Golden Ratio

The Golden Ratio is a value said to provide the most visually pleasing dimensions in areas like art and architecture. In this activity, you will learn more about the Golden Ratio; you will also explore a famous sequence known as the Fibonacci sequence.

<u>Exercise 1</u>

You will each create a sequence. Choose two positive integers.

a. Record the chosen integers as terms #1 and #2 in the table below.

b. Add your two integers to get a third integer (see Figure 1; do not use 6 and 15). Record the sum in the table as term #3.

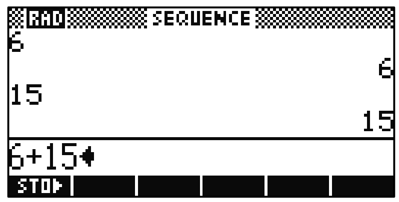

Figure 1

c. Add the 2nd and 3rd integers to get a 4th (Figure 2) and record the result in the table.

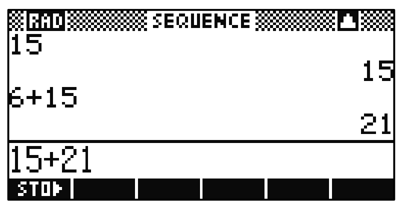

Figure 2

d. Keep adding the last two integers to get a new one until you have recorded a sequence of 16 integers in the table.

Term #	Value
1	
2	
3	
4	
5	
6	
7	
8	

Term #	Value
9	
10	
11	
12	
13	
14	
15	
16	

e. Divide the 16th integer by the 15th integer. Record the quotient in the space below. Be ready to share your result with the class. Do you believe your teacher can guess your answer?

f. The number you have just discovered is an approximation of the Golden Ratio. Write two or three sentences to summarize what you have learned.

Exercise 2

There is a famous sequence similar to the one you created in Exercise 1 called the Fibonacci sequence. The Fibonacci sequence is 1, 1, 2, 3, 5, 8, . . .

a. What are the next two numbers in the sequence? Explain how you found those numbers.

b. Compute the sequence of ratios of Fibonacci numbers. The first two are computed for you. F_1 will represent the first term in the Fibonacci sequence, F_2 will represent the second term, and so on.

$$\frac{F_2}{F_1} = \frac{1}{1} = 1 \qquad \frac{F_3}{F_2} = \frac{2}{1} = 2$$

Continue this process until you find $\frac{F_{10}}{F_9}$. Record your answers in the table below.

Quotients
$\frac{F_2}{F_1} = \frac{1}{1} = 1$
$\frac{F_3}{F_2} = \frac{2}{1} = 2$

c. Compare these ratios to the golden ratio. What do you notice?

<u>Exercise 3</u>

Your teacher will give you an Aplet called Golden Ratio (See Figure 3 below).

a. Look at the Fibonacci sequence from Exercise 2. On the HP 39gs, start your new Aplet and press the NUM key. Use the Aplet to confirm that the column U1 contains the same sequence (Figure 4).

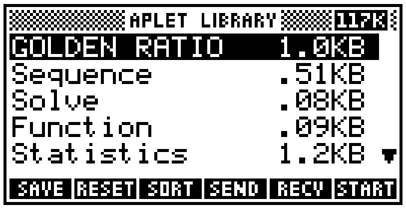

Figure 3

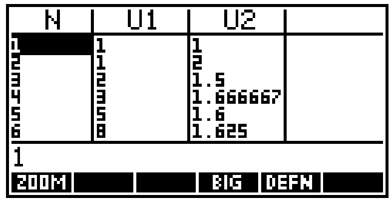

Figure 4

b. In Exercise 2 you computed the sequence of ratios of consecutive Fibonacci numbers. Use the Aplet to confirm that U2 contains the sequence of ratios of terms in the Fibonacci Sequence found in your table. What do you notice about the sequence of ratios as it progresses?

c. Edit the definitions of U1 and U2 so they reflect the integers you chose in Exercise 1. Figure 5 below shows the original definition of the Fibonacci Sequence, with the first two terms both set to 1. In Figure 6, the first two terms have been changed to 6 and 15. Press the SYMB key to see Figure 5 and change the first two terms to match your choices from the table you created in Exercise 1.

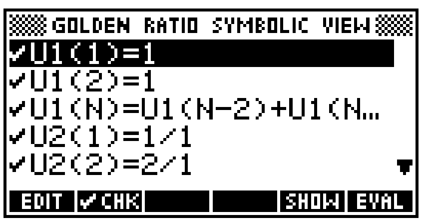

Figure 5

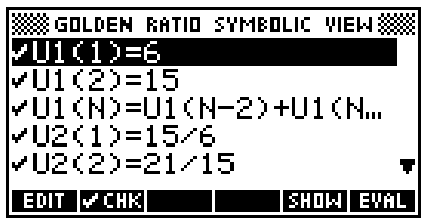

Figure 6

d. Change the first two terms of U2 to match your new U1. In Figure 6, U2(1) has been changed to 15/6; in Figure 7, 21/15 has been entered as U2(2). Make similar changes in your U2 definition to match the numbers you chose in Exercise 1.

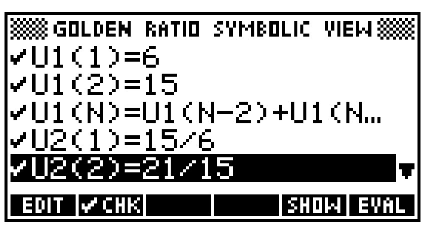

Figure 7

e. Press the NUM key and explore your new U1. Confirm that it matches the numbers in your table from Exercise 1. Then go to U2 and look at the sequence of ratios. What happens to these values as the sequence progresses?