15 Work and Energy

15.1 INTRODUCTION

In the preceding chapter, we solved kinetic problems using Newton's laws of motion. From the second law of motion, we determined the *acceleration* of a body or a system of bodies. Once acceleration is known, we could describe the kinematics of the system, i.e., its *displacement* and *velocity* as functions of time. In this chapter, we will introduce an alternative approach, called **work–energy** method to solve the same type of kinetic problems.

Unlike Newton's second law of motion, which relates force and acceleration, the work–energy method relates *force*, *velocity* and *displacement*. The work–energy method has certain advantages over Newton's method for the following reasons: Firstly, work and energy are *scalar* quantities and hence, they add up algebraically; thus, avoiding the need to deal with directional aspects of force vectors under Newton's method. Secondly, the kinematics of the problem, i.e., displacement and velocity could be determined directly without knowing the acceleration of the system. Displacement and velocity are more real to understand than acceleration, which seems to be an *abstract* quantity. Thirdly, motion of inter connected bodies could be solved without drawing separate free-body diagrams for each body in the system.

In Sections 15.2–15.4, we will define the concept of work in general and works done by a constant force and a variable force. In Section 15.5, we will define power of the driving force, since the rate at which work is done is more important than the work itself. In Sections 15.6 and 15.8, we will define energy, various forms of energy, kinetic and potential energies. In Section 15.7, we will discuss applying work–energy method to solve motion of interconnected bodies.

15.2 WORK DONE BY A FORCE

When a force acting on a particle causes a *displacement* of the particle, the force is then said to have done **work** on the particle. This definition of work is quite different from our daily usage of the word 'work,' which we refer to any activity involving muscular or mental effort. Consider a force $\vec{F}$ acting on a particle at A causing a displacement $\vec{s}$ (from the point A to B) in the direction of the force. We then define work done on the particle as a **product** of magnitudes of **force** and **displacement**. Mathematically, we can write this as

$$W = F\,s \tag{15.1}$$

A ball freely falling under gravity or a block being pulled on a smooth plane by a horizontal force (refer Fig. 15.2) are examples in which the motion of the body is in the direction of the applied force. (It should be noted that as the line of action of the force passes through the centre of gravity of each body, we could idealize each body as a particle). However, we should note that due to *constraints* involved, the displacement of a particle under the action of forces would not always occur in the direction of the force. Consider for instance, a block sliding down a smooth inclined plane due to pull of gravity or a block being pulled on a smooth plane by an inclined force (refer Fig. 15.3). In these two cases, we observe that the direction of motion is *different* to the direction of the force acting, i.e., inclined at an angle θ to the direction of the force.

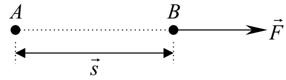
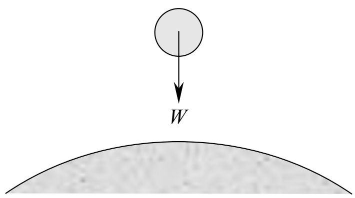
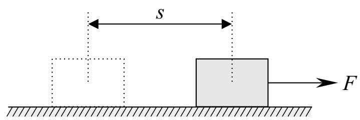


Fig. 15.1 Work done by a force displacing a particle in the direction of the force

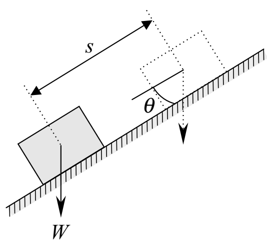


A ball freely falling

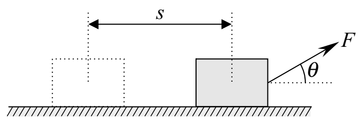


Force acting horizontally

Fig. 15.2



Body sliding down an inclined plane



Force acting at an angle θ

Fig. 15.3

Hence, we can define work done in general, as a product of the **component** of the force in the direction of motion and the displacement. Mathematically, we can write this as

$$W = (F \cos \theta)s \qquad (15.2)$$

As the above expression can also be written as

$$W = (F)\,(s \cos \theta) \qquad (15.3)$$

work done can also be defined as a product of the force and *component* of displacement in the direction of the force.

The unit of work done is dependent on the units of force and displacement. Hence, its S.I. unit is N.m. This is also called Joule, in honour of the British scientist James P. Joule, who worked on the relationship between heat and work. It is defined as the work done by a unit force (1 newton) when it causes a unit displacement (1 metre) along the direction of the force.

For a system of forces acting on a particle, the work done by the system of forces is given by the *algebraic sum* of works done by individual forces. Since work done adds up algebraically, it is a *scalar* quantity, that is, having magnitude, but not direction. However, we see that force and displacement are vectors. The product of two vectors resulting in a scalar quantity can best be represented by the dot product of two vectors. Thus, work done can be written in vector form as

$$W = \vec{F}.\vec{s} = Fs\cos\theta \tag{15.4}$$

If force and displacement are in the same direction, i.e., $\theta = 0°$, then $\cos\theta = 1$ and hence work done is $W = Fs$, which is the same as the Eq. 15.1. If force and displacement are in the opposite direction, i.e., $\theta = 180°$, then $\cos\theta = -1$ and hence work done is $W = -Fs$. We saw in Chapter 6 that the force of friction always acts in the direction *opposite* to that of the motion; hence, work done by the force of friction is always **negative**. In general, if the component of force is in the *direction* of displacement then θ is an *acute* angle and $\cos\theta$ is *positive*. Hence, work done in such a case is *positive*. If the component of force is in the *direction opposite* to that of displacement then θ is an *obtuse* angle and $\cos\theta$ is *negative*. Hence, work done in such a case is *negative*.

We also come across situations in which the work done by a force or a system of forces is **zero**. These are discussed below in detail.

(i) When the displacement [s] is zero Even though forces may act on a particle, if there is **no** displacement of the particle then *no* work is done on the particle. Consider a block resting on a table. In its free-body diagram, we see that even though its weight W and normal reaction R are acting, they do *no* work on the block, as there is *no* displacement of the block. Similarly, in the case of a ball suspended by a string and a beam supported at one end by a hinge support, no work is done by the forces acting, as there is no displacement involved.

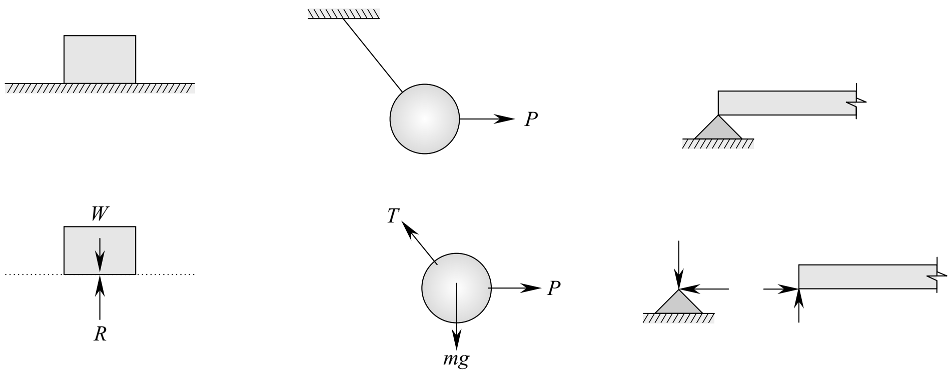


Fig. 15.4 No work is done by forces as there is no displacement

(ii) When the motion is at right angles to the direction of the force When the motion is at right angles to the direction of the force, we see that $\theta = 90°$ and hence, $\cos\theta = 0$. Thus, work done is **zero** as per the Eq. 15.4. Consider a block moving along a horizontal plane as shown in Fig. 15.5. Since the displacement is at right angles to the direction of the forces, namely, its weight and normal reaction, the two forces do *no* work on the block. Similarly, when a body is moving in a circle, the centripetal force does *no* work on the body.

(iii) Total work done is zero When a particle is in *static equilibrium* then the resultant force acting on it is **zero**. Hence, the **total work done** by the system of forces is also **zero**. Consider a system of concurrent forces $\vec{F}_1, \vec{F}_2, \ldots \vec{F}_n$ acting on a particle causing a displacement $\Delta\vec{s}$. Then we can write the total work done on the particle as

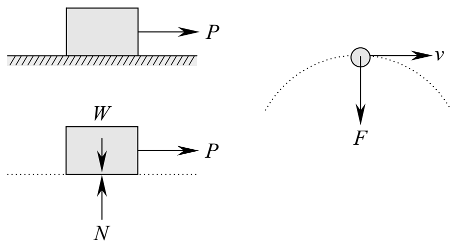


Fig. 15.5 No work is done by forces acting at right angles to the direction of motion

$$W = \vec{F}_1.\Delta\vec{s} + \vec{F}_2.\Delta\vec{s} + \ldots + \vec{F}_n.\Delta\vec{s}$$
$$= [\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n].\Delta\vec{s}$$
$$= \sum\vec{F}.\Delta\vec{s}$$
$$= 0 \text{ [since the resultant of the forces } \sum\vec{F} \text{ is zero]} \qquad (15.5)$$

Corollary The way we have defined work done is quite different from our colloquial usage of the word "work." For instance, a passenger carrying a heavy luggage and waiting for a train is said to do *no* work, even though he has to exert effort to hold the luggage. This is because his effort is causing *no* displacement of the luggage. However, when the train arrives and he lifts the luggage, boards the train and places the luggage on the rack, he is said to do work as it causes displacement of the luggage.

15.3 WORK DONE BY A VARIABLE FORCE

In the previous section, we considered the force as constant, i.e., constant in magnitude and direction and thus we defined work done by the force. In this section, we will discuss work done by a *varying* force acting on a particle.

Consider a variable force $\vec{F}$ acting on a particle as shown in Fig. 15.6. As force is a vector, it can vary in magnitude, direction or both. In such a case, the displacement in general will be along a **curvilinear** path. As shown in the figure, we see that the force vector changes in magnitude as well as in the direction as it moves from the point A to the point B along the path of motion.

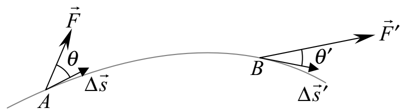


Fig. 15.6 Work done by a variable force

As we let B approach A, the displacement becomes *infinitesimally* small, that is $d\vec{s}$. Then over this infinitesimally small displacement, we can assume the force to remain *constant* in magnitude and direction. Hence, we can write work done over this infinitesimally small displacement $d\vec{s}$ as

$$dW = \vec{F}.d\vec{s} \qquad (15.6)$$

Therefore, work done over the entire path is obtained by integrating the above expression between limits, i.e.,

$$W_{AB} = \int_A^B dW = \int_A^B \vec{F}.d\vec{s}$$
$$= \int_A^B F\cos\theta\, ds \qquad (15.7)$$

The above integral is difficult to evaluate as both F and θ vary from point to point along the path. If we express $\vec{F}$ and $d\vec{s}$ in terms of the rectangular components, i.e.,

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \tag{15.8}$$

and
$$d\vec{s} = dx\vec{i} + dy\vec{j} + dz\vec{k} \tag{15.9}$$

then work done is given as

$$W_{AB} = \int_A^B [F_x\vec{i} + F_y\vec{j} + F_z\vec{k}].\,[dx\vec{i} + dy\vec{j} + dz\vec{k}]$$

$$= \int_A^B [F_x dx + F_y dy + F_z dz] \tag{15.10}$$

The above integral, which is evaluated over the curvilinear path of motion of the particle, is termed as **line integral**.

15.4 WORK DONE IN STRETCHING A SPRING

As a special case of varying force, consider a force varying in magnitude but constant in direction, acting on a particle. Then the resulting displacement is along a *rectilinear* path. A spring, whose one end is fixed and the other end stretched is an example for a variable force constant in direction but varying in magnitude.

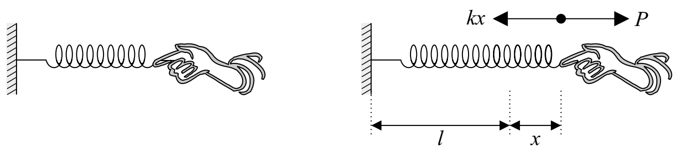


Fig. 15.7 Work done by a variable force

Consider a force P applied to an unstretched spring of length l. Let it cause a displacement x of the free end of the spring, where x is measured from the unstretched position. Due to elastic nature of the spring, a restoring force is developed in the spring, which tries to regain its original unstretched position. Within elastic limit, Hooke's law says that this restoring force is proportional to the elongation and opposite to the direction of displacement. Mathematically,

$$F \propto -x \tag{15.11}$$

Introducing a constant of proportionality, we have

$$F = -kx \tag{15.12}$$

where the constant of proportionality k is called **stiffness** of spring or **spring constant**.

If the spring is stretched slowly, such that it is not accelerated then the restoring force must be equal and opposite to the applied force for equilibrium to be maintained, i.e.,

$$P = kx \tag{15.13}$$

The relationship between the applied force P and elongation x can be represented graphically as shown in Fig. 15.8.

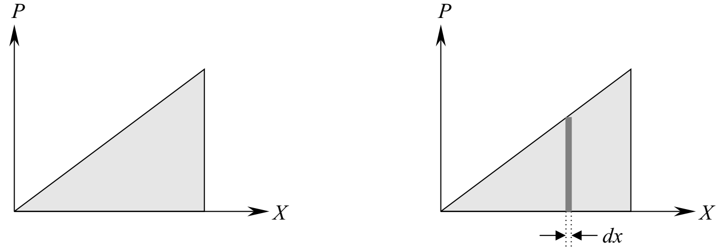


Fig. 15.8 Graphical representation of applied force and elongation

We see that the force and the elongation vary *linearly*. Suppose we consider an infinitesimally small elongation dx then over this infinitesimally small elongation, we can assume the force acting on the spring to be *constant*. Hence, work done over this infinitesimally small displacement is given as

$$dW = P\,dx \tag{15.14}$$

Therefore, work done in stretching the spring to an elongation of x_o from its unstretched position is obtained by integrating the above expression between limits.

$$W = \int_0^{x_o} dW = \int_0^{x_o} P\,dx$$

$$= \int_0^{x_o} kx\,dx$$

$$= \frac{1}{2}\,kx_o^2 \tag{15.15}$$

As the term on the right-hand side represents the area of the triangle in Fig. 15.8, we see that the work done in stretching a spring is given by the area under the P–x curve.

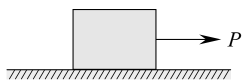


Fig. 15.9

Example 15.1 A block of 5 kg mass is resting on a rough horizontal plane having coefficient of kinetic friction 0.15. It is pulled by a horizontal force P at a constant velocity over a distance of 5 m. Sketch the free-body diagram of the block showing all the forces acting on it. Also, determine (i) the work done by each force acting on the free body, and (ii) the total work done on the body.

Solution The free-body diagram of the block is shown in Fig. 15.9(a), in which the weight of the block, the normal reaction and frictional force together with the externally applied force are shown. As there is no motion along the Y-direction, we know that $\Sigma F_y = 0$. Therefore,

$$N - 5g = 0$$

$$\therefore \quad N = 5g \tag{a}$$

Since the block is moving with constant velocity along the horizontal direction, its acceleration in that direction is zero. Hence, we can write

$$\sum F_x = 0 \Rightarrow$$

$$P - F = 0 \tag{b}$$

Since $F = \mu_k N$, , the above equation can be written as

$$P - \mu_k N = 0$$

$$P - \mu_k (5g) = 0$$

$$\Rightarrow \qquad P = \mu_k(5g)$$

$$= 0.15\,(5 \times 9.81) = 7.36 \text{ N}$$

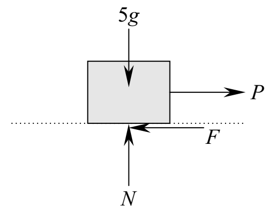


Fig. 15.9(a)

From the equation (b), the force of friction is

$$F = P = 7.36 \text{ N}$$

(i) Work done by each force

Since the displacement of the block is 5 m, work done by the applied force P is

$$W_p = 7.36 \times 5 = 36.8 \text{ J}$$

and work done by the frictional force is

$$W_F = -7.36 \times 5 = -36.8 \text{ J}$$

Note that as the frictional force acts opposite to the direction of displacement of the block, the work done is *negative*. Since the other forces acting on the block, mg and N, are all perpendicular to the direction of displacement of the block, the work done by each of them is zero.

(ii) Hence, the total work done on the block is the algebraic sum of the works done by the forces acting on the block.

$$W = 36.8 - 36.8 = 0$$

Alternatively, we could say that as the block is moving with constant velocity, the resultant force acting on it is zero; hence, the work done on the block is zero.

Example 15.2 A block of 10 kg mass resting on a rough horizontal plane is pulled by an inclined force P as shown in Fig. 15.10, at a constant velocity over a distance of 5 m. The coefficient of kinetic friction between the contact planes is 0.2. Sketch the free-body diagram of the block showing all the forces acting on it. Also, determine (i) the work done by each force acting on the free body, and (ii) the total work done on the block.

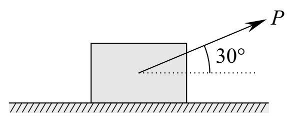


Fig. 15.10

Solution The free-body diagram of the block is shown in Fig. 15.10(a) below. As there is no motion along the Y-direction, we know that $\sum F_y = 0$. Therefore,

$$N + P \sin 30° - 10\,g = 0$$

$$\therefore \qquad N = 10\,g - P \sin 30° \qquad \text{(a)}$$

Since the block is moving with constant velocity along the horizontal direction, its acceleration in that direction is zero. Hence, we can write

$$\sum F_x = 0 \Rightarrow$$

$$P \cos 30° - F = 0 \qquad \text{(b)}$$

Since $F = \mu_k N$, the above equation can be written as

$$P \cos 30° - \mu_k N = 0$$

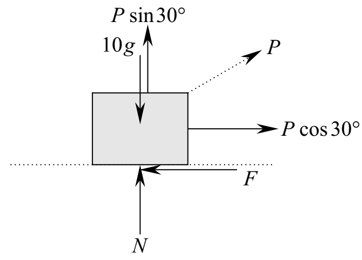


Fig.15.10(a)

Substituting equation (a) in the above equation,

$P \cos 30° - \mu_k (10g - P \sin 30°) = 0$

$$\Rightarrow \qquad P = \frac{10\,\mu_k g}{[\cos 30° + \mu_k \sin 30°]}$$

$$= \frac{10(0.2)(9.81)}{[\cos 30° + (0.2)\sin 30°]}$$

$$= 20.31 \text{ N}$$

Also, force of friction is given as

$$F = P \cos 30°$$

$$= 20.31 \cos 30° = 17.59 \text{ N}$$

(i) Work done by each force

Work done by the horizontal component of P, i.e., $P \cos\theta$ is

$$W_{P\cos\theta} = 20.31 \cos 30° \times 5 = 87.9 \text{ J}$$

and work done by the frictional force is

$$W_F = -17.59 \times 5 = -87.9 \text{ J}$$

Since the other forces acting on the block, $P \sin\theta$, mg and N are all perpendicular to the direction of displacement of the block, the work done by each of them is zero.

(ii) Hence, the total work done on the block is the algebraic sum of works done by each of the forces acting on the block.

$$W = 87.9 - 87.9 = 0$$

Alternatively, we could say that as the block is moving with constant velocity, the resultant force acting on it is zero; hence, the work done on the block is zero.

Example 15.3 Solve the above problem, if the externally applied force P pulls the block at a constant acceleration of 1 m/s^2.

Solution The free body diagram of the block is shown in Fig. 15.11. As before

$$N = 10\,g - P \sin 30° \qquad \text{(a)}$$

Since the block is moving with constant acceleration along the X-axis,

$\sum F_x = ma_x \Rightarrow$

$$P \cos 30° - F = 10a_x$$

$$P \cos 30° - \mu_k N = 10a_x$$

Substituting equation (a) in the above equation,

$P \cos 30° - \mu_k (10g - P \sin 30°) = 10a_x$

$$\Rightarrow \qquad P = \frac{10(a_x + \mu_k g)}{[\cos 30° + \mu_k \sin 30°]}$$

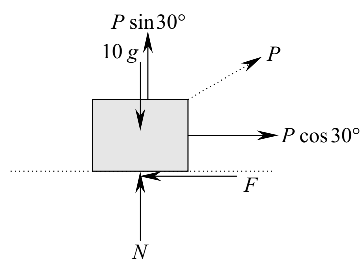


Fig. 15.11

$$= \frac{10[1+(0.2)(9.81)]}{[\cos 30° + (0.2)\sin 30°]}$$

$$= 30.66 \text{ N}$$

Also, force of friction is given as

$$F = P \cos 30° - 10a_x$$

$$= 30.66 \cos 30° - 10(1) = 16.55 \text{ N}$$

(i) Work done by each force

Work done by the horizontal component of P, i.e., $P \cos\theta$ is

$$W_{P\cos\theta} = 30.66 \cos 30° \times 5 = 132.76 \text{ J}$$

and work done by frictional force is

$$W_F = -16.55 \times 5 = -82.75 \text{ J}$$

Since the other forces acting on the body, $P \sin\theta$, mg and N are all perpendicular to the direction of displacement of the body, the work done by each of them is zero.

(ii) Hence, the total work done on the body is the algebraic sum of the works done by each of the forces acting on the body.

$$W = 132.76 - 82.75 \approx 50 \text{ J}$$

Alternatively, we could say that as the body is moving with constant acceleration, the resultant force acting on it is

$$R = ma_x = 10(1) = 10 \text{ N}$$

and the work done by this force is

$$W_R = 10(5) = 50 \text{ J}$$

Hence, we see that the total work done on the body is same as the work done by the resultant force acting on the body as expected.

Example 15.4 A horse pulls a chariot of 200 kg mass at a constant speed. Determine the work done by the horse in pulling the chariot through a distance of 20 m, if the total frictional resistance is 400 N. Also, determine the work done if the horse pulls the chariot at a constant acceleration of 0.5 m/s^2.

Solution *When the horse pulls the chariot at constant speed*

When the horse pulls the chariot at constant speed, the force exerted by the horse on the chariot is equal to the frictional resistance. Hence,

$$F = 400 \text{ N}$$

Therefore, work done by the horse in pulling the chariot through a distance of 20 m is

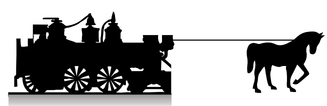


Fig. 15.12

$$W = (400)(20) = 8 \text{ kJ}$$

When the horse pulls the chariot at constant acceleration

When the horse pulls the chariot at a constant acceleration, the total pulling force exerted by the horse is given as

$$F - \text{force of friction} = ma$$

$$\therefore \quad F = \text{force of friction} + \text{ma}$$
$$= 400 + 200(0.5) = 500 \text{ N}$$

Therefore, work done by the horse in pulling the chariot through a distance of 20 m is

$$W = (500)(20) = 10 \text{ kJ}$$

Example 15.5 Find the work done by the force of gravity on a body of 5 kg mass as (i) it falls vertically downwards through a distance of 3 m, and (ii) as it slides down an inclined plane with a slope of 0.75. What do you infer from the result?

Solution

(i) When it falls vertically downwards over a distance of 3 m

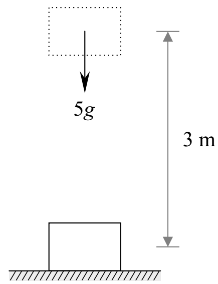


Fig. 15.13(a)

Since the force of gravity and the displacement are in the same direction, the work done by the force of gravity on the body as it descends by a distance of 3 m is given as

$$W_{mg} = (mg)(s)$$
$$= 5 \times 9.81 \times 3 = 147.15 \text{ J}$$

(ii) When it slides down an inclined plane with a slope of 0.75

Since the slope of the plane is 0.75, the sides of the triangle are as indicated in Fig. 15.13(b). The force of gravity can be resolved into two components '$mg \cos\theta$' normal to the plane and '$mg \sin\theta$' along the plane. As the displacement is along the plane, the normal component does not contribute to the work done. Hence,

$$W = (mg \sin\theta)(5)$$

which can also be written as

$$W = (mg)(5 \sin\theta)$$
$$= (mg)(3) = 147.15 \text{ J}$$

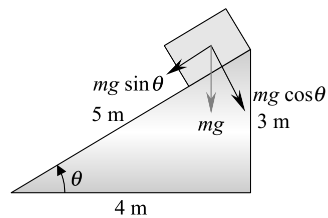


Fig. 15.13(b)

We see that the work done is same as the previous case.

Thus, we see that the work done by the force of gravity is independent of the path traced but only on *initial* and *final* positions of the block in the vertical direction. Such a kind of force is termed *conservative* force.

Example 15.6 A boy lifts water from a 20 m deep well using a bucket-and-pulley arrangement. The mass of the bucket with water is 5 kg. Determine the work done by him (i) if he lifts it at a constant speed, (ii) if he lifts it with a constant acceleration such that the velocity of the bucket at mid-depth is 1.5 m/s.

Solution

(i) When the boy lifts water at constant speed

When he lifts water at a constant speed, the force he exerts is equal to the weight of the bucket with water, i.e.,

$$F = 5(9.81) = 49.05 \text{ N}$$

As the displacement is in the same direction as the force exerted, work done by the boy is

$$W = 49.05(20) = 981 \text{ J}$$

(ii) When the boy lifts water at constant acceleration

When he lifts water at a constant acceleration, the force he exerts is equal to the weight of the bucket with water plus the inertia force, i.e.,

$$F = 49.05 + ma \tag{a}$$

Using the kinematic equation of motion of the bucket,

$$v^2 = v_o^2 + 2as$$

$$\Rightarrow \quad a = \frac{v^2 - v_o^2}{2s}$$

$$= \frac{(1.5)^2 - 0}{2(10)} = 0.1125 \text{ m/s}^2$$

Substituting this value of acceleration in the equation (a), we have

$$F = 49.05 + 5(0.1125) = 49.61 \text{ N}$$

As the displacement is in the same direction as the force exerted, work done by the boy is

$$W = 49.61(20) = 992.2 \text{ J}$$

Example 15.7 A body of 5 kg mass is tied to an inextensible string. Determine the work done by the external agent on the body, if (i) it is lowered down at a constant speed through a distance of 3 m, (ii) if it is lowered down at a constant acceleration of 1 m/s^2 through the same distance, (iii) if it is lifted up at a constant velocity by a distance of 3 m, and (iv) if it is lifted up at a constant acceleration of 1 m/s^2 by the same distance.

Solution *(i) When the body is lowered down at a constant velocity through a distance of 3 m*

Since the body is lowered down at a constant velocity, we know $\sum F_y = 0$. Therefore,

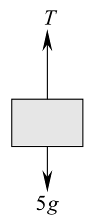


Fig. 15.14

$$5g - T = 0$$

$$\Rightarrow \quad T = 5g = 5(9.81) = 49.05 \text{ N}$$

As this force T acts in the direction opposite to that of the displacement, the work done by the external agent on the body is given as

$$W_T = -(T)(s)$$

$$= -49.05 \times 3 = -147.15 \text{ J}$$

(ii) When the body is lowered down at a constant acceleration through a distance of 3 m

Since the body is lowered down at a constant acceleration of 1 m/s^2, we know

$$\sum F_y = ma_y \Rightarrow$$

$$5g - T = 5(1)$$

$$\therefore \quad T = 5(g - 1) = 5(8.81) = 44.05 \text{ N}$$

As this force T acts in the direction opposite to that of the displacement, the work done by the external agent on the body is given as

$$W_T = -(T)(s)$$
$$= -44.05 \times 3 = -132.15 \text{ J}$$

(iii) *When the body is lifted up at a constant velocity through a distance of 3 m*

Since the body is lifted up at a constant velocity, we know $\Sigma F_y = 0$. Therefore,

$$T - 5g = 0$$
$$\Rightarrow \quad T = 5g = 5(9.81) = 49.05 \text{ N}$$

As this force T acts in the same direction as that of the displacement, the work done by the external agent on the body is given as

$$W_T = (T)(s)$$
$$= 49.05 \times 3 = 147.15 \text{ J}$$

(iv) *When the body is lifted up at a constant acceleration through a distance of 3 m*

Since the body is lifted up at a constant acceleration of 1 m/s^2, we know,

$$\sum F_y = ma_y \Rightarrow$$
$$T - 5g = 5(1)$$
$$\therefore \quad T = 5(g + 1) = 5(10.81) = 54.05 \text{ N}$$

As this force T acts in the same direction as that of the displacement, the work done by the external agent on the body is given as

$$W_T = (T)(s)$$
$$= 54.05 \times 3 = 162.15 \text{ J}$$

Example 15.8 The motion of an automobile of 3 ton mass is represented by v–t graph as shown in Fig. 15.15. If the frictional resistance to the motion is 100 N/ton, determine the work done by the driving force of the automobile in the first 10 seconds and in the next 2 minutes.

Solution From the graph, we see that the acceleration of the automobile is $a_1 = 15/10 = 1.5$ m/s^2 and the distance travelled in the first 10 seconds is $s_1 = \frac{1}{2}(15)(10) = 75$ m. The driving force of the automobile during acceleration is given as

$$F_1 - f = ma_1$$
$$\Rightarrow \quad F_1 = f + ma_1$$
$$= (100 \times 3) + (3000 \times 1.5) = 4.8 \text{ kN}$$

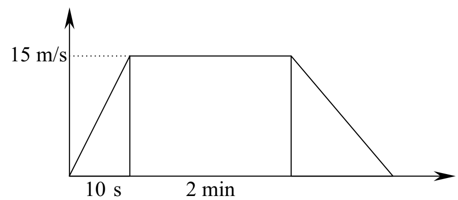


Fig. 15.15

Hence, work done by the driving force is given as

$$W = F_1 s_1 = 4800 \times 75 = 360 \text{ kJ}$$

During the next 2 minutes, it moves at a constant speed. Hence, the driving force is equal to the force of friction, i.e., $F_2 = f = 300$ N. The total distance travelled during this time is $s_2 = (15)(120) = 1800$ m. Hence, the work done by the driving force is given as

$$W = F_2 s_2 = 300 \times 1800 = 540 \text{ kJ}$$

15.5 POWER

In the previous section, we saw work done by a force on a particle. In this section, we will see the *rate* at which this work is done, as it is more important than the work itself. Two different agents may perform the *same* amount of work. However, the one that performs the work faster is of concern to us. For instance, to draw water from a well, if human effort is used, the time taken will be more as compared to that when an electric pump is used. Similarly, a wagon pulled by a horse is slower as compared to that pulled by an engine. Hence, the one doing work faster than the other is said to be more **powerful**.

Thus, **power** is defined as the rate at which work is done. The capacity of an engine or a machine used to do work is normally expressed as its *rated power*. If W is the total work done in a time interval t, then *average* power is given as

$$P_{\text{ave}} = \frac{\text{total work done}}{\text{time taken}} = \frac{W}{t} \tag{15.16}$$

The *instantaneous* power, i.e., power at a particular instant of time is given as

$$P = \frac{dW}{dt} = \frac{d(Fs)}{dt} \tag{15.17}$$

The force can be assumed to be constant over this infinitesimally small time interval dt. Hence, we can write the above expression as

$$P = \frac{Fds}{dt} = Fv \tag{15.18}$$

Thus, power can be defined as product of applied force and velocity of point of application of the force. In S.I system of units, the unit of power is Joule per second (J/s), also called Watt (W), in honor of James Watt, the inventor of steam engine. He also expressed power in terms of power of a horse. Though S.I. system of units is widely used, some of the measuring equipments, engines and machines still measure quantities in older units. For instance, the speedometer records the speed in kilometres per hour (kmph), though the S.I. unit is m/s. Similarly, electrical and mechanical machines still express power in **horsepower**. Hence, it is worthwhile mentioning the relationship between watt and horsepower.

The term horsepower was invented by the engineer James Watt in 1782. He found that a horse could turn a mill wheel (of 12 ft radius) 144 times in an hour or 2.4 times in a minute. Also, a horse could pull with a force of 180 lb. Hence, he defined horsepower as

$$1\text{ hp} = \frac{\text{work done}}{\text{time}} = \frac{(180\text{ lb})(2.4 \times 2\pi \times 12\text{ ft})}{\text{min}} = 32\,572\ \frac{\text{ft.lb}}{\text{min}} \tag{15.19}$$

which is approximated to 33 000 ft.lb/min (or) 745.69987158227022 W. Thus, in the British system of units, one horsepower is given as

$$1\text{ hp} \approx 746\text{ W} \tag{15.20}$$

There is also another unit followed for power, namely, *metric horsepower*. It began in Germany in the nineteenth century and became popular in Europe and Asia. Metric horsepower is defined as roughly 98.6% of mechanical horsepower, i.e., 735.49875 W. Hence, in metric system of units,

$$1\text{ hp} \approx 736\text{ W} \tag{15.21}$$

Example 15.9 Water is to be pumped from a 100 m deep well. If the pump used discharges water at a rate of 0.05 m^3/s, determine metric horsepower of the pump. Weight density of water is 9810 N/m^3.

Solution Discharge of water is defined as the volume of water discharged in unit time. Hence,

$$Q = \frac{\text{volume of water}}{\text{time}} = 0.05 \text{ m}^3/\text{s}$$

Hence, mass of water discharged per unit time is given as

$$m = \frac{\text{density} \times \text{volume of water}}{\text{time}}$$

$$= \text{density} \times Q = \rho Q$$

Therefore, power of the pump is obtained as

$$P = \frac{\text{work done}}{\text{time}}$$

$$= \frac{\text{Force} \times \text{displacement}}{\text{time}}$$

$$= \frac{(\text{mass} \times \text{g}) \times \text{height}}{\text{time}}$$

$$= \frac{\text{mass}}{\text{time}} \times g \times \text{height}$$

$$= \rho Q\, g(\text{height})$$

$$= \gamma Q\ (\text{height}) \qquad [\text{since weight density, } \gamma = \rho g]$$

$$= (9810)(0.05)(100)$$

$$= 49\,050 \text{ W}$$

$$= (49\,050)/736 = 66.64 \text{ metric hp}$$

Example 15.10 A car of 2 ton mass starts from rest and accelerates at a uniform rate to reach a speed of 60 kmph in 20 seconds. If the frictional resistance is 600 N/ton, determine the driving power of the engine when it reaches a speed of 60 kmph.

Solution

Initial speed of car, $v_o = 0$

Final speed of car, $v = 60 \text{ kmph} = 60 \times \frac{5}{18} = 16.67 \text{ m/s}$

For uniform acceleration, we know that the kinematic equation of motion of the car is

$$v = v_o + at$$

Therefore, acceleration of the car is given as

$$a = \frac{v - v_o}{t} = \frac{16.67 - 0}{20} = 0.8335 \text{ m/s}^2$$

The kinetic equation of motion of the car is given as

$$F - f = ma$$

where F is the driving force and f is the force of friction. Therefore,

$$F = f + ma$$

$$= (600)(2) + (2 \times 10^3)(0.8335) = 2867 \text{ N}$$

Therefore, driving power of the engine when the car is moving at 60 kmph is given as

$$P = Fv$$

$$= (2867)(16.67) = 47\ 792.89 \text{ W (or) } 47.8 \text{ kW}$$

Example 15.11 A train of 400 ton mass is pulled by a locomotive of 20 tons along level rails at a constant speed of 80 kmph. If the force of friction is 100 N/ton, determine the driving power of the locomotive.

Solution

Speed of train, $v = 80 \text{ kmph} = 22.22 \text{ m/s}$

Total force of friction is

$$f = (100)\ (400 + 20) = 42\ 000 \text{ N (or) } 42 \text{ kN}$$

As the train runs at constant speed, the pulling force of the locomotive is equal to the force of friction resisting the motion. Hence,

$$F = f = 42 \text{ kN}$$

Therefore, driving power of the locomotive is given as

$$P = Fv$$

$$= (42 \times 10^3)(22.22)$$

$$= 933\ 240 \text{ W (or) } 933.24 \text{ kW}$$

Example 15.12 In the previous problem, determine the maximum speed the train can attain with the same locomotive when it moves on an incline 1 in 100; the frictional resistance being the same. Also, determine the required power of the engine to maintain the same speed of 80 kmph on incline as on level track.

Solution On the incline, the pulling force of the locomotive should be such as to overcome the force of friction f and the component of weight of the train '$mg \sin\theta$' along the incline. Therefore,

$$F = f + mg \sin\theta$$

$$= (42 \times 10^3) + (420 \times 10^3) \times 9.81 \times \frac{1}{100} = 83.2 \text{ kN}$$

Maximum speed attainable

We know power is given as

$$P = Fv$$

$$\therefore \quad v = \frac{P}{F} = \frac{933.24 \times 10^3}{83.2 \times 10^3} = 11.22 \text{ m/s} = 40.4 \text{ kmph}$$

Required power of the engine to maintain the same speed of 80 kmph on incline

If the train has to run at the same speed as on terrain then required power of the locomotive is given as

$$P = Fv$$
$$= 83.2 \times 10^3 \times 22.22$$
$$= 1.85 \text{ MW}$$

Example 15.13 A train of total 300 ton mass descends an incline 1 in 120 with a uniform velocity of 8 m/s. If the frictional resistance is 250 N/ton, determine the power of the engine.

Solution Down the incline, the pulling force of the locomotive and the component of weight of the train down the incline should be such as to overcome the force of friction. Therefore,

$$F + mg \sin\theta - f = 0$$

$$\therefore \quad F = f - mg \sin\theta$$

$$= (250 \text{ N/ton})(300 \text{ ton}) - (300 \times 10^3) \times 9.81 \times \frac{1}{120}$$

$$= 50.5 \text{ kN}$$

Therefore, driving power of the engine is given as

$$P = Fv$$
$$= (50.5 \times 10^3) \times 8$$
$$= 404\,000 \text{ W (or) } 404 \text{ kW}$$

Example 15.14 A block of mass m resting on a rough inclined plane is pulled by a force F acting parallel to the incline. If the block starts from rest and attains a speed of v in t seconds, determine the maximum power exerted on the block. The coefficient of friction between the contact surfaces is μ.

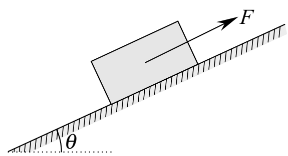


Fig. 15.16

Solution The free-body diagram of the block is shown in Fig. 15.16(a). The kinetic equation of motion of the block along the X and Y directions can be written as

$$\sum F_y = ma_y \Rightarrow$$

$$N - mg \cos\theta = 0$$

$$\therefore \quad N = mg \cos\theta$$

Hence, the force of friction is

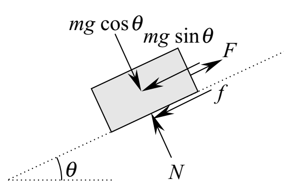


Fig. 15.16(a)

$$f = \mu N = \mu mg \cos\theta$$

$$\sum F_x = ma_x \quad \Rightarrow$$

$$F - mg \sin\theta - f = ma$$

$$\therefore \quad F = ma + mg\sin\theta + f$$

$$= ma + mg\sin\theta + \mu mg\cos\theta \qquad \text{(a)}$$

We know that the kinematic equation of motion of the block is

$$v = v_o + at$$

Since the initial velocity v_o is zero, the acceleration of the block is given as

$$a = v/t$$

Substituting this in the kinetic equation (a), we get

$$F = m\frac{v}{t} + mg\sin\theta + \mu mg\cos\theta$$

$$= mg\left[\frac{v}{gt} + \sin\theta + \mu\cos\theta\right]$$

Therefore, the power exerted on the block is given as

$$P = Fv = mgv\left[\frac{v}{gt} + \sin\theta + \mu\cos\theta\right]$$

Example 15.15 A locomotive exerts its full power to draw a train uphill of slope 1 in n at a constant speed of v. The frictional resistance is 1/mth of the weight of the train. If the same train runs on a level track, prove that the maximum speed that can be attained against the same friction is $v\left[1 + \frac{m}{n}\right]$.

Solution Let W be the total weight of train including the weight of locomotive. When the train is moving uphill at a constant speed v, the driving force of locomotive can be determined as

$$F - f - mg\sin\theta = 0$$

$$\Rightarrow \quad F = f + mg\sin\theta$$

$$= W\left[\frac{1}{m}\right] + W\left[\frac{1}{n}\right]$$

$$= W\left[\frac{m+n}{mn}\right]$$

Therefore, power of the locomotive is

$$P = Fv = Wv\left[\frac{m+n}{mn}\right]$$

Similarly, when the train is running on level track,

$$F' - f = 0$$

$$\Rightarrow \quad F' = f = \frac{W}{m}$$

If v' be the maximum speed of the train on level track then

$$P = F'v'$$

$$Wv\left[\frac{m+n}{mn}\right] = \frac{W}{m}v'$$

$$\Rightarrow \qquad v' = v\left[\frac{m+n}{n}\right] = v\left[1+\frac{m}{n}\right]$$

Example 15.16 The power output of an electric motor is measured using a brake drum arrangement as shown in Fig. 15.17. The brake drum is coupled to the motor and load is applied on the drum by a belt wound over the drum. If the difference in the two spring balance readings is 2.5 kgf, radius of the drum is 0.15 m and the speed of the motor is 1000 rpm, then determine the power output of the motor.

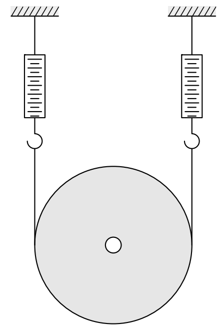


Fig. 15.17

Solution Since the application of tension in the belt retards the motion of the drum coupled with the motor, the retarding torque is given as

$$\tau = (\text{net force acting on the drum}) \times (\text{radius of the drum})$$

Just like the power for linear motion is given as a product of force and linear velocity, the power for angular motion is given as a product of torque and angular velocity, i.e.,

$$P = \tau\omega$$
$$= (\text{net force}) \times (\text{radius of the drum})\omega$$

Note that the product of radius of the drum and angular velocity is equal to the linear velocity of a point on the rim of the drum and thus we see that the power has the same form as that for linear motion.

Net force acting on the drum is the difference in the two spring balance readings. Hence,

$$F = 2.5 \text{ kgf}$$

The conversion factor for force is

$$1 \text{ N force} = (1 \text{ kgf})(9.81 \text{ m/s}^2)$$

Therefore, $$F = 2.5 \times 9.81 = 24.53 \text{ N}$$

The angular velocity of the motor is given as

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1000)}{60} = 104.72 \text{ rad/s}$$

Hence, the power output of the motor is obtained as

$$P = Fr\omega$$
$$= (24.53)(0.15)(104.72)$$
$$= 385.3 \text{ W}$$

Since metric horsepower = watt power/736, the power output of the motor can also be expressed as

$$P = 0.52 \text{ metric hp}$$

15.6 ENERGY

Energy is defined as that property of a body by virtue of which work can be performed or in other words, it is the *capacity* of a body to *do work*. Energy exists in *different forms* such as chemical, electrical, thermal, elastic, nuclear, mechanical, etc. We know from the law of conservation of energy that energy can neither be created nor destroyed, but can be transformed from one form to another. For instance, in the case of firing a bomb, chemical energy is converted into mechanical energy, which makes the bomb move forward and hit the target. Here we see that the chemical composition of the bomb has the energy to do work on the bomb, i.e., to make it move forward. In most of the cases, all the available energy is not converted into useful work, but is lost or dissipated in the form of *heat* energy. Since energy is the capacity of a body to do work, its unit is the same as that of work done, i.e., N.m (or) Joule.

In our study, we are mainly concerned with *mechanical* energy. Mechanical energy is further divided into two types, namely, **kinetic energy** and **potential energy**. Kinetic energy of a body is defined as the energy possessed by virtue of its **motion** and potential energy of a body is defined as the energy possessed by virtue of its **position**. In the following section, we will discuss kinetic energy and in Section 15.8, we will discuss potential energy.

15.6.1 Kinetic Energy

The word 'kinetic' is derived from the Greek word 'kinesis' meaning to *move* and the word 'energy' is the ability to move. A body under motion is said to possess energy called kinetic energy because it is able to do work on any body that comes across its path. This is the reason we do not step in front of a moving vehicle as it has the capacity to do work on us.

Consider an automobile moving with a certain velocity. If the engine is switched off, the automobile will continue to move with the same velocity if there is *no* resistance to its motion. Hence, no work is done by the body as it causes its own displacement. However, we know that the motion is always retarded by frictional resistance, and hence the speed goes on decreasing until the automobile comes to a stop. Here, the body does work against the force of friction in moving forward. Thus, kinetic energy possessed by a body can be measured by the amount of work the moving body will do if brought to rest or by the amount of work originally needed to impart the velocity to it.

15.6.2 Mathematical Expression of Kinetic Energy

When the resultant force acting on a particle is *non-zero*, it causes *acceleration* of the particle. By Newton's second law of motion, the force and the resulting acceleration are related as

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \tag{15.22}$$

Then work done by the force in causing an infinitesimally small displacement $d\vec{r}$ is given as $\vec{F}.d\vec{r}$. Therefore, the total work done in displacing from $\vec{r}_1$ to $\vec{r}_2$ is

$$W = \int_{r_1}^{r_2} \vec{F}.d\vec{r} = \int_{r_1}^{r_2} m\frac{d\vec{v}}{dt}.d\vec{r} \tag{15.23}$$

Multiplying numerator and denominator by *dt*, we change the variable of integration from *dr* to *dt*. In addition, if the particle also started from rest, then

$$W = m\int_0^t \frac{d\vec{v}}{dt}.\frac{d\vec{r}}{dt}dt$$

$$= m\int_0^t \frac{d\vec{v}}{dt}.\vec{v}\ dt$$

$$= \frac{1}{2}m\int_0^t \frac{d}{dt}(\vec{v}.\vec{v})dt$$

$$= \frac{1}{2}m\int_0^t \frac{d}{dt}(v^2)dt$$

$$= \frac{1}{2}m\int_0^v d(v)^2$$

$$= \frac{1}{2}mv^2 \qquad (15.24)$$

Just as we derived the work done in imparting a velocity to a particle at rest, we can also determine in a similar manner the work done in bringing a moving particle to rest. Since kinetic energy is defined as the amount of work done in bringing to rest a moving particle or the amount of work originally needed to impart the velocity to it, we see that kinetic energy can be represented mathematically as

$$\text{K.E} = W = \frac{1}{2}mv^2 \qquad (15.25)$$

We see that the kinetic energy is proportional to *square* of the speed. Thus, a twofold increase in speed will increase the kinetic energy by fourfold, and so on.

15.6.3 Work–Energy Principle

Suppose the net force acting on a particle changes its velocity from v_o to v, then we can derive the work done as before:

$$W = \frac{1}{2}m(v^2 - v_o^2) = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

$$= (\text{K.E})_{\text{final}} - (\text{K.E})_{\text{initial}}$$

$$= \text{change in K.E} \qquad (15.26)$$

Thus, we see that the change in kinetic energy of a particle during any displacement is equal to the work done by the net force acting on it. This is known as **work–energy principle**.

Example 15.17 A block is projected up an inclined plane with an initial velocity of 10 m/s. If coefficients of static and kinetic friction between the contact surfaces are respectively 0.25 and 0.2, determine how far up the plane will the block move before coming to rest.

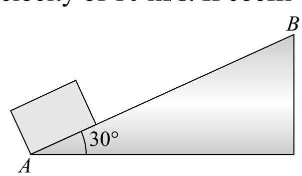


Fig. 15.18

Solution The forces acting on the block are shown in the free-body diagram below: the component of weight '$mg \sin\theta$' acting along the inclined

plane and frictional force F acting in the direction opposite to that of the displacement. As there is no displacement along the Y-direction, we know

$$N - mg\cos\theta = 0$$

$$\Rightarrow \quad N = mg\cos\theta$$

Hence, force of friction is

$$F = \mu_k N = \mu_k mg\cos\theta$$

Let s be the displacement of the block along the plane before coming to rest. Then applying the work–energy equation, we have

$$(\text{K.E})_f - (\text{K.E})_i = \text{work done}$$

$$0 - \frac{1}{2}mv^2 = -(mg\sin\theta)s - (F)s$$

Note that the final kinetic energy is zero as the block comes to rest. In addition, the works done by the forces are negative as they act in the direction opposite to that of the displacement. Therefore,

$$\frac{1}{2}mv^2 = (mg\sin\theta + \mu_k mg\cos\theta)s$$

$$\therefore \quad s = \frac{v^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

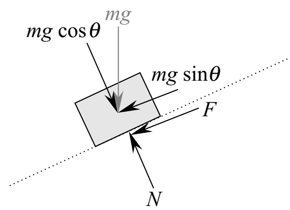


Fig. 15.18(a)

Substituting the values,

$$s = \frac{(10)^2}{2(9.81)(\sin 30° + (0.2)(\cos 30°))} = 7.57 \text{ m}$$

Example 15.18 In the previous problem, state whether the block after coming to rest will slide down under its own weight or not. If so, what will be the velocity of the block as it slides down from the point B and reaches the point A, the bottom of the incline?

Solution After coming to rest, the body will begin to slide only if $\tan\theta > \mu_s$. For given values of 'θ' and 'μ_s,' we see that the condition is satisfied and hence, the block will begin to slide downwards under its own weight. From the free-body diagram of the block as it slides down (shown below), we see that the component of weight $mg\sin\theta$ acts in the direction of displacement and force of friction F acts in the direction opposite to that of displacement. Hence, writing the work–energy equation,

$$[mg\sin\theta - F]s = \frac{1}{2}mv^2 - 0$$

Taking $F = \mu_k N = \mu_k mg\cos\theta$ as before,

$$[mg\sin\theta - \mu_k mg\cos\theta]s = \frac{1}{2}mv^2$$

$$\therefore \quad v = \sqrt{2gs[\sin\theta - \mu_k\cos\theta]}$$

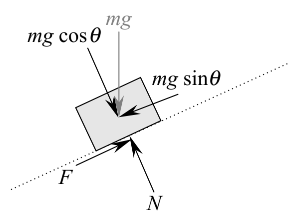


Fig. 15.19

Substituting the values,

$$v = \sqrt{2(9.81)(7.57)[\sin 30° - (0.2)\cos 30°]} = 6.97 \text{ m/s}$$

Example 15.19 A block of 5 kg mass slides down an inclined plane from rest. How far along the horizontal plane, will it reach before coming to rest? The coefficient of kinetic friction between the block and the inclined plane is 0.15 and that between the block and the horizontal plane is 0.2.

Solution This problem can be solved in two ways as explained below:

Method I

We consider two different phases, namely, the motion of the block from A to B and from B to C. The free-body diagram of the block in the two phases of motion is shown in Fig. 15.20(a). Let v be the velocity at B.

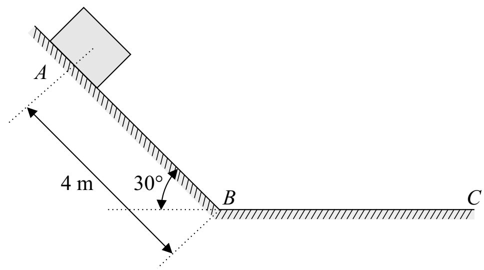


Fig. 15.20

Motion from A to B

Writing the work energy equation for the motion from A to B,

$$\frac{1}{2}mv^2 - 0 = [5g\sin 30° - F_1]4$$

$$\frac{1}{2}5v^2 - 0 = [5g\sin 30° - 0.15(5g\cos 30°)]4 \quad [\text{Since } F_1 = = \mu_{k1} N_1]$$

$$\Rightarrow \qquad v = 5.39 \text{ m/s}$$

Motion from B to C

In this portion of motion, the initial velocity of the block is the velocity at B, i.e., v calculated as above. Writing the work energy equation for the motion from B to C,

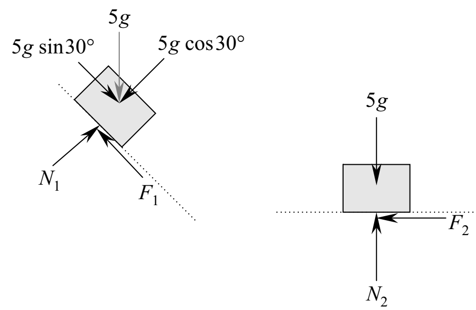


Fig. 15.20(a)

$$0 - \frac{1}{2}mv^2 = -F_2(s)$$

$$-\frac{1}{2}5(5.39)^2 = -0.2(5g)(s) \quad [\text{Since } F_2 = \mu_{k_2} N_2]$$

$$\Rightarrow \qquad s = 7.4 \text{ m}$$

Method II

Instead of considering two different phases, we can also consider the motion directly from A to C. Noting that the initial and final velocities are zero, we can write the work energy equation as

$$0 = (5g\sin 30° - F_1)4 - F_2(s)$$

$$0 = [5g\sin 30° - 0.15(5g\cos 30°)]4 - 0.2(5g)(s)$$

$$\Rightarrow \qquad s = 7.4 \text{ m}$$

Thus, we see that the *same* result is obtained using both the methods.

Example 15.20 A truck of 6 ton mass is moving on a level terrain at a speed of 80 kmph, when brakes are applied. If the braking force is of constant magnitude 4 kN/ton, determine the distance it travels before coming to a stop.

Solution *Given data*

Speed of truck, $v = 80$ kmph $= 80 \times 5/18 = 22.22$ m/s

Total braking force, $F = (4 \times 10^3$ N/ton)(6 tons) $= 24 \times 10^3$ N

The initial kinetic energy of the truck is $(\text{K.E})_i = (1/2)\, mv^2$. As the braking force brings the truck to a stop, its final kinetic energy is zero. It should be noted that the displacement of the truck is opposite to that of the braking force; hence, work done by the braking force is negative. Applying the work–energy equation, we have

$$(\text{K.E})_f - (\text{K.E})_i = \text{work done}$$

$$0 - \frac{1}{2}mv^2 = -(F)(s)$$

$$0 - \frac{1}{2}(6 \times 10^3)(22.22)^2 = -(24 \times 10^3)(s)$$

$$\Rightarrow \quad s = 61.72 \text{ m}$$

Example 15.21 A block of mass m attached to a spring of stiffness k, is pushed to the right with a velocity v when the spring is in the unstretched position. Determine the maximum extension of the spring assuming the contact surfaces to be smooth.

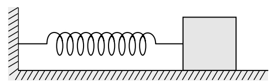


Fig. 15.21

Solution We know that when the block reaches the extreme position, its velocity is zero. Let s be distance of the extreme position with respect to the unstretched position of the spring. Then the restoring force acting on the block is ks. Then applying the principle of work–energy, we have

$$(\text{K.E})_f - (\text{K.E})_i = \text{average work done}$$

$$0 - \frac{1}{2}mv^2 = -\left(\frac{1}{2}ks\right)s$$

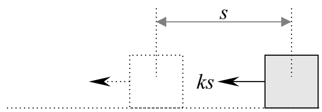


Fig. 15.21(a)

It should be noted that the weight and normal reaction do no work, as the displacement is perpendicular to the direction of these two forces. Therefore,

$$s = \sqrt{\frac{m}{k}}\,v$$

Example 15.22 In the previous problem, suppose the contact surfaces are rough having coefficient of friction μ then determine the maximum extension of the spring.

Solution The free-body diagram of the block is shown in Fig. 15.22. The frictional force acts opposite to that of the displacement. As before applying the principle of work–energy, we have

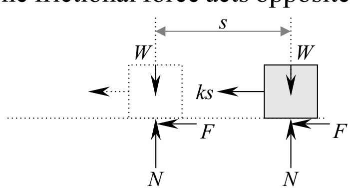


Fig. 15.22

$$(\text{K.E})_f - (\text{K.E})_i = \text{work done}$$

$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}ks^2 - Fs$$

$$mv^2 = ks^2 + 2Fs$$

We know that the frictional force is

$$F = \mu N = \mu mg$$

Therefore, $ks^2 + 2\mu\, mgs - mv^2 = 0$

On solving the quadratic equation, the roots are

$$s = \frac{-2\mu mg \pm \sqrt{4\mu^2 m^2 g^2 + 4kmv^2}}{2k}$$

$$= \frac{-\mu mg \pm \sqrt{\mu^2 m^2 g^2 + kmv^2}}{k}$$

Dividing both numerator and denominator by m, we have

$$s = \frac{-\mu g \pm \sqrt{\mu^2 g^2 + (k/m)v^2}}{(k/m)}$$

As we know displacement has to be positive, we take only positive root, i.e.,

$$s = \frac{-\mu g + \sqrt{\mu^2 g^2 + (k/m)v^2}}{(k/m)}$$

Example 15.23 A 4 kg mass when suspended from a spring extends it by 10 cm. If the same spring is used to stop a block moving horizontally at a speed of 4 m/s, determine the compression of the spring (i) assuming the contact surfaces to be smooth, and (ii) the frictional resistance to motion as 12 N.

Solution

Stiffness of the spring

When a 4 kg mass is suspended from the spring, it extends by 10 cm. Therefore, stiffness of the spring is given as

$$k = \frac{F}{\Delta} = \frac{mg}{\Delta}$$

$$= \frac{(4)(9.81)}{(0.1)} = 392.4 \text{ N/m}$$

Compression of the spring when the contact surfaces are smooth

Initial velocity of block, $v_i = 4$ m/s. Final velocity, v_f of the block after compressing the spring is zero. If x be the compression of the spring, then applying the work–energy equation,

$$(\text{K.E})_f - (\text{K.E})_i = \text{average work done}$$

$$0 - \frac{1}{2}mv_i^2 = -\frac{1}{2}kx^2$$

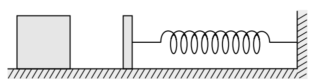


Fig. 15.23

$$\Rightarrow \quad x = \sqrt{\frac{m}{k}}\, v_i$$

$$= \sqrt{\frac{4}{392.4}} \times 4 = 403.9 \text{ mm}$$

Compression of the spring when the contact surfaces are rough

As the force of friction acts opposite to the direction of motion of the block, it does negative work. Hence, applying the work–energy equation as before,

$$0-\frac{1}{2}mv_i^2=-fx-\frac{1}{2}kx^2$$

$$-\frac{1}{2}(4)(4)^2=-12x-\frac{1}{2}(392.4)x^2$$

$$\Rightarrow \quad 196.2x^2+12x-32=0$$

Solving the above quadratic equation for the roots, we get

$$x=\frac{-12\pm\sqrt{(12)^2+4(196.2)(32)}}{392.4}$$

Neglecting the negative value, we have x = 374.4 mm.

Example 15.24 A block of mass m attached to a spring of stiffness k is resting on an inclined plane. If the block is released when the spring is in the unstretched position, derive the expression for maximum extension of the spring. The coefficient of friction between contact surfaces is μ.

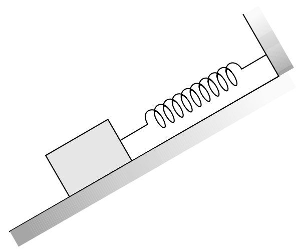


Fig. 15.24

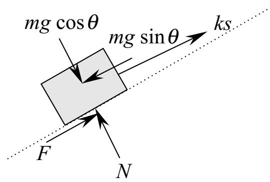


Fig.15.24(a)

Solution Since the block is released from rest, its initial velocity is zero. In addition, as the block stretches the spring to the maximum extent, its final velocity is zero. The normal reaction N and component $mg\cos\theta$ of the weight do no work, as the displacement is perpendicular to the forces. The only forces contributing to the work done are component $mg\sin\theta$ of the weight, force of friction $F=\mu N=\mu mg\cos\theta$ and restoring force ks of the spring. Hence, applying the principle of work–energy, we have

$$(\text{K.E})_f-(\text{K.E})_i=\text{work done}$$

$$0=[mg\sin\theta]s-[\mu\, mg\cos\theta]s-\frac{1}{2}ks^2$$

$$mg[\sin\theta-\mu\cos\theta]s=\frac{1}{2}ks^2$$

Since $s\neq 0$, we have

$$s=\frac{2mg[\sin\theta-\mu\cos\theta]}{k}$$

Example 15.25 A block of 10 kg mass slides down an inclined plane with a slope angle of 35°. It is stopped by a spring of stiffness 1 kN/m. If the block slides down 5 m before hitting the spring then determine the maximum compression of the spring. The coefficient of friction between the block and the inclined plane is 0.15.

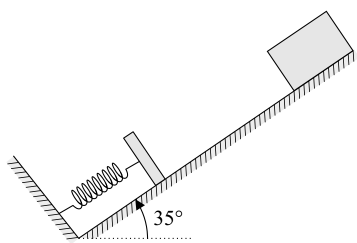


Fig. 15.25

Solution The free-body diagram of the block is shown in Fig. 15.25(a). Since the block is released from rest, its initial velocity is zero. In addition, as the block compresses the spring to the

maximum extent, its final velocity is zero. Let s be the compression of the spring from the unstretched position. Then the total displacement of the block before coming to rest is $(5 + s)$ m. Applying the principle of work–energy, we have

$$[10g \sin 35° - \mu\, 10g \cos 35°][5 + s] - \frac{1}{2} ks^2 = 0$$

$$10g[\sin 35° - (0.15) \cos 35°]\,[5 + s] - \frac{1}{2}(10^3)(s)^2 = 0$$

$$(44.21)(5 + s) - 500s^2 = 0$$

$$500s^2 - 44.21s - 221.05 = 0$$

Upon solving the quadratic equation, we get

$s = 0.711$ m (or) 71.1 cm

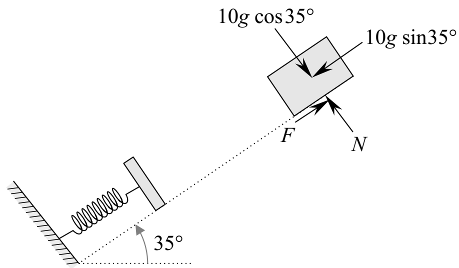


Fig. 15.25(a)

Example 15.26 A car of 2 ton mass powered by an engine of 40 kW capacity, starts from rest and attains maximum speed in 30 seconds. If the frictional resistance to motion is 0.75 kN/ton, determine the maximum speed it can attain. If after attaining the maximum speed, the engine is switched off, determine the distance it would travel before coming to rest.

Solution

Maximum speed attained

If v be the maximum speed attained by the car in 30 seconds, its uniform acceleration is

$$a = v/t = v/30$$

The total frictional resistance to the motion of the car is

$$f = (0.75 \times 10^3 \text{ N/ton})(2 \text{ tons}) = 1500 \text{ N}$$

Since the car is uniformly accelerating, the kinetic equation of motion is given as

$$F - f = ma$$

Therefore, the driving force of the car is obtained as

$$F = f + ma$$

$$= (1500) + (2 \times 10^3)\left[\frac{v}{30}\right]$$

Therefore, maximum power of the engine is given as

$$P = Fv = \left[1500 + \frac{2000}{30}v\right]v$$

$$40 \times 10^3 = 1500v + 66.67v^2$$

$$66.67v^2 + 1500v - 40 \times 10^3 = 0$$

Upon solving the quadratic equation, we get

$$v = \frac{-1500 \pm \sqrt{(1500)^2 + 4(40 \times 10^3)(66.67)}}{2 \times 66.67}$$

$$= 15.7 \text{ m/s (or) } 56.5 \text{ kmph}$$ [neglecting the negative root]

Distance travelled before coming to rest

If after attaining the maximum speed, the engine is switched off, then the final velocity is zero. Hence, applying the work–energy equation,

$$(\text{K.E})_f - (\text{K.E})_i = \text{work done}$$

$$0 - \frac{1}{2}mv^2 = -fs$$

$$\Rightarrow \quad s = \frac{mv^2}{2f}$$

$$= \frac{(2000)(15.7)^2}{2(1500)} = 164.33 \text{ m}$$

Example 15.27 A bullet of 20 g mass moving at 300 m/s pierces a fixed 5 cm thick metal plate and emerges out with a velocity of 50 m/s. Determine the resistance offered by the plate assuming it to be uniform.

Solution

Initial speed of bullet, $v_i = 300$ m/s

Final speed of bullet, $v_f = 50$ m/s

Applying the work–energy equation,

$$(\text{K.E})_f - (\text{K.E})_i = \text{work done}$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -F(s)$$

$$\frac{1}{2}(20 \times 10^{-3})[50^2 - 300^2] = -F[5 \times 10^{-2}]$$

$$\Rightarrow \quad F = 17.5 \text{ kN}$$

15.7 WORK DONE BY INTERNAL FORCES

In this section, we will discuss work done by internal forces such as tension in the string in interconnected bodies. Consider a system of bodies as shown in Fig. 15.26. The internal force, namely, the tension in the string connecting the blocks, acts on the blocks in the opposite directions and hence the net work done for any displacement is *zero*. Thus, this method is advantageous over the Newton's method as separate free-body diagrams need not be drawn for individual members.

Fig. 15.26 Net work done by internal forces is zero

For illustration, consider the system of interconnected bodies shown on the left in Fig. 15.26. Let us isolate each body and analyze their motion using work–energy method assuming that the block-I moves downwards. Since the string is inextensible, the displacement of both the blocks will be same and let it be 's'. Applying the work–energy equation for the block 1,

$$(m_1 g - T)s = \frac{1}{2} m_1 v^2 - 0 \tag{15.27}$$

Similarly, applying the work–energy equation for the block 2,

$$(T - \mu_2 N_2 - m_2 g \sin\theta)s = \frac{1}{2} m_2 v^2 - 0 \tag{15.28}$$

Adding the two equations,

$$(m_1 g - \mu_2 N_2 - m_2 g \sin\theta)s = \frac{1}{2}(m_1 + m_2)v^2 \tag{15.29}$$

We see that the summation of works done by forces acting on both the bodies is equal to summation of kinetic energies of individual bodies. In addition, we see that the work done by the internal force, i.e., tension T is eliminated. Thus, it need not be considered at all in the work–energy equation. Moreover, the kinetic energy of the system is given by the summation of kinetic energies of the individual bodies in the system. This is the advantage of using the work–energy method over Newton's method. The system can be considered as such and there is no need to draw individual free-body diagrams.

Example 15.28 In the system of blocks shown, if $m_1 = 8$ kg and $m_2 = 5$ kg, determine the velocities of the blocks after the block of mass m_2 displaces by 2 m.

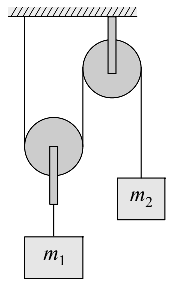


Fig. 15.27

Solution Since $m_1 = 8$ kg, we can see that to maintain its equilibrium, the tension in the string passing over the pulley must be $T = 8g/2 = 4g$. Since this value is less than the weight of the second block, i.e., $5g$, we can conclude that the second block moves down while the first block moves up. Hence, we can write the work–energy equation for the system as

$$m_2 g s_2 - m_1 g s_1 = \frac{1}{2} m_2 v_2^{\,2} + \frac{1}{2} m_1 v_1^{\,2}$$

Note that as the displacement of the block I is opposite to the direction of force of gravity, the work done by the force of gravity is negative. We also know that the displacement of the block I is half that of the displacement of the block II and the velocity of the block I is half that of the velocity of the block II. Hence, if $s_2 = s$ and $v_2 = v$, then $s_1 = s/2$ and $v_1 = v/2$. Therefore, we can write the above expression as

$$m_2 gs - m_1 g\frac{s}{2} = \frac{1}{2}\left[m_2 + \frac{m_1}{4}\right]v^2$$

$$\Rightarrow \qquad v^2 = \frac{4gs[2m_2 - m_1]}{4m_2 + m_1}$$

$$= \frac{4(9.81)(2)[(2\times 5) - 8]}{[(4\times 5) + 8]} = 5.61$$

$$\therefore \qquad v = 2.37 \text{ m/s}$$

Therefore, the velocities of the two blocks are

$$v_2 = 2.37 \text{ m/s} \quad \text{and} \quad v_1 = 1.19 \text{ m/s}$$

Example 15.29 In the system of blocks shown, if $m_1 = 3$ kg and $m_2 = 5$ kg, determine the velocities of the blocks after the block of mass m_2 displaces by 2 m. Take $\mu = 0.15$.

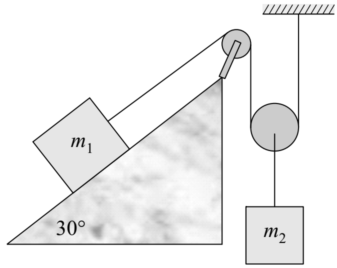


Fig. 15.28

Solution For equilibrium of the block II, we see that the tension in the string passing over the pulley must be such that $2T = m_2 g$ (or) $T = m_2 g/2 = 2.5g$. For equilibrium of the block I, the tension in the string must be such that $T = m_1 g \sin\theta + \mu m_1 g \cos\theta = 1.89g$. Since this value is less than $2.5g$, we conclude that the block II moves down while the block I moves up as shown in Fig. 15.28(a)

Writing the work–energy equation for the system, we have

$$W_2 s_2 - [W_1 \sin\theta + \mu W_1 \cos\theta] s_1 = \frac{1}{2}\frac{W_2}{g} v_2^2 + \frac{1}{2}\frac{W_1}{g} v_1^2$$

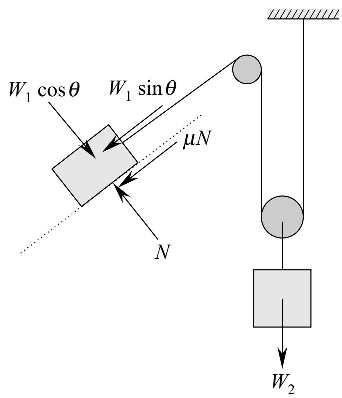


Fig. 15.28(a)

We know that the displacement of the block I is twice that of the displacement of the block II and the velocity of the block I is twice that of the velocity of the block II. If $s_2 = s$ and $v_2 = v$, then $s_1 = 2s$ and $v_1 = 2v$. Hence, we can write the above expression as

$$W_2 s - [W_1 \sin\theta + \mu W_1 \cos\theta]\, 2s = \frac{1}{2}\frac{v^2}{g}[W_2 + 4W_1]$$

$$\Rightarrow \qquad v^2 = \frac{2gs[W_2 - 2W_1(\sin\theta + \mu\cos\theta)]}{4W_1 + W_2}$$

$$= \frac{2gs[m_2 - 2m_1(\sin\theta + \mu\cos\theta)]}{4m_1 + m_2}$$

$$= \frac{2(9.81)(2)\{5 - 2(3)[\sin 30° + (0.15)(\cos 30°)]\}}{4(3) + 5} = 2.82$$

$$\therefore \qquad v = 1.68 \text{ m/s}$$

Therefore, the velocities of the two blocks are

$$v_1 = 3.36 \text{ m/s} \quad \text{and} \quad v_2 = 1.68 \text{ m/s}$$

15.8 POTENTIAL ENERGY

Potential energy of a body is defined as the capacity to do work by virtue of its *position*. There are many types of potential energies such as gravitational, electrical, elastic, etc. In our study, we are mainly concerned with *gravitational* potential energy and *elastic* potential energy.

When a body is placed in a uniform gravitational field, it is attracted by the earth. Thus, work is always required to lift a body **up** against the force of gravity. After lifting, if the force is removed, it is said to posses the capacity to do work. The body allowed to fall down freely would do work on any body that comes across its path. This principle is used in a pile hammer to drive piles. It is also used in a roller

coaster, where the train is initially lifted to a high hill to increase its potential energy. When released from that height, it has more potential and hence the train moves over the coaster with high speed. Though the acceleration due to gravity remains constant at elevations near the earth's surface, the velocity of the body with which it reaches the ground is dependent on the height from which it falls. Hence, we could say that the body has more potential to work at a higher elevation than it has when at the ground level and this is termed potential energy.

To determine the potential energy, a zero height position must be chosen. Typically, the ground is considered to be a position of zero height. Potential energy is measured by the amount of work that has to be done on the body to lift it from ground level to a higher elevation. Thus, to lift a body to a height h from ground level, work required to be done against gravity is

$$W = mgh \tag{15.30}$$

$$\therefore \quad \text{P.E} = W = mgh \tag{15.31}$$

The student should recall the discussion made in Example 15.5 that the work done in a gravitational field is independent of the path followed and thus the force of gravity is termed **conservative force**. In a conservative force field, the total mechanical energy remains constant.

Principle of conservation of mechanical energy If a body is subject to a conservative system of forces, (say gravitational force) then its mechanical energy remains constant for any position in the force field. Consider a body either sliding down a *smooth* incline or freely falling. Since it is initially at rest, all of its energy is potential energy. As it accelerates downwards, some of its potential energy is converted into kinetic energy. At the bottom of the incline or at the ground level, the energy will be purely kinetic, assuming the bottom of the slope or the ground level as the datum for potential energy. By the principle of conservation of energy, we see that the loss in potential energy is equal to the gain in kinetic energy. Mathematically,

$$(\text{P.E})_i - (\text{P.E})_f = (\text{K.E})_f - (\text{K.E})_i \tag{15.32}$$

On rearranging, we have

$$(\text{P.E})_i + (\text{K.E})_i = (\text{P.E})_f + (\text{K.E})_f \tag{15.33}$$

$$\Rightarrow \quad (\text{P.E}) + (\text{K.E}) = \text{constant} \tag{15.34}$$

Thus, we see that the total mechanical energy, i.e., sum of potential and kinetic energies remain constant. This is known as principle of conservation of mechanical energy.

However, we must understand this is true only in the case of conservative force field such as gravitational force. For instance, the force of friction, which is a non-conservative force, when acting on a body, the mechanical energy will not be conserved. Actually, the energy to overcome friction is converted into heat energy.

Elastic potential energy is the energy stored in elastic materials as a result of their stretching or compressing. We already saw in Section 15.4 that work done in stretching in a spring of stiffness k to an elongation x_o is

$$W = \frac{1}{2} k x_o^2$$

This is also true if the spring is compressed. By conservation of work–energy, this work done on the spring is stored in it as what is known as potential energy, because a compressed or stretched spring has

potential to do work. Since this energy is dependent upon the elongation or in other words, its position with respect to its unstretched position, it is known as potential energy.

Example 15.30 A ball is dropped from the top of a tower. If it reaches the ground with a velocity of 30 m/s, determine the height of the tower by the conservation of energy method.

Solution By the principle of conservation of energy, we know that the total mechanical energy remains constant. Hence, the total energy at the top of the tower must be equal to that at the base of the tower, i.e.,

$$(\text{K.E} + \text{P.E})_{\text{top}} = (\text{K.E} + \text{P.E})_{\text{base}}$$

Since the ground surface is taken as the datum, the potential energy at the top is mgh [where h is height of the tower] and that at the bottom is zero. If v is the velocity of the ball at the base, we can write

$$0 + (mg)(h) = \frac{1}{2} mv^2 + 0$$

$$\Rightarrow \qquad h = \frac{v^2}{2g}$$

$$= \frac{(30)^2}{2(9.81)} = 45.87 \text{ m}$$

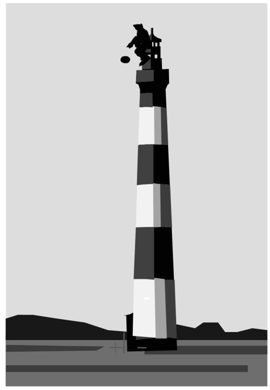


Fig. 15.29

Example 15.31 A small bob suspended by a string of length l oscillates about its mean position. Determine the velocity of the bob at its lowest position if the maximum angle of inclination of the bob with respect to the vertical is θ.

Solution The velocity of the bob at the extreme position is zero and hence, it swings back to the mean position. At the mean position, its velocity is maximum. If we take the lowest position of the bob as the datum then potential energy is maximum at the extreme position. Hence, applying the principle of conservation of energy, we have

$$(\text{K.E} + \text{P.E})_{\text{extreme}} = (\text{K.E} + \text{P.E})_{\text{lowest}}$$

$$0 + mgh = \frac{1}{2} mv^2 + 0$$

$$\Rightarrow \qquad v^2 = 2\text{gh}$$

$$= 2g(l - l\cos\theta)$$

$$= 2gl(1 - \cos\theta)$$

$$\therefore \qquad v = \sqrt{2gl(1-\cos\theta)}$$

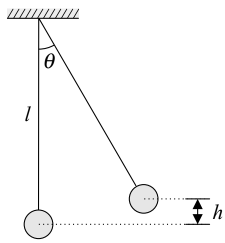


Fig. 15.30

Example 15.32 *ABCDE* is a channel in the vertical plane, *BCDE* being a circular loop with radius r. If a block is released from rest at the point A, determine the minimum height h such that the block completes the loop.

Solution The difference in height between points A and D is $(h - 2r)$. Hence, the loss in potential energy as the block slides from A to D is $mg(h - 2r)$. By the principle of conservation of energy, we know that the reduction in potential energy is equal to the increase in kinetic energy. Applying the conservation of energy,

$$\text{P.E} + \text{K.E} = \text{const}$$

$$mgh + 0 = mg(2r) + \frac{1}{2}mv^2$$

$$\Rightarrow \quad mg(h - 2r) = \frac{1}{2}mv^2$$

$$\therefore \quad v^2 = 2g(h - 2r)$$

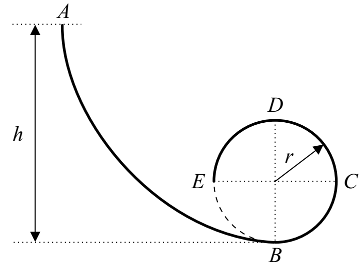


Fig. 15.31

From Chapter 14, we know that at the highest point D, the kinetic equation of motion can be written as

$$R + mg = \frac{mv^2}{r}$$

where R is the normal reaction exerted by the loop on the block. In addition, the normal reaction R is the *least* at the *highest* point. Hence, to determine the minimum height h for the block to stay in the loop, we equate R to zero. Therefore,

$$mg = \frac{mv^2}{r}$$

$$g = \frac{v^2}{r}$$

$$gr = 2g(h - 2r)$$

$$r = 2(h - 2r)$$

$$r = 2h - 4r \quad \Rightarrow \quad h = \frac{5r}{2}$$

Example 15.33 A ball is projected with an initial velocity v_o at an angle of α to the horizontal. Using the principle of conservation of energy, determine the maximum height reached by the ball.

Solution In a projectile motion, we know that the horizontal component of velocity always remains constant and it is equal to $v_o \cos\alpha$. At the highest point, the vertical component of velocity is zero. Applying the principle of conservation of energy, we have

$$(\text{K.E} + \text{P.E})_{\text{point of projection}} = (\text{K.E} + \text{P.E})_{\text{highest point}}$$

$$\frac{1}{2}mv_o^2 + 0 = \frac{1}{2}m(v_o \cos\alpha)^2 + mgh$$

$$mgh = \frac{1}{2}mv_o^2[1 - \cos^2\alpha] = \frac{1}{2}mv_o^2 \sin^2\alpha$$

$$\Rightarrow \quad h = \frac{v_o^2 \sin^2\alpha}{2g}$$

Example 15.34 A pile hammer of 40 kg is lifted to a height of 3 m and dropped onto a pile. If the resistance offered by the soil is uniform equal to 3 kN, determine by how much the pile would advance into the soil.

Solution When the pile is raised to a height of 3 m, its potential energy increases by an amount of

$$\Delta P.E = mgh = 40(9.81)(3) = 1177.2 \text{ J}$$

When it is dropped, the potential energy is converted into kinetic energy due to conservation of energy. Hence, the kinetic energy of the hammer at the instant that it hits the pile is equal to the loss in potential energy, i.e., K.E = 1177.2 J.

To determine by how much the pile would advance into the soil, we apply the work–energy equation. Hence,

$$\Delta K.E = -f\,s$$

where f is the average resistance offered by the soil. Hence,

$$0 - 1177.2 = -(3 \times 10^3)s$$

$$\Rightarrow \qquad s = 0.3924 \text{ m (or) } 392.4 \text{ mm}$$

Example 15.35 A 5 kg block is dropped from a height of 1 m onto a spring of stiffness 3 kN/m as shown. Determine the maximum compression of the spring.

Solution Let s be the compression of the spring. Then the total distance over which the block falls is (1 + s) metre. The change in potential energy of the block is stored in the spring as its potential energy. Therefore,

$$5(9.81)[1 + s] = \frac{1}{2}ks^2$$

$$1500s^2 - 49.05s - 49.05 = 0$$

On solving the quadratic equation for roots, we get

$$s = \frac{49.05 \pm \sqrt{(49.05)^2 + 4(1500)(49.05)}}{3000} = 197.9 \text{ mm}$$

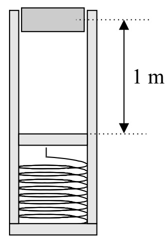


Fig. 15.32

SUMMARY

Work–Energy Method

Work–energy method is an alternative approach to solve kinetic problems. The advantages of this method over Newton's method are that work and energy are scalars; acceleration need not be determined to know the kinematics of the problem; separate free-body diagrams need not be drawn for interconnected bodies.

Work Done by a Constant Force

When a force acting on a particle causes a *displacement* of the particle, the force is then said to have done *work* on the particle. If the displacement $\vec{s}$ is in the direction of the force $\vec{F}$, then work done on the particle is defined as *product* of magnitudes of *force* and *displacement*. Mathematically, this can be written as

$$W = Fs$$

In general, the displacement of a particle under the action of forces would not always occur in the direction of the force due to constraints involved. Hence, work done in general, can be defined as product of component of the force in the direction of motion and the displacement. Alternatively, it can also be defined as product of force and component of displacement in the direction of the force. Mathematically, we can write this as

$$W = (F\cos\theta)s \quad \text{(or)} \quad W = (F)(s\cos\theta)$$

or written in vector form as

$$W = \vec{F}.\vec{s} = Fs\cos\theta$$

Work done is a *scalar* quantity that is having magnitude, but not direction. Hence, work done adds up algebraically. The S.I. unit of work done is N.m (or) Joule.

(i) When $\theta = 0°$, i.e., when force and displacement are in the same direction then $\cos\theta = 1$ and hence work done is $W = Fs$.

(ii) When $\theta = 180°$, i.e., when force and displacement are in the opposite direction then $\cos\theta = -1$ and hence work done is $W = -Fs$. The force of *friction* always acts in the direction opposite to that of the motion; hence, work done by the force of friction is always *negative*.

Work done is *zero* in the following cases:

(i) When the displacement of the particle is zero
(ii) When the motion is at right angles to the direction of the force
(iii) When a particle is in static equilibrium
(iv) Work done by internal forces as in the case of interconnected members

Work Done by a Variable Force

When a variable force acts on a particle, the resulting displacement in general will be along a *curvilinear* path. The work done in causing a displacement from the point A to B is given as

$$W_{AB} = \int_A^B dW = \int_A^B \vec{F}.d\vec{s} = \int_A^B F\cos\theta\, ds$$

Work Done in Stretching a Spring

A spring, whose one end is fixed and the other end stretched is an example for a variable force constant in direction but varying in magnitude. Work done in stretching a spring of stiffness k to an elongation of x_o from its unstretched position is given as

$$W = \int_0^{x_o} dW = \int_0^{x_o} kx\, dx = \frac{1}{2}kx_o^2$$

Power

Power is defined as the rate at which work is done. The capacity of an engine or a machine used to do work is normally expressed as its *rated power*. The one doing work faster than the other is said to be more *powerful*. If W is the total work done for a time interval t, then average power is given as

$$P_{\text{ave}} = \frac{\text{total work done}}{\text{time taken}} = \frac{W}{t}$$

The instantaneous power is given as

$$P = Fv$$

Its S.I unit is Joule per second (J/s), also called Watt (W) and the non-S.I unit is horsepower.

Energy

Energy is defined as that property of a body by virtue of which work can be performed or in other words, it is the *capacity* of a body to *do work*. Since energy is the capacity of a body to do work, its unit is same as that of work done, i.e., N.m (or) Joule.

Kinetic Energy

Kinetic energy of a body is defined as the energy possessed by virtue of its motion. It is measured by the amount of work a moving body will do if brought to rest or by the amount of work originally needed to impart the velocity to it. Mathematically,

$$\text{K.E} = \frac{1}{2} mv^2$$

Work–Energy Principle

The change in kinetic energy of a particle during any displacement is equal to the work done by the net force acting on it. Mathematically,

$$W = \frac{1}{2} mv^2 - \frac{1}{2} mv_o^2 = \text{change in K.E}$$

Potential Energy

Potential energy of a body is defined as the capacity to do work by virtue of its position. It is measured by the amount of work that has to be done on the body to lift it from ground level to a higher elevation. Thus, to lift a body to a height h from ground level, work required to be done against gravity is

$$\text{P.E} = W = mg\,h$$

It should be noted that the work done is independent of the path followed, thus the force of gravity is termed as *conservative* force. In a conservative force field, the total mechanical energy remains constant.

Principle of conservation of mechanical energy

If a body is subject to conservative system of forces, (say gravitational force) then its mechanical energy remains constant.

$$(\text{K.E} + \text{P.E})_1 = (\text{K.E} + \text{P.E})_2$$

EXERCISES

Objective-type Questions

1. Work energy method relates

(a) force, acceleration and time
(b) force, velocity and time
(c) force, velocity and displacement
(d) force, displacement and time

2. When a body is lifted up, the work done by force of gravity is
(a) positive (b) negative (c) zero

3. When a body is freely falling, the work done by force of gravity is
(a) positive (b) negative (c) zero

4. When a body displaces normal to the force of gravity, the work done by force of gravity is
(a) positive (b) negative (c) zero

5. The work done on a body is zero when
(a) there is no displacement of the body
(b) the resultant of forces acting on it is zero
(c) the displacement is perpendicular to the force
(d) all of these

6. The restoring force in a spring is proportional to
(a) the initial unstretched length of the spring
(b) the elongation of the spring
(c) the number of turns of coil in the spring
(d) the cross-sectional area of the spring

7. The work done in stretching a spring of spring constant k by a length Δ is
(a) $k\Delta$ (b) $k\Delta^2$ (c) $k\Delta/2$ (d) $k\Delta^2/2$

8. Two agents A and B do the same amount of work on a body, in which agent A causes a displacement less than that of B. Then state which of the two agents is powerful.
(a) Both the agents are equally powerful.
(b) A is more powerful than B.
(c) B is more powerful than A.
(d) Cannot be determined from the given conditions

9. One metric horsepower is equal to
(a) 1 watt (b) 736 watts (c) 746 watts (d) 1000 watts

10. When the speed of a particle is doubled, its kinetic energy
(a) remains the same (b) increases twofold
(c) increases threefold (d) increases fourfold

11. In a conservative force field,
(a) work done is zero (b) kinetic energy is constant
(c) potential energy is constant (d) total mechanical energy is constant

Answers

1. (c) **2.** (b) **3.** (a) **4.** (c) **5.** (d) **6.** (b) **7.** (d) **8.** (b)
9. (b) **10.** (d) **11.** (d)

Short-answer Questions

1. Define work done on a body (a) by a constant force, and (b) by a varying force.

2. When is the work done upon a body positive and when is it negative?

3. Under what conditions does the work done upon a body become zero?
4. The work done upon a body by a system of forces causing uniform velocity is zero. Discuss.
5. Derive the expression for work done upon stretching a spring without accelerating it.
6. Define power.
7. What is the relationship between Watt power and horsepower?
8. Define energy. What are the various forms of energy?
9. Differentiate between kinetic energy and potential energy.
10. State the work–energy principle.
11. Explain the work done by internal forces in a connected system.
12. Show that the energy of a freely falling body is constant.

Numerical Problems

15.1 A block of 5 kg mass is resting on a rough horizontal plane having a coefficient of kinetic friction of 0.15. It is pulled by a horizontal force P at a constant acceleration of 1 m/s^2 over a distance of 5 m. Sketch the free-body diagram of the block showing all the forces acting on it. Determine the works done by the externally applied force and force of friction.

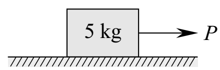


Fig. E.15.1

Ans. 61.8 J, – 36.8 J

15.2 In the Guinness world record, a man pulled a Boeing 747-400 weighing 187 tons, a distance of 91 m in 1 min 27.7 s. If the force of friction is 1 kN/ton then determine the work done by the man and power exerted by him, if he pulled it at a constant speed.

Ans. 17.02 MJ, 194.1 kW

15.3 Water is to be lifted from a well of 80 m depth. If the power of the motor available is 10 kW, determine the discharge of water. The weight density of water is 9810 N/m^3.

Ans. 0.013 m^3/s

15.4 Five men push a bus, which had a breakdown. The mass of the bus is 7 tons and the frictional resistance is 0.25 kN/ton. Determine the work done by the men in pushing the bus at a constant speed over a distance of 15 m.

Ans. 26.25 kJ

15.5 A mass of 5 kg when attached to a spring extends it vertically by 10 cm. Determine the work done in stretching the same spring horizontally by 5 cm.

Ans. 0.613 J

15.6 A man lifts a weight of 500 N through a height of 6 m as shown in Fig. E.15.6. Determine the work done by him if the coefficient of kinetic friction between rope and contact surface is 0.2.

Ans. 2.2 kJ

15.7 In a workshop, a crane lifts a load of 1 ton from the floor to a height of 2 m and it moves horizontally a distance of 5 m and finally drops it onto a platform that is 1 m from the floor level. Determine the total work done by the crane.

Ans. 9810 J

15.8 Compare the works done by external agents in each case: (i) when a body of 20 kg mass is tied to an inextensible string and lifted up by a distance of 3 m at a constant velocity; (ii) when the

same body is pushed up at constant velocity over a smooth inclined plane with a slope of 0.75 to reach the same height. What do you infer from the result?

Ans. (i) 588.6 J; (ii) 588.6 J; work done is same

15.9 A heavy block of 500 kg mass is to be loaded onto a truck. A man pulls it by a rope attached to the block and parallel to the plank as shown in Fig. E.15.9. Determine the work done by him if he pulls it up at a constant speed; the length of plank being 4 m and coefficient of friction between plank and block is 0.2.

Ans. 8.87 kJ

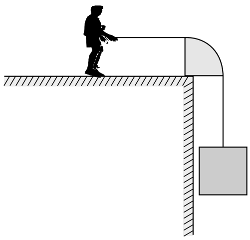


Fig. E.15.6

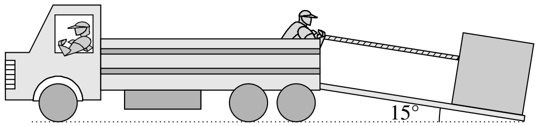


Fig. E.15.9

15.10 A high-speed train of 40 ton mass attains a maximum speed of 270 kmph in 90 seconds. If the frictional resistance is 0.5 kN/ton, determine the power of the engine.

Ans. 4 MW

15.11 A train of 300 ton mass is pulled by a locomotive at a constant speed of 100 kmph. If the resistance due to friction is 2 kN/ton, determine (i) the power of the engine, and (ii) work done by the engine in travelling a distance of 1/2 km.

Ans. 16.67 MW, 300 MJ

15.12 A car of mass m ascends a smooth incline of slope 1 in n at a constant acceleration of a and attains a maximum speed v. Determine the power of the engine.

Ans. $mv[(g/n) + a]$

15.13 A bus of 8 ton mass starts from rest and moves up an incline of 1 in 100. If it attains a maximum speed of 30 kmph in 30 seconds, determine the power of the engine; the frictional resistance being 1.5 kN/ton.

Ans. 125 kW

15.14 A car of 2 ton mass travelling at 45 kmph approaches a traffic junction. When the car is 100 m before the signal, the driver realizes that the green light is about to turn into red in 6 seconds and hence, he accelerates the car uniformly and crosses just before the light turns red. If the frictional resistance is 0.6 kN/ton, determine the power imparted by the engine at that instant.

Ans. 82.9 kW

15.15 A car of 2 ton mass attains a maximum speed of 80 kmph and moves at this speed on a level terrain. If the frictional resistance is 0.8 kN/ton, determine the power of the engine. If the same car moves up an incline of 1 in 110, determine the maximum speed it can attain.

Ans. 35.6 kW, 72.1 kmph

15.16 A cable train used in mountains has 2 bogies, each of 6 ton mass. The frictional resistance to motion is 0.4 kN/ton. Determine the driving power of the engine if the train has to move at 1 m/s (i) up the incline of 1 in 40, and (ii) down the same incline.

Ans. (i) 7.74 kW, (ii) 1.86 kW

15.17 A car of 2.5 ton mass going uphill of grade 1 in 100 can attain a maximum speed of 45 kmph. If the frictional resistance is 0.5 kN/ton, determine the power of the engine. If the same car moves on level terrain, determine the maximum velocity it can attain if the frictional resistance remains the same.

Ans. 18.69 kN, 53.82 kmph

15.18 A bus of 5 ton mass, moving at 60 kmph is stopped by applying brakes in a distance of 40 m. Determine the braking force, assuming it to be constant.

Ans. 17.4 kN

15.19 A bullet of 20 g mass moving at 300 m/s pierces a 3 cm thick metal plate and emerges out with a velocity of 200 m/s. Determine the resistance offered by the plate assuming it to be uniform. Also, determine the minimum number of such plates, each of 3 cm thickness, to be placed together to stop the bullet. Assume the same frictional force to be acting.

Ans. 16.67 kN, 2 plates

15.20 In a toy as shown in Fig. E.15.20, a boy compresses the spring of stiffness 200 N/m by 10 cm and places a ball of 200 g mass over it and releases it. Determine the height to which the ball would rise against gravity.

Ans. 510 mm

15.21 A car of 2 ton mass and moving with a speed of 60 kmph collides with another car of the same mass and at rest. Determine how far both will move together, if the frictional resistance is 0.75 kN/ton. Refer Fig. E.15.21.

Ans. 92.6 m

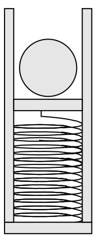


Fig. E.15.20

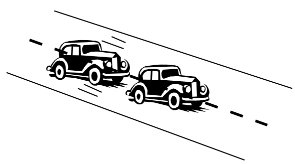


Fig. E.15.21

15.22 An aeroplane of 200 ton mass lands with a speed of 50 m/s. If it has to be brought to a halt within a distance of 500 m, determine (i) the average braking force, and (ii) constant deceleration.

Ans. 500 kN, 2.5 m/s^2

15.23 The driver of a car wishing to save the fuel takes his leg off the accelerator when the car is moving at a maximum speed of 80 kmph. Determine the speed of the car after travelling a distance of 100 m. The mass of car is 2 tons and the force of friction is 0.6 kN/ton.

Ans. 70 kmph

15.24 A lift starts upwards with a uniform acceleration and reaches a speed of 2.4 m/s in 2 seconds. Determine the power of the motor, if the total weight of the lift including the maximum number of occupants is 1 ton. The frictional resistance in the guide rollers is 1 kN.

Ans. 28.8 kW

15.25 In the above problem, if the lift descends down with the same acceleration, determine the power of the motor.

Ans. 18.3 kW

15.26. Using the method of work–energy, determine the acceleration of the system shown in Fig. E.15.26 and tension in the string. Assume the contact surfaces to be smooth.

Ans. $\dfrac{W_2}{W_1+W_2}g, \dfrac{W_1W_2}{W_1+W_2}$

15.27 A skier of 80 kg mass starts sliding down an incline of slope angle 10° as shown in Fig. E.15.27. Determine the speed with which she reaches the bottom of the incline, if the inclined length is 60 m. Assume no resistance is offered to the motion. On reaching the horizontal plain, determine what uniform retarding force she has to exert to bring herself to a halt in a distance of 75 m.

Ans. 14.3 m/s, 109.1 N

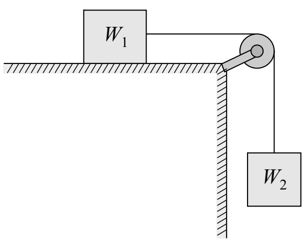


Fig. E.15.26

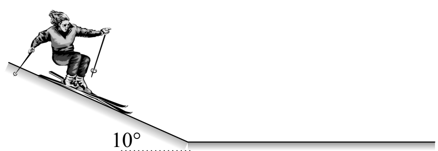


Fig. E.15.27

15.28 A block of 3 kg mass slides down a roller coaster as shown in Fig. E.15.28. If the block starts from rest at the point *A*, determine its velocity when it reaches the (i) point *B*, and (ii) point *C*.

Ans. (i) 12.53 m/s, (ii) 8.86 m/s

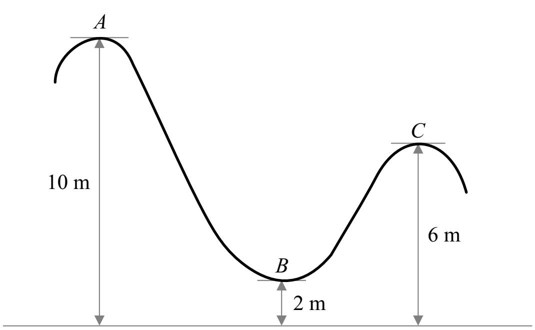


Fig. E.15.28

15.29 A block of 3 kg mass slides down a frictionless loop of 3 m radius and enters a rough horizontal plane as shown in Fig. E.15.29. Determine the distance it travels on the horizontal plane before coming to rest; the coefficient of friction between the block and plane being 0.25.

Ans. 12 m

15.30 A block of 3 kg mass slides down a frictionless loop of 3 m radius and enters a rough horizontal plane and compresses a spring of stiffness 250 N/m as shown in Fig. E.15.30. Determine the compression of the spring; the coefficient of friction between the block and plane being 0.25.

Ans. 456.7 mm

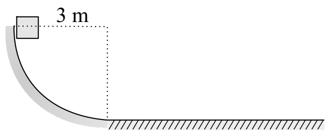


Fig. E.15.29

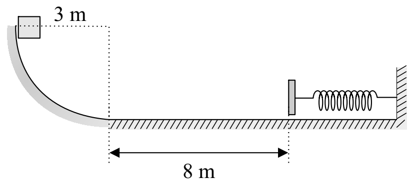


Fig. E.15.30

15.31 A block of 5 kg mass resting on a smooth horizontal plane is attached to a spring as shown in Fig. E.15.31. It is pulled by a distance of 50 cm from the unstretched position of the spring and released; determine the velocity with which the block crosses the unstretched position. The spring extends by 20 cm when a 4 kg mass is suspended from it.

Ans. 3.13 m/s

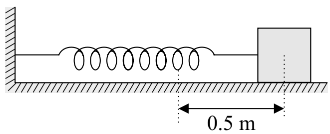


Fig. E.15.31, E.15.32

15.32 In the previous problem, determine the velocity with which the block crosses the unstretched position considering the surface to be rough with coefficient of kinetic friction of 0.2.

Ans. 2.8 m/s