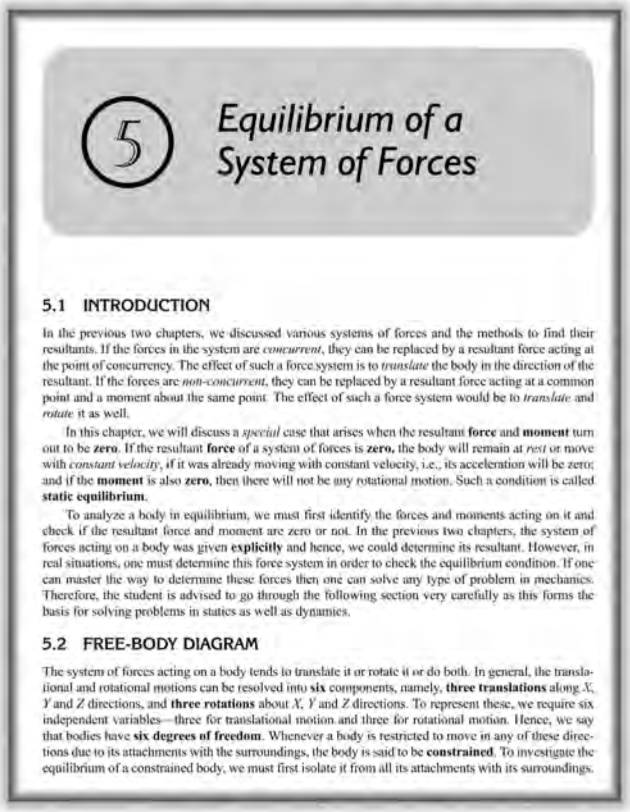
Guided Tour

Introduction

The introduction, at the beginning of each chapter, gives an overview of what is going to be discussed in that chapter. It also forms a bridge between the preceding chapter and the current chapter.



5 Equilibrium of a System of Forces

5.1 INTRODUCTION

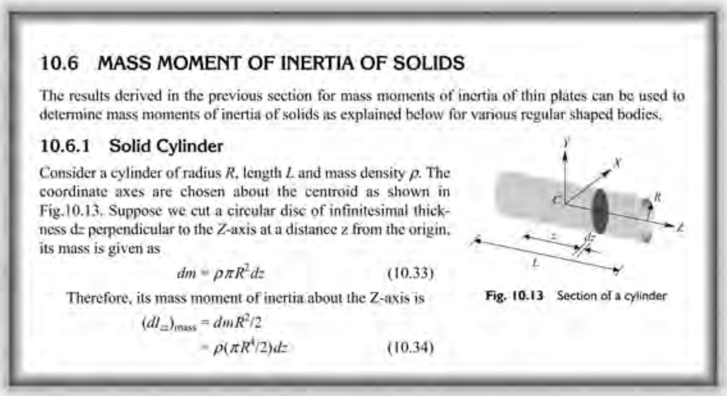
In the previous two chapters, we discussed various systems of forces and the methods to find their resultants. If the forces in the system are *concurrent*, they can be replaced by a resultant force acting at the point of concurrency. The effect of such a force system is to *translate* the body in the direction of the resultant. If the forces are *non-concurrent*, they can be replaced by a resultant force acting at a common point and a moment about the same point. The effect of such a force system would be to *translate* and *rotate* it as well.

In this chapter, we will discuss a *special* case that arises when the resultant **force** and **moment** turn out to be **zero**. If the resultant **force** of a system of forces is **zero,** the body will remain at *rest* or move with *constant velocity*, if it was already moving with constant velocity, i.e., its acceleration will be zero; and if the **moment** is also **zero,** then there will not be any rotational motion. Such a condition is called **static equilibrium**.

To analyze a body in equilibrium, we must first identify the forces and moments acting on it and check if the resultant force and moment are zero or not. In the previous two chapters, the system of forces acting on a body was given **explicitly** and hence, we could determine its resultant. However, in real situations, one must determine this force system in order to check the equilibrium condition. If one can master the way to determine these forces then one can solve any type of problem in mechanics. Therefore, the student is advised to go through the following section very carefully as this forms the basis for solving problems in statics as well as dynamics.

5.2 FREE-BODY DIAGRAM

The system of forces acting on a body tends to translate it or rotate it or do both. In general, the translational and rotational motions can be resolved into **six** components, namely, **three translations** along X, Y and Z directions, and **three rotations** about X, Y and Z directions. To represent these, we require six independent variables—three for translational motion and three for rotational motion. Hence, we say that bodies have **six degrees of freedom**. Whenever a body is restricted to move in any of these directions due to its attachments with the surroundings, the body is said to be **constrained**. To investigate the equilibrium of a constrained body, we must first isolate it from all its attachments with its surroundings.



10.6 MASS MOMENT OF INERTIA OF SOLIDS

The results derived in the previous section for mass moments of inertia of thin plates can be used to determine mass moments of inertia of solids as explained below for various regular shaped bodies.

10.6.1 Solid Cylinder

Consider a cylinder of radius R, length L and mass density ρ. The coordinate axes are chosen about the centroid as shown in Fig.10.13. Suppose we cut a circular disc of infinitesimal thickness dz perpendicular to the Z-axis at a distance z from the origin, its mass is given as

$$dm = \rho \pi R^2 dz \tag{10.33}$$

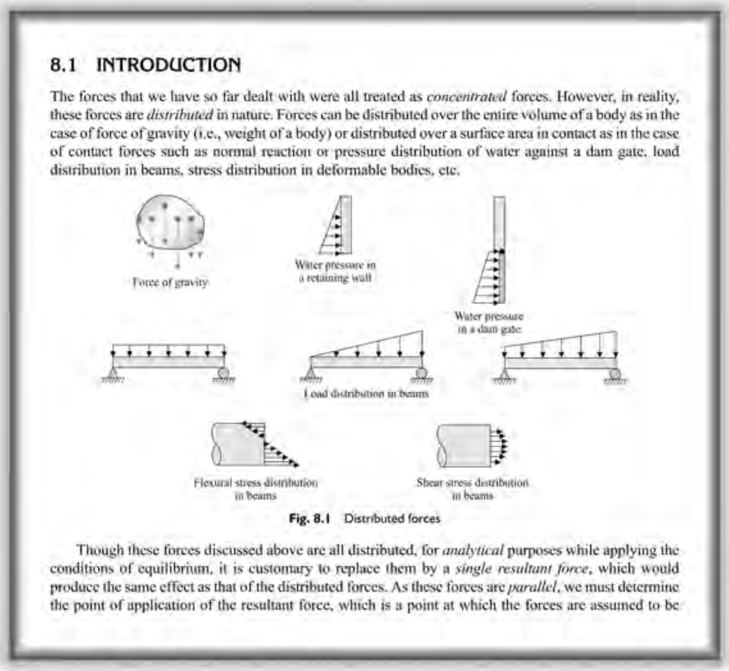
Therefore, its mass moment of inertia about the Z-axis is

$$(dI_{zz})_{mass} = dm R^2/2$$
$$= \rho(\pi R^4/2)dz \tag{10.34}$$

Fig. 10.13 Section of a cylinder

Sections and Sub-sections

Each chapter has been divided into sections and sub-sections to present the basic concepts in a lucid manner.



8.1 INTRODUCTION

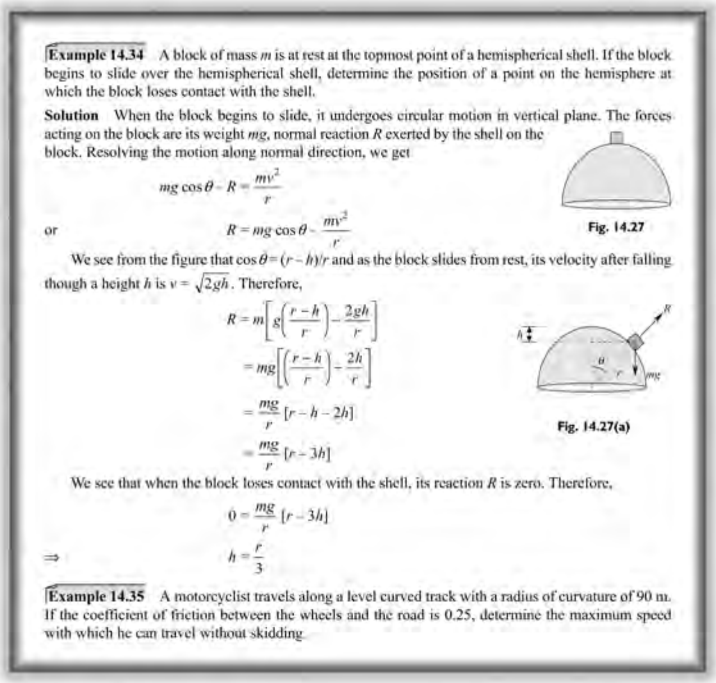
The forces that we have so far dealt with were all treated as *concentrated* forces. However, in reality, these forces are *distributed* in nature. Forces can be distributed over the entire volume of a body as in the case of force of gravity (i.e., weight of a body) or distributed over a surface area in contact as in the case of contact forces such as normal reaction or pressure distribution of water against a dam gate, load distribution in beams, stress distribution in deformable bodies, etc.

Fig. 8.1 Distributed forces

Though these forces discussed above are all distributed, for *analytical* purposes while applying the conditions of equilibrium, it is customary to replace them by a *single resultant force*, which would produce the same effect as that of the distributed forces. As these forces are *parallel*, we must determine the point of application of the resultant force, which is a point at which the forces are assumed to be

Illustrations

Illustrations are a way of presenting the theory for easy and better understanding. Pictures, line diagrams, sketches in two dimensional (plane) and three dimensional (space), tables and charts have been used liberally throughout this book.



Example 14.34 A block of mass m is at rest at the topmost point of a hemispherical shell. If the block begins to slide over the hemispherical shell, determine the position of a point on the hemisphere at which the block loses contact with the shell.

Solution When the block begins to slide, it undergoes circular motion in vertical plane. The forces acting on the block are its weight mg, normal reaction R exerted by the shell on the block. Resolving the motion along normal direction, we get

$$mg\cos\theta - R = \frac{mv^2}{r}$$

or

$$R = mg\cos\theta - \frac{mv^2}{r}$$

Fig. 14.27

We see from the figure that $\cos\theta = (r-h)/r$ and as the block slides from rest, its velocity after falling though a height h is $v = \sqrt{2gh}$. Therefore,

$$R = m\left[g\left(\frac{r-h}{r}\right) - \frac{2gh}{r}\right]$$

$$= mg\left[\left(\frac{r-h}{r}\right) - \frac{2h}{r}\right]$$

$$= \frac{mg}{r}[r-h-2h]$$

$$= \frac{mg}{r}[r-3h]$$

Fig. 14.27(a)

We see that when the block loses contact with the shell, its reaction R is zero. Therefore,

$$0 = \frac{mg}{r}[r-3h]$$

$$\Rightarrow \quad h = \frac{r}{3}$$

Example 14.35 A motorcyclist travels along a level curved track with a radius of curvature of 90 m. If the coefficient of friction between the wheels and the road is 0.25, determine the maximum speed with which he can travel without skidding.

Solved Examples

Solved examples are provided at the end of each major section within a chapter to explain the application of the theory to real world situations. There are as high as 500 solved examples in total.

Solution To draw the free-body diagrams, we detach the cylinder, plank and string separately. Then they are drawn to scale.

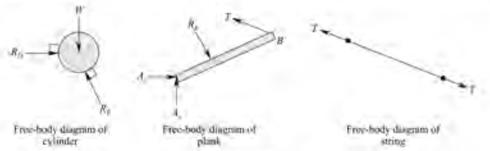


Fig. 5.23(a)

As the cylinder is in contact with the wall at D and with the plank at E, normal reactions R_D (exerted by the wall on the cylinder) and R_E (exerted by the plank on the cylinder) are shown in its free-body diagram. Also, the weight W of the cylinder is placed at the centre of gravity and directed vertically downwards. Since R_E is the force exerted by the plank on the cylinder, the force exerted by the cylinder on the plank will be equal and in the opposite direction. Hence, R_E for the plank is shown in the opposite direction. In addition, at point A, as it is a hinge, the reactions A_x and A_y are shown. At point B, there will be a tension T as it is connected by a string. Note that here we have neglected the weight of the plank. If its weight is to be included, it must be placed at the centre of gravity of the plank. The free-body diagram of the string will have a tension T at both of its ends. Normally, strings are assumed to be weightless.

Free Body Diagrams

Free body diagrams are essential to understand the forces acting in members of a structure or mechanical system. Nearly every solved example is explained with separate free body diagrams for each member constituting a system.

ladder at the end B; contact point C with m along the X and Y directions at the point of impending motion,

$\sum F_x = 0 \Rightarrow$

$$N_C \sin\theta - F_C \cos\theta - F_B = 0$$
$$N_C \sin\theta - \mu N_C \cos\theta - \mu N_B = 0$$
$$\therefore \quad N_C[\sin\theta - \mu\cos\theta] - \mu N_B = 0 \qquad \text{(a)}$$

$\sum F_y = 0 \Rightarrow$

$$N_C \cos\theta + F_C \sin\theta + N_B - W = 0$$
$$N_C \cos\theta + \mu N_C \sin\theta + N_B - W = 0$$
$$\therefore \quad N_C[\cos\theta + \mu\sin\theta] + N_B = W \qquad \text{(b)}$$

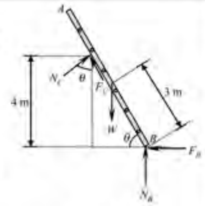


Fig. 6.37(a)

SUMMARY

Any motion that repeats itself after equal intervals of time is termed *periodic* motion. The motion of water waves in seas under the action of wind can be cited as an example. As water waves are free to move, they displace from one place to the other. On the other hand, real civil structures such as transmission cable, diving board in swimming pool, bridge, tall towers, etc., and even mechanical systems such as simple pendulum, string of a musical instrument due to their attachment with the surroundings do not displace from one place to the other upon the action of external forces. Instead, they move back and forth over the *same path*. Such types of periodic motions, which trace the same path in a cyclic manner, are termed *vibratory* or *oscillatory* motions.

For all analytical purposes, we assume all resistance to vibration to be eliminated and such vibrations are termed *undamped free vibrations*. However, this is an ideal condition as we normally observe these vibrations to die out after some time and such vibrations are termed *damped free vibrations*. If the external force continues to act on the structure periodically then the resulting vibrations are termed *forced vibrations*.

Simple Harmonic Motion

Simple harmonic motion is a special case of *rectilinear* motion with *variable* acceleration, in which the acceleration of the particle is proportional to the displacement from the origin and is always directed towards the origin. Most of the structures and mechanical systems are observed to execute this motion under small displacements from the equilibrium position.

The equations of motion as functions of time and displacement for ready reference are shown below:

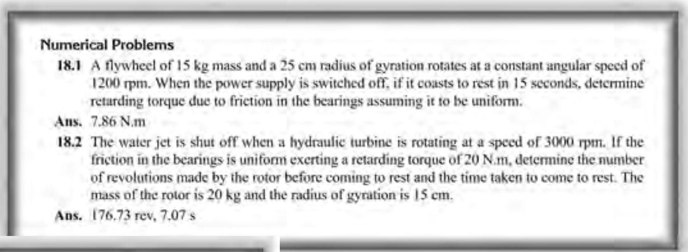
	As function of displacement	*As function of time*	
		$t = 0$, at mean position	*$t = 0$, at extreme position*
Displacement	x	$x = A\sin\omega t$	$x = A\cos\omega t$
Velocity	$v = \pm\omega\sqrt{A^2 - x^2}$	$v = A\omega\cos\omega t$	$v = -A\omega\sin\omega t$
Acceleration	$a = -\omega^2 x$	$a = -A\omega^2\sin\omega t$	$a = -A\omega^2\cos\omega t$

Summary

The summary at the end of each chapter gives a handy description of all definitions and formulas discussed in that chapter. It would be of great help for preparation just before exams.

Numerical Problems with Answers

Each chapter at the end has numerical problems with answers, totaling 639 in the book. They give practice in applying the theory to real world situations and provide further confidence to solve any kind of problem.



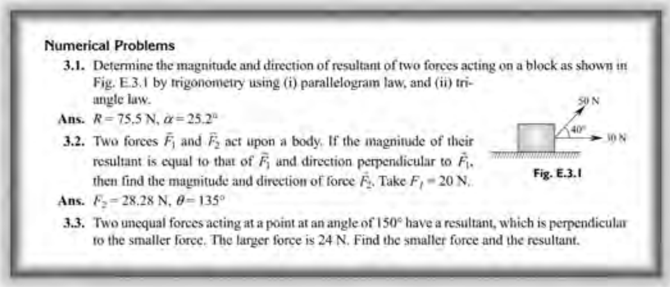
Numerical Problems

18.1 A flywheel of 15 kg mass and a 25 cm radius of gyration rotates at a constant angular speed of 1200 rpm. When the power supply is switched off, if it coasts to rest in 15 seconds, determine retarding torque due to friction in the bearings assuming it to be uniform.

Ans. 7.86 N.m

18.2 The water jet is shut off when a hydraulic turbine is rotating at a speed of 3000 rpm. If the friction in the bearings is uniform exerting a retarding torque of 20 N.m, determine the number of revolutions made by the rotor before coming to rest and the time taken to come to rest. The mass of the rotor is 20 kg and the radius of gyration is 15 cm.

Ans. 176.73 rev, 7.07 s



Numerical Problems

3.1. Determine the magnitude and direction of resultant of two forces acting on a block as shown in Fig. E.3.1 by trigonometry using (i) parallelogram law, and (ii) triangle law.

Ans. $R = 75.5$ N, $\alpha = 25.2°$

3.2. Two forces $\bar{F}_1$ and $\bar{F}_2$ act upon a body. If the magnitude of their resultant is equal to that of $\bar{F}_1$ and direction perpendicular to $\bar{F}_1$, then find the magnitude and direction of force $\bar{F}_2$. Take $F_1 = 20$ N.

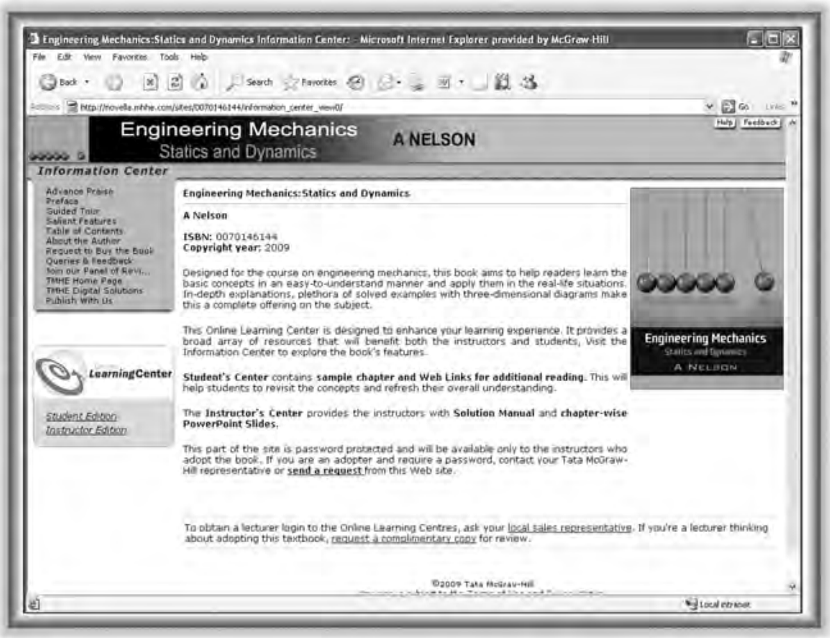
Ans. $F_2 = 28.28$ N, $\theta = 135°$

3.3. Two unequal forces acting at a point at an angle of 150° have a resultant, which is perpendicular to the smaller force. The larger force is 24 N. Find the smaller force and the resultant.

Fig. E.3.1

Online Resources

The quickstep solutions to the numerical problems can also be accessed through the publisher's website.



EXERCISES

Objective-type Questions

1. A rigid body can be idealized as a particle
   (a) only when its size is very minute
   (b) only when the body is at rest
   (c) when there is no translational motion involved
   (d) when there is no rotational motion involved
2. In rectilinear motion, all the particles in the body
   (a) have the same displacement (b) have the same velocity
   (c) have the same acceleration (d) all of these
3. Average velocity is defined as
   (a) average of initial and final velocities
   (b) ratio of change in displacement and elapsed time
   (c) ratio of distance travelled and elapsed time
   (d) average of initial and final speeds
4. When a car moves at a constant speed around a curved path, its velocity
   (a) is zero (b) is constant
   (c) changes in magnitude (d) changes in direction
5. A man walks from one town to another and then comes back. State which of the following statements is true concerning his journey:
   (a) his displacement is zero (b) distance traveled is zero
   (c) average speed is zero (d) time taken is zero
6. A man walks from one town to another at a constant speed of 15 kmph and then returns back at a constant speed of 10 kmph. His average speed for the journey is
   (a) 12.5 kmph (b) 12 kmph (c) 2.5 kmph (d) 25 kmph
7. A particle can move with constant velocity when motion is
   (a) rectilinear (b) curvilinear (c) rotational (d) general motion
8. Uniform motion implies that
   (a) acceleration is constant (b) velocity is constant
   (c) position is constant (d) time is constant
9. The area under an a–t curve represents
   (a) average acceleration (b) instantaneous acceleration
   (c) change in position of the particle (d) change in velocity of the particle
10. The area under a v–t curve represents
    (a) average velocity of the particle (b) instantaneous velocity of the particle
    (c) distance travelled by the particle (d) acceleration of the particle

Objective-type Questions

Objective type questions presented at the end of each chapter are useful not only in competitive exams like GATE but also in the online internal exams of regular engineering courses as followed by some of the universities. There are around 220 questions in total.

Short-answer Questions

1. Distinguish between statics and dynamics.
2. Distinguish between kinematics and kinetics.
3. Distinguish between particle and rigid body.
4. Explain the types of motion with suitable examples.
5. Define position vector and displacement vector.
6. Distinguish between displacement vector and distance travelled.
7. Define velocity of a particle.
8. Define average velocity and instantaneous velocity.
9. Under what conditions is average velocity equal to instantaneous velocity?
10. Define average acceleration and instantaneous acceleration.
11. If a particle moves with constant speed but changes in direction, can there be acceleration?
12. Distinguish between rectilinear motion and curvilinear motion.
13. State the differential equations of motion.
14. Distinguish between uniform motion and uniformly accelerated motion.
15. Derive the x–t, v–t and a–t relationships for uniformly accelerated motion.
16. What are motion curves? What are they used for?
17. Define free fall.
18. What are the assumptions made in free fall?

Short-answer Questions

Each chapter has ample short answer questions for practicing the basic concepts learnt in the theory. There are a total of 255 questions in this book.