To the Student

This *Study Guide and Intervention Workbook* gives you additional examples and problems for the concept exercises in each lesson. The exercises are designed to aid your study of mathematics by reinforcing important mathematical skills needed to succeed in the everyday world. The materials are organized by chapter and lesson, with one Study Guide and Intervention worksheet for every lesson in *IMPACT Mathematics, Course 3*.

Always keep your workbook handy. Along with your textbook, daily homework, and class notes, the completed Study Guide and Intervention Workbook can help you in reviewing for quizzes and tests.

To the Teacher

These worksheets are the same ones found in the Chapter Resource Masters for *IMPACT Mathematics, Course 3*. The answers to these worksheets are available at the end of each Chapter Resource Masters Booklet.

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COURSE 3

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Lesson 1.1 Study Guide and Intervention

Direct Variation

Relationships that have straight-line graphs are called **linear relationships.** In a linear relationship, the change in y and the change in x form a ratio that is constant. A special type of linear equation is called **a direct variation.** The graph of a direct variation always passes through the origin and can be expressed as $y = kx$. The k is called the **constant of variation.** When x doubles in value, y doubles in value, too.

Example 1 A photo printer prints a 3-inch-by-5-inch picture in 3 minutes.

a. Complete the table to show the number of minutes required to print *p* photos.

Two photos take 6 minutes, and 3 photos take 9 minutes.

No. of Photos	No. of Minutes
p	m
0	0
1	3
2	?
3	?

b. As the number of photos printed increases by 1, does the total number of minutes always increase by a constant number? Explain.

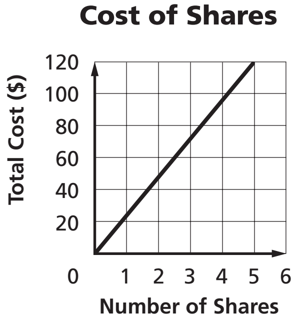
Yes, each photo adds 3 more minutes.

c. Write an equation that shows the number of minutes *m* required to print *p* photos.

The linear relationship between minutes and number of photos is proportional, so we can use the direct variation equation $y = kx$, or $m = kp$. The constant of variation is the number of minutes to make 1 photo. So $k = 3$, and the equation is $m = 3p$.

Exercises

For Exercises 1–2, refer to the graph at the right.



1. Determine whether a proportional linear relationship exists between the two quantities.

2. Write an equation that relates the number of shares and the total cost.

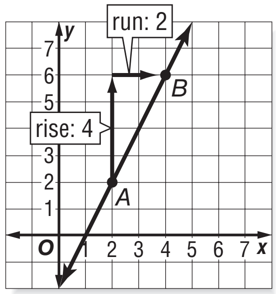
Lesson 1.2 Study Guide and Intervention

Slope

The **slope** of a line is the ratio of the rise, or vertical change, to the run, or horizontal change.

Example 1 **Find the slope of the line in the graph.**

Choose two points on the line. The vertical change from point A to point B is 4 units while the horizontal change is 2 units.



$\text{slope} = \frac{\text{rise}}{\text{run}}$ Definition of slope

$= \frac{4}{2}$ The rise is 4, and the run is 2.

$= 2$ The slope of the line is 2.

Example 2 **The points in the table lie on a line. Find the slope of the line.**

+ 3 + 3 + 3

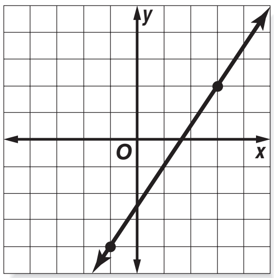
x	−2	1	4	7
y	5	1	−3	−7

− 4 − 4 − 4

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-4}{3} \text{ or } -\frac{4}{3}$ The slope of the line is $-\frac{4}{3}$.

Exercises **Find the slope of each line.**

1.



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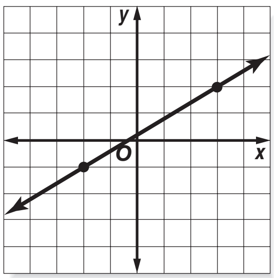
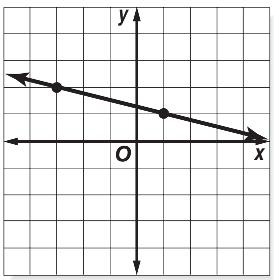


3.



The points given in each table lie on a line. Find the slope of the line.

4.

x	3	5	7	9
y	−1	2	5	8

5.

x	−5	0	5	10
y	4	3	2	1

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Lesson 1.3 Study Guide and Intervention
Write Equations

Linear equations are often written in the form $y = mx + b$. This is called the slope-intercept form. When an equation is written in this form, m is the slope and b is the y-intercept.

Example 1 **State the slope and y-intercept of the graph of $y = x - 3$.**

$y = x - 3$	Write the original equation.
$y = 1x + (-3)$	Write the equation in the form $y = mx + b$.
$\uparrow \quad\quad \uparrow$	
$y = mx + b$	$m = 1, b = -3$
	The slope of the graph is 1, and the y-intercept is -3.

You can use the slope-intercept form of an equation to graph the equation.

Example 2 **Graph $y = 2x + 1$ using the slope and y-intercept.**

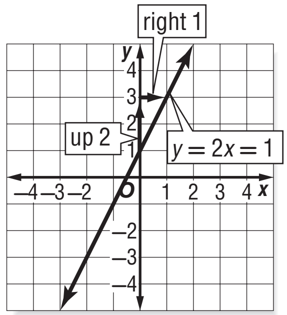
Step 1 Find the slope, 2, and y-intercept, 1.

Step 2 Graph the y-intercept at (0, 1).

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$$m = \frac{2}{1} \begin{array}{l} \leftarrow \text{change in y: up 2 units} \\ \leftarrow \text{change in x: right 1 unit} \end{array}$$

Step 4 Draw a line through the two points.



Exercises

State the slope and y-intercept of the graph of each equation.

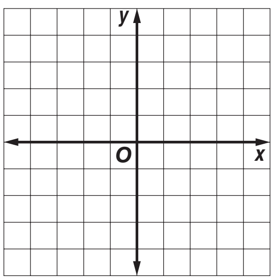
1. $y = x + 1$

2. $y = 2x - 4$

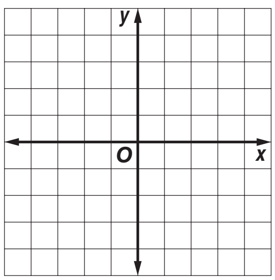
3. $y = \frac{1}{2}x - 1$

Graph each equation using the slope and y-intercept.

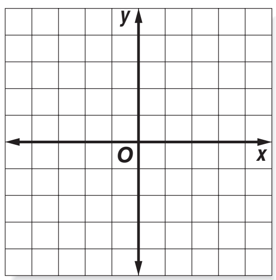
4. $y = 2x + 2$



5. $y = x - 1$



6. $y = \frac{1}{2}x + 2$



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Lesson 2.1 Study Guide and Intervention

Lines

As you learned in the last chapter, linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

Example 1 **State the slope and y-intercept of the graph of $y = 2x - 5$.**

$y = 2x - 5$ Write the original equation.

$y = 2x + (-5)$ Write the equation in the form $y = mx + b$.

$\uparrow \quad \uparrow$

$y = mx + b$ $m = 2, b = -5$

The slope of the graph is 2, and the y-intercept is -5.

You can use the slope-intercept form of an equation to graph the equation.

Example 2 **State whether the equation is linear. If it is linear, identify the values of m and b. If an equation is nonlinear, explain how you know.**

a. $3x + y = 6$

Step 1 Put in slope-intercept form.

$-3x + 3x + y = -3x + 6$

$y = -3x + 6$

Step 2 The equation is linear.

$m = -3, b = 6$

b. $y = x(x + 3) - 7$

Step 1 $y = x^2 + 3x - 7$

Step 2 The equation is not linear because x is squared.

Exercises

State the slope and y-intercept of the graph of each equation.

1. $y = 3x - 2$ **2.** $y = x + 4$ **3.** $y = \frac{1}{3}x - 1$

State whether the equation is linear. If it is linear, identify the values of m and b. If an equation is not linear, explain how you know.

4. $2y = \frac{x}{3} - 1$ **5.** $y = 6x + 4(x - 5)$ **6.** $x(x + 1) = y - 2(x + 6)$

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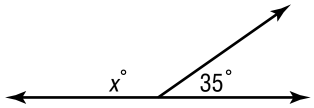
Lesson 2.2 Study Guide and Intervention
Angle Relationships

The relationship between pairs of angles can be used to find missing measures.

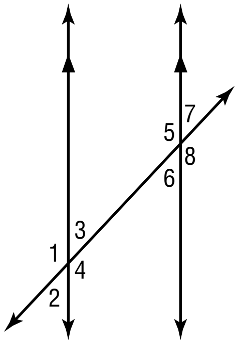
Example 1 **Find the value of x in the figure at the right.**



The two angles are supplementary, so their sum is 180°.

$x + 35 = 180$	Write an equation.
$x - 35 + 35 = 180 - 35$	Subtract 35 from each side.
$x = 145$	Simplify.

Examples **Use the figure at the right.**



2. Find $m\angle 3$ if $m\angle 7 = 70°$.

$\angle 3$ and $\angle 7$ are corresponding angles. Since corresponding angles are congruent, their measures are the same.
$m\angle 3 = m\angle 7$, so $m\angle 3 = 70°$.

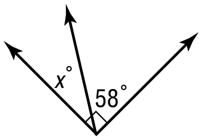
3. Find $m\angle 4$ if $m\angle 5 = 120°$.

$\angle 4$ and $\angle 5$ are alternate interior angles. Since alternate interior angles are congruent, their measures are the same.
$m\angle 4 = m\angle 5$, so $m\angle 4 = 120°$.

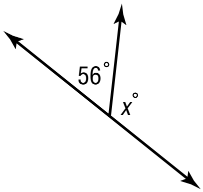
Exercises

Find the value of x in each figure.

1.



2.



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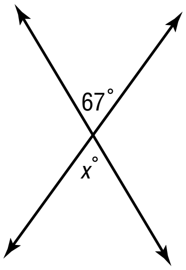


For Exercises 4–8, use the figure at the right.

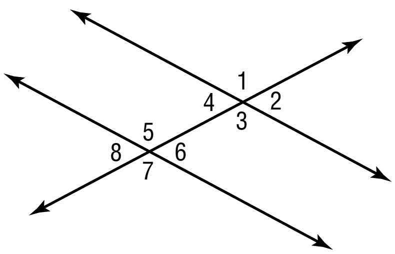
4. Find $m\angle 5$ if $m\angle 3 = 110°$.

5. Find $m\angle 2$ if $m\angle 6 = 75°$.

6. Find $m\angle 1$ if $m\angle 7 = 94°$.

7. Find $m\angle 8$ if $m\angle 4 = 68°$.

8. Find $m\angle 5$ if $m\angle 6 = 71°$.



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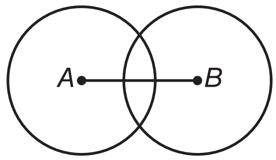
Lesson 2.3 Study Guide and Intervention
Constructions

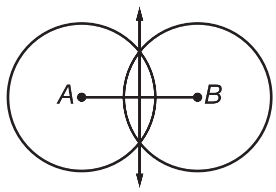
Example 1 **Construct the perpendicular bisector of the line segment *AB*.**

Step 1: Draw a circle, or an arc of a circle, about point *A* with radius more than half the distance between *A* and *B*.

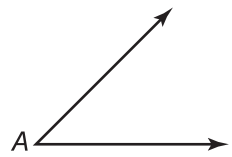
Step 2: Draw a congruent circle, or arc, about *B*.



Step 3: Draw a line through the two points of intersection of the circles. This is the perpendicular bisector of segment *AB*.

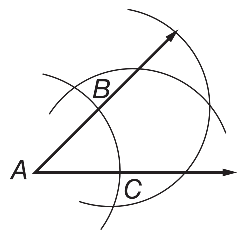


Example 2 **Bisect angle *A*.**

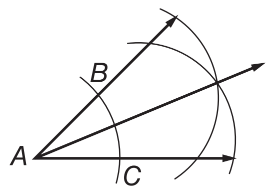


Step 1: Draw an arc through the angle with the compass point on *A*.

Step 2: Draw arcs from points *B* and *C*, where the first arc intersects the sides of the angle. The two new arcs must have the same radius.

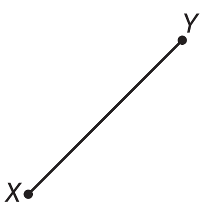


Step 3: Draw a line from *A* through the point where the last two arcs intersect. This is the bisector of angle *A*.

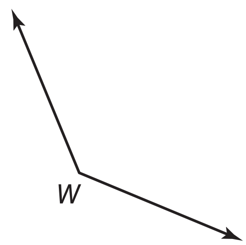


Exercises

1. Construct the perpendicular bisector of segment *XY*.



2. Construct the bisector of angle *W*.



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Lesson 3.1 Study Guide and Intervention
Understand Percents

You can use the percent proportion to find the percent.

$$\frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100} \text{ or } \frac{a}{b} = \frac{n}{100}$$

You can also use the percent proportion to find a missing part or base.

Example 1 **12 is what percent of 60?**

$\frac{a}{b} = \frac{n}{100} \longrightarrow \frac{12}{60} = \frac{n}{100}$	Replace *a* with 12 and *b* with 60.
$12 \cdot 100 = 60 \cdot n$	Find the cross products.
$1{,}200 = 60n$	Multiply.
$\frac{1{,}200}{60} = \frac{60n}{60}$	Divide each side by 60.
$20 = n$	12 is 20% of 60.

Example 2 **What number is 40% of 55?**

$\frac{a}{b} = \frac{n}{100} \longrightarrow \frac{a}{55} = \frac{40}{100}$	Replace *n* with 40 and *b* with 55.
$a \cdot 100 = 55 \cdot 40$	Find the cross products.
$a = 22$	Use similar steps to solve for *a*.

So, 22 is 40% of 55.

Exercises

Write a percent proportion to solve each problem. Then solve. Round to the nearest tenth if necessary.

1. 3 is what percent of 10?

2. What number is 15% of 40?

3. 24 is 75% of what number?

4. 86 is what percent of 200?

5. What number is 65% of 120?

6. 52 is 13% of what number?

7. 35 is what percent of 56?

8. What number is 12.5% of 88?

9. 161 is 92% of what number?

10. 45 is what percent of 66?

11. What number is 31.5% of 200?

12. 81 is 54% of what number?

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Lesson 3.2 Study Guide and Intervention

Work with Percents

To find the percent of increase or decrease, first find the amount of the increase or decrease. Then find the ratio of that amount to the original amount, and express it as a percent.

Example **Two months ago, the bicycle shop sold 50 bicycles. Last month, 55 bicycles were sold. Find the percent of change. State whether the percent of change is an *increase* or a *decrease*.**

Step 1 Subtract to find the amount of change.

$$55 - 50 = 5$$

Step 2 Write a ratio that compares the amount of change to the original number of bicycles.

Express the ratio as a percent.

$\text{percent of change} = \dfrac{\text{amount of change}}{\text{original amount}}$	Definition of percent of change
$= \dfrac{5}{50}$	The amount of change is 5. The original amount is 50.
$= 0.1 \text{ or } 10\%$	Divide. Write as a percent.

The percent of change is 10%. Since the new amount is greater than the original, it is a percent of increase.

Exercises

Find each percent of change. Round to the nearest tenth of a percent if necessary. State whether the percent of change is an *increase* or a *decrease*.

1. Original: 4
New: 5

2. Original: 10
New: 13

3. Original: 15
New: 12

4. Original: 30
New: 18

5. Original: 60
New: 63

6. Original: 160
New: 136

7. Original: 77
New: 105

8. Original: 96
New: 59

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Lesson 4.1 Study Guide and Intervention

Exponents

Expressions containing repeated factors can be written using exponents.

Example 1 **Write $p \cdot p \cdot p \cdot q \cdot q$ using exponents.**

Since p is used as a factor 3 times and q is used as a factor 2 times, $p \cdot p \cdot p \cdot q \cdot q = p^3 \cdot q^2$.

Any nonzero number to the zero power is 1. Any nonzero number to the negative n power is 1 divided by the number to the nth power.

Example 2 **Evaluate 6^2.**

$6^2 = 6 \cdot 6$	Definition of exponents
$= 36$	Simplify.

Example 3 **Evaluate 5^{-3}.**

$5^{-3} = \frac{1}{5^3}$	Definition of negative exponents
$= \frac{1}{125}$	Simplify.

Example 4 **Simplify $2^2 \cdot 2^3$.**

$2^2 \cdot 2^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	Definition of exponents
$= 2^5 = 32$	Simplify.

Exercises

Write each expression using exponents.

1. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ **2.** $4 \cdot 4 \cdot 4 \cdot 4$

Evaluate each expression.

3. 5^3 **4.** $2^3 \cdot 3^2$ **5.** $2^5 \cdot 2^3$ **6.** 3^{-4}

Write each expression using a positive exponent.

7. 6^{-4} **8.** $(-7)^{-8}$ **9.** b^{-6} **10.** n^{-1}

Simplify.

11. $a^2 \cdot a^4$ **12.** $b^{-7} \cdot b^9$ **13.** $(2a)(3a^3)$ **14.** $(-5x^2)(4x^3)$

Lesson 4.2 Study Guide and Intervention

Exponential Relationships

Quantities that are repeatedly multiplied by a number greater than 1 are said to grow exponentially, or to show **exponential increase,** or **exponential growth**.

Example 1 **Fish** Marcus has three fish in an aquarium. Each week the number of fish in the aquarium doubles. The chart shows the number of fish Marcus has after each week.

Number of weeks	0	1	2	3	4
Number of fish	3	6	12	24	48

By what number do you multiply to get from one number of fish to the next?

$3 \cdot 2 = 6$; $6 \cdot 2 = 12$; $12 \cdot 2 = 24$; $24 \cdot 2 = 48$

You multiply by 2 each time.

Quantities that are repeatedly multiplied by a positive number less than 1 are said to decrease exponentially, or to show **exponential decrease,** or **exponential decay.**

Example 2 The equation $y = 5^x$ represents exponential growth. The **growth factor** is 5.

The equation $y = 10 \cdot 0.5^x$ represents exponential decay. The **decay factor** is 0.5.

Exercises

1. For a science experiment, Tanya put one amoeba in a dish. Each day she counted and recorded in a table the number of amoebas in the dish.

Day	0	1	2	3
Bacteria	1	6	36	216

a. How does the number of amoebas change each day?

b. Which expression describes the number of amoebas, $6x$, 6^x, or x^6?

c. How many amoebas will be in the dish on Day 6?

2. Suppose you have 100 milligrams of medicine in your blood stream. Every hour, $\frac{1}{4}$ of the medicine is eliminated.

a. How many milligrams of medicine remain after 1 hour? After n hours? Explain your reasoning.

b. Is this an exponential growth or exponential decay situation?

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Lesson 4.3 Study Guide and Intervention

Radicals

The Square root of ... ________

Examples **Find each square root.**

1. $\sqrt{1}$ Since $1 \cdot 1 = 1$, $\sqrt{1} = 1$.
2. $-\sqrt{16}$ Since $4 \cdot 4 = 16$, $-\sqrt{16} = -4$.
3. $\sqrt{0.25}$ Since $0.5 \cdot 0.5 = 0.25$, $\sqrt{0.25} = 0.5$.
4. $\sqrt{\frac{25}{36}}$ Since $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$, $\sqrt{\frac{25}{36}} = \frac{5}{6}$.

Example 5 **Simplify $\sqrt{180}$.**

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$ Prime factorization of 180

$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 2 \cdot 3 \cdot \sqrt{5}$ Simplify.

$= 6\sqrt{5}$ Simplify.

Example 6 **Simplify $\sqrt{196}$.**

$\sqrt{196} = \sqrt{2 \cdot 2 \cdot 7 \cdot 7}$

$= \sqrt{2^2} \cdot \sqrt{7^2}$

$= 2 \cdot 7$

$= 14.$

Example 7 **Find $\sqrt[3]{27}$.**

Since $3 \cdot 3 \cdot 3 = 27$, $\sqrt[3]{27} = 3$.

Example 8 **Find $\sqrt[6]{64}$.**

Since $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$, $\sqrt[6]{64} = 2$.

Exercises

1. $\sqrt{4}$
2. $\sqrt{9}$
3. $-\sqrt{49}$
4. $-\sqrt{25}$
5. $\sqrt{0.01}$
6. $-\sqrt{0.64}$
7. $\sqrt{\frac{9}{16}}$
8. $-\sqrt{\frac{1}{25}}$

Simplify.

9. $\sqrt{(28)}$
10. $\sqrt{(12)}$
11. $\sqrt{(18)}$
12. $-\sqrt{(50)}$
13. $\sqrt{(9x^4)}$
14. $\sqrt{(100x^3y)}$
15. $\sqrt{(24a^4b^2)}$
16. $\sqrt{(18m^3)}$

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Lesson 5.1 Study Guide and Intervention

Rearrange Algebraic Expressions

Terms that contain the same variables raised to the same powers are called **like terms**.

Example 1 **Simplify $6x - 5 - 2x + 7$.**

$6x - 5 - 2x + 7$	$= 6x + (-5) + (-2x) + 7$	Definition of subtraction
	$= 6x + (-2x) + (-5) + 7$	Commutative Property
	$= [6 + (-2)]x + (-5) + 7$	Distributive Property
	$= 4x + 2$	Simplify

Example 2 **Simplify $4(x + 1) - 5(2x - 3)$.**

$4(x + 1) - 5(2x - 3)$	$= 4x + 4 - 10x + 15$	Distributive Property
	$= 4x - 10x + 4 + 15$	Commutative Property
	$= (4 - 10)x + 4 + 15$	Distributive Property
	$= -6x + 19$	Simplify

Exercises

Simplify each expression.

1. $9m + 3m$

2. $5x - x$

3. $8y + 2y + 3y$

4. $2 + 6a + 4a$

5. $m + 4m + 2m + 5$

6. $3c + 4d - c + 2$

7. $5h - 3g + 2g - h$

8. $3w + 4u - 6$

9. $4r - 5s + 5s - 2r$

10. $4 + m - 3m$

11. $13a + 7a + 2a$

12. $3y + 1 + 5 + 4y$

13. $8d - 4 - d + 5$

14. $10 - 4s + 2s - 3$

15. $2y + 7 - (3y - 5) + y$

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Lesson 5.2 Study Guide and Intervention

Monomials, Binomials, and Trinomials

Example 1 **Multiply $3x(2x + 6)$.**

$3x(2x + 6) = 3x(2x) + 3x(6)$ Distributive property

$= 6x^2 + 18x$ Add exponents when base is the same.

To multiply two binomials, apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method, as illustrated in Example 3.

Example 2 **Find $(x + 3)(x - 4)$ using the horizontal method.**

$(x + 3)(x - 4)$

$= x(x - 4) + 3(x - 4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$

Example 3 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$

$= x^2 + 5x + (-2x) - 10$

$= x^2 + 3x - 10$

Example 4 **Divide $\frac{2x + 6}{2}$.**

$\frac{2x + 6}{2} = \frac{2(x + 3)}{2}$ Distributive property

$= x + 3$ Simplify

Exercises

Multiply or divide.

1. $2a^5 \cdot 6a$

2. $-3t^3 \cdot 2t^8$

3. $4x^2(-5x^6)$

4. $(2w)(3w)$

5. $a(2a + 3)$

6. $5x(4x - 5)$

7. $\frac{8x + 4}{4}$

8. $\frac{4x^2 - 10x}{2x}$

9. $\frac{10x + 25x^2}{5x}$

10. $(x + 3)(x + 4)$

11. $(x - 5)(x + 2)$

12. $(x - 6)(x - 4)$

Name Date

Lesson 5.3 Study Guide and Intervention

Special Products

Perfect Square Trinomials Some pairs of binomials have products that follow specific patterns. One such pattern is called the **square of a sum.** Another is called the **square of a difference.**

Square of a sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3x + 4)(3x + 4)$.**

Use the square-of-a-sum pattern with $a = 3x$ and $b = 4$.

$(3x + 4)(3x + 4)$

$= (3x)^2 + 2(3x)(4) + (4)^2$

$= 9x^2 + 24x + 16$

The product is $9x^2 + 24x + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square-of-a-difference pattern with $a = 2z$ and $b = 9$.

$(2z - 9)(2z - 9)$

$= (2z)^2 - 2(2z)(9) + (9)^2$

$= 4z^2 - 36z + 81$

The product is $4z^2 - 36z + 81$.

There is also a pattern for the product of the sum and the difference of the same two terms. The product is called the **difference of squares.**

Difference of squares	$(a + b)(a - b) = a^2 - b^2$

Example 3 **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b)$	$= a^2 - b^2$	Product of a sum and a difference
$(5x + 3y)(5x - 3y)$	$= (5x)^2 - (3y)^2$	$a = 5x$ and $b = 3y$
	$= 25x^2 - 9y^2$	Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 6)^2$

2. $(3p + 4)^2$

3. $(4x - 5)^2$

4. $(2x - 1)^2$

5. $(2h + 3)^2$

6. $(m + 5)^2$

7. $(x - 4)(x + 4)$

8. $(p + 2)(p - 2)$

9. $(4x - 5)(4x + 5)$

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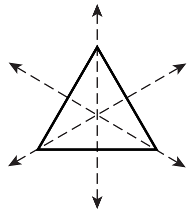
Lesson 6.1 Study Guide and Intervention

Symmetry and Reflection

A figure has **line symmetry** if it can be folded over a line so that one half of the figure matches the other half. This fold line is called the **line of symmetry**. Some figures have more than one line of symmetry.

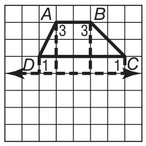
Example 1 **Determine whether the figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.**



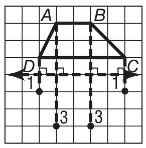
This figure has three lines of symmetry.

Example 2 **Draw the image of quadrilateral *ABCD* after a reflection over the given line.**

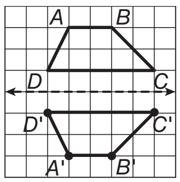
Step 1 Count the number of units between each vertex and the line of reflection.



Step 2 To find the corresponding point for vertex *A*, move along the line through *A* perpendicular to the line of reflection until you are 3 units from the line on the opposite side. Draw a point and label it *A´*. Repeat for each vertex.



Step 3 Connect the new vertices to form quadrilateral *A´B´C´D´*.

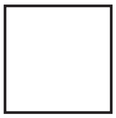


Exercises

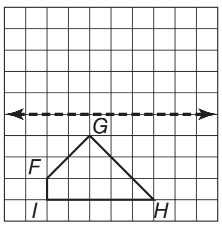
For Exercise 1, draw all lines of symmetry or write *none*.

For Exercises 2–3, draw the image after a reflection over the given line.

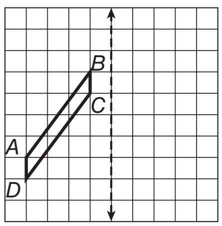
1.



2.



3.



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Lesson 6.2 Study Guide and Intervention
Rotation

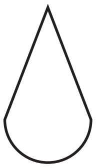
A figure has rotational symmetry if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position. The degree measure of the angle through which the figure is rotated is called the angle of rotation.

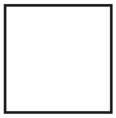
Example **Determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.**

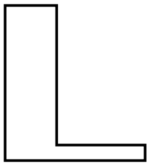
Yes, this figure has rotational symmetry. It matches itself after being rotated 180°.

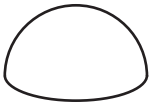
Exercises

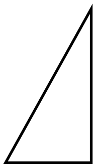
For Exercises 1–9, determine whether the figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angles of rotation.

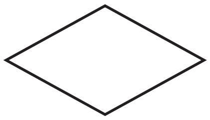
1. 

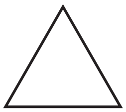
2. 

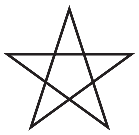
3. 

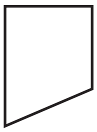
4. 

5. 

6. 

7. 

8. 

9. 

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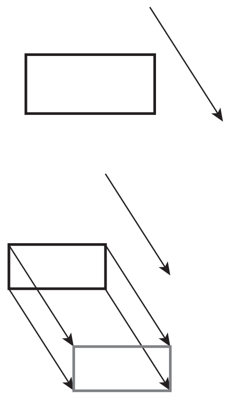
Lesson 6.3 Study Guide and Intervention

Translations, Dilations, and Combined Transformations

When a figure is translated, every point is moved the same distance in the same direction. The translated figure is congruent to the original figure and has the same orientation.

Example 1 **Translate the figure using the given vector.**

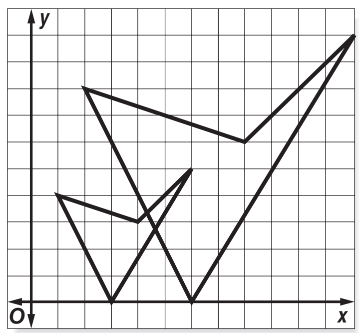


Step 1 Move each vertex the length and direction of the vector.

Step 2 Connect the new vertices to form the translated figure.

A **dilation** creates a figure that is similar but not necessarily congruent to the original figure. The corresponding sides are proportional. This ratio is called the **scale factor**.

Example 2 **Dilate the smaller figure by a scale factor of 2.**

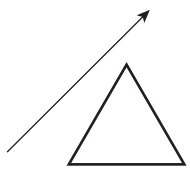


Step 1 Multiply the coordinates of the vertices by 2: (6, 0), (2, 8), (8, 6), and (12, 10).

Step 2 Graph and connect the new points.

Exercises

1. Translate the figure using the given vector.



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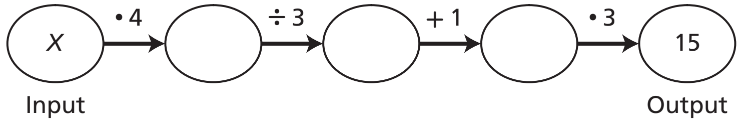
Lesson 7.1 Study Guide and Intervention
Equations

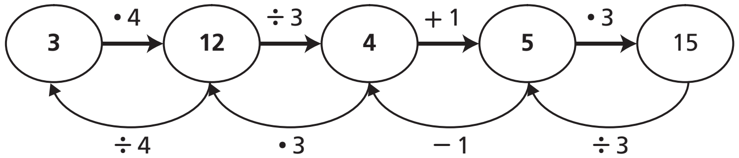
Work Backward Working backward is one of many problem-solving strategies that you can use to solve problems. To work backward, start with the result given at the end of a problem and undo each step to arrive at the beginning number.

Example 1 **Solve $3\left(\frac{4x}{3} + 1\right) = 15$ by backtracking.**

Think of x as the input and 15 as the output. Make a flowchart to show the operations needed to get from the input to the output.



To backtrack, start from the output and work backwards, undoing each operation, until you find the input. The solution is 3.



Example 2 **Solve the equation $12x - 3 = 4x + 13$ by doing the same thing to both sides.**

$12x - 3 = 4x + 13$	Write the equation.
$12x - 4x - 3 = 4x - 4x + 13$	Subtract $4x$ from each side.
$8x - 3 = 13$	Simplify.
$8x - 3 + 3 = 13 + 3$	Add 3 to each side.
$8x = 16$	Simplify.
$x = 2$	Mentally divide each side by 8.

Exercises **Solve each equation by backtracking.**

1. $8(g - 3) = 24$ **2.** $5(x + 3) = 25$ **3.** $7\left(\frac{2c}{3} - 5\right) = 7$

Solve each equation by doing the same thing to both sides.

4. $2x + 1 = x + 11$ **5.** $a + 2 = 5 + 4a$ **6.** $7y + 25 = 2y$

Lesson 7.2 Study Guide and Intervention
Inequalities

An inequality is a mathematical sentence that contains one of the symbols $<$, $>$, $\leq$, or $\geq$.

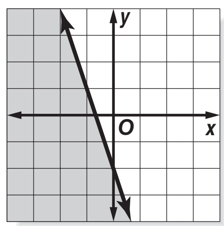
Words	Symbols
m is greater than 7.	$m > 7$
r is less than -4.	$r < -4$
t is greater than or equal to 6.	$t \geq 6$
y is less than or equal to 1.	$y \leq 1$

Example 1 **Solve $v + 3 < 5$. Then graph the solution.**

$$\begin{array}{rcll} v + 3 & < & 5 & \text{Write the inequality.} \\ -3 & = & -3 & \text{Subtract 3 from each side.} \\ \hline v & < & 2 & \text{Simplify.} \end{array}$$

The solution is all numbers less than 2.

Example 2 **Graph $y \leq -3x - 2$**



Graph the equation $y = -3x - 2$.

Since $y \leq -3x - 2$ is the same as "$y < -3x - 2$ and $y = -3x - 2$," the boundary is included in the solution set and the graph should be drawn as a solid line.

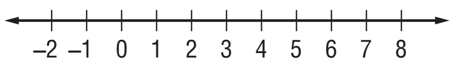
Select a point in each half-plane and test it. Choose (0, 0) and (-2, -2).

$y \leq -3x - 2$	$y \leq -3x - 2$
$0 \leq -3(0) - 2$	$-2 \leq -3(-2) - 2$
$0 \leq -2$ is false.	$-2 \leq 4$ is true.

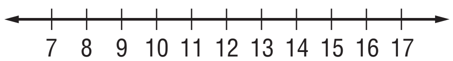
The half-plane that contains (-2, -2) contains the solution. Shade that half-plane.

Exercises **Graph each inequality on the number line or grid.**

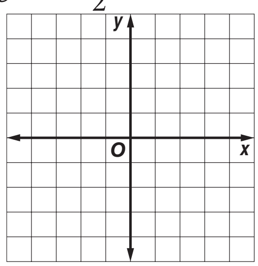
1. $c < 5$



2. $x \geq 10$



3. $y < -\frac{1}{2}x - 3$



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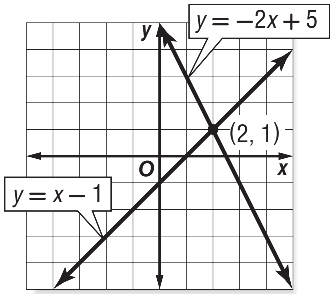
Lesson 7.3 Study Guide and Intervention

Solve Systems of Equations

A set of two or more equations is called a **system of equations.** Solving a system of equations means finding an ordered pair that is a solution of all the equations. You can solve a system of equations by graphing. If you graph the equations on the same coordinate plane, the point where the graphs intersect is the solution of the system of equations.

Example 1 **Solve the system $y = x - 1$ and $y = -2x + 5$ by graphing.**

Both equations are in slope-intercept form. Use the slope and y-intercept of each equation to graph the two equations. The graphs appear to intersect at (2, 1). Check this by substituting the coordinates into each equation.



The solution of the system of equations is (2, 1).

Another way of solving a system of equations is by **substitution.** And a third way of solving a system of equations is **elimination.**

Example 2 **Use substitution to solve the system of equations.**

$y = 2x$ $\quad$ $4x - y = -4$

$$\begin{aligned} 4x - y &= -4 && \text{Second equation} \\ 4x - 2x &= -4 && \text{Substitute } 2x \text{ for } y. \\ 2x &= -4 && \text{Combine like terms.} \\ x &= -2 && \text{Divide by 2 and simplify.} \\ y &= 2x && \text{First equation} \\ y &= 2(-2) && \text{Substitute } -2 \text{ for } x. \\ y &= -4 && \text{Simplify.} \end{aligned}$$

The solution is $(-2, -4)$.

Example 3 **Use elimination to solve the system of equations.**

$x - 3y = 7$ $\quad$ $3x + 3y = 9$

$$\begin{aligned} x - 3y &= 7 && \\ \underline{(+)\ 3x + 3y} &= 9 && \text{Add to eliminate } y. \\ 4x &= 16 && \text{Divide by 4.} \\ x &= 4 && \end{aligned}$$

Solve for x.

$$\begin{aligned} x - 3y &= 7 && \text{First equation.} \\ 4 - 3y &= 7 && \text{Substitute 4 for } x. \\ y &= -1 && \text{Solve for } y. \end{aligned}$$

The solution is $(4, -1)$.

Use elimination to solve each system of equations.

1. $x + y = -4$
$x - y = 2$

2. $2x + 2y = -2$
$3x - 2y = 12$

Use substitution to solve each system of equations.

3. $y = 4x$
$3x - y = 1$

4. $x = 2y - 1$
$x - 3y = -4$

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Lesson 8.1 Study Guide and Intervention
Use Graphs and Tables to Solve Equations

If an object is tossed or projected upward, the formula $h = vt - 16t^2$ approximates the object's height in feet h above its starting point after t seconds when projected at an initial velocity v in feet per second.

Example **Suppose a tennis ball bounces upward at an initial velocity of 32 feet per second from the ground. The height of the ball is given by the equation $h = 32t - 16t^2$.**

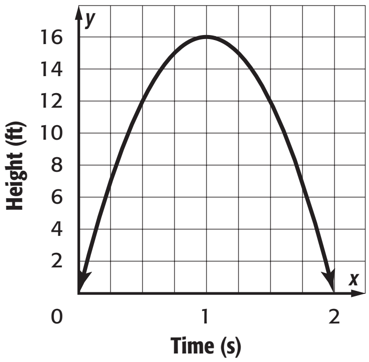
a. How high is the ball after one second?

$h = 32(1) - 16(1)^2$ Substitute 1 for t.

$h = 32 - 16 = 16$ feet Simplify.

b. What is the maximum height the tennis ball reaches?
After how much time does it reach this height?

Step 1: Graph $y = 32x - 16x^2$ on your calculator.



Step 2: Find the coordinates of the highest point on the graph.
The highest point is (1, 16).
The maximum height is 16 feet. This occurs after 1 second.

Exercises

Use the example for Exercises 1–4.

1. How high is the tennis ball after 0.5 seconds?

2. How long does it take the ball to hit the ground?

3. Estimate the time it takes for the ball to first reach a height of 8 feet.

4. How could you use the graph to estimate the solution of $32t - 16t^2 = 4$?

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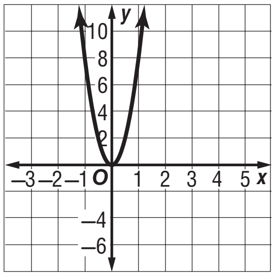
Lesson 8.2 Study Guide and Intervention
Quadratic Relationships

A simple **quadratic equation** is of the form $y = ax^2$. The graph is a symmetric *U*-shaped curve called a **parabola.**

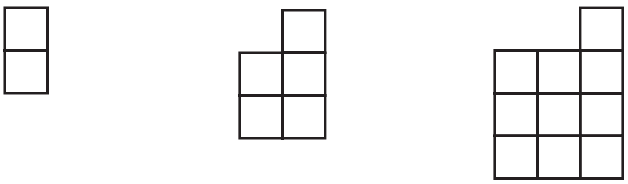
Example 1 **The equation $y = 2x^2$ is a quadratic equation. The graph is a parabola. The line of symmetry is the vertical axis. To draw the graph, complete the table and then plot the points.**

x	y
−2	8
−1	2
0	0
1	2
2	8



Example 2 **The table gives the area A of an $n \times n$ array of squares with one additional square.**

n	1	2	3	4	5	6	7
A	2	5	10	17	26	37	50



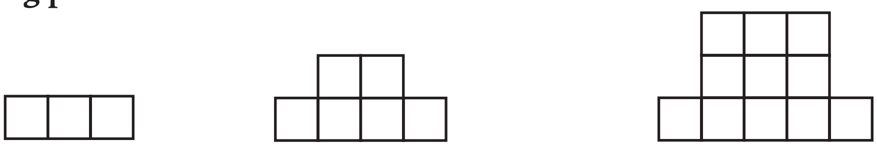
A formula for the area in square units of any figure following this pattern is $A = n^2 + 1$.

Exercises

In Exercises 1–4, tell whether each equation is a quadratic equation.

1. $y = 3x^2$ **2.** $y = 2x + 1$ **3.** $y = 2x^3$ **4.** $y = 0.5x^2$

Use the following patterns in Exercises 5–6.



Stage 1 Stage 2 Stage 3

5. Write a formula for the area in square units A given n.

6. What is the area in square units when $n = 4$?

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Lesson 8.3 Study Guide and Intervention

Families of Quadratics

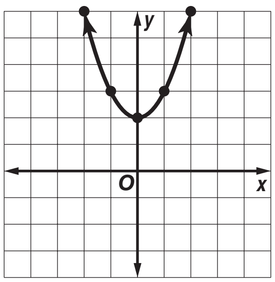
Quadratic Relationship	A relationship described by an equation of the form $y = ax^2 + bx + c$, where $a \neq 0$	**Example:** $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic relationships have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a positive. A parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1

Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
−1	6
0	1
1	−2
2	−3
3	−2
4	1

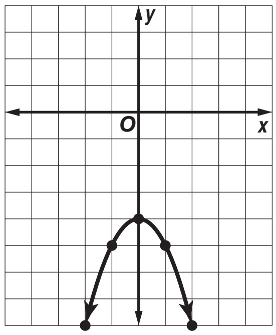


Graph the ordered pairs in the table and connect them with a smooth curve.

Example 2

Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
−6	−7
−5	−2
−4	1
−3	2
−2	1
−1	−2
0	−7

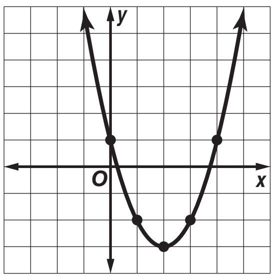


Graph the ordered pairs in the table and connect them with a smooth curve.

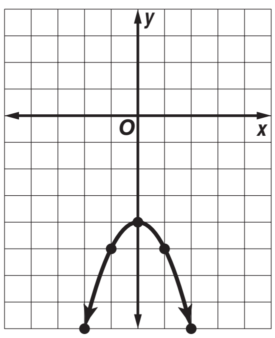
Exercises

Use a table of values to graph each equation.

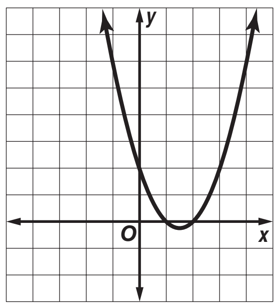
1. $y = x^2 + 2$



2. $y = -x^2 - 4$



3. $y = x^2 - 3x + 2$



Lesson 8.4 Study Guide and Intervention

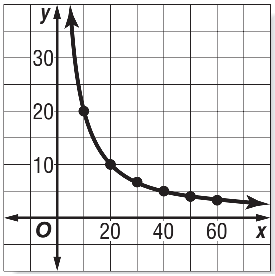
Inverse Variation

When two variables have a constant nonzero product, they are said to be **inversely proportional.** A relationship in which two variables are inversely proportional is called an **inverse variation.**

Example **Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let x = speed in miles per hour and y = time in hours. Graph the variation.**

The equation $xy = 200$ can be used to represent the situation. Use various speeds to make a table.

x	y
10	20
20	10
30	6.7
40	5
50	4
60	3.3

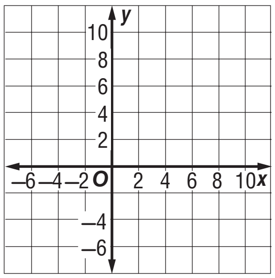


Exercises

1. Complete the table for $xy = 12$.

x	−12	−6	−4	−3	−2	−1	1	2	3	4	6	12
y												

2. Graph $xy = 12$.



3. Louisa wants to fence in a rectangular pen for her pigs. If the area A of the pen is to be 400 square feet, complete the table to show the possible lengths ℓ and widths w of the pen.

Width (ft)	5	10	20	40	50
Length (ft)					

4. Write an equation for the relationship in Exercise 3.

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Lesson 8.5 Study Guide and Intervention
Conjectures

A **conjecture** is an educated guess or generalization that has not yet been proven correct.

Example **Find the differences of the y values and the differences of the differences in this table for $y = x^2 + x + 1$. Is the relationship linear, quadratic, or neither?**

x	1	2	3	4	5	6
y	3	7	13	21	31	43

Differences of y − values: 4 6 8 10 12

Differences of differences: 2 2 2 2

The second differences are constant.

In a linear relationship, the first differences are constant.
In a quadratic relationship, the second differences are constant.

Exercises

Use the method of constant differences to make a conjecture about whether the relationship between x and y is linear, quadratic, or neither. Explain how you decide.

1.

x	1	2	3	4	5	6
y	4	10	18	28	40	54

2.

x	1	2	3	4	5	6
y	−1	2	5	8	11	14

3.

x	1	2	3	4	5	6
y	4	2	−2	−8	−16	−26

4.

x	1	2	3	4	5	6
y	2	9	28	65	126	217

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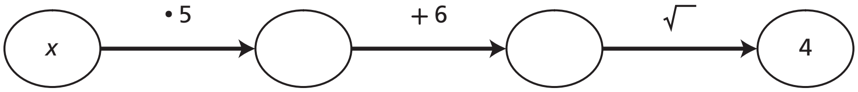
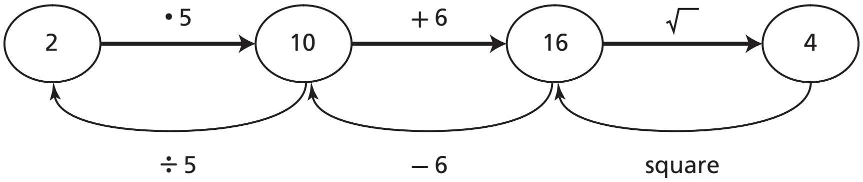
Lesson 9.1 Study Guide and Intervention
Backtracking

Example 1 **Use backtracking to solve the equation $\sqrt{5x + 6} = 4$.**

This is the flowchart.



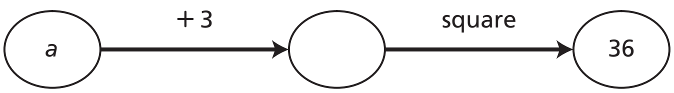
Use backtracking as shown.



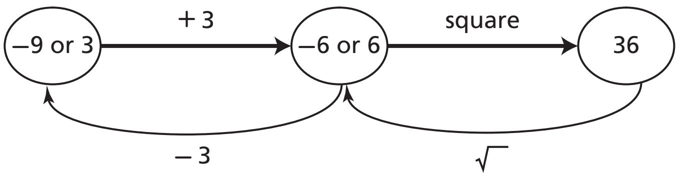
The solution is $x = 2$.

Example 2 **Solve $(a + 3)^2 = 36$ by backtracking.**

This is the flowchart.



Use backtracking as shown.



The solutions are $a = 3$ and $a = -9$.

Exercises

Solve each equation by backtracking.

1. $\frac{5}{x} = 2.5$

2. $\frac{(4 + a)}{3} = 5$

3. $\sqrt{2x + 10} = 4$

4. $\frac{6 - x}{4} = 2$

5. $(c - 7)^2 = 16$

6. $2(y + 3)^2 = 128$

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Lesson 9.2 Study Guide and Intervention

Factoring

To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c.

$x^2 + bx + c = (x + m)(x + n)$, where $m + n = b$ and $mn = c$.

Example 1 **Factor each trinomial.**

a. $\mathbf{x^2 + 7x + 10}$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $n = 5$.

$x^2 + 7x + 10 = (x + 2)(x + 5)$

b. $\mathbf{x^2 - 8x + 7}$

In this trinomial, $b = -8$ and $c = 7$. Notice that $m + n$ is negative and mn is positive, so m and n are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $n = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

To solve an equation by factoring , make one side zero. Use the property that if $ab = 0$, then $a = 0$ or $b = 0$.

Example 2 **Solve $\mathbf{x^2 + 6x = 7}$.**

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $\quad x = -7$	Solve the equation.

Exercises **Factor each trinomial.**

1. $x^2 + 4x + 3$

2. $m^2 + 12m + 32$

3. $r^2 - 3r + 2$

4. $x^2 - x - 6$

5. $x^2 - 4x - 21$

6. $x^2 - 22x + 121$

Solve each equation by factoring.

7. $y^2 - 5y + 4 = 0$

8. $m^2 + 10m + 9 = 0$

9. $x^2 = x + 2$

10. $x^2 - 12x + 36 = 0$

11. $p^2 = 9p - 14$

12. $a^2 = 11a - 18$

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Lesson 9.3 Study Guide and Intervention

Completing the Square

Since few quadratic expressions are factorable, the method of **completing the square** is better for solving some quadratic equations. Use the following steps to solve an equation by completing the square.

Write the equation in the form $x^2 + bx + c = 0$.

Find $\frac{b}{2}$ and then $\left(\frac{b}{2}\right)^2$.

Add and subtract $\left(\frac{b}{2}\right)^2$, the same number, on the left side of the equation.

Find the square root of $x^2 + bx + \left(\frac{b}{2}\right)^2$, and then solve for x.

Example **Solve $x^2 + 6x - 7 = 0$ by completing the square.**

$x^2 + 6x - 7 = 0$	Original equation
$x^2 + 6x + 9 - 16 = 0$	Add and subtract 3^2 on the left side.
$(x + 3)^2 - 16 = 0$	Factor $x^2 + 6x + 9$.
$(x + 3)^2 = 16$	Add 16 to both sides.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -7, 1$	Simplify.

Exercises **Solve each equation by completing the square.**

1. $t^2 - 4t + 3 = 0$

2. $y^2 + 10y + 9 = 0$

3. $y^2 - 8y - 9 = 0$

4. $x^2 - 6x - 16 = 0$

5. $p^2 - 4p - 5 = 0$

6. $x^2 + 4x - 12 = 0$

7. $c^2 + 8c - 20 = 0$

8. $p^2 - 3p - 4 = 0$

9. $x^2 + 20x + 19 = 0$

10. $x^2 - 5x - 14 = 0$

11. $a^2 = 22a + 23$

12. $m^2 - 8m = -7$

13. $x^2 + 10x = 24$

14. $a^2 - 18a = 19$

15. $b^2 + 6b = 16$

16. $4x^2 = 24 + 4x$

17. $m^2 + 2m + 1 = 4$

18. $4k^2 = 40k + 44$

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Lesson 9.4 Study Guide and Intervention

The Quadratic Formula

To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula.**

The formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$.

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form, subtract 3 from each side, and simplify.

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 3 - 3$

$x^2 + 2x - 3 = 0$

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{16}}{2}$$

$$x = \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2}$$

$$= 1 \qquad = -3$$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{44}}{2}$$

$$= 3 + \sqrt{11} \text{ or } 3 - \sqrt{11}$$

$$\approx 6.3 \text{ or } -0.3$$

The solution set is $\{6.3, -0.3\}$.

Exercises **Solve each equation by using the Quadratic Formula, Round to the nearest tenth if necessary.**

1. $x^2 - 3x + 2 = 0$

2. $m^2 - 8m = -16$

3. $16r^2 - 8r = -1$

4. $x^2 + 5x = 6$

5. $3x^2 + 2x = 8$

6. $8x^2 - 8x - 5 = 0$

7. $-4c^2 + 19c = 21$

8. $2p^2 + 6p = 5$

9. $48x^2 + 22x - 15 = 0$

10. $8x^2 - 4x = 24$

11. $2p^2 + 5p = 8$

12. $8y^2 + 9y - 4 = 0$

13. $2x^2 + 9x + 4 = 0$

14. $8y^2 + 17y + 2 = 0$

15. $3z^2 + 5z - 2 = 0$

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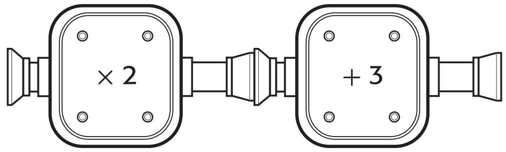
Lesson 10.1 Study Guide and Intervention
Functions

A **function** is a relationship between an input variable and an output variable in which there is only one output for each input. The set of allowable inputs to the function is called the **domain** of the function.

Example 1 **Determine whether the relation {(6, –3), (4, 1), (7, –2), (–3, 1)} is a function. Explain.**

Since each element of the domain is paired with exactly one output element, this relation is a function.

Example 2 **Use the function machine. If the input is 5, find the output.**



First 5 is multiplied by 2, giving 10. Then 3 is added. Since $10 + 3 = 13$, 13 is the output.

In the function rule $f(x) = 2x - 1$, the variable x represents the input, f is the name of the function, and $f(x)$ represents the output. The symbol $f(x)$ is read "f of x."

Example 3 **If $f(x) = 3x - 4$, find each value.**

a. $f(3)$

$f(3) = 3(3) - 4$ $x = 3$
$= 9 - 4$ Multiply.
$= 5$ Simplify.

b. $f(-2)$

$f(-2) = 3(-2) - 4$ $x = -2$
$= -6 - 4$ Multiply.
$= -10$ Simplify.

Exercises

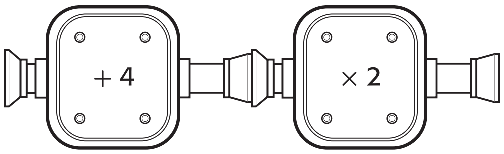
Determine whether each relationship is a function.

1. {(4, 2), (2, 3), (6, 1)}

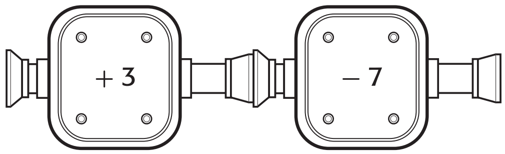
2. {(–3, –3), (–3, 4), (–2, 4)}

Determine the output from each function machine.

3. The input is 2.



4. The input is 3.



If $f(x) = 2x - 4$ and $g(x) = x^2 - 4x$, find each value.

5. $f(4)$

6. $g(2)$

7. $f(-5)$

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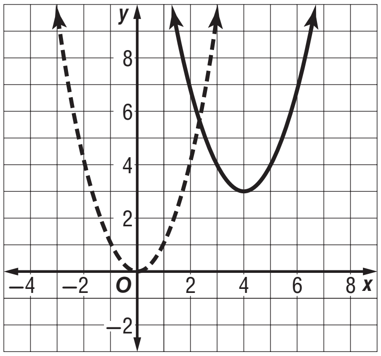
Lesson 10.2 Study Guide and Intervention
Graphs of Functions

Example **The graph of a quadratic equation in the form $f(x) = (x - b)^2 + c$ is a translation of the graph of $f(x) = x^2$. The translation is located b units to the right and c units up.**

Start with a graph of $f(x) = x^2$.

Slide this graph to the right 4 units:
$f(x) = (x - 4)^2$

Then slide it up 3 units:
$f(x) = (x - 4)^2 + 3$



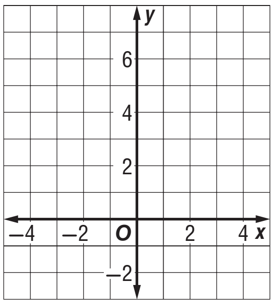
All the possible output values of a function f are called the **range** of the function. For the function graphed above in Example 1, the range is $f(x) \geq 3$. No matter what you substitute for x, the value of $f(x)$ will be greater than or equal to 3.

The turning point, or vertex, is the point on the graph in Example 1 where $x = 4$. $f(x) = (4 - 4)^2 + 3 = 0 + 3 = 3$, so the vertex has coordinates (4, 3).

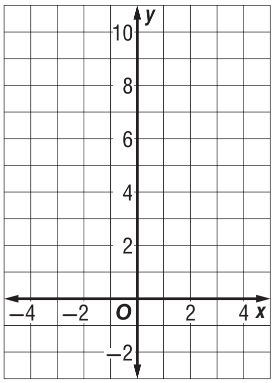
Exercises

Graph each equation

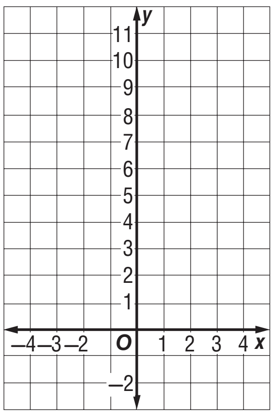
1. $f(x) = x^2 + 1$



2. $f(x) = (x - 1)^2$



3. $f(x) = (x + 1)^2 + 3$



4. Find the range of each function in Exercises 1–3.

5. Find the coordinates of the vertex of each function in Exercises 1–3.

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Lesson 11.1 Study Guide and Intervention
Counting Strategies

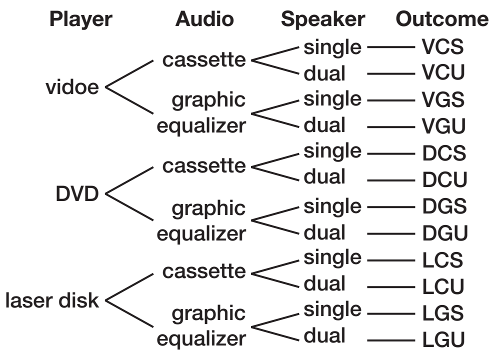
A **probability** is a number between 0 and 1 that indicates how likely something is to happen. Often, to find the probability that something will occur, you need to find all the possible **outcomes**.

Example 1 **A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible. Make an organized list.**

brown/small   brown/large   black/small   black/large

In 2 of the 4 outcomes, the watch is brown. The probability that the watch is brown is $\frac{2}{4}$, or $\frac{1}{2}$.

Example 2 **Suppose you can set up a stereo system with a choice of video, DVD, or laser disk players, a choice of cassette or graphic equalizer audio components, and a choice of single or dual speakers. Draw a tree diagram to show the sample space.**



The tree diagram shows that there are 12 ways to set up the stereo system.

If chosen at random, the probability that the stereo system has a cassette and dual speakers is $\frac{3}{12}$, or $\frac{1}{4}$.

Exercises

In Exercises 1-3, a pizza can be ordered with a choice of sausage, pepperoni, or mushrooms for toppings, a choice of thin or pan for the crust, and a choice of medium or large for the size.

1. Draw a tree diagram to show the sample space.

2. How many different kinds of pizza are possible?

3. What is the probability that the pizza has mushrooms on it?

4. In how many ways can you arrange 4 boxes of cereal on a shelf?

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Lesson 11.2 Study Guide and Intervention

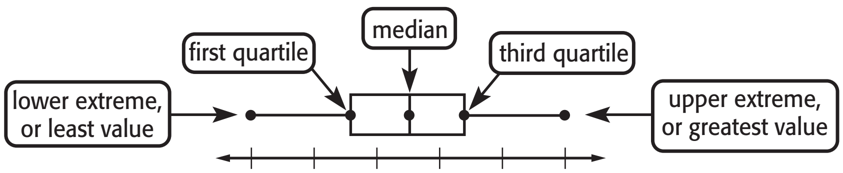
Modeling with Data

Example 1 **Judy was comparing prices of admission at several movie theaters. She found these data. Find the mean price at each theater. Which theater gives the best mean price?**

Theater	Colonial	Discount	Magic	Colonial	Discount	Magic	Colonial	Discount	Magic
Time	noon	noon	noon	4 P.M.	4 P.M.	4 P.M.	7 P.M.	7 P.M.	7 P.M.
Price	\$4.00	\$3.50	\$5.00	\$6.00	\$5.75	\$6.00	\$8.00	\$7.00	\$9.00

The mean price at the Colonial is $\frac{(4 + 6 + 8)}{3} = \$6.00$. The mean price at the Discount is $\frac{(3.5 + 5.75 + 7)}{3} = \$5.42$. The mean price at the Magic is $\frac{(5 + 6 + 9)}{3} = \$6.67$. The Discount Theater offers the best mean price.

A box-and-plot divides a set of data into four parts using the median and quartiles. Each of these parts contains 25% of the data.



Example 2 **Make a box-and-whisker plot of the data below.**

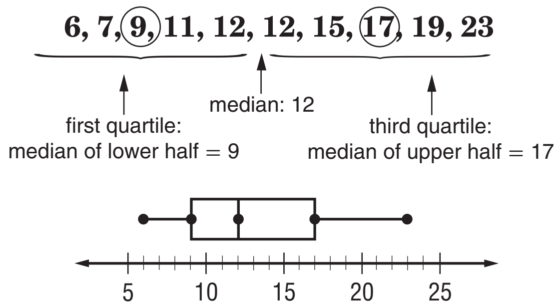
12, 23, 6, 17, 9, 19, 7, 11, 15, 12

Step 1 Order the data from least to greatest.

Step 2 Find the median and the quartiles.

Step 3 Draw a number line, graph the median, the quartiles, and the extremes as points above the line.

Step 4 Draw the box and whiskers.



Exercises

Draw a box-and-whisker plot for each set of data.

1. {17, 5, 28, 33, 25, 5, 12, 3, 16, 11, 22, 31, 9, 11}

2. {\$21, \$50, \$78, \$13, \$45, \$5, \$12, \$37, \$61, \$11, \$77, \$31, \$19, \$11, \$29, \$16}

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Lesson 12.1 Study Guide and Intervention

Work with Algebraic Fractions

An **algebraic fraction** involves division. The denominator cannot equal zero. The expression is undefined for any values of the variable that make the denominator zero. To simplify an algebraic fraction, factor the numerator and denominator. Then divide each by the greatest common factor.

Example 1 **For what value of m is the expression $\frac{(4m-8)}{(m+2)}$ undefined?**

$m + 2 = 0$	The expression is undefined when the denominator equals zero.
$m + 2 - 2 = 0 - 2$	Subtract 2 from each side.
$m = -2$	Simplify.

Therefore, m cannot equal –2.

Example 2 **Simplify $\frac{54x^3}{24xy}$.**

$\frac{54x^3}{24xy} = \frac{(6x)(9x^2)}{(6x)(4y)}$	The GCF of the numerator and denominator is $6x$.
$= \frac{(\cancel{6x})(9x^2)}{(\cancel{6x})(4y)}$	Divide the numerator and denominator by $6x$.
$= \frac{9x^2}{4y}$	Simplify.

Example 3 **Multiply $\frac{5}{n} \cdot \frac{3n}{10}$**

$\frac{5}{n} \cdot \frac{3n}{10} = \frac{5}{n} \cdot \frac{3n}{5 \cdot 2}$	Factor denominator.
$= \frac{\cancel{5}}{\cancel{n}} \cdot \frac{3\cancel{n}}{\cancel{5} \cdot 2}$	Divide the numerator and denominator by $5n$.
$= \frac{3}{2}$	Simplify.

Exercises

Find the values of the variable that make the expression undefined.

1. $\frac{2b}{b-8}$ **2.** $\frac{12-a}{32+a}$ **3.** $\frac{x^2-2}{x+4}$

Simplify each expression.

4. $\frac{12ab}{a^2b^2}$ **5.** $\frac{7n^3}{21n^8}$ **6.** $\frac{x+2}{x^2-4}$

Find each product or quotient.

7. $\frac{mn^2}{3} \cdot \frac{4}{mn}$ **8.** $\frac{n}{4} \div \frac{n}{m}$ **9.** $\frac{x+2}{x-4} \cdot \frac{x-4}{x-1}$

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Lesson 12.2 Study Guide and Intervention

Add and Subtract Algebraic Fractions

To add algebraic fractions with like denominators, add the numerators and then write the sum over the common denominator. Then, if possible, simplify the result.

Example 1 **Find $\frac{5n}{15} + \frac{7n}{15}$.**

$\frac{5n}{15} + \frac{7n}{15} = \frac{5n + 7n}{15}$ Add the numerators.

$= \frac{12n}{15}$ Combine like terms.

$= \frac{4n}{5}$ Simplify.

To add algebraic fractions with unlike denominators, factor each denominator to find the LCD.

Example 2 **Find $\frac{n + 3}{n} + \frac{8n - 4}{4n}$.**

$n = n,\quad 4n = 4 \cdot n,\quad \text{LCD} = 4n$

Only $\frac{n + 3}{n}$ needs to be renamed.

$\frac{n + 3}{n} + \frac{8n - 4}{4n} = \frac{4(n + 3)}{4n} + \frac{8n - 4}{4n}$

$= \frac{4n + 12}{4n} + \frac{8n - 4}{4n}$

$= \frac{12n + 8}{4n}$

$= \frac{3n + 2}{n}$

Exercises

Find the sum.

1. $\frac{3}{a} + \frac{4}{a}$ **2.** $\frac{x^2}{8} + \frac{x}{8}$ **3.** $\frac{2x}{x + 5} + \frac{3x}{x + 5}$

Find the difference.

4. $\frac{3}{a} - \frac{5}{a}$ **5.** $\frac{5x}{8} - \frac{x}{8}$ **6.** $\frac{8t}{w + 6} - \frac{3t}{w + 6}$

Find a common denominator. Then find the sum.

7. $\frac{1}{a} + \frac{7}{3a}$ **8.** $\frac{1}{6x} + \frac{3}{8}$ **9.** $\frac{2}{3c} + \frac{3}{4c}$

Find a common denominator. Then find the difference.

10. $\frac{1}{a} - \frac{9}{4a}$ **11.** $\frac{1}{9x} - \frac{1}{8}$ **12.** $\frac{3}{m} - \frac{3}{4}$

Solve each equation using any method you like. Make sure your answer does not cause any denominator to be zero.

13. $\frac{m}{6} + m = 14$ **14.** $\frac{x - 2}{3} = x - 4$ **15.** $\frac{(s + 2)}{(s - 1)} = \frac{3}{2}$