GCDs and Lowest Terms

Introduction

You already know that two fractions can be equivalent even though they do not look the same. We can be sure of this because of the following:

1) When you multiply two fractions, you multiply the numerators and the denominators to get a new fraction.

2) When the numerator and denominator of a fraction are the same, the fraction is equal to 1.

When you multiply the fractions $\frac{12}{30}$ and $\frac{2}{2}$, the product is $\frac{24}{60}$. You can prove that $\frac{12}{30}$ and $\frac{24}{60}$ are equivalent using the above rules.

STEP 1 $\frac{12}{30} = \frac{12}{30} \cdot 1$ Any number times 1 is equal to itself.

STEP 2 $= \frac{12}{30} \cdot \frac{2}{2}$ $\frac{2}{2} = 1$

STEP 3 $= \frac{24}{60}$ Multiply.

So, $\frac{12}{30}$ is equivalent to $\frac{24}{60}$.

This process can often be reversed. Instead of multiplying the numerator and denominator by the same number, we look for a numb er that evenly divides both the numerator and denominator. Notice that 12 and 30 are both divisible by 3.

$$\frac{12}{30} = \frac{12 \div 3}{30 \div 3} \text{ or } \frac{4}{10}$$

You can prove that $\frac{12}{30}$ and $\frac{4}{10}$ are equivalent using the above rules.

STEP 1 $\frac{4}{10} = \frac{4}{10} \cdot 1$ Any number times 1 is equal to the number.

STEP 2 $= \frac{4}{10} \cdot \frac{3}{3}$ $\frac{3}{3} = 1$

STEP 3 $= \frac{12}{30}$

So, $\frac{12}{30}$ and $\frac{4}{10}$ are equivalent.

The fraction $\frac{4}{10}$ can be reduced even farther. The number 2 is a common divisor to both 4 and 10.

$$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} \text{ or } \frac{2}{5}$$

We say that $\frac{2}{5}$ is in lowest terms (cannot be reduced any further) because 2 and 5 do not have any common divisors.

The number 3 is a common divisor of the numbers 12 and 30 because 12 and 30 are divisible by 3 with no remainder. Can you think of any other common divisors of 12 and 30? One is a common divisor, so are 2 and 6. The largest common divisor is called the *greatest common divisor* or *GCD*. The GCD of 12 and 30 is 6.

The GCD is the key to reducing fractions to lowest terms in one step. For example, to reduce $\frac{12}{30}$ to lowest terms, use the GCD of 12 and 30, 6.

$$\frac{12}{30} = \frac{12 \div 6}{30 \div 6} \text{ or } \frac{2}{5}$$

Exercises

1) What is the GCD of 21 and 28?

2) Reduce $\frac{21}{28}$ to lowest terms using the GCD.

3) What is the GCD of 24 and 64?

4) Reduce $\frac{24}{64}$ to lowest terms using the GCD.

5) What is the GCD of 30 and 90?

6) Reduce $\frac{30}{90}$ to lowest terms using the GCD.

7) What is the GCD of 5 and 12?

8) Can you reduce $\frac{5}{12}$? Explain.

9) When is the GCD of two numbers equal to one of the numbers?

10) When is the GCD of two numbers equal to one?

You are now ready to take the fraction quiz on the calculator. Turn on the HP 39gs and press the APLET key. Look for the HP aplet **FRACTION QUIZ** (you may have to scroll). See your teacher if you don't have the aplet. Once you have **FRACTION QUIZ**, highlight it and press the START menu key. Read the starting note and press the VIEWS key. Choose the option **Find GCD**. Select 10 as the number of questions (unless your teacher tells you a different number) and try them all.

11) How many GCD questions did you get right (out of 10)?

Now choose the option **Reduce**. Select 10 as the number of questions (unless your teacher tells you a different number) and try them all. Keep in mind, each question now has two parts, a GCD and a fraction.

12) How many of each type did you get right (out of 10)?

GCDs:

Fractions:

Notice that when you exit, the HP 39gs summarizes the results of all of the questions since the aplet has been reset. See your teacher before you reset the aplet.