

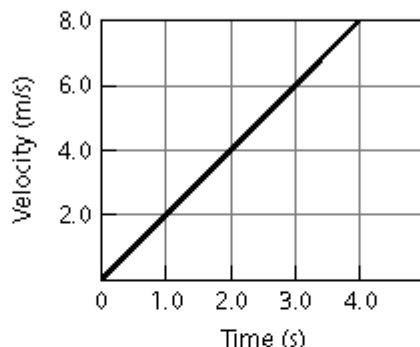
ACTIVITY

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Connecting Math to Physics

Linear Equations and Graphs

Algebraic equations show relationships between variables. Linear equations are a type of algebraic equation commonly used in physics. Examples of linear equations include $v = at$, $F = ma$, and $x = y + 3$. When a linear equation is plotted on a coordinate system, the graph is a line. A graph is a useful tool for visualizing an algebraic equation and for analyzing data. The figure below shows a v - t graph of the equation $v = at$, where $a = 2$.



An equation is linear when the exponent on each variable is 1. (Note: When the exponent on a variable is 1, the variable may be written as either x or x^1 .) The equation $v = 2t$ is a linear equation and is plotted as a line because both v and t have exponents of one.

Determine if each of the following equations is linear.

1. linear $d = vt$

3. linear $F = ma$

2. non-linear $v^2 = at$

4. non-linear $x^2 = y^3 - 2$

The derivative of an equation is the same as the slope of that equation's graph at a point. The slope of a line is constant. Therefore, the derivative of a linear equation is a constant. If the vertical direction of the coordinate system is represented by y and the horizontal direction by x , the derivative is defined as the ratio of an infinitesimal change in the vertical direction, dy , to an infinitesimal change in the horizontal direction, dx . This is often written as dy/dx . The derivative, or steepness, of a line never changes, and therefore, dy/dx is the same for all segments of a line. Note that finding the derivative of a non-linear (for example, a quadratic) equation is much harder, because the slope of a non-linear graph is not constant.

Find the derivative at all points for each of the following lines.

5. Two points on the line are (1, 1) and (5, 9).

$$\frac{dy}{dx} = 8/4 = 2$$

6. Two points on the line are (0, 0) and (-3, 6).

$$\frac{dy}{dx} = 6/-3 = -2$$

7. Refer to the figure on the previous page.

- a. Choose a point on the line and calculate the derivative at that point.

$$2.0 \text{ m/s}^2$$

- b. Choose another point on the line and calculate the derivative.

$$2.0 \text{ m/s}^2$$

- c. Compare the answer in problem 7b with the answer in 7a.

They are the same.

Slope-Intercept Form of a Line

Linear equations may be written into a form where the derivative at all points and the location of the y -intercept may be determined by simply reviewing the equation. This is called the slope-intercept form. For a linear equation with x and y as the variables, the slope-intercept form is $y = mx + b$, where m is the derivative (or slope) and b is the y -coordinate of the y -intercept. (Note: The x -coordinate will be zero for the y -intercept and any other point on the y -axis.) The linear equation $y - 3x = 2$ may be rewritten in slope-intercept form as $y = 3x + 2$. From the equation in slope-intercept form, it is evident that the derivatives at all points is 3 and the y -intercept is (0, 2).

Rearrange the linear equation into slope-intercept form and find the derivative and y -intercept.

8. $y - 1 = 4x$

$$y = 4x + 1; \frac{dy}{dx} = 4; b = 1$$

9. $x + y = 8$

$$y = -x + 8; \frac{dy}{dx} = -1; b = 8$$