Equivalent Expressions

<u>Materials</u>
HP39gs

<u>Introduction</u>
A mathematical expression is any collection of numbers and operations. Some examples are $3 + 2$, 5^2, and $4x - 3xy^2$. Two expressions are *equivalent* when they have the same value. Examples of equivalent expressions are:

$$a + b \text{ and } b + a$$
$$x + x \text{ and } 2x$$
$$(x + 2)(x - 4) \text{ and } x^2 - 2x - 8$$

In this activity, you will investigate equivalent expressions, and learn to identify certain kinds of equivalent expressions. For some of these, you will see a geometric model used to demonstrate why two expressions are equiv alent. You will also see expressions that you might think look to be equivalent, but really are not.
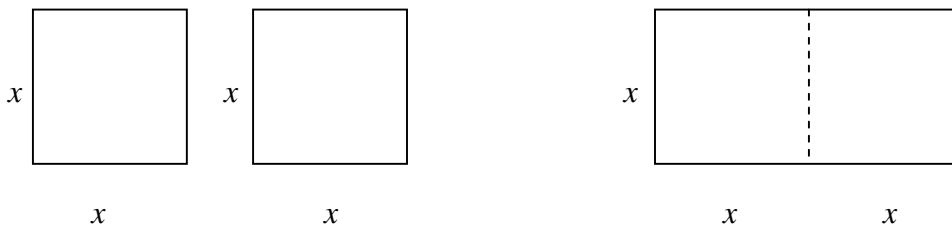
<u>Example 1</u>
A square with side length 3 units has area 3^2 or 9 square units. Take a look at the model below:
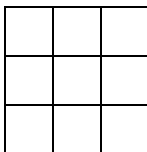
Each side of the square has length 3 units and the area of each smaller square is 1 square unit. There are 3×3 or 9 square units in the big square.

Take a look at the expressions: $x^2 + x^2$ and $2x^2$. Are they equivalent? Consider two squares, each with a side of length x:

Notice that each of the two squares on the left have an area of $x \times x$ or x^2. When we slide one so that it sits right next to the other, we form a rectangle with base $2x$ and height x. Its area is $2x \times x$ or $2x^2$. So, $x^2 + x^2$ and $2x^2$ are equivalent.

Your HP39gs can also provide you with evidence that two expressions are equivalent. Turn your calculator on, press the Aplet key, highlight **Function**, press the RESET menu key, and then the press the START menu key. This restores the Function Aplet to its default state, clearing out any functions you may have defined. Enter X^2+X^2 into F1(X) and $2X^2$ into F2(X), as shown below. Then press the NUM key. Once you see the table of values, press the DEFN menu key, then press the right arrow key. Look at the bottom of the screen. The numbers in the F1 column are values of X^2+X^2, and the numbers in the F2 column are values of $2X^2$.

Repeatedly press the up-or down-arrow key.

1) Can you find a value for x so the values in columns F1 and F2 are different from each other?

Position the cursor in the X column, and enter any number you like for X.

2) Is there a value of X that makes the values of F1 and F2 different from each other?

3) What does this tell you about the expressions $x^2 + x^2$ and $2x^2$?

No matter how hard you try, you'll never find a value of x that makes $x^2 + x^2$ and $2x^2$ different. This gives you evidence that these are equivalent expressions.

Now you will repeat this process with several more pairs of expressions. In each case, enter one expression into F1, and the other into F2. Press NUM, and see if you can find a value for X that makes the expressions evaluate differently. If you can find even one value where they are different, then the expressions are *not* equivalent.

4) $3x + x^2$ and $(3 + x) x$

    a. Are the two expressions equivalent?

    b. Try to draw a geometric model that demonstrates this result.

5) $(x + 2)^2$ and $x^2 + 4$

    a. Are the two expressions equivalent?

    b. Explain your answer.

6) $x^3 + x^3$ and x^6

    a. Are the two expressions equivalent?

    b. Explain your answer.

7) $\dfrac{x + x}{x}$ and 2

    a. Are the two expressions equivalent?

    b. Explain your answer.

8) $\sqrt{x^2}$ and x.

    a. Are the two expressions equivalent?

    b. Explain your answer.

9) $\left(\dfrac{3x^2}{4}\right)^2$ and $\dfrac{27x^6}{64}$

    a. Are the two expressions equivalent?

    b. Explain in words how to get the second expression from the first.

10) $\sqrt{x^2 + 25}$ and $x + 5$

    a. Are the two expressions equivalent?

    b. Explain your answer.