MRP = MRC rule

One of the firm's fundamental problems is to determine the amounts of each type of resource to use in order to obtain the greatest possible profits. This note develops the required conditions both in very general terms and in the case of competitive markets.

Profit is the difference between a firm's revenue and its cost. We must first develop an expression for each of these terms in relation to the firm's choice of inputs. Let's begin by defining the firm's production function: output (Q) is an increasing function of its two inputs, labor (L) and capital (K) as follows—$Q = g(L, K)$. The demand for the firm's product is a function relating price to output, $P = f(Q) = f(g(L, K))$ so that the firm's revenue is given by PQ, or $R(L, K) = f(g(L, K))g(L, K)$.

Next, we describe cost as the function $C(L, K)$, which is increasing in both inputs. We can then write profits as the difference: $\pi(L, K) = R(L, K) - C(L, K)$.

Maximum profit requires that the first partial derivatives with respect to L and K be equal to zero.

$$\frac{\partial \pi(L,K)}{\partial L} = \frac{\partial R(L,K)}{\partial L} - \frac{\partial C(L,K)}{\partial L} = 0, \text{ or } \frac{\partial R(L,K)}{\partial L} = \frac{\partial C(L,K)}{\partial L}$$

$$\frac{\partial \pi(L,K)}{\partial K} = \frac{\partial R(L,K)}{\partial K} - \frac{\partial C(L,K)}{\partial K} = 0, \text{ or } \frac{\partial R(L,K)}{\partial K} = \frac{\partial C(L,K)}{\partial K}$$

The first of these expressions states that the marginal revenue product of labor, $\frac{\partial R(L,K)}{\partial L}$, must equal labor's marginal resource cost, $\frac{\partial C(L,K)}{\partial L}$. The second condition states that the marginal revenue product of capital, $\frac{\partial R(L,K)}{\partial K}$, must equal capital's marginal resource cost, $\frac{\partial C(L,K)}{\partial L}$. This is the basic expression stated in the text, known as the **MRP = MRC rule**.

As $R(L, K) = f(g(L, K))g(L, K)$, we can use the chain rule and the product rule to see that $\frac{\partial R(L,K)}{\partial L} = f(g(L,K))\frac{\partial g(L,K)}{\partial L} + g(L,K)f'(g(L,K))\frac{\partial g(L,K)}{\partial L}$. At this point we can make use of the fact that $g(L, K) = Q$ and factor out the term $\frac{\partial g(L,K)}{\partial K}$ to obtain $\frac{\partial R(L,K)}{\partial L} = (f(Q) + Qf'(Q))\frac{\partial g(L,K)}{\partial L}$. You may recognize the term in parentheses as the firm's marginal revenue. As such, we conclude that labor's marginal revenue product is the product of marginal revenue and the marginal product of labor. A similar result holds for capital: $\frac{\partial R(L,K)}{\partial K} = (f(Q) + Qf'(Q))\frac{\partial g(L,K)}{\partial K}$: capital's marginal revenue product is equal to marginal revenue times the marginal product of capital.

It is useful and instructive to interpret these conditions in the context of competitive markets, for then the firm's marginal revenue is simply the market price. On the cost side, $C(L, K) = wL + rK$ where w and r are the constant wage rate and cost of capital, respectively. In this special case, $\frac{\partial C(L,K)}{\partial L} = w$, and $\frac{\partial C(L,K)}{\partial L} = r$, so that the profit-maximizing conditions simplify to: $P\frac{\partial g(L,K)}{\partial L} = w$ and $P\frac{\partial g(L,K)}{\partial K} = r$. In words, profit maximization in competitive markets requires that the price of the product times the marginal physical product of the resource equals the market price of the resource.