Price discrimination

Suppose a monopolist produces its output at a single location at a cost of $C(Q)$. Once produced, this total output can be sold in either of two submarkets, with no possibility of resale between them. The firm's goal is to maximize profit by selecting the amount of output to sell in each market. Call these two amounts q_1 and q_2, with $Q = q_1 + q_2$.

Let demand in the first market be given by $P_1 = f(q_1)$ and in the second market by $P_2 = g(q_2)$. The firm's profit is the sum of price times quantity in each market minus the total cost of producing the combined amount. That is, $\pi(q_1, q_2) = P_1q_1 + P_2q_2 – C(q_1 + q_2) = f(q_1)q_1 + g(q_2)q_2 – C(q_1 + q_2)$. Maximum profit requires that the partial derivatives of profit with respect to both q_1 and q_2 are equal to zero (and that the relevant second order conditions are met.) That is:

$$\frac{\partial \pi}{\partial q_1} = f(q_1) + q_1 f\,'(q_1) - C\,'(q_1 + q_2) = 0, \text{ and}$$

$$\frac{\partial \pi}{\partial q_2} = g(q_2) + q_2 g\,'(q_2) - C\,'(q_1 + q_2) = 0.$$

You may recognize the terms $f(q_1) + q_1 f\,'(q_1)$ and $g(q_2) + q_2 g\,'(q_2)$ as the marginal revenue from selling in markets 1 and 2, respectively.

A simple rearrangement suggests an alternative interpretation: $f(q_1) + q_1 f\,'(q_1) = g(q_2) + q_2 g\,'(q_2) = C\,'(q_1 + q_2)$. These conditions tell us that marginal revenue in each market should be equal to each other and equal to the cost of producing one more for either market.

As we showed in note 10.2, "Elastic demand at the monopoly price," marginal revenue can be written as $MR = P(1 - 1/E_d)$. Using this result, the above conditions can be rewritten as $P_1(1 - 1/E_1) = P_2(1 - 1/E_2) = C\,'(q_1 + q_2)$, where E_1 and E_2 are the elasticities of demand in markets 1 and 2, respectively. Rearranging the first of these equalities, we have $\frac{P_1}{P_2} = \frac{1 - 1/E_2}{1 - 1/E_1}$.

Suppose that at the profit-maximizing choices, $\frac{P_1}{P_2} > 1$, so that the firm charges a higher price in the first market. From the profit-maximizing condition, this also implies that $\frac{1 - 1/E_2}{1 - 1/E_1} > 1$, or $1 - 1/E_2 > 1 - 1/E_1$. Subtracting one from each side and inverting, this condition will be true only if $-E_2 < -E_1$, or equivalently, if $E_1 < E_2$. Put in words, the price will be higher in the market with the less elastic demand.