The public sector and equilibrium GDP

We can add a public sector to the model of GDP by assuming a fixed level of government expenditures, *G*, and lump-sum taxes, *T*. The addition of taxes to the model introduces another complication: disposable income no longer is identical to GDP. Rather, DI = $Y - T$. Here we again make use of the convention that "*Y*" stands for GDP.

Equilibrium still requires that total production equals total purchases, or in this case, $Y_e = C + I_g + X_n + G$. As before, consumption is assumed to be a linear function of disposable income: $C = a + b\text{DI}$. Substituting $Y - T$ for DI and inserting into the equilibrium relationship, we find $Y_e = a + b(Y_e - T) + I_g + X_n + G$. Finally, we solve for Y_e to find that $Y_e = \left(\frac{1}{1-b}\right) \text{ x } (a - bT + I_g + X_n + G)$.

Following the usual procedure, it is apparent that $\frac{\Delta Y_e}{\Delta G} = \left(\frac{1}{1-b}\right)$ and that $\frac{\Delta Y_e}{\Delta T} = \left(\frac{-b}{1-b}\right)$. That is, the change in equilibrium GDP from a one dollar change in government expenditures is equal to the standard multiplier, while a one dollar increase in lump-sum taxes *decreases* equilibrium GDP by *b* (the MPC) times the standard multiplier. Since the MPC is between zero and one by assumption, it is clear that $\left|\frac{\Delta Y_e}{\Delta T}\right| < \left|\frac{\Delta Y_e}{\Delta G}\right|$. That is, the impact on equilibrium GDP of a change in government spending exceeds the impact of an equal (but in the opposite direction) change in taxes.

Suppose for example that $C = 97.5 + .75\text{DI}$, $I_g = 20$, $X_n = 0$, $G = 20$, and $T = 20$. Following the formula, $Y_e = \left(\frac{1}{1-.75}\right) \text{ x } (97.5 - .75\text{x}(20) + 20 + 0 + 20) = 4 \text{ x } 122.5 = \$490$. A \$10 increase in government spending will increase equilibrium GDP by $\left(\frac{1}{1-.75}\right)\text{x}10 = \$40$, while a \$10 increase in lump-sum taxes will *reduce* equilibrium GDP by $\left(\frac{.75}{1-.75}\right)\text{x}10 = \$30$.