Profit maximization

A firm's profit, π, equals its total revenue, R, minus its total cost, C, all of which are functions of the firm's output. In symbols, $\pi(Q) = R(Q) - C(Q)$. The first-order condition for maximum profit is that the derivative of profit with respect to output is zero. That is, the profit-maximizing level of output solves the equation: $\mathrm{d}\pi(Q)/\mathrm{d}Q = \mathrm{d}R(Q)/\mathrm{d}Q - \mathrm{d}C(Q)/\mathrm{d}Q = 0$. The term $\mathrm{d}R(Q)/\mathrm{d}Q$ is just marginal revenue, while the following term is marginal cost. Accordingly, the condition for maximum profits is MR = MC, or more precisely, that marginal revenue and marginal cost are equal at the profit-maximizing output level. In the specific case of competitive markets, the firm's marginal revenue is equal to price so this condition takes the form $P = MC$.

The second-order condition requires that $\frac{\mathrm{d}^2 R(Q)}{\mathrm{d}Q^2} - \frac{\mathrm{d}^2 C(Q)}{\mathrm{d}Q^2} < 0$, or that marginal cost cuts marginal revenue from below.

While equating marginal revenue with marginal cost provides a maximum value for the firm's profits, this maximum may be only a *local* maximum. Further, the *value* of profit at this local maximum need not even be positive—that is, it is possible that the best the firm can do is to lose as small an amount as possible. To see if the $MR = MC$ rule provides a *global* maximum, we also need to check profits at an output of zero.

If output is zero, then revenue and variable cost are also zero. The firm's profit at zero output equals $\pi(0) = 0 - (FC + 0) = -FC$; that is, a firm choosing to shut down in the short run loses an amount equal to its fixed costs. Naturally, then, the firm would only select this option if its losses at the $MR = MC$ output were greater than its fixed costs. Let Q^* be the output that satisfies the $MR = MC$ rule. Total profit at this point will be $\pi(Q^*) = PQ^* - (FC + VC(Q^*))$. Profit at $Q = 0$ will be higher than at $Q = Q^*$ if and only if $\pi(0) > \pi(Q^*)$, or $-FC > PQ^* - (FC + VC(Q^*))$. Dividing both sides of this inequality by Q^* and rearranging, the firm minimizes its losses at $Q = 0$ if and only if $P < \frac{VC(Q^*)}{Q^*}$.

Note that the right-hand-side term is the firm's average variable cost at the $MR = MC$ output level. Since average variable cost is equal to marginal cost at *minimum* average variable cost, we can state the complete short-run profit-maximizing rule as follows: produce the output for which MR = MC, provided price is greater than minimum average variable cost; otherwise shut down.