

Lesson 4-7

Example 1

Calculate the combinations.

a. ${}_5C_3$

b. ${}_7C_2$

Solution

$$\begin{aligned} \text{a. } {}_5C_3 &= \frac{5!}{(5-3)! 3!} \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} \\ &= \frac{20}{2} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{b. } {}_7C_2 &= \frac{7!}{(7-2)! 2!} \\ &= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)} \\ &= \frac{42}{2} \\ &= 21 \end{aligned}$$

Example 2

DEBATE How many different ways can a 3-person debate team be chosen from 9 team members if there are no restrictions?

Solution

Assume that each person is equally likely to be chosen. So $n = 9$ and $r = 3$ in the combination formula.

$$\begin{aligned} {}_n C_r &= \frac{n!}{(n-r)! r!} \\ {}_9 C_3 &= \frac{9!}{(9-3)! 3!} \\ &= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} \\ &= \frac{504}{6} \\ &= 84 \end{aligned}$$

There are 84 ways to select the debate team.

Example 3

There are 8 students running for 4 student council positions at a school.

- Assuming each student is equally likely to be elected to student council, how many different ways can the council be formed?
- If the names of the students running for council are Donald, Tim, Amy, Nancy, Geraldo, Tyrone, Pamela, and Heather, what is the probability that Tim, Amy, Tyrone, and Heather will be elected?

Solution

- There are eight people to be picked four at a time, and the order is not important. So $n = 8$ and $r = 4$ in the combination formula.

$$\begin{aligned} {}_n C_r &= \frac{n!}{(n-r)! r!} \\ {}_8 C_4 &= \frac{8!}{(8-4)! 4!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} \\ &= \frac{1680}{24} \\ &= 70 \end{aligned}$$

There are 70 different groups of 4 students.

- The group of Tim, Amy, Tyrone, and Heather is one of the 70 possible combinations.

$$\begin{aligned} P(\text{Tim, Amy, Tyrone, Heather}) &= \frac{\text{favorable outcomes}}{\text{possible outcomes}} \\ &= \frac{1}{70} \end{aligned}$$

The probability that these four students will be elected to the council is $\frac{1}{70}$.