The money multiplier

The money multiplier tells us the maximum amount of new demand-deposit money that can be created by a single initial dollar of excess reserves. This multiplier, m, is the inverse of the reserve requirement, R: $m = 1/R$. This note will demonstrate that result.

Suppose some initial amount, d_1, is deposited into the banking system. With a reserve requirement of R, this deposit creates initial excess reserves equal to $E_1 = (1 - R) \times d_1$. Assuming that all of this amount is lent out and redeposited within the system, these excess reserves become new money: $\Delta M_1 = E_1 = (1 - R) \times d_1$. This second deposit creates its own excess reserves equal to $E_2 = (1 - R) \times \Delta M_1 = (1 - R) \times E_1$. As before, E_2 is new money, so that $\Delta M_2 = (1 - R) \times E_1$. Continuing on like this indefinitely, we see a pattern develop:

$\Delta M_1 = E_1$

$\Delta M_2 = E_2 = (1 - R) \times E_1$

$\Delta M_3 = E_3 = (1 - R) \times E_2 = (1 - R)^2 \times E_1$

$\Delta M_4 = E_4 = (1 - R) \times E_3 = (1 - R)^3 \times E_1$

and so on *ad infinitum*.

The total increase in new money (call this "D") can be found by adding up all the successive changes in new money, $D = \Delta M_1 + \Delta M_2 + \Delta M_3 + \ldots$. Then substituting for ΔM_i,
$D = E_1 \times [1 + (1 - R) + (1 - R)^2 + (1 - R)^3 + \ldots]$.

Suppose we multiply both sides by the term $(1 - R)$ and subtract the resulting product from D.
$D - (1 - R) \times D = E_1 \times [1 + (1 - R) + (1 - R)^2 + (1 - R)^3 + \ldots] - E_1 \times [(1 - R) + (1 - R)^2 + (1 - R)^3 + \ldots]$.
All terms on the right-hand side with the exception of the initial $E_1 \times 1$ would cancel out: $D \times [1 - (1 - R)] = D \times R = E_1$. Finally, divide both sides by R to obtain the desired result: $D = E_1 \times \frac{1}{R}$. That is, an initial amount of excess reserves equal to E_1 creates new money equal to this amount multiplied by the inverse of the reserve requirement, or $m = \frac{1}{R}$.