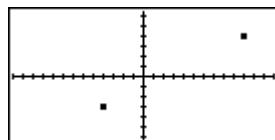


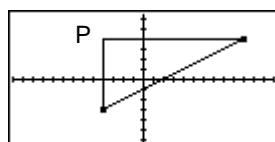
Corner Points and the Distance Formula

Introduction

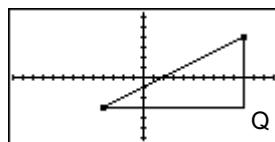
How can the Pythagorean Theorem help to find the distance between the two points $(10, 4)$ and $(-4, -3)$?



The key to answering this question is to understand the idea of a *corner point*. To find a corner point for $(10, 4)$ and $(-4, -3)$, draw a horizontal line on the figure above through the point $(10, 4)$. Draw a vertical line through the point $(-4, -3)$. The two lines intersect at point $P(-4, 4)$, which is a corner point for $(10, 4)$ and $(-4, -3)$. Notice that the three points form a right triangle.



A different corner point, point Q , can be made for $(10, 4)$ and $(-4, -3)$. The right triangle made by three points is shown below:



Exercises

- 1) What are the coordinates of the corner point Q above?

Use the HP 39gs to find a corner point for some randomly generated points.

Press the APLET key and choose **CORNER POINT**. Your teacher can provide you with this if your HP 39gs does not have this aplet in the library. The aplet starts with a note. Read it and then press the VIEWS key. Select **Get Points** and write down the two points you are given.

- 2a) The two points given are $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$ and $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$.

Press OK and you will see the PLOT view with tick marks every one unit. If you want to move the marker around the screen, you will have to turn TRACE off. To do this, press the MENU key, then press TRACE (the little square will disappear to show that trace is

off). You can now move the marker around the screen. If you press the MENU key two more times, you can see the coordinates of the marker on the screen. When you have the marker at a corner point, press the VIEWS key again and select **Enter Answer**. Enter the *x*- and *y*-coordinates of your answer.

2b) What is your answer? (_____ , _____)

The HP 39gs will check your answer and draw a plot showing the triangle made by the two given points and your answer. The goal is to find a point that makes a right triangle. Press VIEWS and select **Get Trip** to start again. If you were correct, you will get two new points. If you were incorrect, start over using the same two points. Continue until you have found five corner points. Record the results.

3) Given points: (_____ , _____) and (_____ , _____).

Corner point: (_____ , _____)

4) Given points: (_____ , _____) and (_____ , _____).

Corner point: (_____ , _____)

5) Given points: (_____ , _____) and (_____ , _____).

Corner point: (_____ , _____)

6) Given points: (_____ , _____) and (_____ , _____).

Corner point: (_____ , _____)

Answer questions 7-11 without the HP 39gs.

7) What is a corner point for the coordinates (8, 3) and (2, 1)? (_____ , _____)

8) What is a different corner point for the coordinates (8, 3) and (2, 1)? (_____ , _____)

9) Given the coordinates (x_1 , 3) and (6, 7), one of the corner points is (6, 3), what is the other corner point?

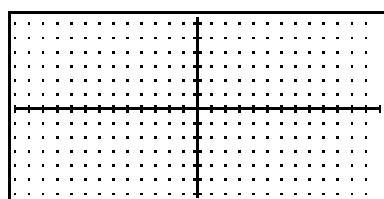
10) Write a general rule for finding a corner point.

11) What is a corner point for the points (x_1, y_1) and (x_2, y_2) ? (_____ , _____)

Once you find the corner points, you can then find the lengths of the legs of the right triangle formed by the three points. This is done by subtracting the coordinates. After finding the lengths of the legs, you can use the Pythagorean Theorem to find the length of the hypotenuse of the right triangle. The length of the hypotenuse is the distance between the two given points.

Find the distance between the points $(8, 3)$ and $(2, 1)$.

12a) Plot the two points and a corner point on the coordinate grid below. Draw the right triangle formed by the three points.



12b) Find the distance from the point $(8, 3)$ to the corner point by subtracting coordinates. Record your results.

12c) Find the distance from the point $(2, 1)$ to the corner point by subtracting coordinates. Record your results.

These distances are the lengths of the legs of the right triangle. You can now use the Pythagorean Theorem to find the length of the hypotenuse. This length is the distance between the points $(8, 3)$ and $(2, 1)$.

12d) What is the distance between the points $(8, 3)$ and $(2, 1)$?

This idea can be generalized into a formula called the distance formula.

13a) Write down a corner point for the points (x_1, y_1) and (x_2, y_2) .

13b) Write an expression that gives the distance from the point (x_1, y_1) to the corner point. This is done by subtracting coordinates.

13c) Write an expression that gives the distance from the point (x_2, y_2) to the corner point. This is done by subtracting coordinates.

13d) Use the Pythagorean Theorem to write an expression for the length of the hypotenuse of the right triangle made by the points (x_1, y_1) , (x_2, y_2) and the corner point.

This expression is the distance formula. It gives the distance between any two points (x_1, y_1) and (x_2, y_2) .