## Elasticity and tax incidence

Consider the adjacent graph, which shows the impact of a unit tax of $\$ T$ in a competitive market. Initially, the equilibrium price is $P$, and the equilibrium quantity is $Q$. The imposition of the tax causes the equilibrium quantity to fall by $\Delta Q$, and the price to consumers increases by $\Delta P_{d}$ while the price to sellers falls by $\Delta P_{s}$. The incidence of the tax measures the shares of the tax that falls on consumers and sellers. Since $\Delta P_{d}+\Delta P_{s}=T$, it is clear that consumers' share is $\frac{\Delta P_{d}}{T}$ and sellers' share is $\frac{\Delta P_{s}}{T}$. Our goal in this note is to relate these two shares to the elasticities of demand and supply.


To begin, recall that the elasticity of demand, $E_{d}$, can be written as $E_{d}=\frac{\Delta Q / Q}{\Delta P_{d} / P}=\frac{\Delta Q}{\Delta P_{d}} \frac{P}{Q}$. (We ignore the minus sign, treating both $\Delta P_{d}$ and $\Delta Q$ as positive amounts.) Suppose we solve this for $\Delta P_{d}$ as follows: $\Delta P_{d}=\frac{P}{Q} \frac{\Delta Q}{E_{d}}$. Likewise, we could find that $\Delta P_{s}=\frac{P}{Q} \frac{\Delta Q}{E_{s}}$.

Next, we make use of the fact that $T=\Delta P_{d}+\Delta P_{s}$, so $T=\frac{P}{Q} \frac{\Delta Q}{E_{d}}+\frac{P}{Q} \frac{\Delta Q}{E_{s}}$. If we multiply and divide the first term in this sum by $E_{s}$ and the second term by $E_{d}$, we get a common denominator and can add the two terms to get $T=\frac{P \Delta Q E_{s}+P \Delta Q E_{d}}{Q E_{d} E_{s}}=\frac{P \Delta Q}{Q}\left(\frac{E_{s}+E_{d}}{E_{d} E_{s}}\right)$.

Making these two substitutions for $\Delta P_{d}$ and $T$, we can find that consumers' share of the tax is:

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\frac{\Delta P_{d}}{T}=\frac{\frac{P \Delta Q}{Q E_{d}}}{\frac{P \Delta Q}{Q}\left(\frac{E_{s}+E_{d}}{E_{d} E_{s}}\right)}=\frac{\frac{1}{E_{d}}}{\left(\frac{E_{s}+E_{d}}{E_{d} E_{s}}\right)}=\left(\frac{E_{s}}{E_{d}+E_{s}}\right) .
$$

A similar calculation shows that sellers' share of the tax is $\frac{\Delta P_{s}}{T}=\left(\frac{E_{d}}{E_{d}+E_{s}}\right)$.
Several conclusions emerge from these two share formulas:

- If the elasticities of demand and supply are equal, consumers' and sellers' share of the tax burden will be equal.
- For a given elasticity of supply, the larger is the elasticity of demand, the larger is the sellers' share, approaching $1(100 \%)$ as supply becomes perfectly elastic.
- For a given elasticity of demand, the larger is the elasticity of supply, the larger is the consumers' share, approaching $1(100 \%)$ as demand becomes perfectly elastic.

