## Similarity

Similar figures in two and three dimensions have many interesting characteristics. This activity will look at similar shapes, starting with perimeter and area of two-dimensional shapes, and then moving on to volume of three-dimensional shapes.

## Exercise 1

For two shapes to be similar, all the lengths of one shape are the same multiple of the corresponding lengths of the other shape and all the corresponding angles are congruent. Two similar squares are shown below.

a. Fill in the table below to see how the perimeters of similar squares are related.

| Side Length (in units) | Perimeter (in units) |
| :---: | :---: |
| Original <br> 2 |  |
| 4 |  |
| 6 |  |
| 8 |  |
| 10 |  |
| 12 |  |
| 14 |  |

b. Explain how each of the perimeters compares to the perimeter of the original square.
c. Pick any perimeter/side pair of numbers and divide the perimeter by the side length, as shown below in Figure 1. Now pick several other pairs and divide the perimeter by the side length. What do you notice?


Figure 1
d. Below are two similar rectangles. Complete the table below to see how the perimeters of similar rectangles are related.


| Length <br> (in units) | Width <br> (in units) | Perimeter (in units) |
| :---: | :---: | :--- |
| Original <br> 2 | Original <br> 1 |  |
| 4 | 2 |  |
| 6 | 3 |  |
| 8 | 4 |  |
| 10 | 5 |  |
| 12 | 6 |  |
| 14 | 7 |  |

e. Explain how each of the perimeters compares to the perimeter of the original rectangle.
f. If you multiply the side lengths of a rectangle by a factor of 30 , by what factor would the perimeter increase?
g. Pick any perimeter/width pair of numbers and divide the perimeter by the side width, as shown below in Figure 2. Now pick several other pairs and divide the perimeter by the width. What do you notice?


Figure 2
h. How do you think your answers for exercises $f$ and $g$ are related?

## Exercise 2

In this exercise, you will again consider similar squares and rectangles, but rather than looking at the perimeter, you will see how the areas are related.
a. Complete the table below for similar rectangles.

| Length <br> (in units) | Width <br> (in units) | Area (in units) |
| :---: | :---: | :--- |
| Original <br> 2 | Original <br> 1 |  |
| 4 | 2 |  |
| 6 | 3 |  |
| 8 | 4 |  |
| 10 | 5 |  |
| 12 | 6 |  |
| 14 | 7 |  |

b. Pick any area/width pair of numbers and divide the area by the width, as shown below in Figure 3. Now pick several other pairs and divide the area by the width. Is the answer the same as it was when you divided perimeter by side length?


Figure 3
c. Complete the table below for similar squares.

| Side Length (in units) | Area (in unit ${ }^{2}$ ) |
| :---: | :---: |
| Original <br> 2 |  |
| 4 |  |
| 6 |  |
| 8 |  |
| 10 |  |
| 12 |  |
| 14 |  |

d. What must you multiply the area of the original figure (either the square or the rectangle) by in order to get the area of the second figure in the table?
e. What must you multiply the area of the original figure (either the square or the rectangle) by in order to get the area of the third figure in the table?
f. If you multiply the side length of a square or rectangle by a factor of 10 , by what factor would the area increase?
g. In exercise b, you made a discovery about the quotients of areas and side lengths that was different from your discovery about the quotients of perimeters and side lengths. How do you think this affected your answers in exercises d through f?

## Exercise 3

Suppose you start with a cube that is 1 cm on each side.

1 cm

a. What happens to the volume if you multiply each dimension by 2 , so that each side is now 2 cm ?

b. Fill in the table below.

| Side Length (in units) | Volume (in unit ${ }^{\mathbf{3}}$ ) |
| :---: | :---: |
| Original <br> 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |

c. What must you multiply the volume of the original cube by in order to get the volume of the second cube in the table?
d. What must you multiply the volume of the original cube by in order to get the volume of the third figure in the table?
e. If you multiply the side length of a cube by a factor of 10 , by what factor would the volume increase?

## Extension:

In 1958, there was a movie called "The Attack of the 50-Foot Woman". Suppose the woman in this movie was originally 5 feet tall and weighed 100 pounds, which represents her volume. How much does she weigh as a 50 -foot woman? How does she compare to the largest land and sea animals?

