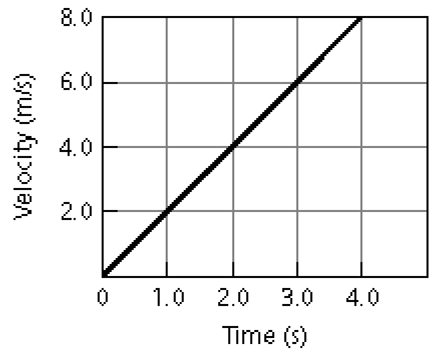
Date ______________ Period ______________ Name ______________________

ACTIVITY 3

Connecting Math to Physics

Linear Equations and Graphs

Algebraic equations show relationships between variables. Linear equations are a type of algebraic equation commonly used in physics. Examples of linear equations include $v = at$, $F = ma$, and $x = y + 3$. When a linear equation is plotted on a coordinate system, the graph is a line. A graph is a useful tool for visualizing an algebraic equation and for analyzing data. The figure below shows a *v-t* graph of the equation $v = at$, where $a = 2$.



An equation is linear when the exponent on each variable is 1. (Note: When the exponent on a variable is 1, the variable may be written as either x or x^1.) The equation $v = 2t$ is a linear equation and is plotted as a line because both v and t have exponents of one.

Determine if each of the following equations is linear.

1. ______________________ $d = \bar{v}t$

2. ______________________ $v^2 = at$

3. ______________________ $F = ma$

4. ______________________ $x^2 = y^3 - 2$

The derivative of an equation is the same as the slope of that equation's graph at a point. The slope of a line is constant. Therefore, the derivative of a linear equation is a constant. If the vertical direction of the coordinate system is represented by y and the horizontal direction by x, the derivative is defined as the ratio of an inifinitesmal change in the vertical direction, dy, to an infinitesmal change in the horizontal direction, dx. This is often written as dy/dx. The derivative, or steepness, of a line never changes, and therefore, dy/dx is the same for all segments of a line. Note that finding the derivative of a non-linear (for example, a quadratic) equation is much harder, because the slope of a non-linear graph is not constant.

Find the derivative at all points for each of the following lines.

5. Two points on the line are (1, 1) and (5, 9).

6. Two points on the line are (0, 0) and (−3, 6).

7. Refer to the figure on the previous page.

a. Choose a point on the line and calculate the derivative at that point.

b. Choose another point on the line and calculate the derivative.

c. Compare the answer in problem 7b with the answer in 7a.

Slope-Intercept Form of a Line

Linear equations may be written into a form where the derivative at all points and the location of the y-intercept may be determined by simply reviewing the equation. This is called the slope-intercept form. For a linear equation with x and y as the variables, the slope-intercept form is $y = mx + b$, where m is the derivative (or slope) and b is the y-coordinate of the y-intercept. (Note: The x-coordinate will be zero for the y-intercept and any other point on the y-axis.) The linear equation $y - 3x = 2$ may be rewritten in slope-intercept form as $y = 3x + 2$. From the equation in slope-intercept form, it is evident that the derivatives at all points is 3 and the y-intercept is (0, 2).

Rearrange the linear equation into slope-intercept form and find the derivative and y-intercept.

8. $y - 1 = 4x$

9. $x + y = 8$