

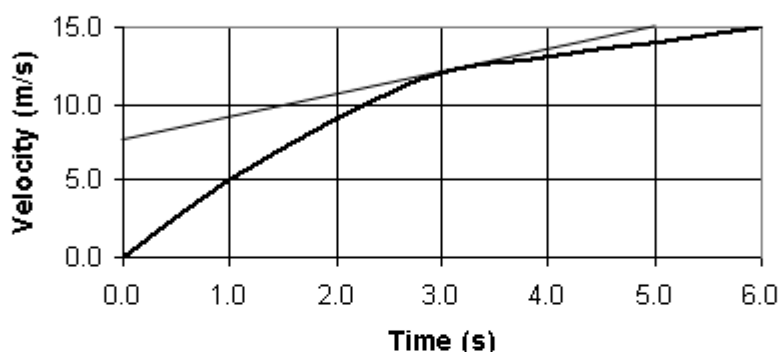
ACTIVITY

3a

Connecting Math to Physics

Non-Linear Equations and Graphs

Often, the relationship between two physical quantities is not linear. For examples, the relationship between kinetic energy and speed is quadratic, and the relationship between gravitational field strength and distance is an inverse square law. Sometimes, you cannot effectively use an equation to model a relationship between two measurements at all. This may be the case when you study the relationship between a car's velocity and time as the car accelerates.

Motion of a Car

In these cases, the slope is not constant. Therefore, the derivative, dy/dx , is not a constant. In the graph above, the derivative at $t = 1.0$ s is different from the derivative at $t = 3.0$ s. To estimate the derivative at a point on this graph, draw a line tangent to the curve and then find the slope of that line. The derivative at $t = 3.0$ s on the graph above is $dv/dt = (8.0 \text{ m/s}) / (5.0 \text{ s}) = 1.6 \text{ m/s}^2$.

In physics, we often re-write the derivative symbols, dy/dx , in terms of the variables actually graphed. Therefore, derivatives on the graph above might be represented as dv/dt . Graphs of displacement v. time might be represented as dd/dt . In each case, the format is $d(\text{y-axis variable}) / d(\text{x-axis variable})$.

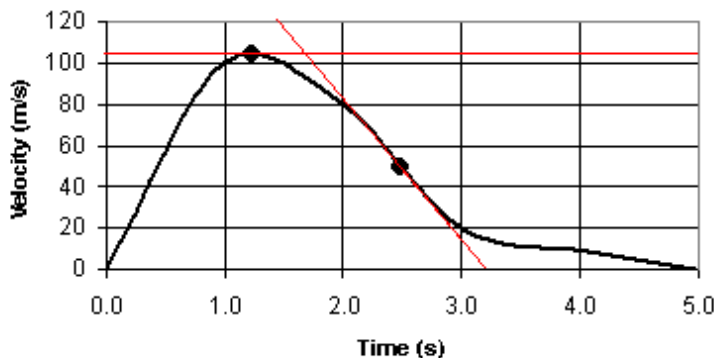
Notice that any derivative on any graph will have the units $(\text{y-axis units}) / (\text{x-axis units})$. The units for any derivative on the graph above are m/s^2 .

3a Connecting Math to Physics

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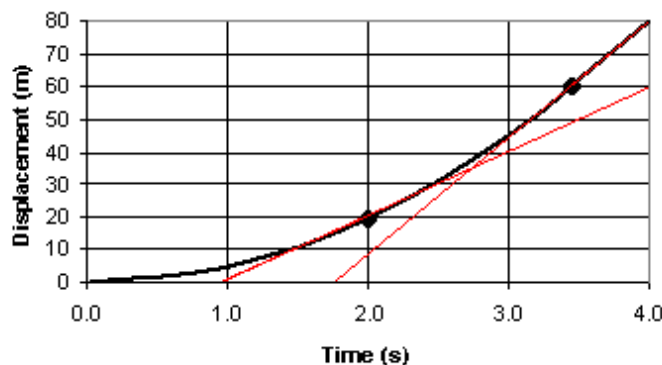
For each of the following exercises, estimate the derivatives at the marked points.

1.



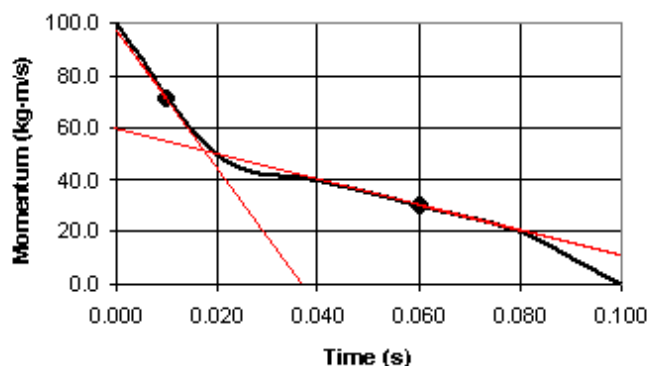
At $t = 1.2$ s, $\frac{dv}{dt} = 0$ m/s².
 At $t = 2.5$ s, $\frac{dv}{dt} = -70$ m/s².

2.



At $t = 2.0$ s, $\frac{dd}{dt} = 20$ m/s.
 At $t = 3.4$ s, $\frac{dd}{dt} = 36$ m/s.

3.



At $t = 0.010$ s,
 $\frac{dp}{dt} = -2900$ kg·m/s² = -2900 N.

At $t = 0.060$ s,
 $\frac{dp}{dt} = -500$ kg·m/s² = -500 N.