followed in an unbalanced delta-connected load.

Taking the phase voltage $V_{RN} = V\angle 0°$ V as reference, and assuming RYB phase sequences, we have the three phase voltages as follows

$$V_{RN} = V\angle 0° \text{ V}, \; V_{YN} = V\angle -120° \text{ V}, \; V_{BN} = V\angle -240° \text{ V}$$

The phase currents are

$$I_R = \frac{V_{RN}}{Z_1} = \frac{V\angle 0°}{Z_1\angle\phi_1} A = \left|\frac{V}{Z_1}\right| \angle -\phi_1 \text{A}$$

$$I_Y = \frac{V_{YN}}{Z_2} = \frac{V \quad 120}{Z_2 \quad \phi_2} \; A = \left| \frac{V}{Z_2} \right| \angle -120^\circ - \phi_2 \text{A}$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{V \quad 240}{Z_3 \quad \phi_3} \; A = \left| \frac{V}{Z_3} \right| \angle -240^\circ - \phi_3 \text{A}$$

Incidentally, I_R, I_Y and I_B are also the line currents; the current in the neutral wire is the vector sum of the three line currents.

Example 9.19 An unbalanced four-wire, star-connected load has a balanced voltage of 400 V, the loads are

$$Z_1 = (4 + j8)\ \Omega;\ Z_2 = (3 + j4)\ \Omega;\ Z_3 = (15 + j20)\ \Omega$$

Calculate the (i) line currents (ii) current in the neutral wire and (iii) the total power.

Solution $Z_1 = (4 + j8)\ \Omega;\ Z_2 = (3 + j4)\ \Omega;\ Z_3 = (15 + j20)\ \Omega$

$Z_1 = 8.94\ \angle 63.40^\circ\ \Omega;\ Z_2 = 5\ \angle 53.1^\circ\ \Omega;\ Z_3 = 25\ \angle 53.13^\circ\ \Omega$

Let us assume *RYB* phase sequence.

The phase voltage $V_{RN} = \dfrac{400}{\sqrt{3}} = 230.94$ V.

Taking V_{RN} as the reference phasor, we have

$$V_{RN} = 230.94\ \angle 0^\circ \text{ V},\ V_{YN} = 230.94\ \angle -120^\circ \text{ V}$$

$$V_{BN} = 230.94\ \angle -240^\circ \text{ V}$$

The three line currents are

(i) $I_R = \dfrac{V_{RN}}{Z_1} = \dfrac{230.94\ \angle 0^\circ}{8.94\ \angle 63.4^\circ}$ A $= 25.83\ \angle -63.4^\circ$ A

$$I_Y = \frac{V_{YN}}{Z_2} \quad \frac{230.94 \quad 120}{5 \quad 53.1} \text{ A} = 46.188\ \angle -173.1^\circ \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_3} \quad \frac{230.94 \quad 240}{25 \quad 53.13} \text{ A} = 9.23\ \angle -293.13^\circ \text{ A}$$

(ii) To find the neutral current, we must add the three line currents. The neutral current must then be equal and opposite to this sum.

Thus, $I_N = -(I_R + I_Y + I_B)$

$= -(25.83\ \angle -63.4^\circ + 46.188\ \angle -173.1^\circ + 9.23\ \angle -293.13^\circ)$ A

$I_N = -[(11.56 - j23.09) + (-45.85 - j5.54) + (3.62 + j8.48)]$ A.

$I_N = -[(-30.67 - j20.15)]$ A $= (30.67 + j20.15)$ A

$I_N = 36.69\ \angle 33.30^\circ$ A

It's phase with respect to V_{RN} is 33.3°, the disposition of all the currents is shown in Fig. 9.36.

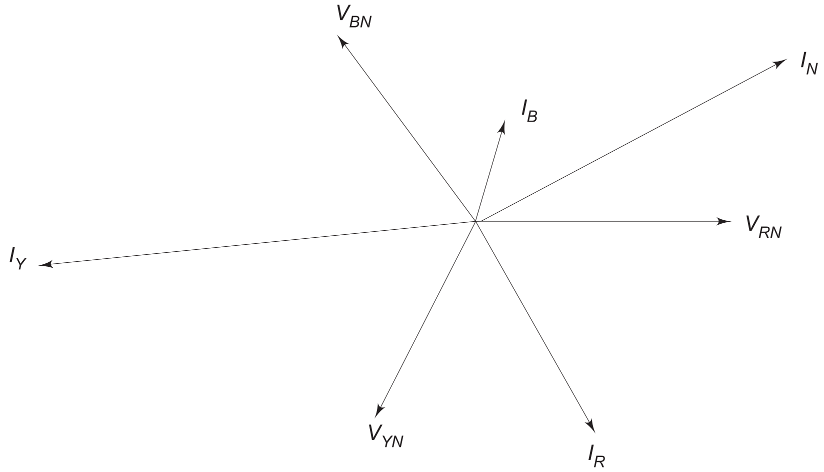


Fig. 9.36

(iii) Power in R phase $= I_R^2 \times R_R = (25.83)^2 \times 4 = 2668.75$ W

Power in Y phase $= I_Y^2 \times R_Y = (46.18)^2 \times 3 = 6397.77$ W

Power in B phase $= I_B^2 \times R_B = (9.23)^2 \times 15 = 1277.89$ W

Total power absorbed by the load

$$= 2668.75 + 6397.77 + 1277.89 = 10344.41 \text{ W}$$

9.10.4 Unbalanced Three Wire Star-Connected Load

In a three-phase, four-wire system if the connection between supply neutral and load neutral is broken, it would result in an unbalanced three-wire star-load. This type of load is rarely found in practice, because all the three wire star loads are balanced. Such as system is shown in Fig. 9.37. Note that the supply star point (N_S) is isolated from the load star point (N_L). The potential of the load star point is different from that of the supply star point. The result is that the load phase voltages is not equal to the supply phase voltage; and they are not only unequal in magnitude, but also subtend angles other than 120° with one another. The magnitude of each phase voltage depends upon the individual phase loads. The potential of the load neutral point changes according to changes in the impedances of the phases, that is why sometimes the load neutral is also called a floating neutral point. All star-connected, unbalanced loads supplied from polyphase systems without a neutral wire have floating neutral point. The phasor sum of the three unbalanced line currents is zero. The phase voltage of the load is not $1/\sqrt{3}$ of the line voltage. The unbalanced three-wire star load is difficult to deal with. It is because load phase voltages cannot be determined directly from the given supply line voltages. There are many methods to solve such unbalanced Y-connected loads. Two frequently used methods are presented here. They are

(i) Star-delta conversion method, and
(ii) The application of Millman's theorem

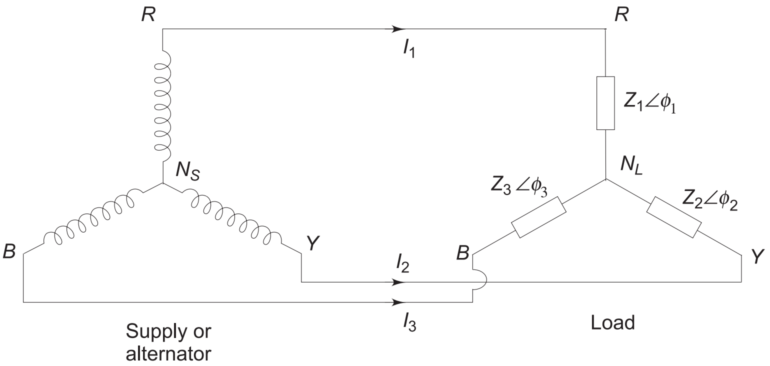


Fig. 9.37

9.10.5 Star-Delta Method to Solve Unbalanced Load

Figure 9.38(a) shows an unbalanced wye-connected load. It has already been shown in Section 9.6 that a three phase star-connected load can be replaced by an equivalent delta-connected load. Thus, the star load of Fig. 9.38(a) can be replaced by equivalent delta as shown in Fig. 9.38(b), where the impedances in each phase is given by

$$Z_{RY} = \frac{Z_R\,Z_Y + Z_Y\,Z_B + Z_B\,Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R\,Z_Y \quad Z_Y\,Z_B \quad Z_B\,Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R\,Z_Y \quad Z_Y\,Z_B \quad Z_B\,Z_R}{Z_Y}$$

The problem is then solved as an unbalanced delta-connected system. The line currents so calculated are equal in magnitude and phase to those taken by the original unbalanced wye (Y) connected load.

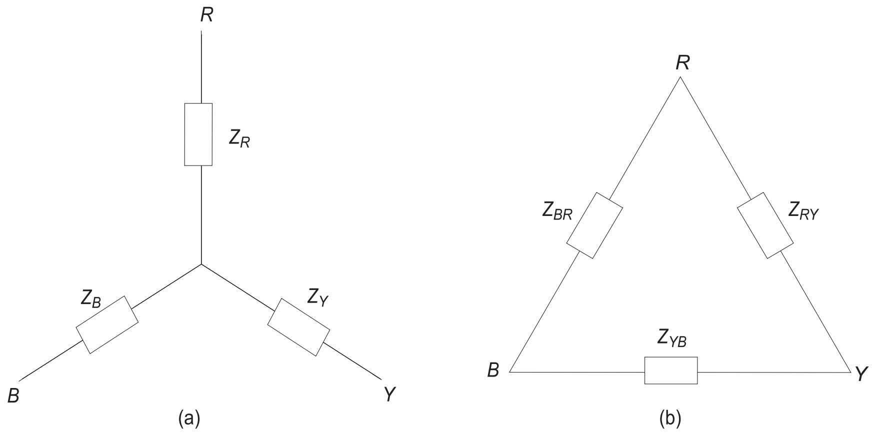


Fig. 9.38

Example 9.20 A 400 V, three-phase supply feeds an unbalanced three-wire, star-connected load. The branch impedances of the load are $Z_R = (4 + j8)\ \Omega$; $Z_y = (3 + j4)\ \Omega$ and $Z_B = (15 + j20)\ \Omega$. Find the line currents and voltage across each phase impedance. Assume *RYB* phase sequence.

Solution The unbalanced star load and its equivalent delta (Δ) is shown in Fig. 9.39(a) and (b) respectively.

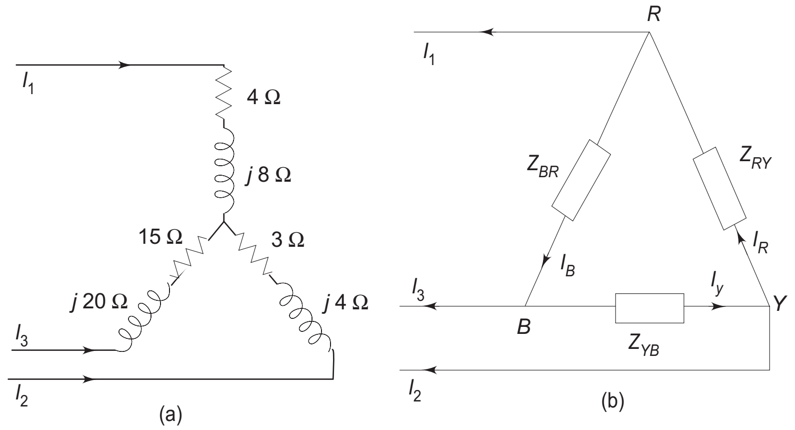


Fig. 9.39

$$Z_R = (4 + j8)\ \Omega = 8.944\ \angle 63.4°\ \Omega$$

$$Z_Y = (3 + j4)\ \Omega = 5\ \angle 53.1°\ \Omega$$

$$Z_B = (15 + j20)\ \Omega = 25\ \angle 53.1\ \Omega$$

Using the expression in Section 9.10.5, we can calculate Z_{RY}, Z_{YB} and Z_{BR}

$$Z_R Z_Y + Z_Y Z_B + Z_B Z_R$$

$$= (8.94\ \angle 63.4°)\ (5\ \angle 53.1°) + (5\ \angle 53.1°)\ (25\ \angle 53.1°)$$
$$+ (25\ \angle 53.1°)\ (8.94\ \angle 63.4°)$$

$$= 391.80\ \angle 113.23°$$

$$Z_{RY} = \frac{Z_R\ Z_Y + Z_Y\ Z_B + Z_B\ Z_R}{Z_B} = \frac{391.80\ \angle 113.23°}{25\ \angle 53.1°} = 15.67\ \angle 60.13°$$

$$Z_{YB} = \frac{Z_R\ Z_Y \quad Z_Y\ Z_B \quad Z_B\ Z_R}{Z_R} \quad \frac{391.80 \quad 113.23}{8.94 \quad 63.4} = 43.83\ \angle 49.83°$$

$$Z_{BR} = \frac{Z_R\ Z_Y + Z_Y\ Z_B + Z_B\ Z_R}{Z_Y} = \frac{391.80\ \angle 113.23°}{5\ \angle 53.1°} = 78.36\ \angle 60.13°$$

Taking V_{RY} as reference, $V_{RY} = 400\ \angle 0$

$$V_{YB} = 400\ \angle -120°;\ V_{BR} = 400\ \angle -240°$$

$$I_R = \frac{V_{RY}}{Z_{RY}} \quad \frac{400 \quad 0}{15.67 \quad 60.13} = 25.52\ \angle -60.13°$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120^\circ}{43.83\angle 49.83^\circ} = 9.12\angle -169.83^\circ$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{400\angle -240^\circ}{78.36\angle 60.13^\circ} = 5.10\angle -300.13^\circ$$

The various line currents in the delta load are

$$I_1 = I_R - I_B = 25.52\angle -60.13^\circ - 5.1\angle -300.13^\circ$$
$$= 28.41\angle -69.07^\circ \text{ A}$$
$$I_2 = I_Y - I_R = 9.12\angle -169.83^\circ - 25.52\angle -60.13^\circ$$
$$= 29.85\angle 136.58^\circ \text{ A}$$
$$I_3 = I_B - I_Y = 5.1\angle -300.13^\circ - 9.12\angle -169.83^\circ$$
$$= 13\angle 27.60^\circ \text{ A}$$

These line currents are also equal to the line (phase) currents of the original star-connected load. The voltage drop across each star-connected load will be as follows.

Voltage drop across $Z_R = I_1 Z_R$

$$= (28.41\angle -69.070^\circ)(8.94\angle 63.4^\circ) = 253.89\angle -5.67^\circ \text{ V}$$

Voltage drop across $Z_Y = I_2 Z_Y$

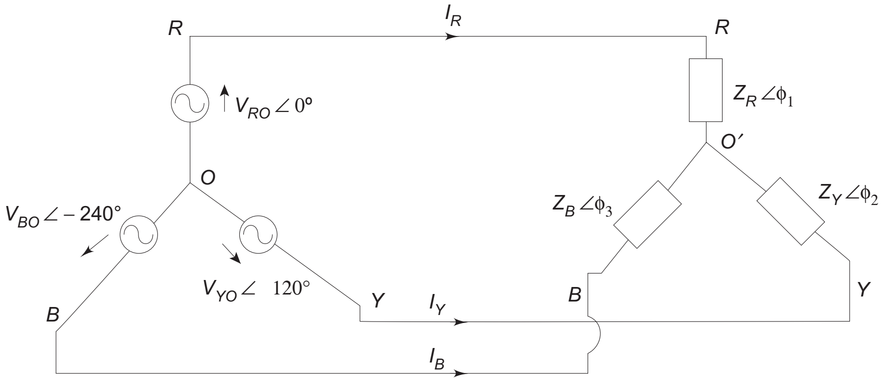
$$= (29.85\angle 136.58^\circ)(5\angle 53.1^\circ) = 149.2\angle 189.68^\circ \text{ V}$$

Voltage drop across $Z_B = I_3 Z_B$

$$= (13\angle 27.60^\circ)(25\angle 53.1^\circ) = 325\angle 80.70^\circ \text{ V}$$

9.10.6 Millman's Method of Solving Unbalanced Load

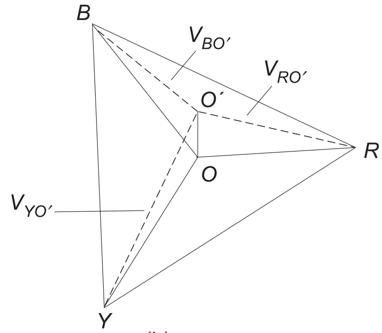
One method of solving an unbalanced three-wire star-connected load by star-delta conversion is described in Section 9.10.5. But this method is laborious and involves lengthy calculations. By using Millman's theorem, we can solve this type of problems in a much easier way. Consider an unbalanced wye (Y) load connected to a balanced three-phase supply as shown in Fig. 9.40(a). V_{RO}, V_{YO} and V_{BO} are the phase voltages of the supply. They are equal in magnitude, but displaced by 120° from one another. $V_{RO'}$, $V_{YO'}$ and $V_{BO'}$ are the load phase voltages; they are unequal in magnitude as well as differ in phase by unequal angles. Z_R, Z_Y and Z_B are the impedances of the branches of the unbalanced wye (Y) connected load. Figure 9.40(b) shows the triangular phasor diagram of the complete system. Distances RY, YB and BR represent the line voltages of the supply as well as load. They are equal in magnitude, but displaced by 120°. Here O is the star-point of the supply and is located at the centre of the equilateral triangle RYB. O' is the load star point. The star point of the supply which is at the zero potential is different from that of the star point at the load, due to the load being unbalanced. O' has some potential with respect to O and is shifted away from the centre of the triangle. Distance $O'O$ represents the voltage of the load star point with respect to the star point of the supply $Vo'o$.



(a)

Fig. 9.40

$V_{o'o}$ is calculated using Millman's theorem. If $V_{o'o}$ is known, the load phase voltages and corresponding currents in the unbalanced wye load can be easily determined.



(b)

Fig. 9.40

According to Millman's theorem, $V_{o'o}$ is given by

$$V_{o'o} = \frac{V_{Ro}\, Y_R + V_{Yo}\, Y_Y + V_{Bo}\, Y_B}{Y_R + Y_Y + Y_B}$$

where the parameters Y_R, Y_Y and Y_B are the admittances of the branches of the unbalanced wye connected load. From Fig. 9.40(a), we can write the equation

$$V_{Ro} = V_{Ro'} + V_{o'o}$$

or the load phase voltage

$$V_{Ro'} = V_{Ro} - V_{o'o}$$

Similarly, $V_{Yo'} = V_{Yo} - V_{o'o}$ and $V_{Bo'} = V_{Bo} - V_{o'o}$ can be calculated. The line currents in the load are

$$I_R = \frac{V_{Ro'}}{Z_R} = (V_{Ro} - V_{o'o})\, Y_R$$

$$I_Y = \frac{V_{Yo'}}{Z_Y} = (V_{Yo} - V_{o'o})\, Y_Y$$

$$I_B = \frac{V_{Bo'}}{Z_B} = (V_{Bo} - V_{o'o})\, Y_B$$

The unbalanced three-wire star-connected loads can also be determined by using Kirchhoff's laws, and Maxwells mesh or loop equation. In general, any method which gives quick results in a particular case should be used.

Example 9.21 To illustrate the application of Millmans method to unbalanced loads, let us take the problem in example given in Section 9.10.5.

Solution The circuit diagram is shown in Fig. 9.41.

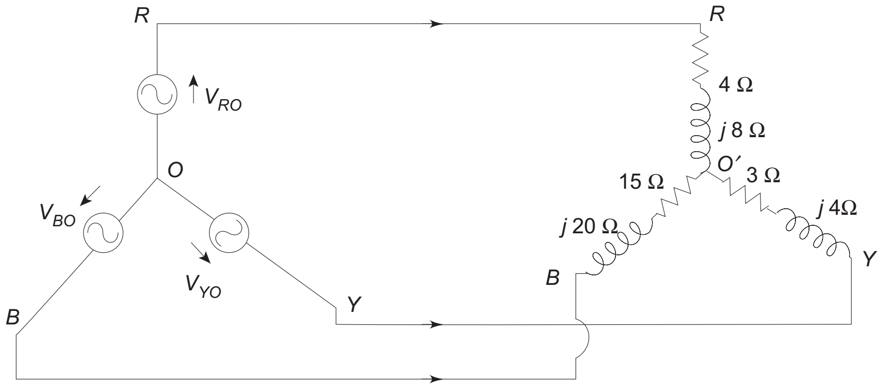


Fig. 9.41

Taking V_{RY} as reference line voltage = 400 $\angle 0°$, phase voltages lag 30° behind their respective line voltages. Therefore, the three phase voltages are

$$V_{Ro} = \frac{400}{\sqrt{3}} \angle -30° \text{ V}$$

$$V_{Yo} = \frac{400}{\sqrt{3}} \angle -150° \text{ V}$$

$$V_{Bo} = \frac{400}{\sqrt{3}} \angle -270° \text{ V}$$

The admittances of the branches of the wye load are

$$Y_R = \frac{1}{Z_R} = \frac{1}{8.94 \angle 63.4°} = 0.11 \angle -63.40° \ \mho$$

$$Y_Y = \frac{1}{Z_Y} \quad \frac{1}{5 \quad 53.1} = 0.2 \angle -53.1° \ \mho$$

$$Y_B = \frac{1}{Z_B} \quad \frac{1}{25 \quad 53.1} = 0.04 \angle -53.1° \ \mho$$

$$V_{Ro}Y_R + V_{Yo}Y_Y + V_{Bo}Y_B = (230.94 \angle -30°)\,(0.11 \angle -63.40°)$$
$$+ (230.94 \angle -150°)\,(0.2 \angle -53.1°) + (230.94 \angle -270°)\,(0.04 \angle -53.1°)$$
$$= 36.68 \angle 182.66°$$

$$Y_R + Y_Y + Y_B = 0.11 \angle -63.4° + 0.2 \angle -53.1° + 0.04 \angle -53.1°$$
$$= 0.35 \angle -56.2° \ \mho$$

Substituting the above values in the Millmans theorem, we have

$$V_{o'o} = \frac{V_{Ro}\, Y_R \quad V_{Yo}\, Y_Y \quad V_{Bo}\, Y_B}{Y_R \quad Y_Y \quad Y_B}$$

$$= \frac{36.68 \quad 182.66}{0.35 \quad 56.2} = 104.8 \angle 238.86°$$

The three load phase voltages are

$$V_{Ro'} = V_{Ro} - V_{o'o}$$
$$= 230.94 \angle -30° - 104.8 \angle 238.86° = 253.89 \angle -5.67° \text{ V}$$
$$V_{Yo'} = V_{Yo} - V_{o'o}$$
$$= 230.94 \angle -150° - 104.8 \angle 238.86° = 149.2 \angle 189.68° \text{ V}$$
$$V_{Bo'} = V_{Bo} - V_{o'o}$$
$$= 230.94 \angle -270° - 104.8 \angle 238.86° = 325 \angle 80.7° \text{ V}$$

9.11 POWER MEASUREMENT IN THREE-PHASE CIRCUITS

9.11.1 Power Measurement in a Single Phase Circuit by Wattmeter

Wattmeters are generally used to measure power in the circuits. A wattmeter principally consists of two coils, one coil is called the current coil, and the other the pressure or voltage coil. A diagramatric representation of a wattmeter connected to measure power in a single phase circuit is shown in Fig. 9.42.

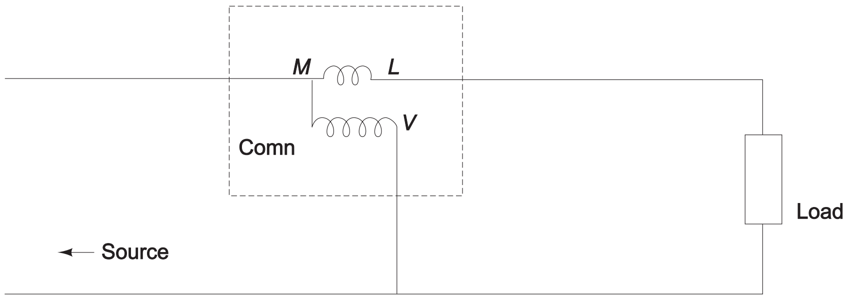


Fig. 9.42

The coil represented with less number of turns between M and L is the current coil, which carries the current in the load and has very low impedance. The coil

with more number of turns between the common terminal (comn) and V is the pressure coil, which is connected across the load and has high impedance. The load voltage is impressed across the pressure coil. The terminal M denotes the mains side, L denotes load side, *common* denotes the common point of current coil and pressure coil, and V denotes the second terminal of the pressure coil, usually selected as per the range of the load voltage in the circuit. From the figure, it is clear that a wattmeter has four terminals, two for current coil and two for potential coil. When the current flow through the two coils, they set up magnetic fields in space. An electromagnetic torque is produced by the interaction of the two magnetic fields. Under the influence of the torque, one of the coils (which is movable) moves on a calibrated scale against the action of a spring. The instantaneous torque produced by electromagnetic action is proportional to the product of the instantaneous values of the currents in the two coils. The small current in the pressure coil is equal to the input voltage divided by the impedance of the pressure coil. The inertia of the moving system does not permit it to follow the instantaneous fluctuations in torque. The wattmeter deflection is therefore, proportional to the average power ($VI \cos \phi$) delivered to the circuit. Sometimes, a wattmeter connected in the circuit to measure power gives downscale reading or backward deflection. This is due to improper connection of the current coil and pressure coil.

To obtain up scale reading, the terminal marked as 'Comn' of the pressure coil is connected to one of the terminals of the current coil as shown in Fig. 9.43. Note that the connection between the current coil terminal and pressure coil terminal is not inherent, but has to be made externally. Even with proper connections, sometimes the wattmeter will give downscale reading whenever the phase angle between the voltage across the pressure coil and the current through the current

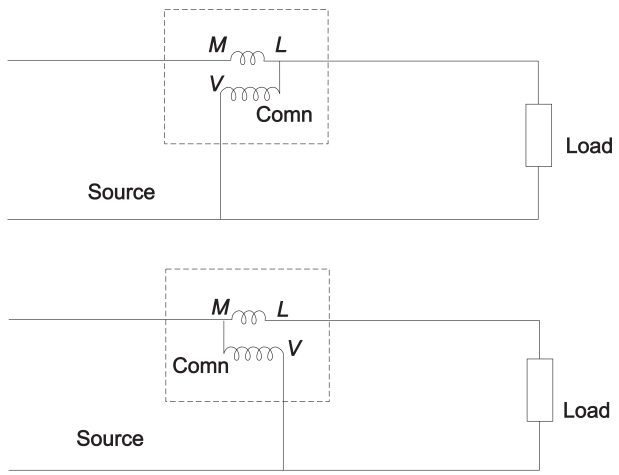


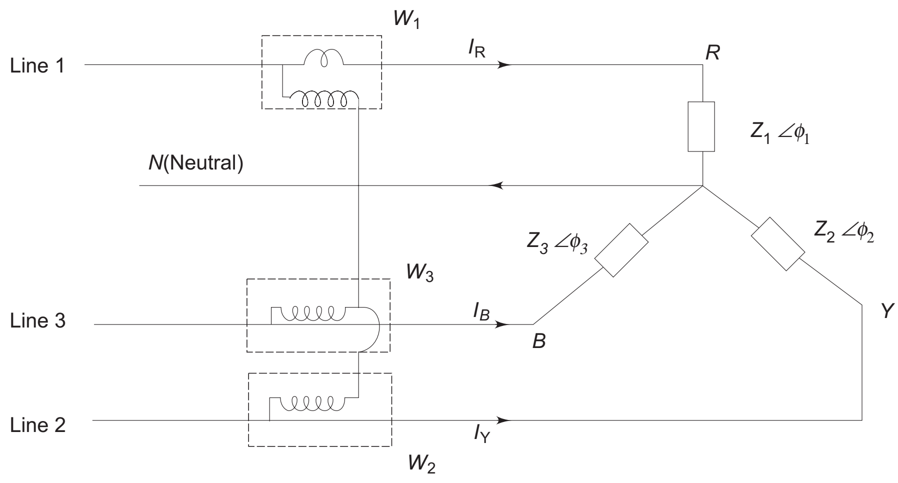
Fig. 9.43

coil is more than 90°. In such a case, connection of either the current coil or the pressure coil must be reversed.

9.11.2 Power in Three-Phase Circuits

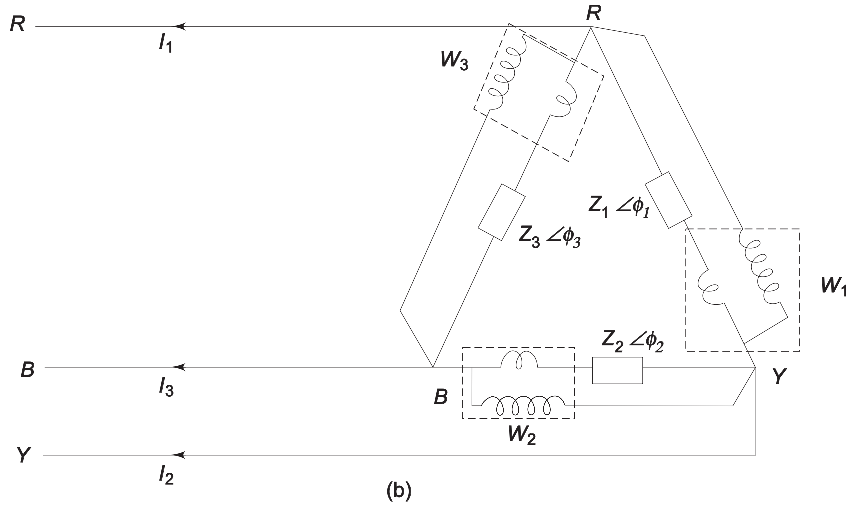
Measurement of power by a wattmeter in a single phase circuit can be extended to measure power in a three-phase circuit. From Section 9.11.1, it is clear that we require three wattmeters, one in each phase to measure the power consumed in a three-phase system. Obviously, the total power is the algebraic sum of the readings of the three wattmeters. In this way we can measure power in balanced and unbalanced loads. In a balanced case it would be necessary to measure power only in one phase and the reading is multiplied by three to get the total power in all the three phases. This is true in principle, but presents a few difficulties in practice. To verify this fact let us examine the circuit diagram in Fig. 9.44(a) and (b).

Observation of Figs 9.44(a) and (b) reveals that for a star-connected load, the neutral must be available for connecting the pressure coil terminals. The current coils must be inserted in each phase for a delta-connected load. Such connections sometimes may not be practicable, because the neutral terminal is not available all the time in a star-connected load, and the phases of the delta-connected load are not accessible for connecting the current coils of the wattmeter. In most of the commercially available practical three-phase loads, only three line terminals are available. We, therefore, require a method where we can measure power in the three-phases with an access to the three lines connecting the source to the load. Two such methods are discussed here.



(a)

Fig. 9.44



(b)

Fig. 9.44

9.11.3 Three Wattmeter and Two Wattmeter Method

In this method, the three wattmeters are connected in the three lines as shown in Fig. 9.45, i.e. the current coils of the three wattmeters are introduced in the three lines, and one terminal of each potential coil is connected to one terminal of the corresponding current coil, the other three being connected to some common point which forms an effective neutral *n*.

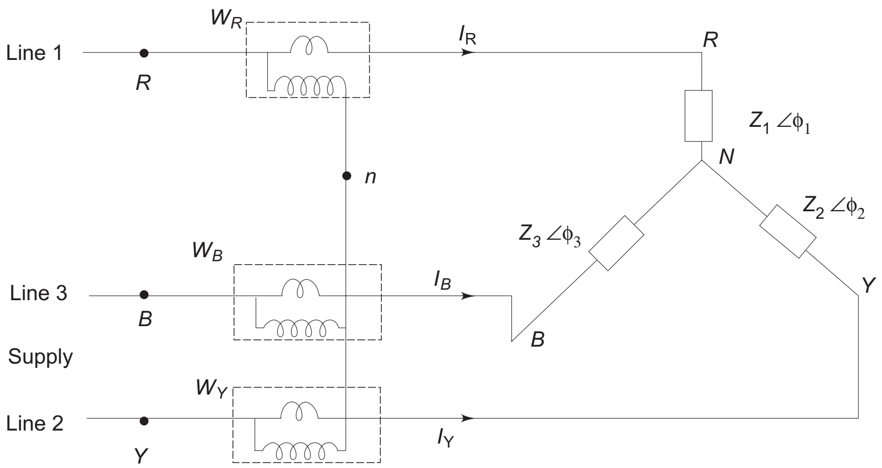


Fig. 9.45

The load may be either star-connected or delta-connected. Let us assume a star-connected load, and let the neutral of this load be denoted by N. Now the reading on the wattmeter W_R will correspond to the average value of the product of the instantaneous value of the current I_R flowing in line 1, with the voltage

drop V_{Rn}, where V_{Rn} is the voltage between points R and n. This can be written as $V_{Rn} = V_{RN} + V_{Nn}$, where V_{RN} is the load phase voltage and V_{Nn} is the voltage between load neutral, N, and the common point, n. Similarly, $V_{Yn} = V_{YN} + V_{Nn}$, and $V_{Bn} = V_{BN} + V_{Nn}$. Therefore, the average power, W_R indicated by the wattmeter is given by

$$W_R = \frac{1}{T}\int_o^T V_{Rn}\, I_R\, dt$$

where T is the time period of the voltage wave

$$W_R = \frac{1}{T}\int_o^T (V_{RN} + V_{Nn})I_R\, dt$$

Similarly,
$$W_Y = \frac{1}{T}\int_o^T V_{Yn}\, I_Y\, dt$$

$$= \frac{1}{T}\int_o^T (V_{YN} + V_{Nn})I_Y\, dt$$

and
$$W_B = \frac{1}{T}\int_o^T V_{Bn}\, I_B\, dt$$

$$= \frac{1}{T}\int_o^T (V_{BN} + V_{Nn})I_B\, dt$$

Total average power $= W_R + W_Y + W_B$

$$= \frac{1}{T}\int_o^T (V_{RN}\, I_R + V_{YN}\, I_Y + V_{BN}\, I_B)\, dt + \frac{1}{T}\int_o^T V_{Nn}\,(I_R + I_Y + I_B)\, dt$$

Since the system in the problem is a three-wire system, the sum of the three currents I_R, I_Y and I_B at any given instant is zero. Hence, the power read by the three wattmeters is given by

$$W_R + W_Y + W_B = \frac{1}{T}\int_o^T (V_{RN}\, I_R + V_{YN}\, I_Y + V_{BN}\, I_B)\, dt$$

If the system has a fourth wire, i.e. if the neutral wire is available, then the common point, n is to be connected to the system neutral, N. In that case, V_{Nn} would be zero, and the above equation for power would still be valid. In other words, whatever be the value of V_{Nn}, the algebraic sum of the three currents I_R, I_Y and I_B is zero. Hence, the term $V_{Nn}\,(I_R + I_Y + I_B)$ would be zero. Keeping this advantage in mind, suppose the common point, n, in Fig. 9.45 is connected to line B. In such case, $V_{Nn} = V_{NB}$; then the voltage across the potential coil of wattmeter

W_B will be zero and this wattmeter will read zero. Hence, this can be removed from the circuit. The total power is read by the remaining two wattmeters, W_R and W_Y.

$\therefore$ Total power = $W_R + W_Y$

Let us verify this fact from Fig. 9.46.

The average power indicated by wattmeter W_R is

$$W_R = \frac{1}{T}\int_o^T V_{RB}\, I_R\, dt$$

and that by
$$W_Y = \frac{1}{T}\int_o^T V_{YB} \cdot I_Y\, dt$$

Also
$$V_{RB} = V_{RN} + V_{NB}$$
$$V_{YB} = V_{YN} + V_{NB}$$

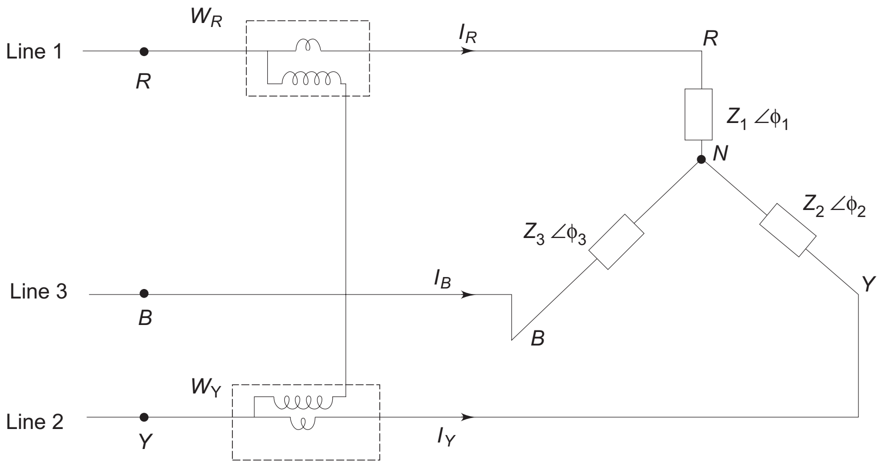


Fig. 9.46

$$W_R + W_Y = \frac{1}{T}\int_o^T (V_{RB} \cdot I_R + V_{YB} \cdot I_Y)\, dt$$

$$= \frac{1}{T}\int_o^T \{(V_{RN} + V_{NB})\, I_R + (V_{YN} + V_{NB})\, I_Y\}\, dt$$

$$= \frac{1}{T}\int_o^T \{(V_{RN}\, I_R + V_{YN}\, I_Y) + (I_R + I_Y)\, V_{NB}\}\, dt$$

We know that
$$I_R + I_Y + I_B = 0$$
$$I_R + I_Y = -\, I_B$$

Substituting this value in the above equation, we get

$$W_R + W_Y = \frac{1}{T}\int_o^T \{(V_{RN}\, I_R + V_{YN}\, I_Y) + (-I_B)\, V_{NB}\}\; dt$$

$$V_{NB} = -V_{BN}$$

$$W_R + W_Y = \frac{1}{T}\int_o^T \{(V_{RN}\, I_R + V_{YN} \cdot I_Y + V_{BN} \cdot I_B)\}\; dt$$

which indicates the total power in the load.

From the above discussion it is clear that the power in a three-phase load, whether balanced or unbalanced, star-connected or delta-connected, three-wire or four wire, can be measured with only two wattmeters as shown in Fig. 9.46. In fact, the two wattmeter method of measuring power in three-phase loads has become a universal method. If neutral wire is available in this method it should not carry any current, or the neutral of the load should be isolated from the neutral of the source.

The current flowing through the current coil of each wattmeter is the line current, and the voltage across the pressure coil is the line voltage. In case the phase angle between line voltage and current is greater than 90°, the corresponding wattmeter would indicate downscale reading. To obtain upscale reading, the connections of either the current coil, or the pressure coil has to be interchanged. Reading obtained after reversal of coil connection should be taken as negative. Then, the algebraic sum of the two wattmeter readings gives the total power.

9.11.4 Power Factor by Two Wattmeter Method

When we talk about the power factor in three-phase circuits, it applies only to balanced circuits, since the power factor in a balanced load is the power factor of any phase. We cannot strictly define the power factor in three-phase unbalanced circuits, as every phase has a separate power factor. The two wattmeter method, when applied to measure power in a three-phase balanced circuits, provides information that help us to calculate the power factor of the load.

Figure 9.47 shows the vector diagram of the circuit shown in Fig. 9.46. Since the load is assumed to be balanced, we can take $Z_1 \angle\phi_1 = Z_2 \angle\phi_2 = Z_3 \angle\phi_3 = Z \angle\phi$ for the star-connected load. Assuming RYB phase sequence, the three rms load phase voltages are V_{RN}, V_{YN} and $V_{BN} \cdot I_R$, I_Y and I_B are the rms line (phase) currents. These currents will lag behind their respective phase voltages by an angle ϕ. (An inductive load is considered).

Now consider the readings of the two wattmeters in Fig. 9.46. W_R measures the product of effective value of the current through its current coil I_R, effective value of the voltage across its pressure coil V_{RB} and the cosine of the angle

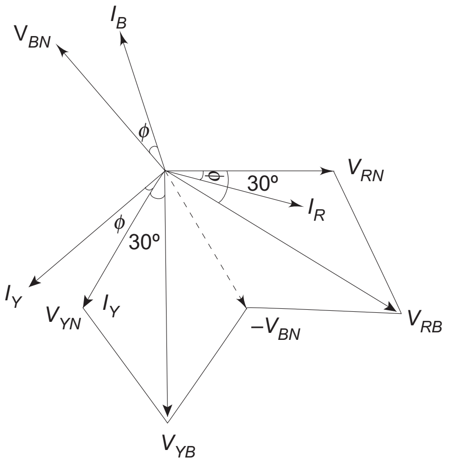


Fig. 9.47

between the phasors I_R and V_{RB}. The voltage across the pressure coil of W_R is given as follows.

$$V_{RB} = V_{RN} - V_{BN} \text{ phasor difference}$$

It is clear from the phasor diagram that the phase angle between

$$V_{RB} \text{ and } I_R \text{ is } (30° - \phi)$$

$$\therefore \qquad W_R = V_{RB} \cdot I_R \cos (30 - \phi)$$

Similarly, W_Y measures the product of effective value of the current through its current coil I_Y, the effective value of the voltage across its pressure coil, V_{YB} and the cosine of the angle between the phasors V_{YB} and I_Y.

$$V_{YB} = V_{YN} - V_{BN}$$

From Fig. 9.47, it is clear that the phase angle between V_{YB} and I_Y is $(30° + \phi)$.

$$\therefore \qquad W_Y = V_{YB} \cdot I_Y \cos (30° + \phi)$$

Since the load is balanced, the line voltage $V_{RB} = V_{YB} = V_L$ and the line current $I_R = I_Y = I_L$

$$W_R = V_L \cdot I_L \cos (30° - \phi)$$

$$W_Y = V_L I_L \cos (30° + \phi)$$

Adding W_R and W_Y gives total power in the circuit, thus

$$W_R + W_Y = \sqrt{3}\, V_L I_L \cos \phi$$

From the two wattmeter readings, it is clear that for the same load angle ϕ, wattmeter W_R registers more power when the load is inductive. It is also connected in the leading phase as the phase sequence is RYB. Therefore, W_R is higher reading wattmeter in the circuit of Fig. 9.46. In other words, if the load is capacitive, the wattmeter connected in the leading phase reads less for the same load angle. So, if we know the nature of the load, we can easily identify the phase sequence of the system. The higher reading wattmeter always reads positive. By

proper manipulation of two wattmeter readings, we can obtain the power factor of the load.

$$W_R = V_L I_L \cos (30° - \phi) \qquad \text{(Higher reading)}$$
$$W_Y = V_L I_L \cos (30° + \phi) \qquad \text{(Lower reading)}$$

$$W_R + W_Y = \sqrt{3}\ V_L I_L \cos \phi$$
$$W_R - W_Y = V_L I_L \sin \phi$$

Taking the ratio of the above two values, we get

$$\frac{W_R - W_Y}{W_R + W_Y} = \frac{\tan \phi}{\sqrt{3}}$$

or
$$\tan \phi = \sqrt{3} \left[\frac{W_R - W_Y}{W_R + W_Y} \right]$$

$$\phi = \tan^{-1} \sqrt{3} \left[\frac{W_R - W_Y}{W_R + W_Y} \right]$$

Thereafter, we can find $\cos \phi$

Example 9.22 The two wattmeter method is used to measure power in a three-phase load. The wattmeter readings are 400 W and – 35 W. Calculate (i) total active power (ii) power factor, and (iii) reactive power.

Solution From the given data, the two wattmeter readings W_R = 400 W (Higher reading wattmeter) W_Y = – 35 *W* (Lower reading wattmeter).

(i) Total active power = $W_1 + W_2$

$$= 400 + (-35) = 365\ W$$

(ii) $$\tan \phi = \sqrt{3}\, \frac{W_R - W_Y}{W_R + W_Y} = \sqrt{3}\, \frac{400 - (-35)}{400 + (-35)} = \sqrt{3} \times \frac{435}{365} = 2.064$$

$$\phi = \tan^{-1} 2.064 = 64.15°;\ \text{P.F} = 0.43$$

(iii) Reactive power = $\sqrt{3}\ V_L I_L \sin \phi$

We know that $W_R - W_Y = V_L I_L \sin \phi$

$$\therefore \quad W_R - W_Y = 400 - (-35) = 435$$

Reactive power = $\sqrt{3} \times 435$ = 753.44 VAR

Example 9.23 The input power to a three-phase load is 10 kW at 0.8 Pf. Two wattmeters are connected to measure the power, find the individual readings of the wattmeters.

Solution Let W_R be the higher reading wattmeter and W_Y the lower reading wattmeter

$$W_R + W_Y = 10 \text{ kW} \qquad (9.1)$$

$$\phi = \cos^{-1} 0.8 = 36.8°$$

$$\tan \phi = 0.75 = \sqrt{3}\,\frac{W_R \quad W_Y}{W_R \quad W_Y}$$

or $$W_R - W_Y = \frac{(0.75)}{\sqrt{3}}\,(W_R + W_Y) = \frac{0.75}{\sqrt{3}} \times 10 \text{ kW}$$

$$= 4.33 \text{ kW} \qquad (9.2)$$

From Eqs (9.1) and (9.2) we get

$$W_R + W_Y = 10 \text{ kW}$$

$$\frac{W_R \quad W_Y \quad 4.33 \text{ kW}}{2\,W_R \quad 14.33 \text{ kW}}$$

or $$W_R = 7.165 \text{ kW}$$

$$W_Y = 2.835 \text{ kW}$$

9.11.5 Variation in Wattmeter Readings with Load Power Factor

It is useful to study the effect of the power factor on the readings of the wattmeter. In Section 9.11.4, we have proved that the readings of the two wattmeters depend on the load power factor angle ϕ, such that

$$W_R = V_L I_L \cos (30 - \phi)^\circ$$

$$W_Y = V_L I_L \cos (30 + \phi)^\circ$$

We can, therefore, make the following deductions

(i) When ϕ is zero, i.e. power factor is unity, from the above expressions we can conclude that the two wattmeters indicate equal and positive values.

(ii) When ϕ rises from 0 to 60°, i.e. upto power factor 0.5, wattmeter W_R reads positive (since it is connected in the leading phase); whereas wattmeter W_Y reads positive, but less than W_R. When $\phi = 60°$, $W_Y = 0$ and the total power is being measured only by wattmeter W_R.

(iii) If the power factor is further reduced from 0.5, i.e. when ϕ is greater than 60°, W_R indicates positive value, whereas W_Y reads down scale reading in such case. As already explained in Section 9.11.3 the connections of either the current coil, or the pressure coil of the corresponding wattmeter have to be interchanged to obtain an upscale reading, and the reading thus obtained must be given a negative sign. Then the total power in the circuit would be $W_R + (-W_Y) = W_R - W_Y$. Wattmeter W_Y reads downscale for the phase angle between 60° and 90°. When the power factor is zero (i.e. $\phi = 90°$), the two wattmeters will read equal and opposite values.

i.e. $$W_R = V_L I_L \cos (30 - 90)^\circ = 0.5\, V_L I_L$$

$$W_Y = V_L I_L \cos (30 + 90)^\circ = -0.5\, V_L I_L$$

9.11.6 Leading Power Factor Load

The above observations are made considering the lagging power factor. Suppose the load in Fig. 9.46(a) is capacitive, the wattmeter connected in the leading

phase would read less value. In that case, W_R will be the lower reading wattmeter, and W_Y will be the higher reading wattmeter. Figure 9.48 shows the phasor diagram for the leading power factor.

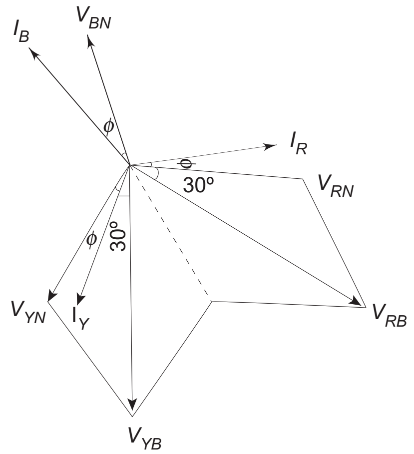


Fig. 9.48

As the power factor is leading, the phase currents, I_R, I_Y and I_B are leading their respective phase voltage by an angle ϕ. From Fig. 9.48, the reading of the wattmeter connected in the leading phase is given by

$$W_R = V_{RB} \cdot I_R \cos (30 + \phi)^\circ$$
$$= V_L I_L \cos (30 + \phi)^\circ \text{ (lower reading wattmeter)}$$

Similarly, the reading of the wattmeter connected in the lagging phase is given by

$$W_Y = V_{YB}\, I_Y \cos (30 - \phi)^\circ$$
$$= V_L I_L \cos (30 - \phi)^\circ \text{ (higher reading wattmeter)}$$

Again the total power is given by

$$W_R + W_Y = \sqrt{3}\ V_L I_L \cos \phi$$
$$W_Y - W_R = V_L I_L \sin \phi$$

Hence $$\tan \phi = \sqrt{3}\, \frac{W_Y - W_R}{W_Y + W_R}$$

A comparison of this expression with that of lagging power factor reveals the fact that the two wattmeter readings are interchanged, i.e. for lagging power factor, W_R is the higher reading wattmeter, and W_Y is the lower reading wattmeter; where as for leading power factor, W_R is the lower reading wattmeter and W_Y is the higher reading wattmeter. While using the expression for power factor, whatever may be the nature of the load, the lower reading is to be subtracted from the higher reading in the numerator. The variation in the wattmeter reading with the capacitive load follows the same sequence as in inductive load, with a change in the roles of wattmeters.

Example 9.24 The readings of the two wattmeters used to measure power in a capacitive load are – 3000 W and 8000 W, respectively. Calculate (i) the input power, and (ii) the power factor at the load. Assume *RYB* sequence.

Solution

(i) Total power = $W_R + W_Y = -3000 + 8000 = 5000$ W

(ii) As the load is capacitive, the wattmeter connected in the leading phase gives less value.

$\therefore \qquad W_R = -3000$

Consequently $\qquad W_Y = 8000$

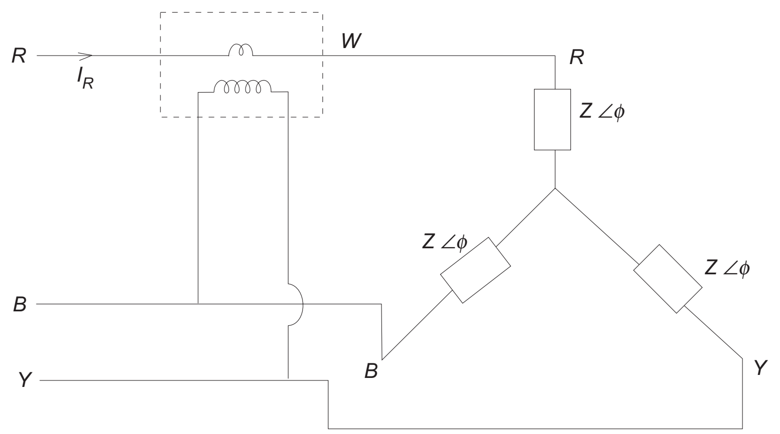
$$\tan\phi = \sqrt{3}\,\frac{W_Y - W_R}{W_Y + W_R} = \sqrt{3}\,\frac{(8000 - (-3000))}{5000} = 3.81$$

$\therefore \qquad \phi = 75.29°$ (lead); $\cos\phi = 0.25$

9.11.7 Reactive Power with Wattmeter

We have already seen in the preceding section that the difference between higher reading wattmeter and lower reading wattmeter yields $V_L I_L \sin\phi$. So, the total reactive power = $\sqrt{3}\, V_L I_L \sin\phi$. Reactive power in a balanced three-phase load can also be calculated by using a single wattmeter.

As shown in Fig. 9.49(a), the current coil of the wattmeters is connected in any one line (*R* in this case), and the pressure coil across the other two lines (between *Y* and *B* in this case). Assuming phase sequence *RYB* and an inductive load of angle ϕ, the phasor diagram for the circuit in Fig. 9.49(a) is shown in Fig. 9.49(b).

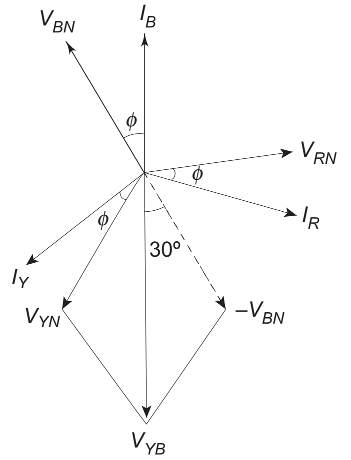


(a)

Fig. 9.49

From Fig. 9.49(a), it is clear that the wattmeter power is proportional to the product of current through its current coil, I_R, voltage across its pressure coil, V_{YB}, and cosine of the angle between V_{YB} and I_R.

or $$V_{YB} = V_{YN} - V_{BN} = V_L$$



(b)

Fig. 9.49

From the vector diagram the angle between V_{YB} and I_R is $(90 - \phi)°$

$\therefore$ Wattmeter reading $= V_{YB} I_R \cos (90 - \phi)°$

$= V_L I_L \sin \phi$ VAR

If the above expression is multiplied by $\sqrt{3}$, we get the total reactive power in the load.

Example 9.25 A single wattmeter is connected to measure reactive power of a three-phase, three-wire balanced load as shown in Fig. 9.49(a). The line current is 17 A and the line voltage is 440 V. Calculate the power factor of the load if the reading of the wattmeter is 4488 VAR.

Solution We know that wattmeter reading is equal to $V_L I_L \sin \phi$

$\therefore$ $4488 = 440 \times 17 \sin \phi$

$\sin \phi = 0.6$

Power factor $= \cos \phi = 0.8$

9.12 EFFECTS OF HARMONICS

The relationship between line and phase quantities for wye and delta connections as derived earlier are strictly valid only if the source voltage is purely sinusoidal. Such a waveform is an ideal one. Modern alternations are designed to give a terminal voltage which is almost sinusoidal. But it is nearly impossible to realise an ideal waveform in practice. All sinusoidally varying alternating waveforms deviate to a greater or lesser degree, from an ideal sinusoidal shape. Due to non-uniform distribution of the field flux and armature reaction in a.c. machines, the current and voltage waves may get distorted. Such waveforms are referred to as

non-sinusoidal or complex waveforms. In modern machines this distortion is relatively small. All non-sinusoidal waves can be broken up into a series of sinusoidal waves whose frequencies are integral multiples of the frequency of the fundamental wave. The sinusoidal components of a complex wave are called *harmonics*. It is therefore necessary to consider the effect of certain harmonics on currents and voltages in the phase of three-phase wye and delta systems in effecting the line and phase quantities.

The fundamental wave is called the basic wave or first harmonic. The second harmonic has a frequency of twice the fundamental, the third harmonic frequency is three times the fundamental frequency and so on. Each harmonic is a pure sinusoid. Waves having $2f$, $4f$, $6f$, etc. are called even harmonics and those having frequencies $3f$, $5f$, $7f$, etc. are called odd harmonics. Since the negative half of the wave is a reproduction of the positive half, the even harmonics are absent. Therefore, a complex wave can be represented as a sum of fundamental and odd harmonics.

9.12.1 Harmonic Effect in Wye

Let us consider a wye connected generator winding, whose arrangement is shown diagrammatically in Fig. 9.50. The voltage induced in phase a of the three-phase symmetrical system, including odd harmonics is given by

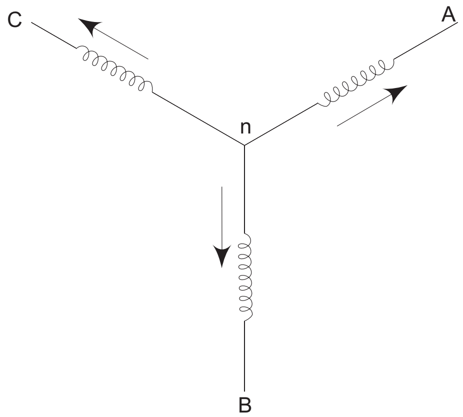


Fig. 9.50

$$V_{na} = E_{m_1} \sin(\omega t + \theta_1) + E_{m_3} \sin(3\omega t + \theta_3) + E_{m_5} \sin(5\omega t + \theta_5) + \cdots \quad (9.3)$$

Where E_{m_1}, E_{m_3}, E_{m_5}, etc. are the peak values of the fundamental and other harmonics and θ_1, θ_3, θ_5, etc. are phase angles. Assuming *abc* phase sequence. The voltage in phase *b* will be

$$v_{nb} = E_{m_1} \sin(\omega t + \theta_1 - 120°) + E_{m_3} \sin(3\omega t - 360° + \theta_3)$$
$$+ E_{m_5} \sin(5\omega t - 600° + \theta_5) + \ldots$$

$$= E_{m_1} \sin(\omega t + \theta_1 - 120°) + E_{m_3} \sin(3\omega t + \theta_3)$$
$$+ E_{m_3} \sin(5\omega t + \theta_5 - 240°) + \ldots \qquad (9.4)$$

The voltage in phase c will be

$$v_{nc} = E_{m_1} \sin(\omega t + \theta_1 - 240°) + E_{m_3} \sin(3\omega t + \theta_3 - 720°)$$
$$+ E_{m_5} \sin(5\omega t + \theta_5 - 1200°) + \ldots$$
$$= E_{m_1} \sin(\omega t + \theta_1 - 240°) + E_{m_3} \sin(3\omega t + \theta_3)$$
$$+ E_{m_5} \sin(5\omega t + \theta_5 - 120°) + \ldots \qquad (9.5)$$

Equations 9.3, 9.4, and 9.5 show that all third harmonics are in time phase with each other in all the three phases as shown in Fig. 9.51(a). The same applies to the, ninth, fifteenth, twenty first… harmonics, i.e. all odd multiples of 3. Other than odd multiples of 3, the fifth, seventh, eleventh… and all other harmonics are displaced 120° in time phase mutually with either the same phase sequence or opposite phase sequence compared with that of the fundamentals. Fifth harmonics and seventh harmonic sequences are shown in Figs 9.5(c) and 9.5(d) respectively.

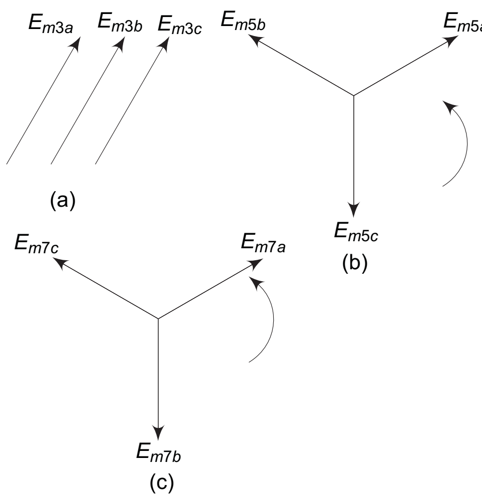


Fig. 9.51

Summarising the above facts, the fundamental and all those harmonics obtained by adding a multiple of 6, i.e. 1, 7, 13, 19,…, etc. will have the same sequence. Similarly, the fifths and all harmonics obtained by adding a multiple of 6, i.e. 5, 11, 17, 23,…, etc. will have sequence opposite to that of the fundamental.

Voltage Relations

The voltage between lines ab in the wye connected winding in Fig. 9.50 may be written as

$$e_{ab} = e_{an} + e_{nb}$$

Adding of each harmonic separately is shown in Fig. 9.52.

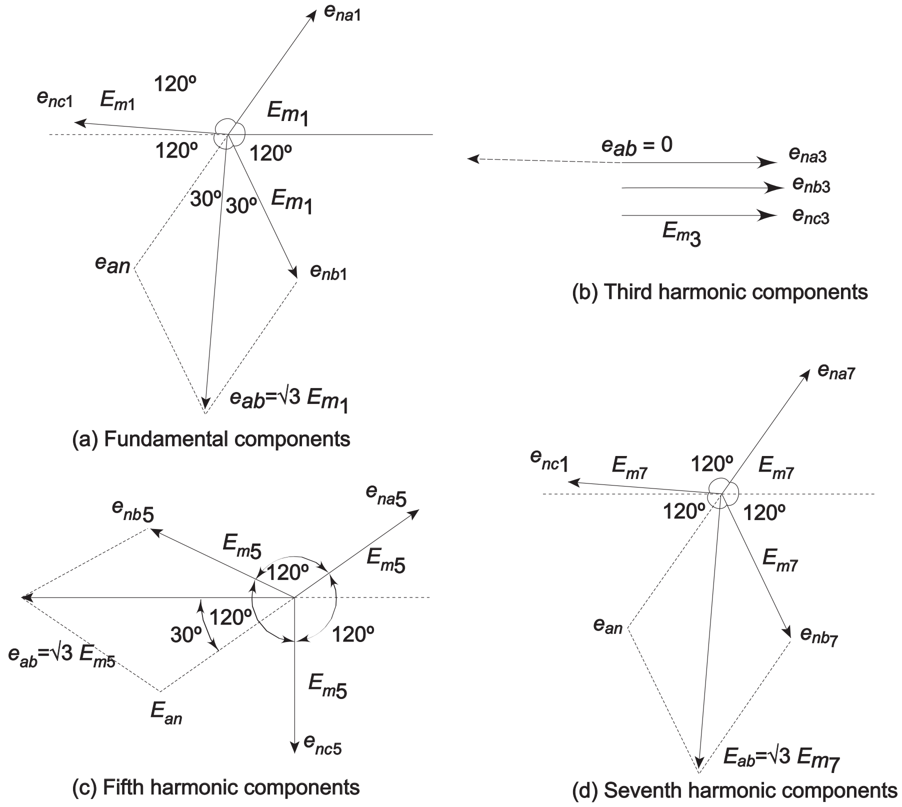


Fig. 9.52

It is seen from the vector diagrams of Fig. 9.52 that there is no third harmonic component in the line voltage. Hence, the rms value of the line voltage is given by

$$E_{ab} = \sqrt{3}\sqrt{\frac{E_{m_1}^2 + E_{m_5}^2 + E_{m_7}^2 + \dots}{2}} \tag{9.6}$$

and the rms value of the phase voltage is

$$E_{na} = \sqrt{\frac{E_{m_1}^2 + E_{m_3}^2 + E_{m_5}^2 + E_{m_7}^2 + \dots}{2}} \tag{9.7}$$

It is seen from the above equations that in a wye connected system, the line voltage is not equal to $\sqrt{3}$ times the phase voltage if harmonics are present. This is true only when the third harmonics are absent.

Current Relations

Similar to the complex voltage wave, the instantaneous value of the complex current wave can be written as

$$i = I_{m_1} \sin(\omega t + \phi_1) + I_{m_3} \sin(3\omega t + \phi_3) + I_{m_5} \sin(5\omega t + \phi_5) \quad (9.8)$$

where I_{m_1}, I_{m_3}, I_{m_5}, etc. are the peak values of fundamental and other harmonics; $(\phi_1 - \phi_1)$ is the phase difference between fundamental component of the harmonic voltage and current and $(\phi_3 - \phi_3)$ is the phase difference between 3rd harmonics and so on. Applying *KCL* for the three phase wye connected winding in Fig. 9.50.

$$I_{na} + i_{nb} + i_{nc} = 0 \quad (9.9)$$

The equations for i_{na}, i_{nb} and i_{nc} can be obtained by replacing currents in the place of voltages in equations 9.3, 94 and 9.5 under balanced conditions the sum of the three currents is equal to zero, only when they have equal magnitudes and displaced by 120° apart in time phase in the three phases. All harmonics fulfil the above condition except the third harmonics and their odd multiples as they are in the same phase as shown in Fig. 9.51(a) or 9.52(b). Hence, the resultant of $i_{na} + i_{nb} + i_{nc}$ consists of the arithmetic sum of the third harmonics in the three phases. Hence, there must be a neutral wire or fourth wire to provide a return path for the third harmonic. We can summarise the above facts as follows. In a balanced three-wire wye connection, all harmonics are present except third and its odd multiples. In a four-wire wye connection, i.e. with a neutral wire, all harmonics will exist.

9.12.2 Harmonic Effect in Delta

Let the three windings of the generator be delta-connected as shown in Fig. 9.53. Let v_{na}, v_{nb} and v_{nc} be the phase emfs and v_{na}, v_{nb} and v_{nc} be the terminal voltages of the three phases *a*, *b* and *c* respectively. According to *KVL* the algebraic sum of the three terminal voltages in the closed loop is given by

$$v_{na} + v_{nb} + v_{nc} = v_{ca} + v_{ab} + v_{bc} = 0 \quad (9.10)$$

There will be a circulating current in the closed loop due to the resultant third harmonic and their multiple induced emfs. This resultant emf is dropped in the closed loop impedance. Hence, the third harmonic voltage does not appear across the terminals of the delta. Hence, the terminal voltages in delta connection v_{ca}, v_{ab} and v_{bc} are given by equations 1, 2 and 3 respectively without third harmonic terms.

Current Relations

The three phase windings in Fig. 9.53, carry all the harmonics internally and are given by

$$\begin{aligned} i_{na} = i_{ca} &= I_{m_1} \sin(\omega t + \theta_1) + I_{m_2} \sin(3\omega t + \theta_3) \\ &\quad + I_{m_5} \sin(5\omega t + \theta_5) + \ldots \qquad (9.11) \\ i_{nb} = i_{ab} &= I_{m_1} \sin(\omega t + \theta_1 - 120°) + I_{m_3} \sin(3\omega t + \theta_3 - 360°) \\ &\quad + I_{m_5} \sin(5\omega t + \theta_5 - 600°) + \ldots \end{aligned}$$

$$= I_{m_1} \sin(\omega t + \theta_1 - 120°) + I_{m_3} \sin(3\omega t + \theta_3)$$
$$+ I_{m_5} \sin(5\omega t + \theta_5 - 240°) \ldots \quad (9.12)$$
$$i_{nc} = i_{bc} = I_{m_1} \sin(\omega t + \theta_1 - 240°) + I_{m_3} \sin(3\omega t + \theta_3 - 720°)$$
$$+ I_{m_5} \sin(5\omega t + \theta_5 - 1200°)$$
$$= I_{m1} \sin(\omega t + \theta_1 - 240_{3}) + I_{m_3} \sin(3\omega t + \theta_3)$$
$$+ I_{m_5} \sin(5\omega t + \theta_5 - 120°) \quad (9.13)$$

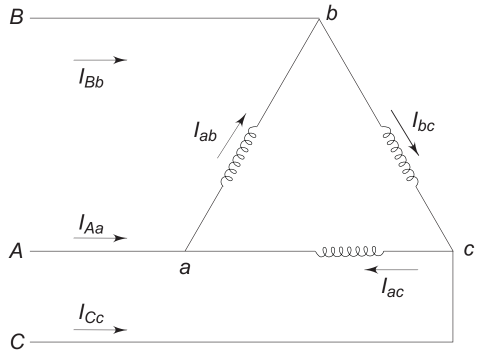


Fig. 9.53

Equations 9.11, 9.12 and 9.13 represent the phase currents in the delta connection. The line currents I_{Aa} I_{Bb} and I_{Cc} can be obtained by applying KCL at the three junctions of the delta in Fig. 9.53. The current vector diagrams are similar to the voltage vector diagrams shown in Fig. 9.52 except that the voltages are to be replaced with currents. The line currents can be obtained in terms of phase currents by applying KCL at three junctions as follows

$$I_{Aa} = i_{ab} - i_{Ca}$$
$$= I_{m_1} \sin(\omega t + \theta_1 - 120°) + I_{m_5} \sin(5\omega t + \theta_5 - 240°)$$
$$- I_{m_1} \sin(\omega t + \theta_1) - I_{m_5} \sin(5\omega t + \theta_5) \quad (9.14)$$
$$i_{Bb} = i_{bc} - i_{ab}$$
$$= I_{m1} \sin(\omega t + \theta_1 - 240°) + I_{m_5} \sin(5\omega t + \theta_5 - 120°)$$
$$- I_{m1} \sin(\omega t + \theta_1 - 120°) - I_{m_5} \sin(5\omega t + \theta_5 - 240°) \quad (9.15)$$
$$i_{Cc} = i_{ca} - i_{bc}$$
$$= I_{m1} \sin(\omega t + \theta_1) + I_{m_5} \sin(5\omega t + \theta_5) - I_{m1} \sin(\omega t + \theta_1 - 240°)$$
$$- I_{m_5} \sin(5\omega t + \theta_5 - 120°) \quad (9.16)$$

Equations 9.14, 9.15 and 9.16 indicate that no third harmonic currents can exist in the line currents of a delta connection.

The rms value of the line current from the above equation is

$$I_L = \sqrt{3} \sqrt{\frac{I_{m_1}^2 + I_{m_5}^2 + \ldots}{2}} \quad (9.17)$$

The rms value of the phase current from Equations 9.11, 9.12 and 9.13 is given by

$$I_{\mathrm{ph}} = \sqrt{\frac{I_{m_1}^2 + I_{m_3}^2 + I_{m_5}^2 + \dots}{2}} \tag{9.18}$$

It is seen from Equations 9.17 and 9.18, that in a delta-connected system the line current is not equal to $\sqrt{3}$ times the phase current. It is only true when there are no third harmonic currents in the system.

9.13 EFFECTS OF PHASE-SEQUENCE

The effects of phase sequence of the source voltages are not of considerable importance for applications like lighting, heating, etc. but in case of a three-phase induction motor, reversal of sequence results in the reversal of its direction. In the case of an unbalanced polyphase system, a reversal of the voltage phase sequence will, in general, cause certain branch currents to change in magnitude as well as in phase position. Even though the system is balanced, the readings of the two wattmeters in the two wattmeter method of measuring power interchange when subjected to a reversal of phase sequence when two or more three phase generators are running parallel to supply a common load, reversing the phase sequence of any one machine cause severe damage to the entire system. Hence, when working on such systems, it is very important to consider the phase sequence of the system. Unless otherwise stated, the term "phase sequence" refers to voltage phase sequence. The line currents and phase currents follow the same sequence as the system voltage. The phase sequence of a given system, is a small meter with three long connecting leads in side which it has a circular disc. The rotation of which previously been checked against a known phase sequence. In three-phase systems, only two different phase sequences are possible. The three leads are connected to the three lines whose sequence is to be determined, the rotation of the disc can be used as an indicator of phase sequence.

9.14 POWER FACTOR OF AN UNBALANCED SYSTEM

The concept of power factor in three-phase balanced circuits has been discussed in Section 9.11.4. It is the ratio of the phase watts to the phase volt-amperes of any one of the three phases. We cannot strictly define the power factor in three-phase unbalanced circuits, as each phase has a separate power factor. Generally for three-phase unbalanced loads, the ratio of total watts ($\sqrt{3}\, V_L I_L \cos\theta$) to total volt-amperes ($\sqrt{3}\, V_L I_L$) is a good general indication of the power factor.

Another recognised definition for an unbalanced polyphase system is called the vector power factor, given by

$$\text{Power factor} = \frac{\sum VI\cos\theta}{\sum VI}$$

Where $\sum VI \cos\theta$ is the algebraic sum of the active powers of all individual phases given by

$$\sum VI \cos\theta = V_a I_a \cos\theta_a + V_b I_b \cos\theta_b + V_c I_c \cos\theta_c + \ldots$$

and
$$\sum VI = \sqrt{\left(\sum VI \cos\theta\right)^2 + \left(\sum VI \sin\theta\right)^2}$$

$\sum VI \sin\theta$ is the algebraic sum of the individual phase reactive volt-amperes. The following example illustrates the application of vector power factor for unbalanced loads.

Consider Example 9.19 where the phase voltage and currents have been already calculated. Here $V_{RN} = 230.94 \angle 0°$ V, $V_{YN} = 230.94 \angle -120°$V, $V_{BN} = 230.94 \angle -240°$V

$I_R = 24.83 \angle -63.4°$ A; $I_Y = 46.188 \angle -173.1°$ A; $I_B = 9.23 \angle -293.13°$ A.

Active power of phase $R = 230.94 \times 25.83 \times \cos 63.4°$ = 2.6709 kW

Active power of phase $Y = 230.94 \times 46.188 \times \cos 53.1°$ = 6.4044 kW

Active power of phase $B = 230.94 \times 9.23 \times \cos 53.13°$ = 1.2789 kW

10.3542 kW

$$\sum VI \cos\theta = 10.3542 \text{ kW}$$

Reactive power of phase $R = 230.94 \times 25.83 \times \sin 63.4°$ = 5.3197 KVAR

Reactive power of phase $Y = 230.94 \times 46.188 \times \sin 53.1°$ = 8.5299 KVAR

Reactive power of phase $B = 230.94 \times 9.23 \times \sin 53.13°$ = 1.7052 KVAR

15.5548 KVAR

$$\sum VI \sin\theta = 15.5548 \text{ KVAR}$$

$$\text{Power factor} = \frac{10.3542}{\sqrt{(15.5548)^2 + (10.3542)^2}} = 0.5541$$

Additional Solved Problems

Problem 9.1 The phase voltage of a star-connected three-phase ac generator is 230 V. Calculate the (i) line voltage (ii) active power output if the line current of the system is 15 A at a power factor of 0.7 and (iii) active and reactive components of the phase currents.

Solution The supply voltage (generator) is always assumed to be balanced

$\therefore$ $V_{Ph} = 230$ V; $I_L = I_{Ph} = 15$ A, $\cos\phi = 0.7$, $\sin\phi = 0.71$

(i) In a star-connected system $V_L = \sqrt{3}\, V_{Ph} = 398.37$ V

(ii) Power output $= \sqrt{3}\, V_L I_L \cos\phi$

$$= \sqrt{3} \times 398.37 \times 15 \times 0.7 = 7244.96 \text{ W}$$

(iii) Active component of the current $= I_{Ph} \cos\phi$

$$= 15 \times 0.7 = 10.5 \text{ A}$$

Reactive component of the current $= I_{Ph} \sin\phi$

$$= 15 \times 0.71 = 10.65 \text{ A}$$

Problem 9.2 A three-phase delta-connected RYB system with an effective voltage of 400 V, has a balanced load with impedances $3 + j4\ \Omega$. Calculate the (i) phase currents (ii) line currents and (iii) power in each phase.

Solution

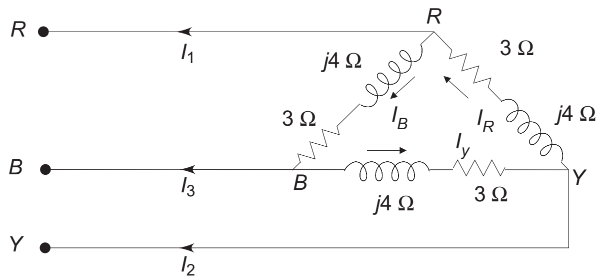


Fig. 9.54

$$V_L = V_{Ph} = 400 \text{ V}$$

Assuming RYB phase sequence, we have

$$V_{RY} = 400\ \angle 0°;\ V_{YB} = 400\ \angle -120°;\ V_{BR} = 400\ \angle -240°$$

$$Z = 3 + j4 = 5\ \angle 53.1°$$

The three phase currents

$$I_R = \frac{V_{RY}}{Z} = \frac{400\ \angle 0°}{5\ \angle 53.1°} = 80\ \angle -53.1°$$

$$I_Y = \frac{V_{YB}}{Z} = \frac{400\ \angle -120°}{5\ \angle 53.1°} = 80\ \angle -173.1°$$

$$I_B = \frac{V_{YB}}{Z} = \frac{400\ \angle -240°}{5\ \angle 53.1°} = 80\ \angle -293.1°$$

$$I_R = 80\ \angle -53.1° = 48.03 - j63.97$$

$$I_Y = 80\ \angle -173.1° = -79.42 - j9.61$$

$$I_B = 80\ \angle -293.1° = 31.38 + j73.58$$

The three line currents are

$$I_1 = I_R - I_B = 138.55\ \angle -83.09°$$

$$I_2 = I_Y - I_R = 138.55\ \angle 156.9°$$

$$I_3 = I_B - I_Y = 138.55 \angle 36.89°$$

$$\cos\phi = \frac{R_{Ph}}{Z_{Ph}} = \frac{3}{5} = 0.6$$

(iii) Power consumed in each phase $= V_{Ph}\, I_{Ph} \cos\phi$

$$= 400 \times 80 \times 0.6 = 19200 \text{ W}$$

$$\text{Total power} = 3 \times 19200 = 57600 \text{ W}$$

Problem 9.3 The load in Problem 9.2 is connected in star with the same phase sequence across the same system. Calculate (i) the phase and line currents (ii) the total power in the circuit, and (iii) phasor sum of the three line currents.

Solution The circuit is shown in Fig. 9.55.

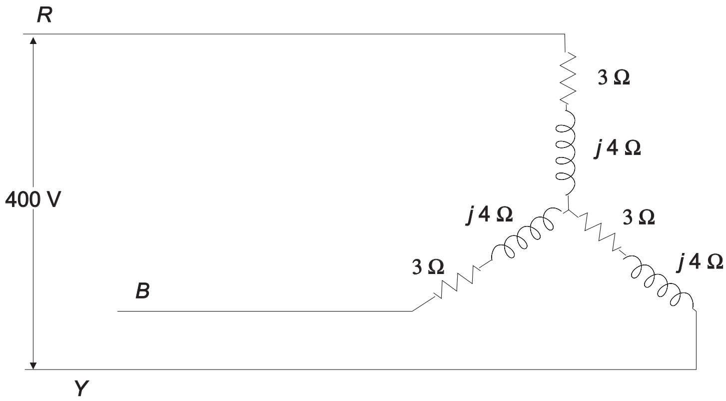


Fig. 9.55

Assuming RYB phase sequence, since

$$V_L = 400 \text{ V}$$

$$V_{Ph} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

Taking V_{RN} as reference, the three phase voltages are $V_{RN} = 230.94 \angle 0°$; $V_{YN} = 230.94 \angle -120°$; and $V_{BN} = 230.9 \angle -240°$.

The three line voltages, V_{RY}, V_{YB} and V_{BR} are 30° ahead of their respective phase voltages.

$$I_{Ph} = I_L;\ Z_{Ph} = 3 + j4 = 5 \angle 53.1°$$

The three phase currents are

$$I_R = \frac{V_{RN}}{Z_{Ph}} = \frac{230.94 \angle 0°}{5 \angle 53.1°} = 46.18 \angle -53.1°$$

$$I_Y = \frac{V_{YN}}{Z_{Ph}} = \frac{230.09 \angle -120°}{5 \angle 53.1°} = 46.18 \angle -173.1°$$

$$I_B = \frac{V_{BN}}{Z_{Ph}} = \frac{230.09 \angle -240°}{5 \angle 53.1°} = 46.18 \angle -293.1°$$

(ii) Total power $= \sqrt{3}\, V_L\, I_L \cos \phi$

$$= \sqrt{3} \times 400 \times 46.18 \times 0.6 = 19196.6 \text{ W}$$

Thus, it can be observed that the power consumed in a delta load will be three times more than that in the star connection

(iii) Phasor sum of the three line currents

$$= I_R + I_Y + I_B$$
$$= 46.18 \angle -53.1° + 46.18 \angle -173.1° + 46.18 \angle -293.1° = 0.$$

Problem 9.4 A three-phase balanced delta-connected load with line voltage of 200 V, has line currents as $I_1 = 10 \angle 90°$; $I_2 = 10 \angle -150°$ and $I_3 = 10 \angle -30°$. (i) What is the phase sequence? (ii) What are the impedances?

Solution Figure 9.56 (a) represents all the three line currents in the phasor diagram.

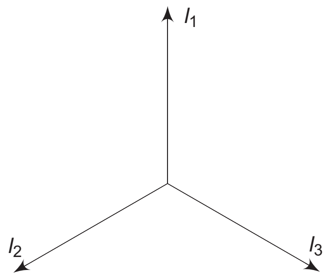


Fig. 9.56 (a)

(i) It can be observed from Figs 9.56 (a) and (b) that the current flowing in line B, i.e. I_3 lags behind I_1 by 120°, and the current flowing in line Y, i.e. I_2 lags behind i_3 by 120°. ∴ The phase sequence is RBY.

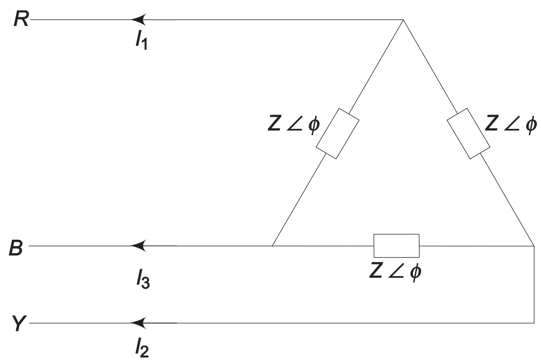


Fig. 9.56 (b)

(ii) $V_{Ph} = V_L = 200$

$$I_{Ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{200\sqrt{3}}{10} = 34.64\ \Omega$$

Problem 9.5 Three equal inductors connected in star take 5 kW at 0.7 Pf when connected to a 400 V, 50 Hz three-phase, three-wire supply. Calculate the line currents (i) if one of the inductors is disconnected, and (ii) If one of the inductors is short circuited.

Solution

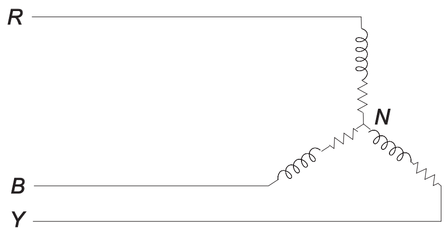


Fig. 9.57(a)

Total power when they are connected to 400 V supply

$$P = \sqrt{3}\, V_L I_L \cos\phi = 5000\ \text{W}$$

$$I_{Ph} = I_L = \frac{5000}{\sqrt{3} \times 400 \times 0.7} = 10.31\ \text{A}$$

$$\text{Impedance/phase} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{\sqrt{3} \times 10.31} = 22.4\ \Omega$$

$$R_{Ph} = Z_{Ph} \cos\phi = 22.4 \times 0.7 = 15.68\ \Omega$$

$$X_{Ph} = Z_{Ph} \sin\phi = 22.4 \times 0.714 = 16\ \Omega$$

(i) If phase Y is disconnected from the circuit, the other two inductors are connected in series across the line voltage of 400 V as shown in Fig. 9.57(a).

$$I_R = I_B = \frac{400}{2 \times Z_{Ph}} = 8.928\ \text{A}$$

$$I_Y = 0$$

(ii) If phase Y and N are short circuited as shown in Fig. 9.57(b), the phase voltages V_{RN} and V_{BN} will be equal to the line voltage 400 V.

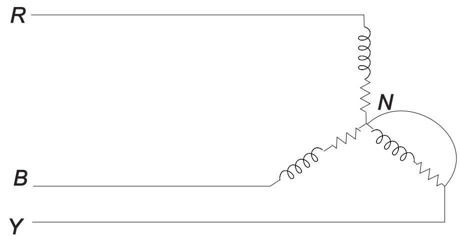


Fig. 9.57(b)

$$I_{Ph} = I_R = I_B = \frac{400}{Z_{Ph}} = \frac{400}{22.4} = 17.85 \text{ A}$$

The current in the Y phase is equal to the phasor sum of the R and B.

$$\therefore \quad I_Y = 2 \times I_{Ph} \cos\left(\frac{60}{2}\right) = 30.91 \text{ A}$$

Problem 9.6 For the circuit shown in Fig. 9.58, calculate the line current, the power and the power factor. The value of R, L and C in each phase are 10 ohms, 1 H and 100 μF, respectively.

Solution Let us assume RYB sequence.

$$V_{RN} = \frac{400}{\sqrt{3}} \angle 0° = 231 \angle 0°; \; V_{YN} = 231 \angle -120°; \; V_{BN} = 231 \angle -240°$$

$$\text{Admittance of each phase } Y_{Ph} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{10} + \frac{1}{j314} + j314 \times 100 \times 10^{-6}$$

$$Y_{Ph} = 0.1 + j28.22 \times 10^{-3}$$

$$= 0.103 \angle 15.75° \; \mho$$

$$I_{Ph} = V_{Ph} Y_{Ph}$$

$$= (231 \angle 0°)(0.103 \angle 15.75°)$$

$$= 23.8 \angle 15.75° \text{ A}$$

$$\text{Power} = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 23.8 \cos 15.75°$$

$$= 15869.57 \text{ W}$$

$$\text{Power factor} = \cos 15.75° = 0.96 \text{ leading}$$

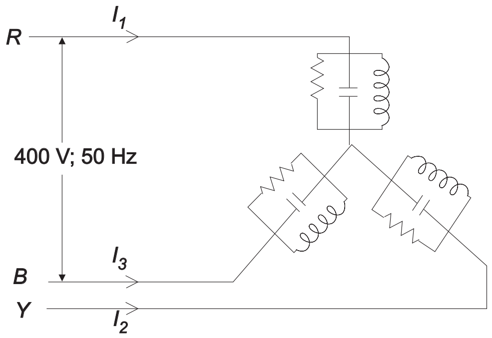


Fig. 9.58

Problem 9.7 For the circuit shown in Fig. 9.59, an impedance is connected across *YB*, and a coil of resistance 3 Ω and inductive reactance of 4 Ω is connected across *RY*. Find the value of *R* and *X* of the impedance across *YB* such that $I_2 = 0$. Assume a balanced three-phase supply with RYB sequence.

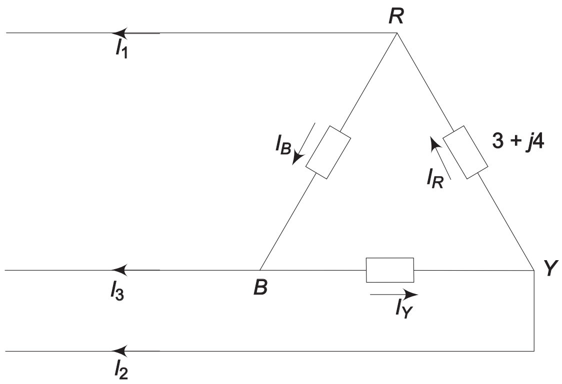


Fig. 9.59

Solution As usual I_R, I_Y and I_B are phase currents, and I_1, I_2 and I_3 are line currents.

Applying KCL at node *Y*, we have

$$I_2 = I_Y - I_R$$

Since $$I_2 = 0$$

$$I_Y = I_R$$

$\therefore$ $$I_R = \frac{V_{RY}}{3 + j4}, \; I_Y = \frac{V_{YB}}{Z_{YB}}$$

$$V_{RY} = V\angle 0°, \; V_{YB} = V\angle -120°$$

$$\frac{V\angle 0°}{3 + j4} = \frac{V\angle -120°}{Z_{YB}}$$

$$Z_{YB} = \frac{V\angle -120^\circ}{V\angle 0^\circ}\ (3 + j4)$$

$$= 1.96 - j4.6$$

$\therefore$ $R = 1.96\ \Omega;\ X = 4.6\ \Omega$ (capacitive reactance)

Problem 9.8 A symmetrical three-phase 440 V system supplies a balanced delta-connected load. The branch current is 10 A at a phase angle of 30°, lagging. Find (i) the line current (ii) the total active power, and (iii) the total reactive power. Draw the phasor diagram.

Solution (i) In a balanced delta-connected system

$$I_L = \sqrt{3}\, I_{Ph} = \sqrt{3} \times 10 = 17.32 \text{ A}$$

(ii) Total active power

$$= \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times 440 \times 17.32 \times \cos 30^\circ = 11.431 \text{ kW}$$

(iii) Total reactive power

$$= \sqrt{3}\, V_L\, I_L \sin\phi$$

$$= \sqrt{3} \times 440 \times 17.32 \times \sin 30^\circ = 6.5998 \text{ KVAR}$$

The phasor diagram is as under

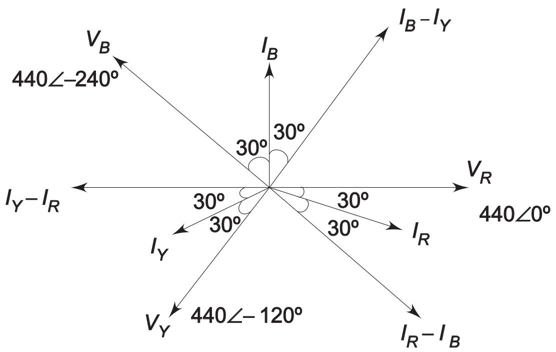


Fig. 9.60

V_R, V_Y and V_B are phase voltages, and are equal to the line values. I_R, I_Y and I_B are the phase currents, and lag behind their respective phase voltages by 30°. Line currents $(I_R - I_B)$, $(I_Y - I_R)$ and $(I_B - I_Y)$ lag behind their respective phase currents by 30°.

Problem 9.9 Find the line currents and the total power consumed by the unbalanced delta-connected load shown in Fig. 9.61.

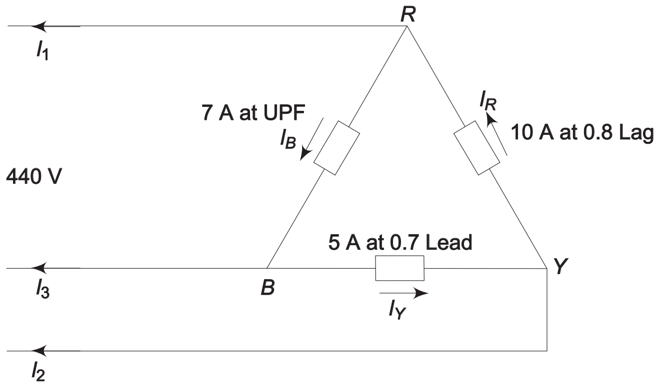


Fig. 9.61

Solution Assuming RYB phase sequence, from the given data

$$I_R = 10 \angle -36.88°;\ I_Y = 5 \angle 45.57°;\ I_B = 7 \angle 0°$$

Line currents are

$$I_1 = I_R - I_B = 6.08 \angle -80°$$
$$I_2 = I_Y - I_R = 10.57 \angle 11.518°$$
$$I_3 = I_B - I_Y = 5 \angle -45.56°$$

Total power is the sum of the powers consumed in all the three phases.

$\therefore$ Power in $RY = V_{RY} \times I_R \times 0.8$
$= 400 \times 10 \times 0.8 = 3200$ W

Power in $YB = V_{YB} \times I_Y \times 0.7$
$= 400 \times 5 \times 0.7 = 1400$ W

Power in $BY = V_{BR} \times I_B \times 1$
$= 400 \times 7 \times 1 = 2800$ W

Total power $= 3200 + 1400 + 2800 = 7400$ W

Problem 9.10 A delta-connected three-phase load has 10 Ω between R and Y, 6.36 mH between Y and B, and 636 μF between B and R. The supply voltage is 400 V, 50 Hz. Calculate the line currents for RBY phase sequence.

Solution

$$Z_{RY} = 10 + j0 = 10 \angle 0°;\ Z_{YB} = 0 + jX_L = 0 + jX_L$$
$$= 0 + j2\pi fL = 2 \angle 90°$$

$$Z_{BR} = 0 - jX_C = 0 - \frac{j}{2\pi fC} = 5 \angle -90°$$

Since the phase sequence is RBY, taking V_{RY} as reference voltage, we have

$$V_{RY} = 400 \angle 0°;\ V_{BR} = 400 \angle -120°;\ V_{YB} = 400 \angle -240°$$

$$I_R = \frac{V_{RY}}{10 \angle 0°} = \frac{400 \angle -0°}{10 \angle 0°} = 40 \angle 0°$$

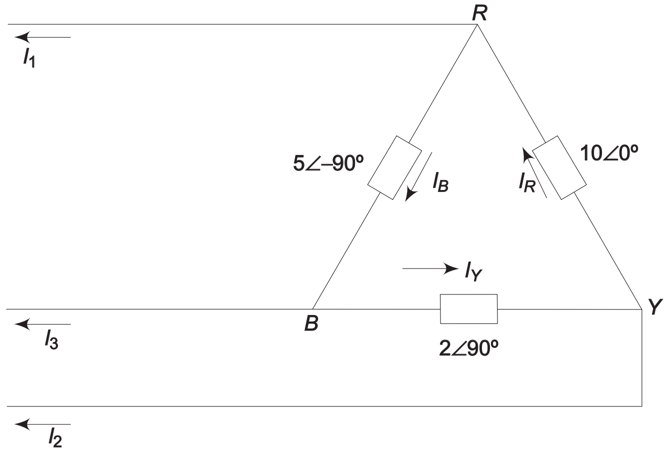


Fig. 9.62

$$I_Y = \frac{V_{YB}}{2\angle 90^\circ} = \frac{400\angle -240^\circ}{2\angle 90^\circ} = 200\angle -330^\circ$$

$$I_B = \frac{V_{BR}}{5\angle -90^\circ} = \frac{400\angle -120^\circ}{5\angle -90^\circ} = 80\angle -30^\circ$$

The three line currents are

$$I_1 = I_R - I_B = 40\angle 0^\circ - 80\angle -30^\circ = 49.57\angle 126.2^\circ$$
$$I_2 = I_Y - I_R = 200\angle -300^\circ \angle -40\angle 0^\circ = 166.56\angle 36.89^\circ$$
$$I_3 = I_B - I_Y = 80\angle -30^\circ - 200\angle -330^\circ = 174.35\angle 233.41^\circ$$

Problem 9.11 The power consumed in a three phase balanced star-connected load is 2 kW at a power factor of 0.8 lagging. The supply voltage is 400 V, 50 Hz. Calculate the resistance and reactance of each phase.

Solution Phase voltage $= \dfrac{400}{\sqrt{3}}$

Power consumed = 2000 W = $\sqrt{3}\, V_L I_L \cos\phi$

Phase current or line current $I_L = \dfrac{2000}{\sqrt{3}\times 400\times 0.8} = 3.6$ A

Impedance of each phase

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{\sqrt{3}\times 3.6} = 64.15 \text{ A}$$

Since the power-factor of the load is lagging, the reactance is inductive reactance. From the impedance triangle shown in Fig. 9.63, we have

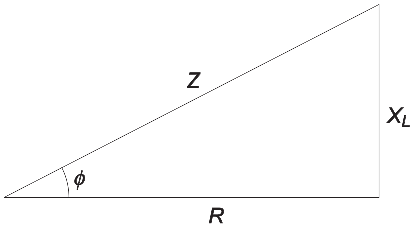


Fig. 9.63

Resistance of each phase $R_{Ph} = Z_{Ph} \cos \phi$

$= 64.15 \times 0.8 = 51.32\ \Omega$

Reactance of each phase $X_{Ph} = Z_{Ph} \sin \phi$

$= 64.15 \times 0.6 = 38.5\ \Omega$

Problem 9.12 A symmetrical three-phase 100 V; three-wire supply feeds an unbalanced star-connected load, with impedances of the load as $Z_R = 5\ \angle 0°\ \Omega$, $Z_Y = 2\ \angle 90°\ \Omega$ and $Z_B = 4\ \angle -90°\ \Omega$. Find the (i) line currents, (ii) voltage across the impedances and (iii) the displacement neutral voltage.

Solution As explained earlier, this type of unbalanced *Y*-connected three-wire load can be solved either by star-delta conversion method or by applying Millman's theorem.

(a) *Star-Delta Conversion Method*

The unbalanced star-connected load and its equivalent delta load are shown in Fig. 9.64 (a) and (b).

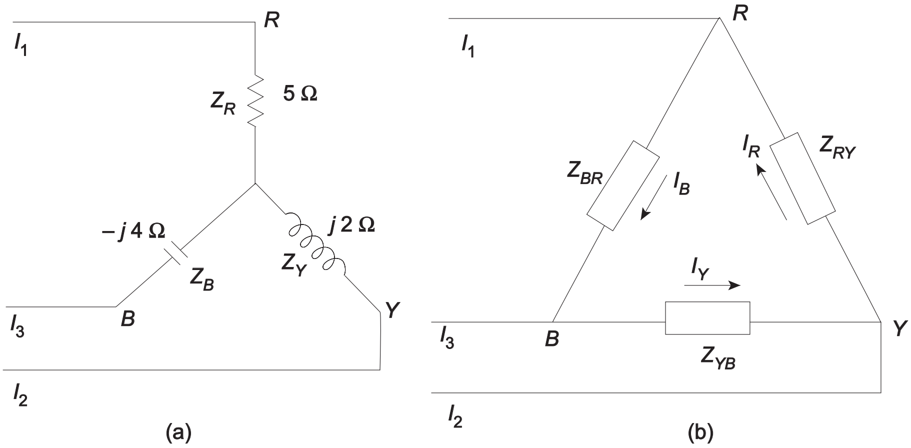


Fig. 9.64

$$Z_R Z_Y + Z_Y Z_B + Z_B Z_R = (5\ \angle 0°)\ (2\ \angle 90°) + (2\ \angle 90°)\ (4\ \angle -90°)$$
$$+ (4\ \angle -90°)\ (5\ \angle 0°) = 8 - j10 = 12.8\ \angle -51.34°$$

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = \frac{12.8\ \angle -51.34°}{4\ \angle -90°} = 3.2\ \angle 38.66°$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} = \frac{12.8 \angle -51.34°}{5 \angle 0°} = 2.56 \angle -51.34°$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} = \frac{12.8 \angle -51.34°}{2 \angle 90°} = 6.4 \angle -141.34°$$

Taking V_{RY} as the reference, we have

$$V_{RY} = 100 \angle 0°,\ V_{YB} = 100 \angle -120°;\ V_{BR} = 100 \angle -240°$$

The three phase currents in the equivalent delta load are

$$I_R = \frac{V_{RY}}{Z_{RY}} = \frac{100 \angle 0°}{3.2 \angle 38.66°} = 31.25 \angle -38.66°$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{100 \angle -120°}{2.56 \angle -51.34°} = 39.06 \angle -68.66°$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{100 \angle -240°}{6.4 \angle -141.34°} = 15.62 \angle -98.66°$$

The line currents are

$$\begin{aligned} I_1 &= I_R - I_B = 31.25 \angle -38.66° - 15.62 \angle -98.66° \\ &= (24.4 - j19.52) - (-2.35 + j15.44) = (26.75 - j4.08) \\ &= 27.06 \angle -8.671° \end{aligned}$$

$$\begin{aligned} I_2 &= I_Y - I_R = 39.06 \angle -68.66° - 31.25 \angle -38.66° \\ &= (14.21 - j36.38) - (24.4 - j19.52) = (-10.19 - j16.86) \\ &= 19.7 \angle 238.85° \end{aligned}$$

$$\begin{aligned} I_3 & \\ &= I_B - I_Y = 15.62 \angle -98.66° - 39.06 \angle -98.66° \\ &= (-2.35 - j15.44) - (14.21 - j36.38) = (-16.56 + j20.94) \\ &= 26.7 \angle 128.33° \end{aligned}$$

These line currents are also equal to the line (phase) currents of the original star connected load.

(ii) Voltage drop across each star-connected load will be as under.

Voltage across $Z_R = I_1 \times Z_R$

$$= (27.06 \angle -8.671°)\ (5 \angle 0°) = 135.3 \angle -8.67°$$

Voltage across $Z_Y = I_2 \times Z_Y$

$$= (19.7 \angle 238.85°)\ (2 \angle 90°) = 39.4 \angle 328.85°$$

Voltage across $Z_B = I_3 \times Z_B$

$$= (26.7 \angle 128.33°)\ (4 \angle -90°) = 106.8° \angle 38.33°$$

(b) *BY Applying Millman's Theorem*

Consider Fig. 9.64(c), taking V_{RY} as reference line voltage = 100 $\angle 0°$.

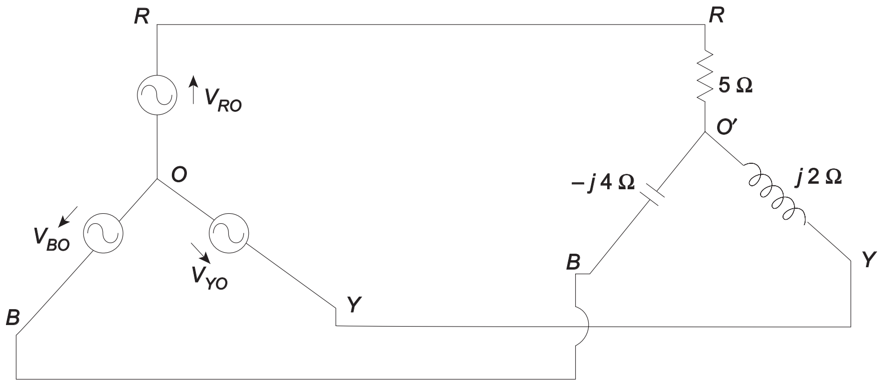


Fig. 9.64 (c)

Phase voltages log 30° behind their respective line voltages. Therefore, the three phase voltages are

$$V_{RO} = \frac{100}{\sqrt{3}} \angle -30^\circ$$

$$V_{YO} = \frac{100}{\sqrt{3}} \angle -150^\circ$$

$$V_{BO} = \frac{100}{\sqrt{3}} \angle -270^\circ$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5 \angle 0^\circ} = 0.2 \angle 0^\circ$$

$$Y_Y = \frac{1}{Z_Y} = \frac{1}{2 \angle 90^\circ} = 0.5 \angle -90^\circ$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{4 \angle -90^\circ} = 0.25 \angle 90^\circ$$

$$\begin{aligned}
V_{RO}Y_R + V_{YO}Y_Y + Y_{BO}Y_B &= (57.73 \angle -30^\circ)(0.2 \angle 0^\circ) \\
&\quad + (57.73 \angle -150^\circ)(0.5 \angle -90^\circ) \\
&\quad + (57.73 \angle -270^\circ)(0.25 \angle 90^\circ) \\
&= 11.54 \angle -30^\circ + 28.86 \angle -240^\circ + 14.43 \angle -180^\circ \\
&= (10 - j5.77) + (-14.43 + j25) + (-14.43 + j0) \\
&= -18.86 + j19.23 = 26.93 \angle 134.44^\circ \\
Y_R + Y_Y + Y_B &= 0.2 + 0.5 \angle -90^\circ + 0.25 \angle 90^\circ \\
&= 0.32 \angle -51.34^\circ \\
V_{O'O} &= \frac{V_{RO}Y_R + V_{YO}Y_Y + V_{BO}Y_B}{Y_R + Y_Y + Y_B} = \frac{26.93 \angle 134.44^\circ}{0.32 \angle -51.34^\circ} \\
&= 84.15 \angle 185.78^\circ
\end{aligned}$$

The three load phase voltages are

$$V_{RO'} = V_{RO} - V_{O'O}$$
$$= 57.73\ \angle -30° - 84.15\ \angle 185.78°$$
$$= (50 - j28.86) - (-83.72 - j8.47)$$
$$= (133.72 - j20.4) = 135.26\ \angle -8.67°$$
$$V_{YO'} = V_{YO} - V_{O'O}$$
$$= 57.73\ \angle -150° - 84.15\ \angle 185.78°$$
$$= (-50 - j28.86) - (-83.72 - j8.47)$$
$$= 33.72 - j20.4 = 39.4\ \angle -31.17° \text{ or } 39.4\ \angle 328.8°$$
$$V_{BO'} = V_{BO} - V_{O'O}$$
$$= 57.73\ \angle -270° - 84.15\ \angle 185.78°$$
$$= 0 + j57.73 + 83.72 + j8.47$$
$$= 83.72 + j66.2 = 106.73\ \angle 38.33°$$
$$I_R = \frac{135.26\ \angle -8.67°}{5\ \angle 0°} = 20.06\ \angle -8.67°$$
$$I_Y = \frac{39.4\ \angle 328.80°}{2\ \angle 90°} = 19.7\ \angle 238.8°$$
$$I_B = \frac{106.73\ \angle 38.33°}{4\ \angle -90°}$$
$$= 26.68\ \angle 128.33°$$

Problem 9.13 A three phase three-wire unbalanced load is star-connected. The phase voltages of two of the arms are

$$V_R = 100\ \angle -10°;\ V_Y = 150\ \angle 100°$$

Calculate voltage between star point of the load and the supply neutral.

Solution As shown in Fig. 9.65

$$V_{RO} = V_{RO'} + V_{O'O}$$

or $$V_{O'O} = V_{RO} - V_{RO'} \tag{9.19}$$

Also $$V_{O'O} = V_{YO} - V_{YO'} \tag{9.20}$$

Let $$V_{RO} = V\ \angle 0°$$

Assuming RYB phase sequence

$$V_{YO} = V\ \angle -120°$$

Substituting in Eqs 9.19 and 9.20, we have

$$V_{O'O} = V\ \angle 0° - 100\ \angle -10° \tag{9.21}$$
$$V_{O'O} = V\ \angle -120° - 150\ \angle 100° \tag{9.22}$$

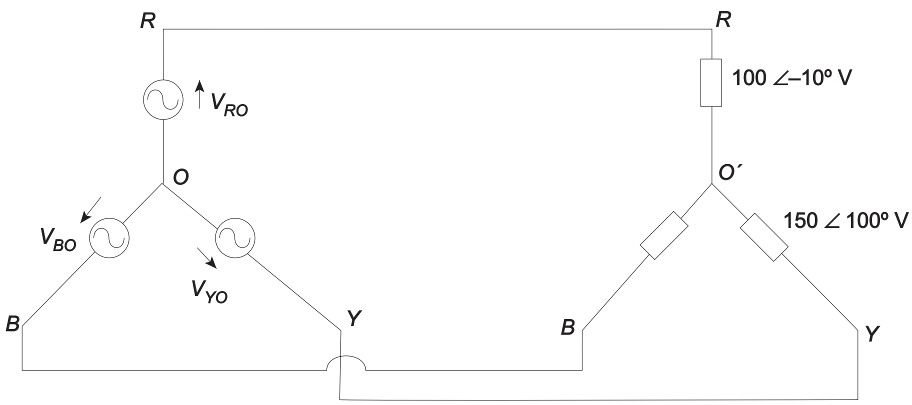


Fig. 9.65

Subtracting Eq. (9.22) from Eq. (9.21), we get

$$O = [(V + jO) - (98.48 - j17.36)] - [(0.5V + j0.866V) - (-26.04 + j147.72)]$$

$$O = 1.5\,V - j0.8666V - 124.52 + j165.08$$

$$= V\,(1.5 - j0.866) = 124.52 - j165.08$$

$$V = \frac{124.52 - j165.08^\circ}{1.5 - j0.866} = \frac{206.77\angle -52.97^\circ}{1.732\angle -30^\circ}$$

$$V = 119.38\angle -22.97^\circ$$

Voltage between $O'O = V_{RO} - V_{RO'}$

$$V_{O'O} = 119.38\angle -22.97^\circ - 100\angle -10^\circ$$

$$= 109.91 - j46.58 - 98.48 + j17.36$$

$$= 11.43 - j29.22 = 31.37\angle -68.63^\circ$$

Problem 9.14 Find the reading of a wattmeter in the circuit shown in Fig. 9.66(a). Assume a symmetrical 400 V supply with RYB phase sequence and draw the vector diagram.

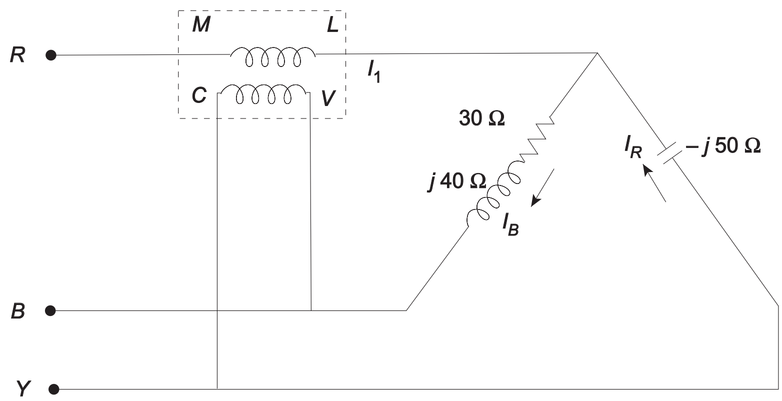


Fig. 9.66(a)

Solution The reading in the wattmeter is equal to the product of the current through the current coil I_1 voltage across its pressure coil V_{YB} and cos of the angle between the V_{YB} and I_1.

$$I_R = \frac{V_{RY}}{-j50} = \frac{400\angle 0^\circ}{50\angle -90^\circ} = 8\angle 90^\circ$$

$$I_B = \frac{V_{BR}}{30 + j40} = \frac{400\angle -240^\circ}{50\angle 53.13^\circ} = 8\angle -293.13^\circ \text{ or } 8\angle 66.87^\circ$$

Line current

$$\begin{aligned} I_1 &= I_R - I_B \\ &= 8\angle 90^\circ - 8\angle -293.13^\circ \\ &= 0 + j8 - 3.14 - j7.35 = -3.14 + j0.65 = 3.2\angle 168.3^\circ \end{aligned}$$

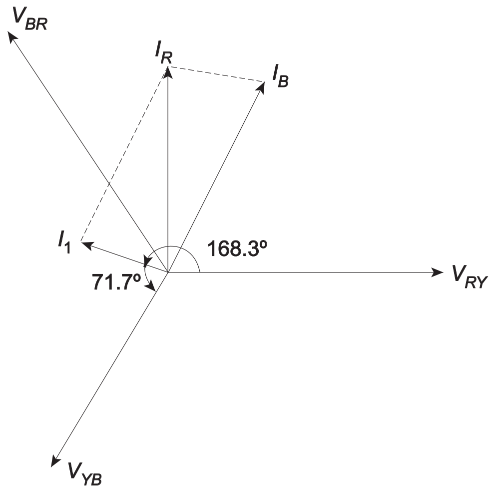


Fig. 9.66(b)

From the vector diagram in Fig. 9.66(b), it is clear that the angle between V_{YB} and I_1 is 71.7°.

$\therefore$ Wattmeter reading is equal to $V_{YB} \times I_1 \cos 71.7^\circ$

$$= 400 \times 3.2 \times \cos 71.7 = 402 \text{ W}$$

Problem 9.15 Calculate the total power input and readings of the two wattmeters connected to measure power in a three-phase balanced load, if the reactive power input is 15 KVAR, and the load pf is 0.8.

Solution Let W_1 be the lower reading wattmeter and W_2 the higher reading wattmeter

$$\cos\phi = 0.8$$

$$\phi = 36.86^\circ$$

$$\tan\phi = \frac{\text{Reactive power}}{\text{Active power}}$$

or
$$= \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1}$$

$$\text{Reactive power} = \sqrt{3}\,(W_2 - W_1) = 15000$$

$$= W_2 - W_1 = 8660.508 \text{ W} \qquad (9.23)$$

$$\therefore \quad 0.75 = \sqrt{3}\,\frac{15000}{W_2 + W_1}$$

or Total power input $W_2 + W_1 = 34641.01$ W (9.24)

From Eqs. (9.23) and (9.24) we get

$$W_2 = 21650.76 \text{ W}$$

$$W_1 = 12990.24 \text{ W}$$

Problem 9.16 Two wattmeters are connected to measure power in a three-phase circuit. The reading of the one of the meter is 5 kW when the load power factor is unity. If the power factor of the load is changed to 0.707 lagging, without changing the total input power, calculate the readings of the two wattmeters.

Solution Both wattmeters indicate equal values when the power factor is unity

$$\therefore \quad W_1 + W_2 = 10 \text{ kW (Total power input)} \qquad (9.25)$$

Let W_2 be the higher reading wattmeter

Then W_1 is the lower reading wattmeter

$$\cos\phi = 0.707 \quad \therefore\ \phi = 45°$$

$$\tan\phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} \quad 1 = \sqrt{3}\,\frac{W_2 - W_1}{10}$$

$$\therefore \quad W_2 - W_1 = \frac{10}{\sqrt{3}} = 5.773 \text{ kW} \qquad (9.26)$$

From Eqs. (9.25) and (9.26),

$$W_2 = 7.886 \text{ kW}$$

$$W_1 = 2.113 \text{ kW}$$

Problem 9.17 The line currents in a balanced six-phase mesh connected generator are 35.35 A. What is the magnitude of the phase current?

Solution From Section 9.8.5

$$I_L = 2I_{Ph}\sin\frac{180°}{n}$$

$$I_{Ph} = \frac{35.35}{2\sin\frac{180}{6}} = 35.35$$

Problem 9.18 Find the voltage between the adjacent lines of a balanced six-phase star-connected system with a phase voltage of 132.8 volts.

Solution From Section 9.7.5 $E_L = 2E_{Ph} \sin \dfrac{180°}{n}$

$$E_L = 2 \times 132.8 \times \sin \frac{180°}{6} = 132.8 \text{ V}$$

Problem 9.19 In the wye connected system shown in Fig. 9.67, it is assumed that only fundamental and third harmonic voltages are present when the voltages are measured with a voltmeter between *na* and *ba*. They are given by 230 and 340 volts respectively. Calculate the magnitude of the third harmonic voltages in the system.

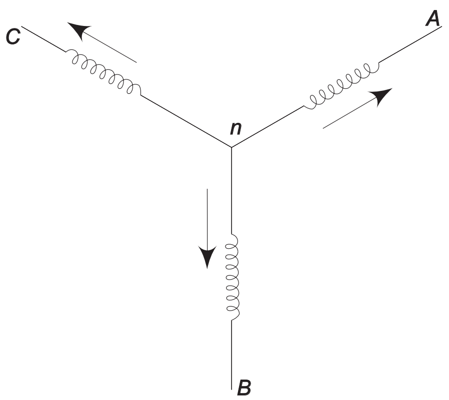


Fig. 9.67

Solution Only phase voltage V_{na} of the system shown in Fig. 9.67 contains 3rd harmonic whereas line voltage V_{ba} contains only 1st harmonic. Hence,

Fundamental component of the phase $= \dfrac{340}{\sqrt{3}}$

Third harmonic component $= \sqrt{220^2 - \left(\dfrac{340}{\sqrt{3}}\right)^2} = 99.33$ V

Problem 9.20 Illustrate the effect of reversal of voltage sequence up on the magnitudes of the currents in the system shown in Example 9.20.

Solution The line currents for RYB sequence have already been calculated. $I_1 = 28.41 \angle -69.07°$, $I_2 = 29.85 \angle 136.58°$ and $I_3 = 13 \angle 27.60°$ A.

If the phase sequence is reversed by RBY then

$$I_R = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0°}{15.67 \angle 60.13°} = 25.52 \angle -60.13° \text{ A}$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -240°}{43.83 \angle 49.83°} = 9.12 \angle -289.83° \text{ A}$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{400 \angle -120°}{78.36 \angle 60.13°} = 5.1 \angle -180.13° \text{ A}$$

Various line currents are given by

$$I_1 = I_R - I_B = 25.52 \angle -60.13° - 5.1 \angle -180.13° = 28.41 \angle -51.189° \text{ A}$$

$$I_2 = I_Y - I_R = 9.12 \angle -289.83° - 25.52 \angle -60.13° = 32.175 \angle 107.37° \text{ A}$$

$$I_3 = I_B - I_Y = 5.1 \angle -180.13° - 9.12 \angle -289.83° = 11.85 \angle 46.26° \text{ A}$$

From the above calculations, it can be verified that the magnitudes of the line currents are not same when the phase sequence is changed.

PSpice Problems

Problem 9.1 A 3-phase Δ connected RYB system is shown in Fig. 9.68 with an effective voltage of 400 V, has a balanced load with impedances $3 + j4\Omega$. Using PSpice calculate (i) phase currents (ii) line currents and (iii) power in each phase.

$L = 12.73$ mH

$R = 3\ \Omega$

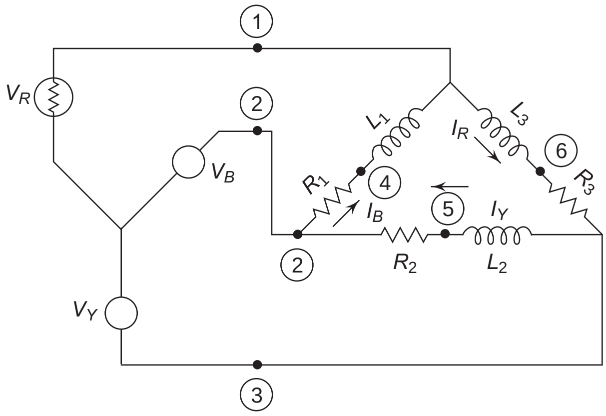


Fig. 9.68 (a)

```
*  3 PHASE BALANCED ANALYSIS
VR    1    0    AC 230.94 -30
VY    3    0    AC 230.94 -150
VB    2    0    AC 230.94 90
R1    2    4    3
L1    1    4    12.732 M
```

```
R2   2   5   3
L2   5   3   12.732 M
R3   3   6   3
L3   1   6   12.732 M
.AC LIN 1 50 50
.PRINT AC IM(VR) IP(VR) IM(VY) IP(VY) IM(VB) IP(VB)
+ IM(R1) IP(R1) IM(R2) IP(R2) IM(R3) IP(R3)
.PROBE
.END
```

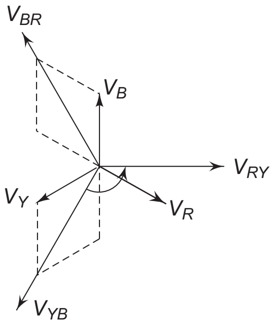


Fig. 9.68 (b)

**** AC ANALYSIS   TEMPERATURE = 27.000 DEG C

****

FREQ	IM(VR)	IP(VR)	IM(VY)	IP(VY)	IM(VB)
5.000E + 01	1.386E + 02	9.687E + 01	1.386E + 02	–2.313E + 01	1.386E + 02

FREQ	IP(VB)	IM(R1)	IP(R1)	IM(R2)	IP(R2)
5.000E + 01	–1.431E + 02	8.000E + 01	6.687E + 01	8.000E + 01	6.871E + 00

FREQ	IM(R3)	IP(R3)
5.000E + 01	8.000E + 01	1.269E + 02

Result:

IR = –I(R3) = 48.03 – J63.97

IY = –I(R2) = –79.42 – J9.61

IB = I(R1) = 31.38 + J73.58

I1 = IR – IB = 138.55∠–83.09

I2 = IY – IR = 138.55∠156.9

I3 = IB – IY = 138.55∠36.89

POWER CONSUMED IN EACH PHASE = VPH IPH COS ϕ = $400 \times 80 \times 0.6$

= 19.2 KW

TOTAL POWER = 3×19.2 K = 57.6 KW.

Problem 9.2 The AC circuit of following Fig. 9.69(a) is supplied from a three- phase balanced supply. Use PSpice to calculate RMS magnitudes and phase angles of currents, I_R, I_Y, I_B and I_N.

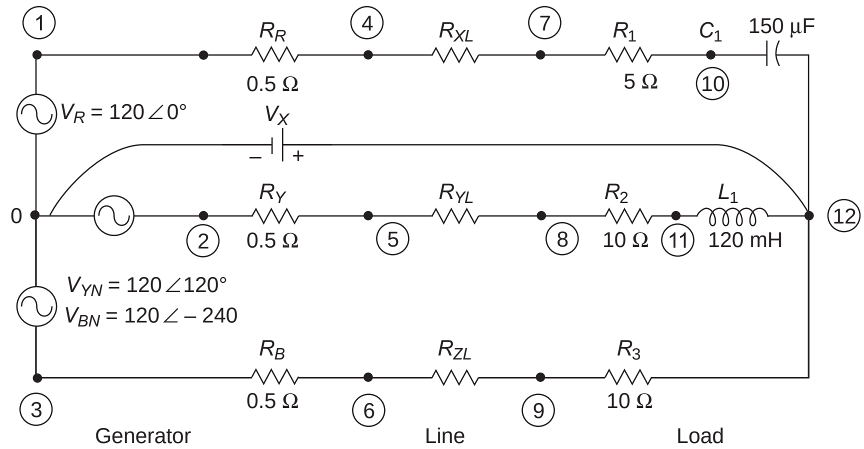


Fig. 9.69 (a)

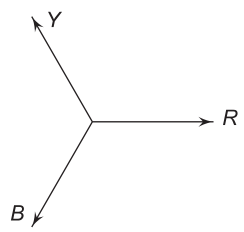


Fig. 9.69 (b)

```
9.2    THREE PHASE CIRCUIT
VRN    1     0     AC 120 0
VYN    2     0     AC 120 120
VBN    3     0     AC 120 -120
RR     1     4     0.5
RY     2     5     0.5
RB     3     6     0.5
RXL    4     7     1
RYL    5     8     1
RZL    6     9     1
R1     7     10    5
R2     8     11    10
R3     9     12    10
C1     10    12    150 UF
L2     11    12    120MH
VX     12    0     DC 0V
.AC LIN 1 50 100
.PRINT AC IM(RR) IP(RR) IM(RY) IP(RY) IM(RB) IP(RB)
+ VM(7,12) VP(7,12) VM(8,12) VP(8,12) VM(9,12) VP(9,12)
.PRINT AC IM(VX) IP(VX)
.END
```

**** AC ANALYSIS TEMPERATURE = 27.000 DEG C
****

FREQ	IM(RR)	IP(RR)	IM(RY)	IP(RY)	IM(RB)
5.000E + 01	5.407E + 00	7.297E + 01	3.045E + 00	4.696E + 01	1.043E + 01

FREQ	IP(RB)	VM(7,12)	VP(7,12)	VM(8,12)	VP(8,12)
5.000E + 01	–1.200E + 02	1.179E + 02	–3.772E + 00	1.187E + 02	1.221E + 02

FREQ	VM(9,12)	VP(9,12)
5.000E + 01	1.043E + 02	–1.200E + 02

FREQ	IM(VX)	IP(VX)
5.000E + 01	2.262E + 00	–1.335E + 02

Practice Problems

9.1 Three non-reactive resistors of 5, 10 and 15 Ω are star-connected to *R*, *Y* and *B* phase of a 440 V symmetrical system. Determine the current and power in each resistor and the voltage between star point and neutral; assume the phase sequence as RYB.

9.2 A three-phase, three-wire symmetrical 440 V source is supplying power to an unbalanced, delta-connected load in which $Z_{RY} = 20 \angle 30° \ \Omega$, $Z_{YB} = 20 \angle 0° \ \Omega$ and $Z_{BR} = 20 \angle -30° \ \Omega$. If the phase sequence is RYB, calculate the line currents.

9.3 Three equal resistances connected in star across a three-phase balanced supply consume 1000 W. If the same three resistors were reconnected in delta across the same supply, determine the power consumed.

9.4 The currents in R_Y, Y_B and B_R branches of a mesh connected system with symmetrical voltages are 20 A at 0.7 lagging power factor, 20 A at 0.8 leading power factor, and 10 A at UPF respectively. Determine the current in each line. Phase sequence is RYB. Draw a phasor diagram.

9.5 A three-phase, four-wire symmetrical 440 V; RYB system supplies a star-connected load in which $Z_R = 10 \angle 0° \ \Omega$, $Z_Y = 10 \angle 26.8° \ \Omega$ and $Z_B = 10 \angle -26.8° \ \Omega$. Find the line currents, the neutral current and the load power.

9.6 A balanced three-phase, star-connected voltage source has $V_{RN} = 230 \angle 60° \ \Omega \ V_{\text{rms}}$ with *RYB* phase sequence, and it supplies a balanced delta-connected three-phase load. The total power drawn by the load is 15 kW at 0.8 lagging power factor. Find the line currents, load and phase currents.

9.7 Three identical impedances $10 \angle 30° \ \Omega$ in a delta-connection, and three identical impedances $5 \angle 35° \ \Omega$ in a star-connection are on the same three-phase, three-wire 173 V system. Find the line currents and the total power.

9.8 Three impedances of $(7 + j4) \ \Omega$; $(3 + j2) \ \Omega$ and $(9 + j2) \ \Omega$ are connected between neutral and the red, yellow and blue phases, respectively of a three-phase, four-wire system; the line voltage is 440 V. Calculate (i) the current in each line, and (ii) the current in the neutral wire.

9.9 Three capacitors, each of 100 μF are connected in delta to a 440 V, three-phase, 50 Hz supply. What will be the capacitance of each of the three capacitors if the same three capacitors are connected in star across the same supply to draw the same line current.

9.10 A 400 V, three-phase supply feeds an unbalanced three-wire, star-connected load, consisting of impedances $Z_R = 7 \angle 10° \ \Omega$, $Z_Y = 8 \angle 30° \ \Omega$ and $Z_B = 8 \angle 50° \ \Omega$. The phase sequence is RYB. Determine the line currents and total power taken by the load.

9.11 The power taken by a 440 V, 50 Hz, three-phase induction motor on full load is measured by two wattmeters, which indicate 250 W and 1000 W, respectively. Calculate (i) the input (ii) the power factor (iii) the current, and (iv) the motor output, if the efficiency is 80%.

9.12 In the two wattmeter method of power measurement, the power registered by one wattmeter is 3500 W, while the other reads down scale. After reversing the later, it reads 300 W. Determine the total power in the circuit and the power factor.

9.13 Three non-inductive resistances of 25 Ω, 10 Ω and 15 Ω are connected in star to a 400 V symmetrical supply. Calculate the line currents and the voltage across the each load phase.

9.14 Three impedances, $Z_R = (3 + j2) \ \Omega$; $Z_Y = j9 \ \Omega$ and $Z_B = 3 \ \Omega$ are connected in star across a 400 V, 3-wire system. Find the loads on the equivalent delta-connected system phase-sequence RYB.

9.15 Consider the unbalanced Δ – Δ circuit shown in Fig. 9.70. Use Pspice to find generator currents, the line currents and phase currents.

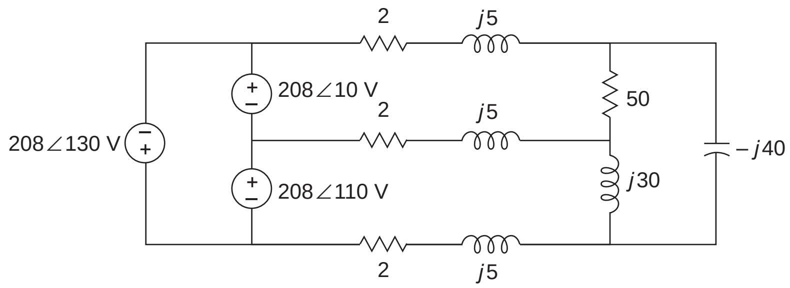


Fig. 9.70

9.16 For the unbalanced circuit shown in Fig. 9.71, calculate
(a) the line currents
(b) the real power absorbed by the load
(c) the total complex power supplied by the source. Use PSpice.

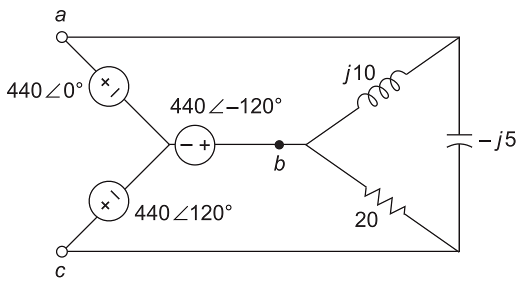


Fig. 9.71

9.17 The circuit shown in Fig. 9.72 is supplied from a $3 - \phi$ balanced supply. Use PSpice to calculate rms magnitudes and phase angles of currents: I_a, I_b, I_c and I_N.

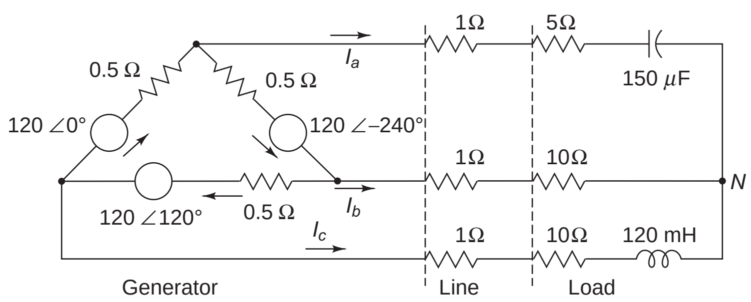


Fig. 9.72

Objective Type Questions

9.1 The resultant voltage in a closed balanced delta circuit is given by
  (a) three times the phase voltage
  (b) $\sqrt{3}$ times the phase voltage
  (c) zero

9.2 Three coils A, B, C, displaced by 120° from each other are mounted on the same axis and rotated in a uniform magnetic field in clockwise direction. If the instantaneous value of emf in coil A is $E_{\max} \sin \omega t$, the instantaneous value of emf in B and C coils will be
  (a) $E_{\max} \sin\left(\omega t - \frac{2\pi}{3}\right); E_{\max} \sin\left(\omega t - \frac{4\pi}{3}\right)$
  (b) $E_{\max} \sin\left(\omega t + \frac{2\pi}{3}\right); E_{\max} \sin\left(\omega t + \frac{4\pi}{3}\right)$
  (c) $E_{\max} \sin\left(\omega t - \frac{2\pi}{3}\right); E_{\max} \sin\left(\omega t + \frac{4\pi}{3}\right)$

9.3 The current in the neutral wire of a balanced three-phase, four-wire star connected load is given by
 (a) zero
 (b) $\sqrt{3}$ times the current in each phase
 (c) 3 times the current in each phase

9.4 In a three-phase system, the volt ampere rating is given by
 (a) $3V_L\, I_L$
 (b) $\sqrt{3}\, V_L\, I_L$
 (c) $V_L\, I_L$

9.5 In a three-phase balanced star connected system, the phase relation between the line voltages and their respective phase voltage is given as under
 (a) the line voltages lead their respective phase voltages by 30°.
 (b) the phase voltages lead their respective line voltage by 30°.
 (c) the line voltages and their respective phase voltages are in phase.

9.6 In a three-phase balanced delta connected system, the phase relation between the line currents and their respective phase currents is given by
 (a) the line currents lag behind their respective phase currents by 30°.
 (b) the phase currents lag behind their respective line currents by 30°.
 (c) the line currents and their respective phase currents are in phase.

9.7 In a three-phase unbalanced, four-wire star-connected system, the current in the neutral wire is given by
 (a) zero
 (b) three times the current in individual phases
 (c) the vector sum of the currents in the three lines

9.8 In a three-phase unbalanced star-connected system, the vector sum of the currents in the three lines is
 (a) zero
 (b) not zero
 (c) three times the current in the each phase

9.9 Wattmeter deflection in ac circuit is proportional to the
 (a) maximum power in the circuit
 (b) instantaneous power in the circuit
 (c) average power in the circuit

9.10 Three wattmeter method of power measurement can be used to measure power in
 (a) balanced circuits
 (b) unbalanced circuits
 (c) both balanced and unbalanced circuits

9.11 Two wattmeter method of power measurement can be used to measure power in
 (a) balanced circuits
 (b) unbalanced circuits
 (c) both balanced and unbalanced circuits

9.12 In two wattmeter methods of power measurements, when the pf is 0.5
- (a) the readings of the two wattmeters are equal and positive
- (b) the readings of the two wattmeters are equal and opposite
- (c) the total power is measured by only one wattmeter

9.13 The reading of the wattmeter connected to measure the reactive power in a three phase circuit is given by zero, the line voltage is 400 V and line current 15 A; then the pf of the circuit is
- (a) zero
- (b) unity
- (c) 0.8