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Study Guide

Writing Expressions and Equations

If the cost of one CD is $15 and you want to buy three CDs, you know that your total cost will be $15 × 3, or $45. The expression 15×3 is called a **numerical expression**.

You could use a letter such as n to represent the number of CDs you might buy. Then the expression $15 \times n$, or $15n$, would represent the cost of buying n CDs. The expression $15n$ is called an **algebraic expression** because it contains a variable. A variable such as n is a letter used to represent a number. You can use variables to write verbal expressions as algebraic expressions.

Verbal Expressions	Algebraic Expression
2 plus c c more than 2	$2 + c$
k minus 5 k decreased by 5	$k - 5$
two times x the product of 2 and x	$2x$
q divided by the sum of 6 and b the quotient of q and the sum of 6 and b	$\frac{q}{6 + b}$

Expressions do not contain equal signs, but tell only which operations to perform. Equations always contain an equal sign.

Example: Write an equation for each sentence.

a. Eight multiplied by 2 equals 16.
$8 \times 2 = 16$

b. Three less than 7 times a number n is 24.
$7n - 3 = 24$

Write an algebraic expression for each verbal expression.

1. the difference of r and 10

2. increase the product of 3 and a by 1

Write a verbal expression for each algebraic expression.

3. $\frac{p}{7}$

4. $2x + 5$

5. $\frac{1}{2}(4 + w)$

Write an equation for each sentence.

6. Seven minus a number z is the same as 15.

7. Ten more than twelve times a number h equals 25.

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Study Guide

Order of Operations

Read this sentence: *Jason said Leona is smart.* You need punctuation to tell you whether the sentence means *Jason said, "Leona is smart."* or *"Jason," said Leona, "is smart."*

The meaning of a mathematical expression such as $20 - 2 \times 3$ can also be confusing unless you know which numbers and operations should be grouped together. The order of operations at the right tells you that $20 - 2 \times 3$ means $20 - (2 \times 3)$ or 14.

Order of Operations

1. Find the values of the expressions inside grouping symbols, such as parentheses () and brackets [], and as indicated by fraction bars.
2. Do all multiplications and divisions from left to right.
3. Do all additions and subtractions from left to right.

Example 1: Find the value of $16 \div 8 \times 5$.

$$16 \div 8 \times 5 = 2 \times 5 = 10$$

Multiply and divide from left to right.

Example 2: Find the value of $7(10 - 3)$.

$$7(10 - 3) = 7 \times 7 = 49$$

Simplify within parentheses first.

Example 3: Find the value of $\frac{10 - (2 \times 3)}{5 \div 5}$.

$$\frac{10 - (2 \times 3)}{5 \div 5} = \frac{10 - 6}{5 \div 5}$$

Simplify within parentheses first.

$$= \frac{4}{1}$$

Evaluate the numerator and the denominator separately.

$$= 4$$

Find the value of each expression.

1. $3 + 4 - 2$

2. $6 + 3 \times 7$

3. $1 + 15 \div 5 \times 7$

4. $(7 + 6) \times 5$

5. $2 + 8 \times 3 - 1$

6. $\frac{7 + 1}{2}$

7. $(2 + 8) \times 3 - 1$

8. $\frac{12 + 6}{12 - 6}$

9. $\frac{5 \times 4}{4 + 6}$

10. $5 \times (11 - 7)$

11. $\frac{7 - 5}{10 + 5}$

12. $\frac{1 + 5}{2 \times 9}$

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Study Guide

Commutative and Associative Properties

Carlos makes salad dressings with olive oil and balsamic vinegar. Sometimes he adds the olive oil first and other times he adds the vinegar first. The salad dressing is always the same, so the order doesn't matter.

The order in which any two numbers are either added or multiplied doesn't change the sum or product. Addition and multiplication are said to be *commutative*.

Commutative Property of Addition	For any two numbers a and b, $a + b = b + a$.
Commutative Property of Multiplication	For any two numbers a and b, $a \cdot b = b \cdot a$.

Example 1: $3x + 4y + 5x$
$= 3x + 5x + 4y$ *Commutative (+)*
$= 8x + 4y$

Example 2: $5 \times 11 \times 2$
$= 5 \times 2 \times 11$ *Commutative (×)*
$= 10 \times 11$
$= 110$

You can also regroup numbers when you are adding or multiplying without changing the sum or product. Addition and multiplication are said to be *associative*.

Associative Property of Addition	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication	For any numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Example 3: $(10 + 5) + 8$
$= 10 + (5 + 8)$ *Associative (+)*
$= 23$

Example 4: $(100 \cdot 4) \cdot 5$
$= 100 \cdot (4 \cdot 5)$ *Associative (×)*
$= 2000$

Simplify each expression.

1. $(d + 7) + 3$

2. $2x + 7 + 5x$

3. $2 \times 7k \times 5$

4. $(4a + 2b) + (a + b)$

5. $7 \cdot y \cdot 3$

6. $(8 \times m) \times 4$

Name the property shown by each statement.

7. $29 + b = b + 29$

8. $2(4 \cdot 6) = (2 \cdot 4)6$

9. $(3 + 21) + 7 = (3 + 7) + 21$

10. $42 \cdot 8 = 8 \cdot 42$

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Study Guide

Distributive Property

Judi buys a cup of juice and a bagel for Hans and herself at the cafeteria. Juice costs \$1 and a bagel costs \$0.50.

To find the total, Judi finds the total for herself and doubles it. 2(\$1 + \$0.50) = 2 × \$1.50 = \$3	To find the total, Hans finds the cost of 2 bagels and 2 juices and adds them. 2(\$1) + 2(\$0.50) = \$2 + \$1 = \$3

Judi and Hans find the same total.
2(\$1 + \$0.50) = 2(\$1) + 2(\$0.50).
This is an example of the Distributive Property.

Distributive Property

For all numbers a, b, and c,
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$

You can use the Distributive Property to simplify expressions.

Example 1: Simplify $3(x + y) + 4x$.

$$\begin{aligned} 3(x + y) + 4x &= 3x + 3y + 4x && \textit{Distributive Property} \\ &= 3x + 4x + 3y && \textit{Commutative Property} \\ &= 7x + 3y && \textit{Substitution Property} \end{aligned}$$

Example 2: Simplify $7(m + p) + 2(m - p)$.

$$\begin{aligned} 7(m + p) + 2(m - p) &= 7m + 7p + 2m - 2p && \textit{Distributive Property} \\ &= 7m + 2m + 7p - 2p && \textit{Commutative Property} \\ &= 9m + 5p && \textit{Substitution Property} \end{aligned}$$

Simplify each expression.

1. $3(u + v)$

2. $5(k - 2)$

3. $4(2 + 5s) + 3$

4. $7(1 - 2h)$

5. $1(a + 2j + 12k)$

6. $17(c - 2d)$

7. $15(ab + 3c)$

8. $4(2w + 3) + 2$

9. $2(a + 2b) + 3(2a - b)$

10. $7(x + 2y)$

11. $3(e - 4f - ef)$

12. $2(3m + 1) + 4m$

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Study Guide

A Plan for Problem Solving

A problem-solving plan can help you identify and organize the information in a problem, then plan and execute a solution. A problem-solving plan should include these steps.

1. **Explore** Read the problem carefully. Identify the information that is given and determine what you need to find.
2. **Plan** Select a strategy for solving the problem. If possible, estimate what you think the answer should be before solving the problem.
3. **Solve** Use your strategy to solve the problem. You may have to choose a variable for the unknown, and then write an expression.
4. **Examine** Check your answer. Does it make sense? Is it reasonably close to your estimate?

Example: A tree in your yard grows 7 inches a year and is now 92 inches tall. In about how many years will the tree be 122 inches tall?

Explore: The tree is already 92 inches tall and it grows 7 inches a year. You need to find how many years it will take the tree to grow to 122 inches.

Plan: Since the tree needs to grow about 30 more inches and $30 \div 7 \approx 4$, you can estimate that it will take more than 4 years for the tree to reach 122 inches.

Solve:

Number of Years From Present	Height of Tree in Inches
1	$92 + 7 = 99$
2	$99 + 7 = 106$
3	$106 + 7 = 113$
4	$113 + 7 = 120$
5	$120 + 7 = 127$

Examine: Your table shows you that the tree will be 122 inches in a little over 4 years. Since the answer matches your estimate, the answer is reasonable.

Solve the problem.

Janine is selling subscriptions to an Internet service. She began by selling one subscription the first day. On the second day she sold two more subscriptions, and on the third day she sold 3 more. If she continues to sell subscriptions according to this pattern, how many will she have sold at the end of one week?

1–6

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Study Guide

Collecting Data

A large ski area surveyed 50 skiers to find out how long they waited in a lift line during a busy period. Their responses are in the chart below.

Time in Lift Line (minutes)									
20	23	20	16	25	26	18	18	19	21
25	24	20	22	19	15	23	22	22	19
18	24	23	16	18	17	17	24	23	23
15	12	22	21	25	24	15	23	24	17
23	22	24	16	16	20	19	23	21	26

A frequency table is one way to organize data so you can draw conclusions more easily from the data. In a frequency table, you use tally marks to record how frequently events occur.

Example: Make a frequency table to organize the survey data. Waiting times vary from 12 minutes to 26 minutes. If you group the waiting times in sets of three, your table will not be too long. The groups are called *intervals*. This table has intervals of three minutes. Each result from the survey is recorded in the tally chart. The total number of tallies is recorded in the Frequency column.

Lift Line Wait

Time (min)	Tally	Frequency
12–14	|	1
15–17	~~||||~~ ~~||||~~	10
18–20	~~||||~~ ~~||||~~ ||	12
21–23	~~||||~~ ~~||||~~ ~~||||~~ |	16
24–26	~~||||~~ ~~||||~~ |	11

Make a frequency table to organize the data in the table. Use intervals of $4.

Profits ($)					
92	95	94	91	100	101
90	90	92	105	101	102
99	94	100	95	93	92
102	103	91	97	105	92

1–7

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Study Guide

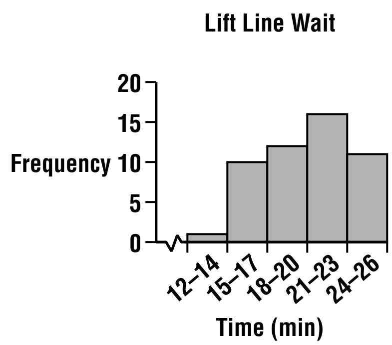
Displaying and Interpreting Data

Data can be easier to analyze if they are presented in the form of a graph. There are many ways to graph data, such as line graphs, histograms, and stem-and-leaf plots. A histogram is a graph of the data in a frequency table.

Example: The frequency table shows the amount of time skiers waited in a lift line. Construct a histogram for the data.

Lift Line Wait		
Time (min)	**Tally**	**Frequency**
12–14	|	1
15–17	𝍸 𝍸	10
18–20	𝍸 𝍸 ||	12
21–23	𝍸 𝍸 𝍸 |	16
24–26	𝍸 𝍸 |	11

- The horizontal axis displays the time intervals from the table.
- The vertical axis displays equal intervals of 1.
- For each time interval, draw a bar. The height of the bar is equal to its frequency.
- Label the two axes and title the histogram.



Make a histogram of the data in the frequency table.

Students Visiting Museum		
Ages	**Tally**	**Frequency**
8–9	𝍸 |	6
10–11	𝍸 𝍸	10
12–13	𝍸 𝍸 𝍸	15
14–15	𝍸 𝍸 ||	12
16–17	||||	4

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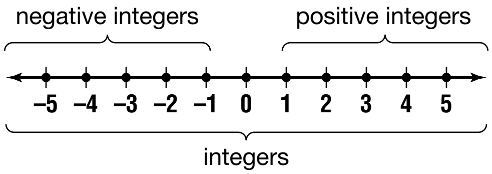
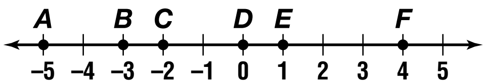
2–1 Study Guide

Graphing Integers on a Number Line

The numbers displayed on the number line below belong to the set of **integers**. The arrows at both ends of the number line indicate that the numbers continue indefinitely in both directions. Notice that the integers are equally spaced.

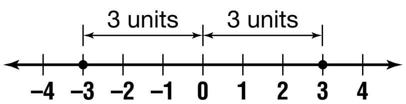


Use dots to graph numbers on a number line. You can label the dots with capital letters.



The coordinate of B is -3 and the coordinate of D is 0.

Because 3 is to the right of -3 on the number line, $3 > -3$. And because -5 is to the left of 1, $-5 < 1$. Because 3 and -3 are the same distance from 0, they have the same **absolute value**, 3. Use two vertical lines to represent absolute value.



$|3| = 3$ *The absolute value of 3 is 3.*

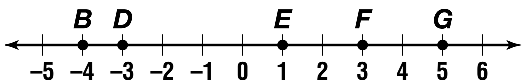
$|-3| = 3$ *The absolute value of* -3 *is 3.*

Example: Evaluate $|-12| + |10|$.

$$|-12| + |10| = 12 + 10 \qquad |-12| = 12 \text{ and } |10| = 10$$
$$= 22$$

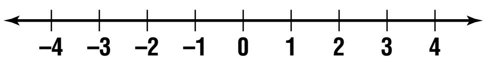
Name the coordinate of each point.

1. B **2.** D **3.** G

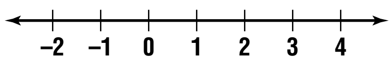


Graph each set of numbers on a number line.

4. $\{-3, 2, 4\}$



5. $\{-1, 0, 3\}$



Write < or > in each blank to make a true sentence.

6. -7 ______ 5 **7.** -3 ______ -8 **8.** $|-1|$ ______ 0

Evaluate each expression.

9. $|9|$ **10.** $|-15|$ **11.** $|-20| - |10|$

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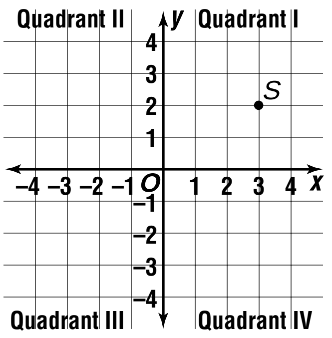
Study Guide

The Coordinate Plane

The two intersecting lines and the grid at the right form a **coordinate system**. The horizontal number line is called the ***x*-axis,** and the vertical number line is called the ***y*-axis**. The *x*- and *y*-axes divide the coordinate plane into **four quadrants**. Point *S* in Quadrant I is the graph of the **ordered pair** (3, 2). The ***x*-coordinate** of point *S* is 3, and the ***y*-coordinate** of point *S* is 2.



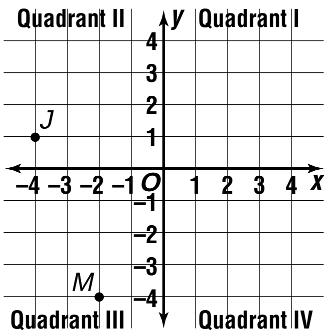
The point at which the axes meet has coordinates (0, 0) and is called the **origin**.

Example 1: What is the ordered pair for point *J*? In what quadrant is point *J* located?

You move 4 units to the left of the origin and then 1 unit up to get to *J*. So the ordered pair for *J* is (−4, 1). Point *J* is located in Quadrant II.

Example 2: Graph *M*(−2, − 4) on the coordinate plane.

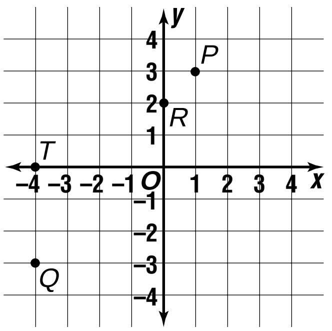
Start at the origin. Move left on the *x*-axis to −2 and then down 4 units. Draw a dot here and label it *M*.



Write the ordered pair that names each point.

1. *P* **2.** *Q*

3. *R* **4.** *T*

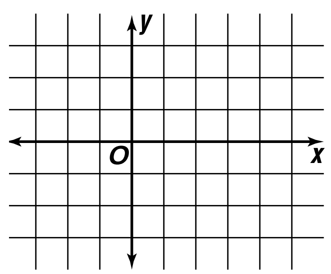


Graph each point on the coordinate plane. Name the quadrant, if any, in which each point is located.

5. *A*(5, −1) **6.** *B*(−3, 0)

7. *C*(−3, 1) **8.** *D*(0, 1)

9. *E*(3, 3) **10.** *F*(−1, −2)



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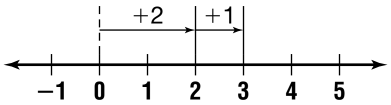
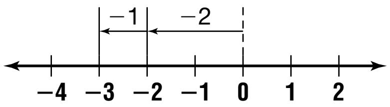
2–3 Study Guide

Adding Integers

You can use a number line to add integers. Start at 0. Then move to the right for positive integers and move to the left for negative integers.



$2 + 1 = 3$

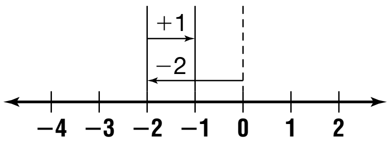


$-2 + (-1) = -3$

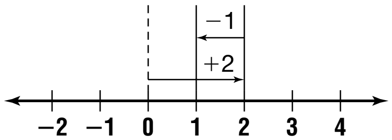
Both integers are positive.
First move 2 units right from 0.
Then move 1 more unit right.

Both integers are negative.
First move 2 units left from 0.
Then move 1 more unit left.

When you add one positive integer and one negative integer on the number line, you change directions, which results in one move being subtracted from the other move.



$-2 + 1 = -1$



$2 + (-1) = 1$

Move 2 units left, then 1 unit right.

Move 2 units right, then 1 unit left.

Use the following rules to add two integers and to simplify expressions.

Rule	Examples
To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.	$7 + 4 = 11$ $-8 + (-2) = -10$ $-5x + (-3x) = -8x$
To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.	$9 + (-6) = 3$ $1 + (-5) = -4$ $-2x + 9x = 7x$ $3y + (-4y) = -y$

Find each sum.

1. $5 + 8$ **2.** $-8 + (-9)$ **3.** $12 + (-8)$ **4.** $-16 + 5$

5. $5 + (-8) + (-5)$ **6.** $-8 + (-8) + 20$ **7.** $12 + 5 + (-1)$

Simplify each expression.

8. $3x + (-6x)$ **9.** $-5y + (-7y)$ **10.** $2m + (-4m) + (-2m)$

2–4

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Study Guide

Subtracting Integers

If the sum of two integers is 0, the numbers are **opposites** or **additive inverses**.

Example 1: **a.** -3 is the opposite of 3 because $-3 + 3 = 0$

b. 17 is the opposite of -17 because $17 + (-17) = 0$

Use this rule to subtract integers.

To subtract an integer, add its opposite or additive inverse.

Example 2: Find each difference.

a. $5 - 2$

$5 - 2 = 5 + (-2)$ *Subtracting 2 is the same as adding its opposite, -2.*

$= 3$

b. $-7 - (-1)$

$-7 - (-1) = -7 + 1$ *Subtracting -1 is the same as adding its opposite, 1.*

$= -6$

Example 3: Evaluate $c + d - e$ if $c = -1$, $d = 7$, and $e = -3$.

$c + d - e = -1 + 7 - (-3)$ *Replace c with -1, d with 7, and e with -3.*

$= -1 + 7 + 3$ *Write $7 - (-3)$ as $7 + 3$.*

$= 6 + 3$ *$-1 + 7 = 6$*

$= 9$ *$6 + 3 = 9$*

Find each difference.

1. $5 - 8$ **2.** $-8 - (-9)$ **3.** $-2 - 8$ **4.** $-4 - (-5)$

5. $16 - 8$ **6.** $10 - (-10)$ **7.** $0 - 10$ **8.** $0 - (-18)$

Simplify each expression.

9. $3x - 9x$ **10.** $-4y - (-6y)$ **11.** $2m - 8m - (-2m)$

Evaluate each expression if $x = -1$, $y = 2$, and $z = -4$.

12. $x - y$ **13.** $y - z - 5$ **14.** $z - y - (-2)$

15. $9 - x$ **16.** $x - z - z$ **17.** $0 - y$

2–5

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Study Guide

Multiplying Integers

Use these rules to multiply integers and to simplify expressions.

The product of two positive integers is positive. The product of two negative integers is positive. The product of a positive integer and a negative integer is negative.

Example 1: Find each product.

a. $7(12)$

$7(12) = 84$ — *Both factors are positive, so the product is positive.*

b. $-5(-9)$

$-5(-9) = 45$ — *Both factors are negative, so the product is positive.*

c. $-4(8)$

$-4(8) = -32$ — *The factors have different signs, so the product is negative.*

Example 2: Evaluate $-3ab$ if $a = 3$ and $b = -5$.

$-3ab = -3(3)(-5)$	*Replace a with 3 and b with −5.*
$= -9(-5)$	*$-3 \cdot 3 = -9$*
$= 45$	*Both factors are negative.*

Example 3: Simplify $-12(4x)$.

$-12(4x) = (-12 \cdot 4)(x)$	*Associative Property*
$= -48x$	*$-12 \cdot 4 = -48$*

Find each product.

1. $3(8)$ **2.** $(-7)(-9)$ **3.** $12(-1)$ **4.** $-6(5)$

5. $4(-1)(-5)$ **6.** $(-8)(-8)(-2)$ **7.** $2(-5)(10)$

Evaluate each expression if a = 3, b = −2, and c = −3.

8. $5c$ **9.** $2ab$ **10.** abc **11.** $3b - c$

Simplify each expression.

12. $3(-6x)$ **13.** $-5(-7y)$ **14.** $(2p)(-4q)$

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Study Guide

Dividing Integers

Example 1: Use the multiplication problems at the right to find each quotient.

$3 \cdot 5 = 15$
$-3(-5) = 15$
$-3 \cdot 5 = -15$
$3(-5) = -15$

a. $15 \div 5$
Since $3 \cdot 5 = 15$, $15 \div 5 = 3$.

b. $15 \div (-5)$
Since $-3 \cdot (-5) = 15$, $15 \div (-5) = -3$.

c. $-15 \div 5$
Since $-3 \cdot 5 = -15$, $-15 \div 5 = -3$.

d. $-15 \div (-5)$
Since $3 \cdot (-5) = -15$, $-15 \div (-5) = 3$.

Use these rules to divide integers.

The quotient of two positive integers is positive. The quotient of two negative integers is positive. The quotient of a positive integer and a negative integer is negative.

Example 2: Evaluate $\frac{-3r}{s}$ if $r = 8$ and $s = -2$.

$\frac{-3r}{s} = \frac{-3 \cdot 8}{-2}$ *Replace r with 8 and s with −2.*

$= \frac{-24}{-2}$ $\quad -3 \cdot 8 = -24$

$= 12$ $\quad -24 \div (-2) = 12$

Find each quotient.

1. $36 \div 9$ **2.** $-63 \div (-7)$ **3.** $25 \div (-1)$ **4.** $-60 \div 5$

5. $\frac{20}{-5}$ **6.** $\frac{-18}{-3}$ **7.** $\frac{-1}{-1}$ **8.** $\frac{-56}{8}$

Evaluate each expression if k = −1, m = 3, and n = −2.

9. $-21 \div m$ **10.** $\frac{2n}{k}$ **11.** $m \div k$ **12.** $\frac{m+5}{n}$

3–1

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Study Guide

Rational Numbers

The chart below shows the fraction and decimal forms of some rational numbers.

Rational Number	5	$\frac{1}{6}$	$-2\frac{1}{4}$	0.75	$-0.83\overline{3}$
Fraction Form	$\frac{5}{1}$	$\frac{1}{6}$	$-\frac{9}{4}$	$\frac{3}{4}$	$-\frac{5}{6}$
Decimal Form	5.0	$0.16\overline{6}$	-2.25	0.75	$-0.83\overline{3}$

You can compare numbers using a number line or using cross products.

Example: Write <, >, or = in each blank to make a true sentence.

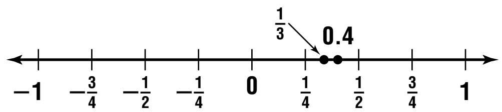
a. ______ $\frac{3}{7}$

$2(7)$ ______ $3(5)$ *Cross multiply.*

$14 < 15$

So $\frac{2}{5} < \frac{3}{7}$.

b. 0.4 ______ $\frac{1}{3}$



Since 0.4 is to the right of $\frac{1}{3}$, $0.4 > \frac{1}{3}$.

Write <, >, or = in each blank to make a true statement.

1. $\frac{2}{3}$ ______ $\frac{1}{2}$

2. $\frac{1}{3}$ ______ $\frac{3}{6}$

3. $\frac{3}{4}$ ______ $\frac{5}{8}$

4. 0.8 ______ $\frac{2}{3}$

5. 1.5 ______ $1\frac{1}{2}$

6. $\frac{2}{3}$ ______ $\frac{2}{5}$

7. $\frac{3}{5}$ ______ $\frac{5}{7}$

8. 0.63 ______ $\frac{2}{3}$

9. $0.\overline{3}$ ______ $\frac{1}{3}$

10. -4 ______ $-\frac{8}{2}$

11. 1.25 ______ $\frac{5}{4}$

12. 0.445 ______ $0.44\overline{4}$

Write the numbers in each set from least to greatest.

13. $\frac{1}{3}, \frac{1}{8}, \frac{1}{2}$

14. $\frac{3}{8}, 0.4, -\frac{2}{5}$

15. $\frac{3}{4}, \frac{1}{5}, 0.8$

16. $-\frac{1}{2}, -\frac{2}{3}, -2$

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3–2 Study Guide

Adding and Subtracting Rational Numbers

During track practice, Sheila recorded the time it took her to run two consecutive miles. She ran the first mile in 7.26 seconds. She ran the second mile in 7.01 seconds. The net change in times from the first mile to the second mile is $7.01 - 7.26$ or -0.25 seconds. Sheila ran the second mile 0.25 seconds faster.

To add and subtract rational numbers, you use the same rules that you learned for adding and subtracting integers.

Adding Rational Numbers	To add rational numbers with the *same* sign, add their absolute values. Give the result the same sign as the rational numbers. To add rational numbers with *different* signs, subtract their absolute values. Give the result the same sign as the number with the greater absolute value.
Subtracting Rational Numbers	To subtract a rational number, add its opposite.

Examples: Find each sum or difference.

a. $-7.4 + (-10.3)$
$= -(|-7.4| + |-10.3|)$
$= -(7.4 + 10.3)$
$= -17.7$

b. $-5.2 - 9.1$
$= -5.2 + (-9.1)$
$= -14.3$

c. $-8.2 + 5.2 + (-9.1)$
$= [-8.2 + 5.2] + (-9.1)$
$= -3 + (-9.1)$
$= -|3 + 9.1|$
$= -12.1$

Find each sum or difference.

1. $3.1 + 1.2$
2. $-1.4 + 5.6$
3. $4.2 - 1.7$

4. $8.4 + 36.8$
5. $-6.3 + (-0.12)$
6. $13.5 + (-10.2)$

7. $-4.3 - 16.8$
8. $75.25 - 125.55$
9. $18.12 - (-5.66)$

10. $-11.89 + 25.1$
11. $14.6 + 23.4 + (-3.6)$
12. $\frac{1}{2} + \frac{1}{3}$

13. $\frac{2}{3} - \frac{1}{4}$
14. $\frac{3}{8} + \frac{2}{7}$
15. $-\frac{3}{10} + \frac{3}{4}$

Evaluate each expression if $a = \frac{1}{5}$, $b = -2\frac{1}{2}$, $c = -9.5$, and $d = 15.6$.

16. $a + b$
17. $c - d$
18. $d - c$

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Study Guide

Mean, Median, Mode, and Range

For his Social Studies class, Carlos surveyed five gas stations and recorded these prices for 1 gallon of gasoline.

$1.32 $1.28 $1.43 $1.32 $1.30

The mean, the median, the mode, and the range of these prices can all be used to describe the prices.

mean	The mean, or *average*, of a set of data is the sum of the data divided by the number of pieces of data.
median	The median of a set of data is the middle number when the data in the set are arranged in numerical order. If there are two middle numbers, the median is the mean of those two numbers.
mode	The mode of a set of data is the number that occurs most often in the set. A set can have no mode or more than one mode.
range	The range of a set of data is the difference between the greatest and the least values of the set.

Example: Find the mean, the median, the mode, and the range of the gasoline prices that Carlos recorded.

Find the mean of the data.

$$\frac{1.32 + 1.28 + 1.43 + 1.32 + 1.30}{5} = 1.33$$

The mean is $1.33.

Find the median of the data.
First arrange the data in order.
Then identify the middle number.
1.28, 1.30, **1.32**, 1.32, 1.43
The median is $1.32.

Find the mode of the data.
Since $1.32 occurs most often, $1.32 is the mode.

Find the range of the data.
Subtract the greatest and least values.
$\$1.43 - \$1.28 = \$0.15$
The range of the data is $0.15.

Find the mean, median, mode, and range of each set of data.

1. 48, 25, 29, 42, 36, 36

2. 5.1, 2.7, 2.7, 2.7

3. 101, 113, 98

4. 18.2, 20.4, 18.2, 11.6, 20.4

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Study Guide

Equations

Jason has won 3 gold medals in swim meets this year. Next Saturday he will swim in 3 events. How many events will he win if he has 5 gold medals at the end of the meet?

Let m = the number of gold medals Jason wins next Saturday. Then the equation $3 + m = 5$ models the number of gold medals Jason will have at the end of the meet. The replacement set for m is $\{1, 2, 3\}$. Since 2 is the only number from the replacement set that makes the equation true, 2 is the solution of the equation.

Example 1: Find the solution of $x + 10 = -21$ if the replacement set is $\{-30, -31, -32\}$.

Find the value in the replacement set that makes the equation true.

$x + 10 = -21$	$x + 10 = -21$	$x + 10 = -21$
$-30 + 10 = -21$	$-31 + 10 = -21$	$-32 + 10 = -21$
false	*true*	*false*

Since -31 makes the equation $x + 10 = -21$ true, the solution is -31.

Sometimes you can solve equations by applying the order of operations.

Example 2: Solve each equation.

a. $b = 27 - 2(3)$ *Multiply.*
$b = 27 - 6$ *Subtract.*
$b = 21$
The solution is 21.

b. $\frac{5 + 1}{8(2) - 14} - 8 = h$ *Evaluate the fraction.*
$\frac{6}{2} - 8 = h$ *Divide.*
$3 - 8 = h$ *Subtract.*
$-5 = h$
The solution is -5.

Find the solution of each equation if the replacement sets are $x = \{1, 2, 3\}$, $y = \{-5, -4, -3\}$, and $z = \{-2, 0, 2\}$.

1. $x + 1 = 4$
2. $5 + z = 3$
3. $-6 = -3z$
4. $7y - 2 = -37$
5. $x + 12x = 26$
6. $1 = 7z - (-1)$
7. $\frac{y}{5 - 7} = 2$
8. $\frac{10}{x} + x = 7$
9. $8z + 5 = z - 9$

Solve each equation.

10. $\frac{49}{7} = g$
11. $-6 + 9 = p$
12. $n = 5 - 24 \div 3$
13. $\frac{2 \cdot 4 - 8}{1 - 2} = d$
14. $\frac{8 - 9}{4(5)} = w$
15. $s = \frac{4 + 15 \div 3}{-4(1) + 1}$

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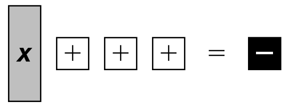
Study Guide

Solving Equations by Using Models

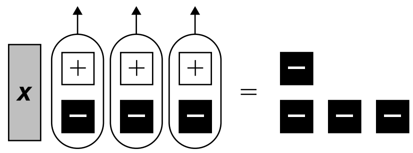
You can use algebra tiles to model and solve equations.

Example: Use algebra tiles to solve $x + 3 = -1$. Model $x + 3 = -1$ by placing 1 x-tile and 3 one-tiles on one side of the mat to represent $x + 3$. Place 1 negative one-tile on the other side of the mat to represent -1.



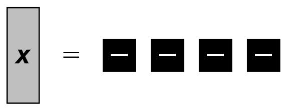
$x + 3 = -1$

To get the x-tile by itself, add 3 negative one-tiles to each side. Then remove the zero pairs.



$x + 3 + (-3) = -1 + (-3)$

The x-tile is matched with 4 negative one-tiles. Therefore, $x = -4$.



$x = -4$

Solve each equation. Use algebra tiles if necessary.

1. $y + 4 = 5$

2. $b - 3 = -2$

3. $6 = 4 + a$

4. $r + (-3) = -5$

5. $1 = h + 6$

6. $8 + m = -6$

7. $n - (-4) = -3$

8. $5 = p - 8$

9. $c + 4 = -2$

10. $-3 = x - 3$

11. $k - 2 = -4$

12. $7 = x - (-3)$

13. Yolanda scored 3 points higher on her math test than she scored on her previous test. If her grade was 83 on this test, what was her score on the previous test?

a. Write an equation that can be used to find Yolanda's score on the previous test.

b. What was Yolanda's score on the previous test?

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Study Guide

Solving Addition and Subtraction Equations

You can use the addition and subtraction properties of equality to solve equations.

Addition Property of Equality	If you add the same number to each side of an equation, the two sides remain equal. *Example:* If $x = 5$, then $x + 2 = 5 + 2$.
Subtraction Property of Equality	If you subtract the same number from each side of an equation, the two sides remain equal. *Example:* If $x = -1$, then $x - 7 = -1 - 7$.

Example: Solve each equation. Check your solution.

a. $b - 3 = -5$

$$\begin{aligned} b - 3 &= -5 \\ b - 3 + 3 &= -5 + 3 && \textit{Add 3 to each side.} \\ b &= -2 && -3 + 3 = 0 \end{aligned}$$

Check: $b - 3 = -5$

$$\begin{aligned} -2 - 3 &\stackrel{?}{=} -5 && \textit{Replace } b \textit{ with } -2. \\ -5 &= -5 && \checkmark \end{aligned}$$

The solution is -2.

b. $x + 4 = 1$

$$\begin{aligned} x + 4 &= 1 \\ x + 4 - 4 &= 1 - 4 && \textit{Subtract 4 from each side.} \\ x &= -3 && 4 - 4 = 0 \end{aligned}$$

Check: $x + 4 = 1$

$$\begin{aligned} -3 + 4 &\stackrel{?}{=} 1 && \textit{Replace } x \textit{ with } -3. \\ 1 &= 1 && \checkmark \end{aligned}$$

The solution is -3.

Solve each equation. Check your solution.

1. $x + 15 = 18$ **2.** $n - 6 = -9$ **3.** $p - (-5) = 1$

4. $27 = k + -10$ **5.** $d + (-16) = 12$ **6.** $2 + s = -15$

7. $-7 + w = -2$ **8.** $38 = 11 + v$ **9.** $-44 = c - 10$

10. $2.7 = x + 5.8$ **11.** $y - (-6.1) = 20.5$ **12.** $-9.9 + a = -25$

13. $m + \frac{2}{3} = -\frac{5}{6}$ **14.** $-\frac{1}{2} = y - \frac{3}{4}$ **15.** $-a + \frac{1}{4} = \frac{7}{8}$

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3–7 Study Guide

Solving Equations Involving Absolute Value

Some equations involve absolute value.

Example 1: Solve $|x - 5| = 2$. Check your solution.

$|x - 5| = 2$ means $x - 5 = 2$ or $x - 5 = -2$. Solve both equations.

$$x - 5 = 2 \qquad\qquad x - 5 = -2$$
$$x - 5 + 5 = 2 + 5 \quad \textit{Add 5 to each side.} \quad x - 5 + 5 = -2 + 5$$
$$x = 7 \qquad\qquad x = 3$$

Check: $|7 - 5| \stackrel{?}{=} 2$; $|2| \stackrel{?}{=} 2$; $2 = 2$ ✓

Check: $|3 - 5| \stackrel{?}{=} 2$; $-2 \stackrel{?}{=} 2$; $2 = 2$ ✓

The solution set is {3, 7}.

Example 2: Solve $3 + |x| = 8$. Check your solution.

$3 - 3 + |x| = 8 - 3$ *Simplify by subtracting 3 from each side.*

$|x| = 5$

$x = -5$ or 5

Check: $3 + |-5| \stackrel{?}{=} 8$; $3 + 5 \stackrel{?}{=} 8$; $8 = 8$ ✓

Check: $3 + |5| \stackrel{?}{=} 8$; $3 + 5 \stackrel{?}{=} 8$; $8 = 8$ ✓

The solution set is {−5, 5}.

Example 3: Solve $|x| - 12 = -16$. Check your solution.

$|x| - 12 + 12 = -16 + 12$ *Simplify by adding 12 to each side.*

$|x| = -4$

This sentence can never be true. The solution is the empty set or $\varnothing$.

Solve each equation. Check your solution.

1. $|x| = 7$ **2.** $|m| = -6$ **3.** $|x| + 1 = 5$

4. $2 + |z| = 11$ **5.** $|d - 2| = 10$ **6.** $|r + 2| = 0$

7. $|p - 1| = 5$ **8.** $|c + 12| = -12$ **9.** $|r - (-3)| = 6$

10. Ashley said it was too cold to go snowboarding because the temperature was only 3°F away from 0°F. What are the two possible temperatures?

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4–1 Study Guide

Multiplying Rational Numbers

Use the following rules to multiply two rational numbers and to simplify expressions.

Rules		Examples
Rational Numbers with Different Signs	The product of two rational numbers having different signs is negative.	$5(-2.5) = -12.5$ $-6(4.8) = -28.8$ $0.4(-1.5x) = -0.6x$
Rational Numbers with the Same Sign	The product of two rational numbers having the same sign is positive.	$(-4.2)(-8) = 33.6$ $12(3.2) = 38.4$ $(3w)(2.1y) = 6.3wy$
Fractions	To multiply fractions, multiply the numerators and multiply the denominators.	$-\frac{3}{5} \cdot \frac{1}{4} = -\frac{3}{20}$ $-\frac{1}{2} \cdot \left(-2\frac{1}{3}\right) = -\frac{1}{2} \cdot \left(-\frac{7}{3}\right)$ $= \frac{7}{6}$ or $1\frac{1}{6}$

Find each product.

1. $3.8(-4)$

2. $-6(-1.2)$

3. $6(6.1)$

4. $-0.5(-5.2)$

5. $(-8.4) \cdot 0$

6. $-2.4 \cdot (10.5)$

7. $\frac{1}{7} \cdot \frac{2}{3}$

8. $-2 \cdot \left(-\frac{2}{5}\right)$

9. $-\frac{2}{3} \cdot \frac{2}{9}$

10. $-\frac{5}{3} \cdot \left(-\frac{2}{7}\right)$

11. $0 \cdot \left(-\frac{1}{5}\right)$

12. $\frac{1}{3}\left(-1\frac{1}{4}\right)$

Simplify each expression.

13. $3(1.4a)$

14. $-0.6x\ (-3y)$

15. $\frac{1}{4} \cdot \left(-\frac{1}{2}b\right)$

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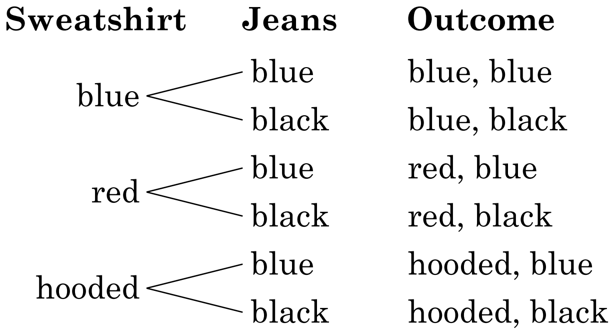
4–2 Study Guide

Counting Outcomes

Jeremy has three sweatshirts and two pairs of jeans. One way to find the number of different outfits is to draw a tree diagram, as shown below.

Sweatshirts	Jeans
blue red hooded	blue black



Since there are six outcomes, there are six outfits possible.

The **Fundamental Counting Principle** can also be used to count outcomes. It states that if an event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by event N can occur in $m \times n$ ways.

$$3 \text{ sweatshirts} \times 2 \text{ jeans} = 3 \times 2 \text{ or } 6 \text{ outfits}$$

Find the number of possible outcomes by drawing a tree diagram.

1. sandwich with one condiment

Menu Item	Condiment
hamburger cheeseburger hot dog	ketchup mustard pickle onion

2. computer with one peripheral

Computer	Peripheral
desk model laptop	DVD drive CD-Rom drive floppy disk drive

Find the number of possible outcomes by using the Fundamental Counting Principle.

3. A certain game includes cards with 3 different pictures, 4 colors, and 6 numbers. Find the number of cards in the game.

4. Find the number of possible ways of answering a true-false question followed by a multiple choice question with 4 choices.

5. Suppose you can order sweaters from a catalog in 8 sizes, 3 colors, and 2 delivery methods. Find the number of possible sweater orders.

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Study Guide

Dividing Rational Numbers

Use the following rules to divide rational numbers or fractions.

Rule or Property		Examples
Dividing Rational Numbers	The quotient of two numbers having different signs is negative The quotient of two numbers having the same sign is positive.	$-4.8 \div 6 = -0.8$ $4.8 \div (-6) = -0.8$ $2.8 \div (0.4) = 7$ $-2.8 \div (-0.4) = 7$
Multiplicative Inverse Property	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.	$-\frac{3}{4}$ and $-\frac{4}{3}$ are multiplicative inverses.
Dividing Fractions	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b, c, d \neq 0$	$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3}$ or $\frac{8}{15}$ $\frac{3}{4} \div (-6) = \frac{3}{4} \cdot \left(-\frac{1}{6}\right)$ $= -\frac{3}{24}$ or $-\frac{1}{8}$

Find each quotient.

1. $-8 \div 2.5$

2. $-1.6 \div (-2)$

3. $3.6 \div 0.6$

4. $5.5 \div (-5.5)$

5. $0 \div -0.6$

6. $-18.7 \div 5.5$

7. $-42 \div (-0.5)$

8. $0 \div \frac{2}{7}$

9. $-\frac{4}{5} \div \frac{4}{5}$

10. $\frac{1}{7} \div \frac{2}{5}$

11. $-\frac{2}{5} \div \frac{1}{2}$

12. $\frac{1}{9} \div (-4)$

13. $-\frac{3}{8} \div \frac{11}{8}$

14. $-\frac{3}{7} \div \left(-\frac{7}{3}\right)$

15. $\frac{3}{4} \div \frac{1}{5}$

4–4

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Study Guide

Solving Multiplication and Division Equations

Use the following rules to solve algebraic equations.

Rule or Property		Examples
Division Property of Equality	For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.	$4x = -24$ $\frac{4x}{4} = \frac{-24}{4}$ $x = 6$
Multiplication Property of Equality	For any numbers a, b, and c, if $a = b$, then $ac = bc$.	$\frac{1}{4}x = 12$ $4 \cdot \frac{1}{4}x = 4 \cdot 12$ $x = 48$

Solve each equation.

1. $8y = 48$

2. $-6x = 42$

3. $3.6 = 12n$

4. $1.5 = -3w$

5. $0 = -0.6y$

6. $18 = 0.5x$

7. $-6 = -8y$

8. $\frac{7}{4}n = 1$

9. $\frac{1}{5}x = 10$

10. $\frac{3}{4}y = 12$

11. $-\frac{3}{5}y = -3$

12. $4 = -\frac{1}{2}x$

13. $8 = \frac{x}{-2}$

14. $-\frac{4}{5}n = -\frac{1}{4}$

15. $\frac{1}{2} = -\frac{5}{8}x$

16. $1.2y = 9$

17. $-4x = 12.8$

18. $-\frac{n}{3} = \frac{1}{3}$

4–5

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Study Guide

Solving Multi-Step Equations

Some equations require more than one step to solve. To solve a problem that has more than one step, the best strategy is to undo the operations in reverse order. Always check your solution.

Example 1: Solve $3x - 8 = 31$.

$$3x - 8 = 31$$
$$3x - 8 + 8 = 31 + 8$$
$$3x = 39$$
$$\frac{3x}{3} = \frac{39}{3}$$
$$x = 13$$

Check: $3x - 8 = 31$
$$3(13) - 8 \stackrel{?}{=} 31$$
$$39 - 8 \stackrel{?}{=} 31$$
$$31 = 31 \checkmark$$

Example 2: Solve $\frac{x}{4} + 6 = 12$.

$$\frac{x}{4} + 6 = 12$$
$$\frac{x}{4} + 6 - 6 = 12 - 6$$
$$\frac{x}{4} = 6$$
$$4 \cdot \frac{x}{4} = 4 \cdot 6$$
$$x = 24$$

Check: $\frac{x}{4} + 6 = 12$
$$\frac{24}{4} + 6 \stackrel{?}{=} 12$$
$$6 + 6 \stackrel{?}{=} 12$$
$$12 = 12 \checkmark$$

Solve each equation. Check your solution.

1. $6y - 4 = 50$ **2.** $-8x - 2 = 38$ **3.** $-15 = -12n + 9$

4. $-2m + 6 = 22$ **5.** $-4y - 8 = 30$ **6.** $-1.5x + 6 = -54$

7. $8.2y + 4 = 65.5$ **8.** $-1.5 = 3w + 6$ **9.** $-8 = -2.5x + 5$

10. $\frac{x}{2} + 7 = 9$ **11.** $\frac{x}{3} - 12 = -2$ **12.** $\frac{x}{-6} - 12 = 2$

13. $3x + 7 = 7$ **14.** $-8 = 1.5y + 4$ **15.** $\frac{y}{2} + 2 = 25$

4–6

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Study Guide

Variables on Both Sides

Some equations contain variables on both sides and require more than one step to solve. To solve these equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side. Then solve and check.

Example 1: Solve $2x - 6 = x + 4$.

$$\begin{aligned} 2x - 6 &= x + 4 \\ 2x - 6 - x &= x + 4 - x \\ x - 6 &= 4 \\ x - 6 + 6 &= 4 + 6 \\ x &= 10 \end{aligned}$$

Check: $2x - 6 = x + 4$

$$\begin{aligned} 2(10) - 6 &\stackrel{?}{=} 10 + 4 \\ 20 - 6 &\stackrel{?}{=} 14 \\ 14 &= 14 \checkmark \end{aligned}$$

Example 2: Solve $\frac{1}{4}x - 12 = \frac{3}{4}x$.

$$\begin{aligned} \tfrac{1}{4}x - 12 &= \tfrac{3}{4}x \\ \tfrac{1}{4}x - 12 - \tfrac{1}{4}x &= \tfrac{3}{4}x - \tfrac{1}{4}x \\ -12 &= \tfrac{1}{2}x \\ 2 \cdot (-12) &= 2 \cdot \tfrac{1}{2}x \\ -24 &= x \end{aligned}$$

Check: $\frac{1}{4}x - 12 = \frac{3}{4}x$

$$\begin{aligned} \tfrac{1}{4}(-24) - 12 &\stackrel{?}{=} \tfrac{3}{4}(-24) \\ -6 - 12 &\stackrel{?}{=} -18 \\ -18 &= -18 \checkmark \end{aligned}$$

Solve each equation. Check your solution.

1. $6m - 40 = m$

2. $-5y - 2 = y + 10$

3. $-15n = -12n + 9$

4. $-4y + 6 = -3y + 12$

5. $6y - 8 = 6y - 6 - 2$

6. $-15x + 8 = -15x - 7$

7. $4.2y + 4.4 = 3.1y$

8. $w = 3.8w - 7$

9. $-8 - m = -3.5m + 5$

10. $\frac{1}{5}x + 12 = \frac{2}{5}x$

11. $\frac{1}{3}x - 8 = -\frac{1}{3}x$

12. $\frac{3}{7}x - 14 = -\frac{5}{7}x + 2$

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Study Guide

Grouping Symbols

The first step in solving any equation that contains grouping symbols is to remove the parentheses by using the Distributive Property. Then solve and check.

Example 1: Solve $6(2x + 1) = 42$.

$$\begin{aligned} 6(2x + 1) &= 42 \\ 12x + 6 &= 42 \\ 12x + 6 - 6 &= 42 - 6 \\ 12x &= 36 \\ x &= 3 \end{aligned}$$

Check:

$$\begin{aligned} 6(2x + 1) &= 42 \\ 6(2(3) + 1) &\stackrel{?}{=} 42 \\ 6(6 + 1) &\stackrel{?}{=} 42 \\ 6(7) &\stackrel{?}{=} 42 \\ 42 &= 42 \checkmark \end{aligned}$$

Example 2: Solve $\frac{3}{4}(12x + 8) = -21$

$$\begin{aligned} \tfrac{3}{4}(12x + 8) &= -21 \\ 9x + 6 &= -21 \\ 9x + 6 - 6 &= -21 - 6 \\ 9x &= -27 \\ x &= -3 \end{aligned}$$

Check:

$$\begin{aligned} \tfrac{3}{4}(12x + 8) &= -21 \\ \tfrac{3}{4}(12 \cdot (-3) + 8) &\stackrel{?}{=} -21 \\ \tfrac{3}{4}(-36 + 8) &\stackrel{?}{=} -21 \\ \tfrac{3}{4}(-28) &\stackrel{?}{=} -21 \\ -21 &= -21 \checkmark \end{aligned}$$

Solve each equation. Check your solution.

1. $3(n - 2) = 12$

2. $-24 = 4(y - 2)$

3. $-12 = -2(4x + 2)$

4. $6 + 2(x - 5) = 2x - 4$

5. $-3x = 2(3x + 9)$

6. $x = 3(-2x + 9) + 8$

7. $4.2(y - 2) = 14.7$

8. $w = 3.5(w - 6)$

9. $3.6 - 1.2n = -2.8(n - 5)$

10. $\frac{1}{3}(x - 9) = 18$

11. $\frac{1}{2}(8x + 24) = 4$

12. $\frac{3}{4}(-12x - 8) = x + 34$

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Study Guide

Solving Proportions

An equation stating that two ratios are equal is called a **proportion**. For example, $\frac{8}{3} = \frac{16}{6}$ is a proportion. Use the Property of Proportions and other algebraic properties you know to solve proportions.

Property of Proportions
The cross products of a proportion are equal. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.

Example 1: Solve $\frac{6}{5} = \frac{18}{n}$.

$$\frac{6}{5} = \frac{18}{n}$$

$6n = 5(18)$ *Find the cross products.*

$6n = 90$ *Simplify.*

$n = 15$

Check: $\frac{6}{5} \stackrel{?}{=} \frac{18}{15}$

$6(15) \stackrel{?}{=} 5(18)$

$90 = 90$ ✓

Example 2: Solve $\frac{p}{12} = \frac{p-4}{6}$.

$$\frac{p}{12} = \frac{p-4}{6}$$

$$6p = 12(p - 4)$$

$$6p = 12p - 48$$

$$-6p = -48$$

$$p = 8$$

Check: $\frac{8}{12} \stackrel{?}{=} \frac{8-4}{6}$

$8(6) \stackrel{?}{=} 12(8 - 4)$

$48 = 48$ ✓

Solve each proportion.

1. $\frac{g}{3} = \frac{12}{4}$

2. $\frac{7}{4} = \frac{35}{m}$

3. $\frac{2}{3} = \frac{a}{60}$

4. $\frac{5}{8} = \frac{x}{60}$

5. $\frac{5}{b} = \frac{65}{39}$

6. $\frac{c}{38} = \frac{2}{4}$

7. $\frac{d}{2} = \frac{2}{8}$

8. $\frac{k}{10} = \frac{1}{1000}$

9. $\frac{8}{x-2} = \frac{24}{7}$

10. $\frac{y}{9} = \frac{y+2}{3}$

11. $\frac{x}{x-3} = \frac{18}{6}$

12. $\frac{5}{x-2} = \frac{8}{x+1}$

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Study Guide

Scale Drawings and Models

A scale drawing or scale model is used to represent an object that is too large or too small to be drawn or built at its actual size.

Example 1: The Statue of Liberty is about 150 feet tall. A model is 15 inches tall. Find the scale used.

The scale used is the ratio of the length of the model to the actual length, or 15 in. = 150 feet. Reduce the ratio to get a scale of 1 in. = 10 feet.

Another method is to convert 150 feet to inches, or 150 ft = 150(12) in. = 1800 in. The scale is 15 in. = 1800 in. Reduce the ratio to get 1:120. The units do not need to be included if they are the same.

Example 2: The scale of a map of Florida is 1 inch = 20 miles. Find the actual distance between Miami and St. Petersburg if the distance between them on the map is 13 inches.

$$\frac{1 \text{ inch}}{20 \text{ miles}} = \frac{13 \text{ inches}}{x \text{ miles}} \qquad \textit{Use a proportion.}$$

$$1 \cdot x = 20(13)$$

$$x = 260$$

The distance from Miami to St. Petersburg is 260 miles.

Find each scale or distance.

1. On the blueprint of a house, the kitchen is 5 inches long. If the actual kitchen is 20 feet long, find the scale of the blueprint.

2. On a map, the scale is 1 inch = 25 miles. Find the actual distance for each map distance.

From	To	Map Distance
Portsmouth, OH	Springfield, OH	5.2 inches
Chicago, IL	Lawrenceville, IL	10 inches
Santa Fe, NM	Clovis, NM	$8\frac{2}{5}$ inches

3. The Sears Tower is 1450 feet tall. If a model is 25 inches tall, find the scale.

4. In an HO scale model of a train, the length of the engine is 6 inches. If the HO scale is 1 : 87, find the actual length. Write the answer in feet.

5. Las Vegas, Nevada, is 445 miles from Reno, Nevada. If the distance on the map is $11\frac{1}{8}$ inches, find the scale used for the map.

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Study Guide

The Percent Proportion

Percent is a ratio that compares a number to 100. For example, 6 out of 100 can be expressed as $\frac{6}{100}$, 6:100, or 6%. Use the percent proportion to find percents.

> If P is the percentage, B is the base, and r is the percent, the percent proportion is $\frac{P}{B} = \frac{r}{100}$.

Example 1: 80% of what number is 336?

$$\frac{P}{B} = \frac{r}{100}$$

$$\frac{336}{B} = \frac{80}{100}$$

$$336(100) = 80B \quad \textit{Cross multiply.}$$

$$33600 = 80B \quad \textit{Simplify.}$$

$$\frac{33,600}{80} = \frac{80B}{80}$$

$$420 = B$$

So, 80% of 420 = 336.

Example 2: 75 is what percent of 300?

$$\frac{P}{B} = \frac{r}{100}$$

$$\frac{75}{300} = \frac{r}{100}$$

$$75(100) = 300r$$

$$7500 = 300r$$

$$\frac{7500}{300} = \frac{300r}{300}$$

$$25 = r$$

So, 75 is 25% of 300.

Use the percent proportion to find each number.

1. 40 is what percent of 80?

2. 8 is what percent of 25?

3. Find 10% of 170.

4. 18 is 12% of what number?

5. What number is 150% of 12?

6. 32 is 400% of what number?

7. 30 is what percent of 150?

8. Find 84% of 500.

9. 9% of what number is 8.1?

10. What number is 15% of 42?

11. 50% of 0.75 is what number?

12. Find 140% of 82.

13. 60% of what number is 2.1?

14. Find 0.5% of 8.

15. 600 is what percent of 240?

16. 1 is what percent of 250?

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Study Guide

The Percent Equation

The percent proportion $\frac{P}{B} = \frac{r}{100}$ can also be written as the equation $P = RB$, where P is the percentage, B is the base, and R is the rate expressed as a decimal.

Example 1: Find 60% of 225.

$P = RB$
$R = 0.6$ *Write R as decimal.*
$P = 0.6(225)$
$P = 135$

So, 60% of 225 = 135.

Example 2: 18 is 40% of what number?

$P = RB$
$R = 0.4$
$18 = 0.4(B)$
$\frac{18}{0.4} = \frac{0.4B}{0.4}$ *Divide by 0.4.*
$45 = B$

So, 18 is 40% of 45.

Use the percent equation to find each number.

1. 72 is what percent of 90?

2. Find 10% of 240.

3. 90 is what percent of 360?

4. Find 8% of 120.

5. Find 32% of 600.

6. 14% of what number is 9.8?

7. What number is 24% of 90?

8. 150% of 0.6 is what number?

9. 750 is what percent of 150?

10. 48 is 300% of what number?

11. 2 is what percent of 500?

12. Find 0.5% of 12.

13. Find 160% of 85.

14. What number is 265% of 24?

15. 150 is what percent of 25?

16. 98 is 5% of what number?

17. Marcia bought $80 in groceries. What is her final cost after sales tax of 6.5% is added?

18. Mr. Perez paid 14% of his income in taxes. If his taxes were $7686, what was his income?

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Study Guide

Percent of Change

The median price of a home increased from \$90,600 in 1988 to \$134,600 in 1998. The change in the price can be written as a percent. Since the price increased, it is called the **percent of increase**. Likewise, the percent that an amount decreases is known as the **percent of decrease**.

To find the percent of increase or decrease, write a ratio that compares the amount of increase or decrease to the original amount.

Example: Find the percent of increase in the median price of a home.

original price: \$90,600 new price: \$134,600

The amount of increase $= 134{,}600 - 90{,}600 = 44{,}000$. Substitute the amount of increase for P. You are comparing it to the original price, so substitute \$90,600 for B.

Method 1

$$\begin{aligned} \frac{P}{B} &= \frac{r}{100} \\ \frac{44{,}000}{90{,}600} &= \frac{r}{100} \\ 44{,}000(100) &= 90{,}600r \\ 4{,}400{,}000 &= 90{,}600r \\ \frac{4{,}400{,}000}{90{,}600} &= \frac{90{,}600r}{90{,}600} \\ 48.6 &= r \end{aligned}$$

Method 2

$$\begin{aligned} P &= RB \\ 44{,}000 &= R \cdot 90{,}600 \\ \frac{44{,}000}{90{,}600} &= R \cdot \frac{90{,}600}{90{,}600} \\ 0.486 &= R \end{aligned}$$

Divide each side by 90,600.

So the percent of increase was about 49%.

Find the percent of change. Round to the nearest percent.

1. original: 25
new: 26

2. original: \$20,000
new: \$17,500

3. original: 96
new: 54

4. original: 15
new: 33

5. In January, gasoline cost \$1.05 per gallon. In July, gasoline cost \$1.20 per gallon. Find the percent of increase.

6. The original price of a CD player is \$105. A discount of \$21 is given. Find the percent of decrease.

7. The original cost of a computer is \$900. If the price is decreased by 15%, what is the sale price?

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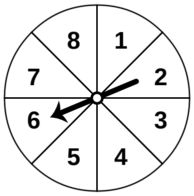
Study Guide

Probability and Odds

You can measure the chances of an event happening with probability, a number that is always between 0 and 1, inclusive. You can express the probability of an event as a fraction, as a decimal, or as a percent, using the following relationship.

> The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.
>
> $$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Example 1: The spinner at the right is spun. Find the probability of spinning a number greater than 5.



There are 3 favorable outcomes, spinning a 6, 7, or an 8. There are 8 possible outcomes.

Therefore $P(\text{a number greater than 5}) = \frac{3}{8}$.

Another way to measure the chance of an event is with odds.

> The odds of an event is a ratio that compares the number of favorable outcomes to the number of unfavorable outcomes.
>
> $$\text{Odds} = \frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$$

Example 2: Refer to the spinner above. Find the odds of spinning an even number.

There are 4 favorable outcomes: 2, 4, 6, or 8. The number of unfavorable outcomes is $8 - 4 = 4$. Therefore the odds of spinning an even number are $4:4$ or $1:1$.

Refer to the spinner above. Find the probability of each outcome.

1. a 3

2. an even number

3. a number less than 3

4. a number greater than 4

Refer to the spinner above. Find the odds of each outcome.

5. an odd number

6. a multiple of 3

7. not a 6

8. a number greater than 4

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Study Guide

Compound Events

To win a game, you have to roll either an even number or a 3 with one die. What is the probability that you will win? The probability of tossing an even number = $P(\text{even number}) = \frac{1}{2}$, and the probability of tossing a 3 is $P(3) = \frac{1}{6}$. These events cannot occur at the same time and so they are **mutually exclusive**. This is a compound event because there is more than 1 event occurring. You find the probability of mutually exclusive events by adding the probabilities of each event.

$$P(\text{even number or a 3}) = P(\text{even number}) + P(3)$$
$$= \frac{1}{2} + \frac{1}{6} \text{ or } \frac{2}{3}$$

Compound events are called **independent** if the outcome of one event does not affect the outcome of the other event. For example, if two dice are rolled, rolling an odd number with the first die does not affect rolling a 4 on the second die. You find the probability of two independent events by multiplying the probabilities of each event.

$$P(\text{odd number and a 4}) = P(\text{odd number}) \cdot P(4)$$
$$= \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12}$$

Probability of Mutually Exclusive Events *A* and *B*	$P(A \text{ or } B) = P(A) + P(B)$
Probability of Independent Events *A* and *B*	$P(A \text{ and } B) = P(A) \cdot P(B)$

A die is rolled. Find the probability of each compound event.

1. P(a 4 or a 2)

2. P(an odd number or a 6)

3. P(a 2 or a 3)

4. P(a 1 or a 6)

5. P(an even number or a 5)

6. P(a 1 or a multiple of 2)

7. P(a 1 or a number greater than 4)

8. P(an odd number or an even number)

Two dice are rolled. Find the probability of each compound event.

9. P(an odd number and a 1)

10. P(an odd number and an even number)

11. P(two odd numbers)

12. P(a 6 and a number greater than 4)

13. P(two numbers less than 3)

14. P(an even number and a 5)

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Study Guide

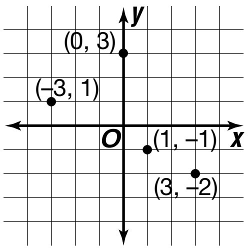
Relations

A set of ordered pairs is called a **relation**. The set of all first coordinates of the ordered pairs is the **domain** of the relation. The set of all second coordinates is the **range**. You can use a table or a graph to represent a relation.

Example 1: Express the relation $\{(-3, 1), (0, 3), (1, -1), (3, -2)\}$ as a table and as a graph. Then determine the domain and range.

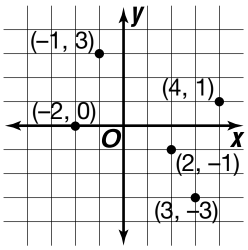
The domain is $\{-3, 0, 1, 3\}$ and the range is $\{1, 3, -1, -2\}$.

x	*y*
−3	1
0	3
1	−1
3	−2



Example 2: Express the relation shown on the graph as a set of ordered pairs. Then find the domain and the range.

The set of ordered pairs for the relation is $\{(-2, 0), (-1, 3), (2, -1), (3, -3), (4, 1)\}$. The domain is $\{-2, -1, 2, 3, 4\}$ and the range is $\{0, 3, -1, -3, 1\}$.



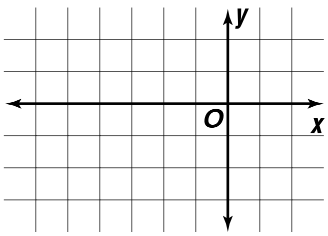
Identify the domain and the range of each function.

1. $\{(-6, 0), (-2, 3), (4, -1)\}$

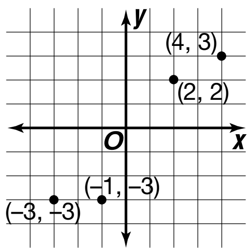
2.

x	*y*
−4	−3
−3	−3
−2	1
1	4
2	6

3. Express the relation $\{(-4, -1), (-2, -2), (0, 0)\}$ as a table and as a graph. Then determine the domain and the range.



4. Express the relation shown on the graph as as a set of ordered pairs and in a table. Then determine the domain and the range.



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Study Guide

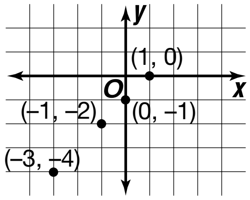
Equations as Relations

Since $x + 5 = 9$ is true when $x = 4$, we say that 4 is a solution of the equation. Equations with two variables have solutions that may include one or more ordered pairs. The ordered pair $(-3, 0)$ is a solution of the equation $y = x + 3$ because $0 = -3 + 3$. But $(-4, 1)$ is not solution because $1 \neq -4 + 3$.

Example: Solve $y = -1 + x$ if the domain is $\{-3, -1, 0, 1\}$. Graph the solution.

Substitute each value of x into the equation to find the corresponding y-value. Then graph the ordered pairs.

x	y	(x, y)
-3	$-1 + (-3) = -4$	$(-3, -4)$
-1	$-1 + (-1) = -2$	$(-1, -2)$
0	$-1 + 0 = -1$	$(0, -1)$
1	$-1 + 1 = 0$	$(1, 0)$



The solution set is $\{(-3, -4), (-1, -2), (0, -1), (1, 0)\}$.

Which ordered pairs are solutions of each equation?

1. $y = x - 4$ **a.** $(-1, -5)$ **b.** $(-1, -3)$ **c.** $(0, -4)$ **d.** $(5, 9)$

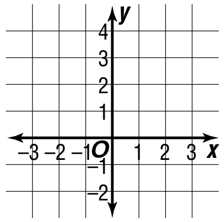
2. $b = 2a + 9$ **a.** $(7, 23)$ **b.** $(-2, 5)$ **c.** $(1, 11)$ **d.** $(-6, -3)$

3. $4g + h = -6$ **a.** $(-4, -10)$ **b.** $(-10, 34)$ **c.** $(0, 6)$ **d.** $(3, -18)$

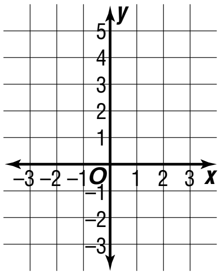
4. $-3x - y = 5$ **a.** $(2, 11)$ **b.** $(2, -11)$ **c.** $(-4, -17)$ **d.** $(-10, -35)$

Solve each equation if the domain is {−2, −1, 0, 1, 2}. Graph the solution set.

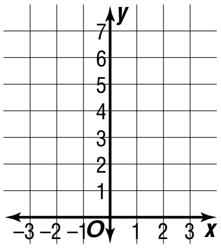
5. $y = x + 2$



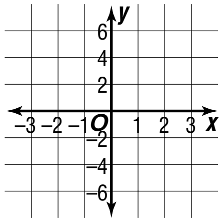
6. $y = -2x + 1$



7. $y = 5 - x$



8. $y = 3x$



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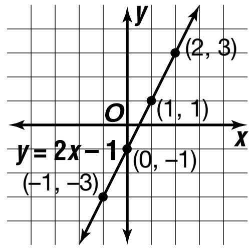
Study Guide

Graphing Linear Relations

The solution set of the equation $y = 2x - 1$ contains an infinite number of ordered pairs. A few of the solutions are shown in the table at the right.

x	y	(x, y)
-1	$2(-1) - 1 = -3$	$(-1, -3)$
0	$2(0) - 1 = -1$	$(0, -1)$
1	$2(1) - 1 = 1$	$(1, 1)$
2	$2(2) - 1 = 3$	$(2, 3)$



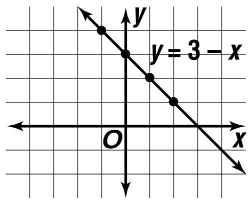
Graphing the ordered pairs indicates that the graph of the equation is a straight line. An equation whose graph is a straight line is a **linear equation**.

Example: Graph $y = 3 - x$.

Select at least three values for x. Determine the values for y.

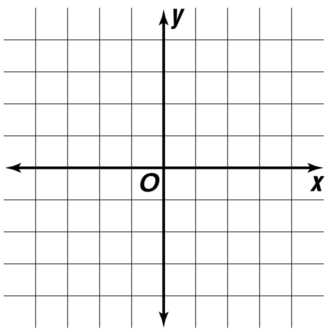
Graph the ordered pairs and use them to draw a line.

x	y	(x, y)
-1	$3 - (-1) = 4$	$(-1, 4)$
0	$3 - 0 = 3$	$(0, 3)$
1	$3 - 1 = 2$	$(1, 2)$
2	$3 - 2 = 1$	$(2, 1)$

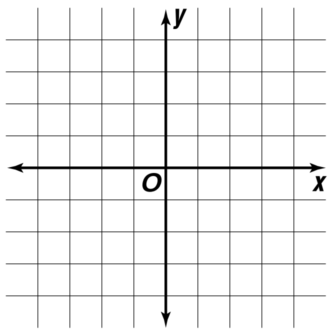


Graph each equation.

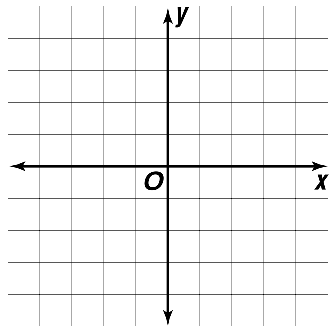
1. $y = x - 2$



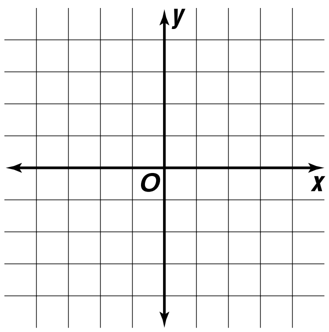
2. $y = -3x$



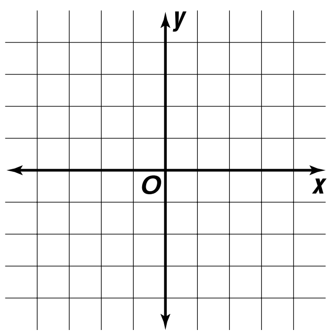
3. $y = -1$



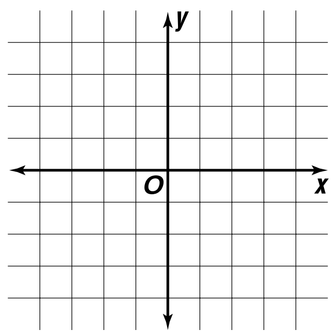
4. $y = x$



5. $x = -3$



6. $y = 1 + 2x$



6–4

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Study Guide

Functions

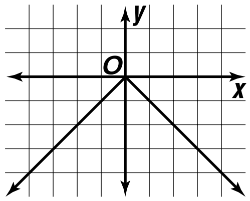
A **function** is a special kind of relation in which each member of the domain is paired with exactly one member of the range. Study these examples.

$\{(-4, 7), (0, 1), (5, 1)\}$

This relation is a function because every first coordinate is matched with exactly one second coordinate.

x	y
7	0
-2	3
7	-8

This relation is *not* a function because 7 is paired with two y-values, 0 and -8.



This relation is a function because every x-value on the graph is paired with exactly one y-value.

Equations that are functions can be written in **functional notation**. Notice that $f(x)$ replaces y in this example.

Equation	Functional Notation
$y = 2 - 5x$	$f(x) = 2 - 5x$

We read $f(x)$ as "f of x." If $x = 3$, then $f(3) = 2 - 5(3)$ or -13.

Example: If $f(x) = 3x - 1$, find $f(-2)$.

$f(x) = 3x - 1$
$f(-2) = 3(-2) - 1$ *Replace x with −2.*
$f(-2) = -7$ *Simplify.*

Determine whether each relation is a function.

1. $\{(3, 1), (-2, 0), (-3, 5), (5, 6)\}$

2. $\{(0, 1), (2, 1), (3, -1)\}$

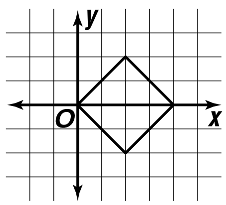
3. $\{(-8, -4), (3, 2), (-8, -1), (7, 0)\}$

4. $\{(9, 10), (-9, 10), (-4, 5), (-5, 4)\}$

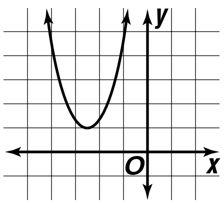
5.

x	y
5	−2
0	−4
5	2
−3	1

6.



7.



If f(x) = −2x + 3 and g(x) = x − 5, find each value.

8. $f(1)$ **9.** $f(-6)$ **10.** $g(2)$ **11.** $g(-5)$

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Study Guide

Direct Variation

A linear function that can be written in the form $y = kx$, where $k \neq 0$, is called a **direct variation**. In a direct variation, *y varies directly as x.*

Because a direct variation is a linear function whose solution contains (0, 0), the graph of a direct variation equation is a straight line that passes through the origin.

Example 1: Determine whether each function is a direct variation.

a. $y = 10x$

The function is a linear function. Since $0 = 10(0)$, (0, 0) is a solution of $y = 10x$. Therefore, the function is a direct variation and the graph will pass through the origin.

b. $y = 2x + 1$

The function is a linear function. But since $0 \neq 2(0) + 1$, (0, 0) is not a solution of $y = 2x + 1$. Therefore, the function is not a direct variation and the graph will not pass through the origin.

Example 2: Assume that y varies directly as x and $y = 30$ when $x = 5$. Find x when $y = -12$.

Step 1 Find the constant of variation, k.

$y = kx$	
$30 = k(5)$	*When $y = 30$, $x = 5$.*
$6 = k$	*Solve for k.*

Step 2 Use $k = 6$ to find x when $y = -12$.

$y = kx$	
$y = 6x$	*Substitute 6 for k.*
$-12 = 6x$	*Substitute −12 for y.*
$x = -2$	*Solve for x.*

When $y = -12$, $x = -2$.

Determine whether each equation is a direct variation.

1. $y = x$ **2.** $y = x + 2$ **3.** $y = 6$ **4.** $y = 4 - x$

5. $\frac{y}{x} = 8$ **6.** $y = -7x$ **7.** $x = -1$ **8.** $\frac{y}{2x} = 5$

Solve. Assume that y varies directly as x.

9. Find x when $y = 24$ if $y = 18$ when $x = 6$.

10. Find x when $y = 6$ if $y = -8$ when $x = 4$.

6–6

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Study Guide

Inverse Variation

Shauna wants to improve her running speed. As her speed increases, her time decreases.

rate · time = distance					
r	***t***	***d***	***r***	***t***	***d***
$\frac{1}{9}$ mile/minute	9 minutes	1 mile	$\frac{1}{7}$ mile/minute	7 minutes	1 mile
$\frac{1}{8}$ mile/minute	8 minutes	1 mile	$\frac{1}{6}$ mile/minute	6 minutes	1 mile

The equation $rt = d$ is an example of an **inverse variation**. An inverse variation is described by an equation of the form $xy = k$, where $k \neq 0$. We say that *y varies inversely as x*.

Example: Suppose y varies inversely as x and $y = 3$ when $x = 4$. Find y when $x = -12$.

Step 1 Find the constant of variation, k.

$$xy = k$$
$$3(4) = k \quad \textit{When } y = 3, x = 4.$$
$$12 = k \quad \textit{Solve for } k.$$

Step 2 Use $k = 12$ to find y when $x = -12$.

$$xy = k$$
$$xy = 12 \quad \textit{Substitute 12 for } k.$$
$$-12y = 12 \quad \textit{Substitute } -12 \textit{ for } x.$$
$$y = -1 \quad \textit{Solve for } y.$$

When $x = -12$, $y = -1$.

Determine if each equation is an* inverse *variation or a* direct *variation.

1. $y = 5x$ **2.** $ab = 5$ **3.** $xy = -1$

Solve. Assume that y varies inversely as x.

4. Find y when $x = -9$ if $y = 3$ when $x = 6$.

5. Suppose $y = 5$ when $x = 16$. Find x when $y = 10$.

6. If $y = 4.5$ when $x = 6$, find y when $x = 3$.

7. Find y when $x = 0.125$ if $y = 1.5$ when $x = 2.5$.

8. Find x when $y = 0.9$, if $y = 1.5$ when $x = 0.3$.

9. Suppose $x = -8$ when $y = 6$. Find y when $x = 16$.

10. If $y = \frac{1}{4}$ when $x = 8$, find x when $y = \frac{2}{5}$.

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Study Guide

Slope

Slope is the ratio of the rise, or the vertical change, to the run, or the horizontal change. A greater ratio indicates a steeper slope. A typical ski mountain has a slope of about $\frac{1}{4}$, while a car windshield may have a slope of 3.

For any two points (x_1, y_1) and (x_2, y_2),

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples: Find the slope of the line containing each pair of points.

a. $(4, -2)$ and $(-3, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - (-2)}{-3 - 4}$$

$$m = \frac{-9}{7}$$

b. $(-3, 6)$ and $(-3, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 6}{-3 - (-3)}$$

$$m = \frac{-7}{0}$$

Since you cannot divide by 0, the slope is *undefined*.

c. $(4, 5)$ and $(-2, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 5}{-2 - 4}$$

$$m = \frac{0}{-6} \text{ or } 0$$

Determine the slope of the line passing through the points whose coordinates are listed.

1. $(-2, 1)$ and $(4, 2)$

2. $(0, 3)$ and $(4, 1)$

3. $(-3, -5)$, $(5, 7)$

4. $(4, 3)$ and $(4, -1)$

5. $(8, -2)$ and $(-3, -2)$

6. $(5, 1)$ and $(-1, -5)$

7. $(7, -1)$ and $(6, 6)$

8. $(5, -2)$ and $(-5, 2)$

9. $(7, -7)$ and $(-6, 6)$

10. $(4, -4)$ and $(0, 3)$

11. $(-2, 4)$ and $(-2, 9)$

12. $(0, 8)$ and $(-3, 8)$

7–2

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Study Guide

Writing Equations in Point-Slope Form

You can write the equation of a line if you know its slope and the coordinates of one point or if you know the coordinates of two points on the line. Use the point-slope form.

> **Point-Slope Form**
>
> For a nonvertical line through the point at (x_1, y_1) with slope m, the point-slope form of a linear equation is

Examples: Write the point-slope form of an equation for each line.

a. the line passing through the point at $(-4, 2)$ and having a slope of $\frac{2}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{2}{3}(x - (-4))$$

$$y - 2 = \frac{2}{3}(x + 4)$$

b. the line passing through points at $(4, -4)$ and $(-3, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \textit{Find } m.$$

$$m = \frac{1 - (-4)}{-3 - 4} \text{ or } -\frac{5}{7}$$

$$y - (-4) = -\frac{5}{7}(x - 4) \qquad \textit{Substitute } m.$$

$$y + 4 = -\frac{5}{7}(x - 4)$$

Write the point-slope form of an equation for each line, given either the coordinates of a point and the slope or the coordinates of two points.

1. $(-2, -1)$, $m = 2$

2. $(4, -1)$, $m = -\frac{1}{2}$

3. $(-3, -5)$, $m = \frac{3}{2}$

4. $(4, 3)$, $m = \frac{1}{5}$

5. $(8, -2)$, $m = 0$

6. $(5, 1)$, $m = -\frac{2}{3}$

7. $(7, -1)$ and $(6, 6)$

8. $(5, -2)$ and $(-5, 2)$

9. $(7, -7)$ and $(-6, 6)$

10. $(4, -4)$ and $(0, 3)$

11. $(-2, -4)$ and $(-12, 9)$

12. $(0, 8)$ and $(-3, 8)$

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Study Guide

Writing Equations in Slope-Intercept Form

Given the slope m and the y-intercept b of a line, the slope-intercept form of an equation of the line is $y = mx + b$. Sometimes it is more convenient to express the equation of a line in slope-intercept form instead of point-slope form. This form is especially useful when graphing lines.

Examples: Write the point-intercept form of an equation for each line.

a. the line with $m = -\frac{2}{3}$ and $b = 4$

$y = mx + b$

$y = -\frac{2}{3}x + 4$

b. the line passing through points at $(4, -2)$ and $(6, 2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Find m.*

$m = \frac{2 - (-2)}{6 - 4} = \frac{4}{2} = 2$

$y - 2 = 2(x - 6)$

$y - 2 = 2x - 12$ *Multiply.*

$y - 2 + 2 = 2x - 12 + 2$ *Add 2 to each side.*

$y = 2x - 10$ *Simplify.*

Write the slope-intercept form of an equation for each line, given either the slope and the y-intercept or the coordinates of two points.

1. $m = 2, b = -3$

2. $m = \frac{1}{2}, b = 5$

3. $m = \frac{7}{4}, (0, -3)$

4. $m = -\frac{1}{5}, b = 0$

5. $m = \frac{1}{6}, b = -1$

6. $m = 0, b = 5$

7. $(5, -3)$ and $(-4, -3)$

8. $(-5, 1)$ and $(-1, 5)$

9. $(0, -1)$ and $(6, 5)$

10. $(-2, -2)$ and $(-4, 2)$

11. $(-2, -4)$ and $(-12, 6)$

12. $(0, 8)$ and $(-3, 4)$

13. $(0, 8)$ and $(-2, 7)$

14. $(0, -3)$ and $(6, 5)$

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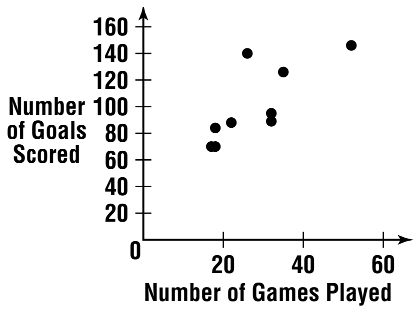
7–4 Study Guide

Scatter Plots

The scatter plot below is a graph of the number of games played in the highest-scoring World Cup finals compared to the number of goals scored. The data are listed in the table.

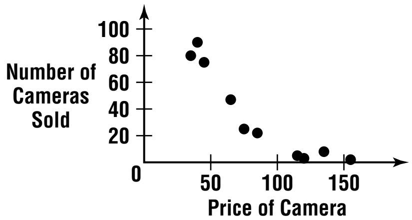


Year	Number of Games Played	Number of Goals Scored
1954	26	140
1938	18	84
1934	17	70
1950	22	88
1930	18	70
1958	35	126
1970	32	95
1982	52	146
1962	32	89
1966	32	89

You can use the scatter plot to draw conclusions about the data. The data points fall roughly along a line with a positive slope. Therefore, we say there is a positive relationship between the number of games played and the number of goals scored. A line with a negative slope would indicate a negative relationship, and no line would indicate no relationship between the variables. A valid conclusion from this scatter plot is that as the number of games played increases, more goals are scored.

Determine whether each scatter plot has a* positive *relationship,* negative *relationship, or* no *relationship. If there is a relationship, describe it.

1.



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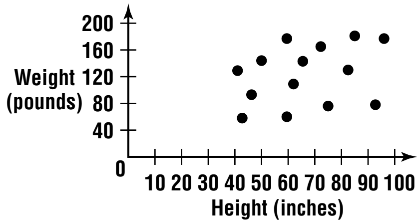

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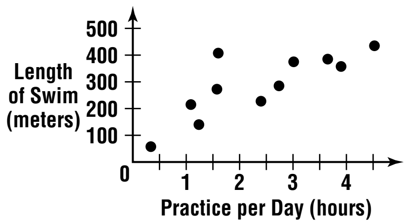

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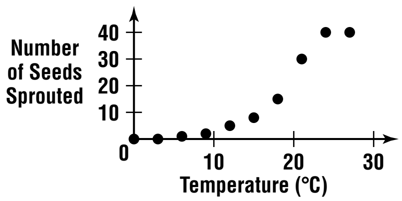

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Study Guide

Graphing Linear Equations

You may use the slope-intercept form to graph linear equations, as shown in the example below.

Example: Graph $2x + 3y = 6$.

Rewrite in slope-intercept form.

$$2x + 3y = 6$$

$$2x + 3y - 2x = 6 - 2x \qquad \textit{Subtract 2x from each side.}$$

$$3y = -2x + 6$$

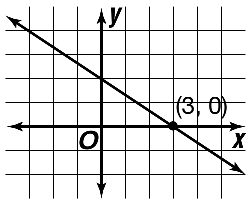
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3} \qquad \textit{Divide each side by 3.}$$

$$y = -\frac{2}{3}x + 2$$

To graph $y = -\frac{2}{3}x + 2$, plot a point at the y-intercept, 2.

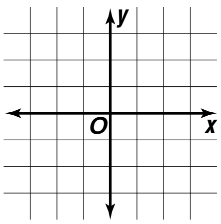
Then use the slope, $-\frac{2}{3}$. From (0, 2), go down 2 units.

Then go right 3 units. Graph a point at (3, 0).
Draw the line through the points.

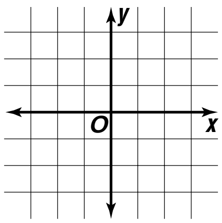


Graph each equation.

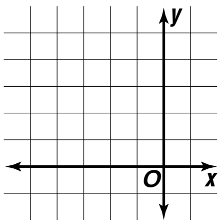
1. $y = -x - 1$



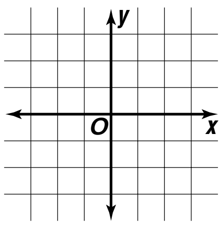
2. $y = \frac{4}{3}x - 2$



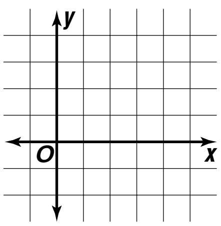
3. $y = \frac{1}{2}x + 4$



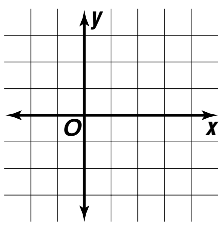
4. $y = -3$



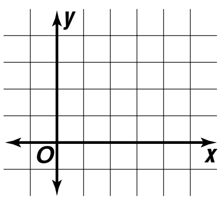
5. $2x + 4y = 8$



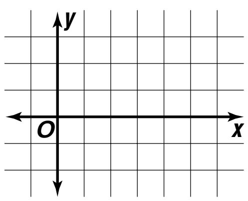
6. $2x - 3y = 6$



7. $x + 2y = 4$



8. $x - 5y = 5$



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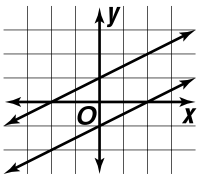
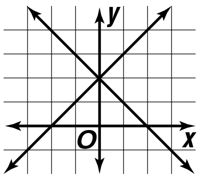
Study Guide

Families of Linear Graphs

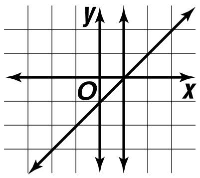
Graphs of linear equations that have at least one characteristic in common are called **families of graphs**. Graphs are families if they have the same slope, the same y-intercept, or the same x-intercept. An example of each is shown below.



same slope



same y-intercept

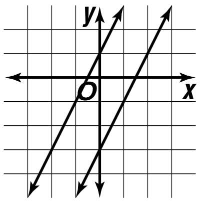


same x-intercept

Example: Graph the pair of equations. Explain why they are a family of graphs.

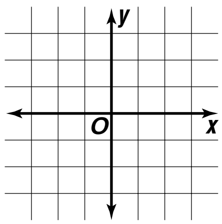
$y = 2x - 3$
$y = 2x + 1$

Since both graphs have the same slope, this is a family of graphs.

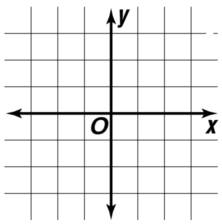


Graph each pair of equations. Explain why they are a family of graphs.

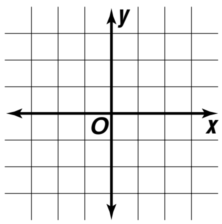
1. $y = -x - 1$
$y = -x + 2$



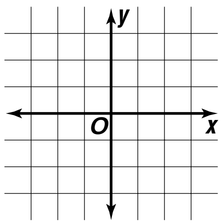
2. $y = \frac{4}{3}x - 2$
$y = \frac{3}{4}x - 2$



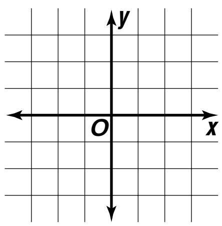
3. $y = x - 1$
$y = -x + 1$



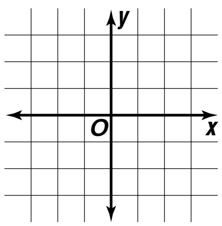
4. $2y = 3x - 6$
$4y = 6x$



5. $3y = x - 3$
$x = 3$



6. $2x - y = 2$
$y = x - 2$



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Study Guide

Parallel and Perpendicular Lines

Lines that have the same slope are **parallel**. The graphs of the equations of two lines are a family of graphs because they have the same slope. For example, the graphs of $y = 2x$ and $y = 2x - 3$ are parallel because their corresponding equations have the same slope, 2.

Lines whose slopes are negative reciprocals are **perpendicular**. That is, the product of the slopes is -1. For example, the graphs of $y = 2x$ and $y = -\frac{1}{2}x - 1$ are perpendicular because the product of the slopes, 2 and $-\frac{1}{2}$, is -1.

Parallel Lines	same slope, different y-intercept	$y = \frac{3}{4}x + 2$ $y = \frac{3}{4}x - 5$
Perpendicular Lines	Product of slopes is -1.	$y = -5x + 1$ $y = \frac{1}{5}x - 9$

***Determine whether the graphs of the equations are* parallel, perpendicular, *or* neither.**

1. $y = -x + 2$
$y = -x - 3$

2. $y = \frac{3}{4}x + 4$
$y = \frac{3}{4}x - 2$

3. $y = -\frac{1}{2}x + 3$
$y = 2x - 4$

4. $y = \frac{3}{2}x - 5$
$y = x + 3$

5. $y = \frac{3}{2}x + 2$
$y = -\frac{2}{3}x + 5$

6. $y = 3x + 2$
$-6x + 2y = -8$

7. $y = \frac{3}{2}x + 2$
$2x - y = 8$

8. $y = -2x + 4$
$x - 2y = 8$

9. $4y - 3x = 1$
$2y = \frac{3}{2}x - 14$

10. $y = 3 - x$
$2y - x = 8$

8–1

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Study Guide

Powers and Exponents

The product of a number with itself is called a **perfect square**. For example, 64 is a perfect square because $64 = 8 \times 8$. The **exponent** 2 can be used to write 8×8 as 8^2. Likewise, the exponent 4 can be used to write $8 \times 8 \times 8 \times 8$ as 8^4. Numbers that are expressed using exponents are called **powers**.

$$\text{base} \longrightarrow 8^{4} \longleftarrow \text{exponent}$$

Example 1: Write $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ using exponents.
The base is 3. There are 5 factors of 3, so the exponent is 5.
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

Example 2: Write $x \cdot x \cdot x \cdot x$ using exponents.
The base is x. There are 4 factors of x, so the exponent is 4.
$x \cdot x \cdot x \cdot x = x^4$

Example 3: Write $5x^2y^3$ as a multiplication expression.
There is 1 factor of 5, 2 factors of x, and 3 factors of y.
$5 \cdot x \cdot x \cdot y \cdot y \cdot y$

Write each expression using exponents.

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

2. $(-3)(-3)(-3)(-3)$

3. $x \cdot x \cdot x \cdot x \cdot x$

4. $(-2)\ a \cdot a \cdot a \cdot a \cdot b$

5. $10 \cdot 10 \cdot 10 \cdot 10$

6. $5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

Write each power as a multiplication expression.

7. 7^3

8. $-5y^4$

9. d^4e^3

10. $9ab^3$

11. wx^2y^3

12. $(-2)^5$

Evaluate each expression if x = −3, y = 2, and z = −1.

13. zx^3y^2

14. $y(x^2 - z)$

8–2

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Study Guide

Multiplying and Dividing Powers

You can multiply powers by using the following rules.

Product of Powers	Examples
To multiply powers with the same base, add the exponents. $a^m \cdot a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6 $x^3 \cdot x^2 = x^{3+2}$ or x^5 $(ab^2) \cdot (a^3b^4) =$ $(a \cdot a^3)(b^2 \cdot b^4)$ or a^4b^6

You can divide powers by using the following rules.

Quotient of Powers	Examples
To divide powers with the same base, subtract the exponents. $\frac{a^m}{a^n} = a^{m-n}$	$\frac{10^4}{10^3} = 10^{4-3}$ or 10^1 $\frac{x^5}{x^2} = x^{5-2}$ or x^3 $\frac{b^8c^4}{b^2c} = b^{8-2} \cdot c^{4-1}$ or b^6c^3

Simplify each expression.

1. $6^3 \cdot 6^2$

2. $(-3)\ x^2 \cdot x^3$

3. $y^8 \cdot y^9$

4. $(5m^3)(3m^2n^4)$

5. $(ab^2)(a^2b^3)$

6. $(-10x^4y^2)(3x^2y)$

7. $\frac{8^9}{8^7}$

8. $\frac{w^6}{w^3}$

9. $\frac{a^7b^2}{a^6b}$

10. $\frac{9j^5}{3j^2}$

11. $\frac{36x^9y^5}{18x^3y^2}$

12. $\frac{-24m^6n^3}{6m^2n^2}$

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Study Guide

Negative Exponents

In science and technology, negative exponents are sometimes used to represent very small numbers. For example, the diameter of an atom is expressed as 10^{-10} meter. This is the decimal 0.0000000001. This number expressed as a fraction is $\frac{1}{10{,}000{,}000{,}000}$.

When simplifying an expression with a negative exponent, you may need to use the Quotient of Powers rule.

Negative Exponents	Examples
$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2}$ or $\frac{1}{9}$ $x^{-3} = \frac{1}{x^3}$ $\frac{b^2}{b^{-3}} = b^{2-(-3)}$ or b^5

Remember that a negative exponent is used to write a reciprocal, not to represent a negative number.

Simplify each expression.

1. 5^{-3}

2. 3^{-2}

3. y^{-8}

4. $5m^{-3}$

5. $(a^{-2})(b^3)$

6. $-6x^{-4}y^6$

7. $\frac{3^4}{3^{-2}}$

8. $\frac{k^{-3}}{k^5}$

9. $\frac{12x^5}{4x^{-2}}$

10. $a^2b^{-2}c^{-1}$

11. $\frac{-24m^6n^3}{3m^2n}$

12. $\frac{36x^9y^5z^{-2}}{9x^3y^2}$

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Study Guide

Scientific Notation

In science and in other applications, **scientific notation** is often used to represent very small or very large numbers. For example, the speed of light is about 3×10^8 meters per second. This represents the number 300,000,000 meters per second.

In scientific notation, a number is expressed in the form $a \times 10^n$, where a is a number greater than 1 and less than 10 and n is an integer.

Scientific Notation	Examples
1. Place the decimal point after the first non-zero digit in the given number. **2.** Find the power of 10 by counting decimal places. When the given number is one or greater, the power of 10 is positive. When the given number is between zero and one, the exponent of 10 is negative.	$6{,}200{,}000 = 6.2 \times 10^6$ $0.000056 = 5.6 \times 10^{-5}$

Express each number in standard form.

1. 6.1×10^4

2. 4.8×10^{-2}

3. 8.12×10^3

4. 5×10^7

5. 9×10^{-5}

6. 1.1×10^{-7}

7. 2.15×10^5

8. 5.1651×10^3

Express each number in scientific notation.

9. 8400

10. 3,000,000

11. 0.05

12. 14.2

13. 0.00048

14. 82,000,000,000

15. 0.0000725

16. 6

8–5

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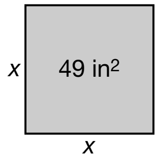
Study Guide

Square Roots

Suppose you know that the area of the square below is 49 square inches. What is the length of a side? Since $7 \times 7 = 49$, each side is 7 inches long.



Also, since $7 \times 7 = 49$, we say that the **square root** of 49 is 7. A shorter way to write this is with the symbol $\sqrt{}$, a **radical sign**. Write $\sqrt{49} = 7$. Use the following rules to simplify square roots.

Square Roots	Examples
1. The square root of a number is one of its equal factors.	$\sqrt{49} = 7$
2. The square root of a product is equal to the product of each square root. $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{225} = \sqrt{9} \cdot \sqrt{25} = 3 \cdot 5 = 15$
3. The square root of a quotient is equal to the quotient of each square root. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{25}{144}} = \frac{\sqrt{25}}{\sqrt{144}} = \frac{5}{12}$

Simplify.

1. $\sqrt{81}$

2. $-\sqrt{25}$

3. $\sqrt{100}$

4. $-\sqrt{400}$

5. $\sqrt{625}$

6. $-\sqrt{1156}$

7. $\sqrt{\frac{81}{49}}$

8. $\sqrt{\frac{25}{36}}$

9. $-\sqrt{\frac{16}{100}}$

10. $\sqrt{0.25}$

11. $-\sqrt{0.0016}$

12. $-\sqrt{\frac{0.16}{0.09}}$

8–6

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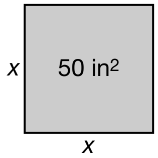
Study Guide

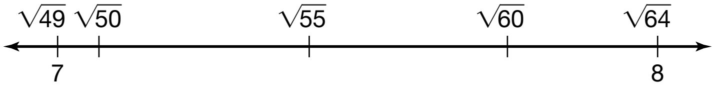
Estimating Square Roots

Suppose you know that the area of the square below is 50 square inches. What is the length of a side?



Since there is no rational number whose square is 50, you need to estimate the answer. If you use a calculator to find $\sqrt{50}$, it will return an approximate value of 7.071067812. This represents an **irrational number**, a decimal number that does not repeat or terminate.

You can use perfect squares to estimate irrational square roots. Since 50 is close to 49, $\sqrt{50} \approx 7$, so the length of the side of the square is about 7 inches. Likewise, if the area of a square is 60 square inches, the side length would be $\sqrt{60} \approx 8$ inches, since the actual value is close to $\sqrt{64}$, or 8.



Estimate each square root to the nearest whole number.

1. $\sqrt{90}$ **2.** $\sqrt{134}$ **3.** $\sqrt{17}$

4. $\sqrt{500}$ **5.** $\sqrt{1000}$ **6.** $\sqrt{98}$

7. $\sqrt{320}$ **8.** $\sqrt{5}$ **9.** $\sqrt{75}$

10. $\sqrt{84.5}$ **11.** $\sqrt{128.9}$ **12.** $\sqrt{0.025}$

13. $\sqrt{0.0075}$ **14.** $\sqrt{10.01}$ **15.** $\sqrt{0.9988}$

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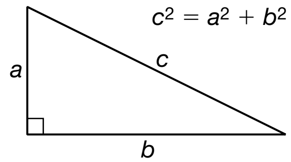
8–7 Study Guide

The Pythagorean Theorem

One of the most famous theorems in mathematics gives the relationship among the sides of a right triangle. The Pythagorean Theorem states that the square of the length of the **hypotenuse** of a right triangle is equal to the sum of the squares of the lengths of the other two sides, also known as **legs**. Use the model shown for a right triangle with hypotenuse c.



This relationship is powerful because it is true for *any* right triangle.

Example 1: Find the length of the hypotenuse of a right triangle with side lengths 5 centimeters and 12 centimeters.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144 = 169$$
$$c = \sqrt{169}$$
$$c = 13$$

The length of the hypotenuse is 13 centimeters.

Example 2: Find the length of a side of a right triangle if the length of the hypotenuse is 25 feet and the length of one leg is 7 feet.

$$c^2 = a^2 + b^2$$
$$25^2 = 7^2 + b^2$$
$$625 = 49 + b^2$$
$$625 - 49 = 49 + b^2 - 49$$
$$576 = b^2$$
$$b = \sqrt{576}$$
$$b = 24$$

The length of the leg is 24 feet.

If c is the measure of the hypotenuse of a right triangle and a and b are the measures of the legs, find each missing measure.

1. $a = 3, b = 4, c = ?$

2. $a = 7, b = 24, c = ?$

3. $a = 9, b = 40, c = ?$

4. $a = 5, b = ?, c = 13$

5. $a = 20, b = ?, c = 29$

6. $a = ?, b = 8, c = 10$

7. $a = ?, b = 48, c = 50$

8. $a = 9, b = ?, c = 15$

9. $a = 60, b = 80, c = ?$

10. $a = ?, b = 36, c = 45$

11. $a = 40, b = 42, c = ?$

12. $a = 25, b = ?, c = 65$

9–1

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Study Guide

Polynomials

The expressions y, $-6x$, $5a^2$, and $10cd^3$ are examples of **monomials**. A monomial is a number, a variable, or a product of numbers and variables. Any exponents in a monomial are positive integers. The exponents cannot be variables.

Example: Which of the following expressions are monomials?

$-8g$ $\quad 2z^{-4}$ $\quad 17t^5v$ $\quad 3 - a$ $\quad \frac{9}{pq}$ $\quad 6^y$

$-8g$ and $17t^5v$ are monomials because they are products of numbers and variables.

$2z^{-4}$ is not a monomial because it has a negative exponent.

$3 - a$ is not a monomial because it involves subtraction.

$\frac{9}{pq}$ is not a monomial because it involves division.

6^y is not a monomial because its exponent is a variable.

The sum of two or more monomials is called a **polynomial**. Each monomial is a term of the polynomial. Polynomials with two or three terms have special names.

$15r^4 + 1$ is a **binomial**. It has two terms, $15r^4$ and 1.

$-9 + g - 4g^2$ is a **trinomial**. It has three terms, -9, g, and $-4g^2$.

Determine whether each expression is a monomial. Explain why or why not.

1. $18x + 2$

2. $-21s^4t^2$

3. w^{-2}

4. $\frac{4}{5}a^3b$

***State whether each expression is a polynomial. If it is a polynomial, identify it as a* monomial, binomial, *or* trinomial.**

5. $\frac{8}{x}$

6. $-7r + 9s - 3$

7. $abc^3 - a^3bc$

8. $35u^5v^6$

9. $5 + 5^k$

10. $8d - 9e \div f$

11. $16x - 16y$

12. $8j^2 + 3j - 7$

13. $3m^3 + \frac{1}{3}m$

14. $-14p + p^{-14}$

9–2

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Study Guide

Adding and Subtracting Polynomials

To add polynomials, group the **like terms** together and then find the sum. $3x^2$ and x^2 are like terms. x^2 and x, and x^2 and y^2 are unlike terms.

Example 1: Find $(3x^2 + 2x - 5) + (x^2 + 4x + 4)$.

$(3x^2 + 2x - 5) + (x^2 + 4x + 4)$
$= (3x^2 + 2x + (-5)) + (x^2 + 4x + 4)$ *Rewrite subtraction.*
$= (3x^2 + x^2) + (2x + 4x) + (-5 + 4)$ *Regroup like terms.*
$= (3 + 1)x^2 + (2 + 4)x + (-5 + 4)$ *Distributive property*
$= 4x^2 + 6x - 1$ *Simplify.*

You can subtract a polynomial by adding its additive inverse.

Example 2: Find the additive inverse of $5b^2 - 3$.

The additive inverse is $-(5b^2 - 3)$ or $-5b^2 + 3$.

Example 3: Find $(4m^3 - 6) - (7m^3 - 9)$.

$(4m^3 - 6) - (7m^3 - 9)$
$= (4m^3 - 6) + (-7m^3 + 9)$ *The additive inverse of $7m^3 - 9$ is $-7m^3 + 9$.*
$= (4m^3 - 7m^3) + (-6 + 9)$ *Regroup like terms.*
$= (4 - 7)m^3 + (-6 + 9)$ *Distributive property*
$= -3m^3 + 3$ *Simplify.*

Find each sum or difference.

1. $(2a + 3) + (5a + 1)$

2. $(8w^2 + w) + (7w^2 - 3w)$

3. $(-5c^4 + 2c^2 - 6) + (6c^4 - 2c^2 + 5)$

4. $(12m - 5n) + (12m + 5n)$

5. $(4g + h^3) + (-9g - 4h^3)$

6. $(2 - 16x^2) + (8 - 16x^2)$

7. $(18 + 5xy) + (-6 - 10xy)$

8. $(35a^2 + 15a - 20) + (10a^2 + 25)$

9. $(6d + 3) - (4d + 5)$

10. $(14 - 3t) - (2 + 7t)$

11. $(-18s^2 + s) - (6s^2 - 8s)$

12. $(26g - 13gh) - (-2g + gh)$

13. $(7y^2 + 2y + 21) - (9y^2 + 6y + 11)$

14. $(-5m^2 + 2n - 1) - (7m^2 + 16n - 8)$

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Study Guide

Multiplying a Polynomial by a Monomial

You can use the Distributive Property to multiply a polynomial by a monomial.

Example 1: Multiply $6(x + 7)$.

$$\begin{aligned} 6(x + 7) &= 6(x) + 6(7) && \textit{Distributive Property.} \\ &= 6x + 42 && \textit{Simplify.} \end{aligned}$$

Example 2: Multiply $-2(g^2 + 3g - 5)$.

$$\begin{aligned} -2(g^2 + 3g - 5) &= -2(g^2) + (-2)(3g) + (-2)(-5) \\ &= -2g^2 - 6g + 10 \end{aligned}$$

Example 3: Multiply $9a(a + 1)$.

$$\begin{aligned} 9a(a + 1) &= 9a(a) + 9a(1) \\ &= 9a^2 + 9a \end{aligned}$$

Some equations require that you multiply polynomials.

Example 4: Solve $5(x + 3) = 25$.

$$\begin{aligned} 5(x + 3) &= 25 \\ 5x + 15 &= 25 && \textit{Distributive Property} \\ 5x + 15 - 15 &= 25 - 15 && \textit{Subtract 15 from each side.} \\ 5x &= 10 && \textit{Combine like terms.} \\ x &= 2 && \textit{Divide each side by 5.} \end{aligned}$$

Find each product.

1. $8(x + 2)$

2. $-3(x + 5)$

3. $10(2a - b)$

4. $4(v^2 - 4v + 9)$

5. $-3y(y - 1)$

6. $2r(-2r^2 + 6r - 5)$

7. $0.3(2p + 4)$

8. $4.5(m^3 + m^2)$

9. $\frac{1}{2}(z + 10)$

Solve each equation.

10. $2(y + 3) = 10$

11. $-5(x + 8) = 5$

12. $7(3s - 1) = -49$

13. $-4(-2w + 7) = -20$

14. $6(d + 1) - 6 = 18$

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Study Guide

Multiplying Binomials

The Distributive Property can be used to multiply binomials.

Example 1: Multiply $(x + 2)(x + 5)$.

$$\begin{aligned}(x + 2)(x + 5) &= x(x + 5) + 2(x + 5) && \textit{Distributive Property}\\ &= x(x) + x(5) + 2(x) + 2(5) && \textit{Distributive Property}\\ &= x^2 + 5x + 2x + 10 && \textit{Simplify.}\\ &= x^2 + 7x + 10 && \textit{Combine like terms.}\end{aligned}$$

Example 2: Multiply $(3x + 1)(x - 4)$.

$$\begin{aligned}(3x + 1)(x - 4) &= 3x(x - 4) + 1(x - 4) && \textit{Distributive Property}\\ &= 3x(x) + 3x(-4) + 1(x) + 1(-4) && \textit{Distributive Property}\\ &= 3x^2 - 12x + x - 4 && \textit{Simplify.}\\ &= 3x^2 - 11x - 4 && \textit{Combine like terms.}\end{aligned}$$

You can also use a shortcut called the **FOIL method** to multiply two binomials. Find the four products indicated by the letters in the word FOIL. Then add the like terms.

Example 3: Multiply $(y + 4)(y - 3)$.

	F		O		I		L
	First terms	+	**O**uter terms	+	**I**nner terms	+	**L**ast terms
$(y + 4)(y - 3) =$	$y(y)$	+	$y(-3)$	+	$4(y)$	+	$4(-3)$

$= y^2 - 3y + 4y - 12$ *Add the like terms.*

$= y^2 + y - 12$

Find each product.

1. $(x + 6)(x + 3)$

2. $(y + 4)(y - 2)$

3. $(m - 3)(m - 1)$

4. $(h - 10)(h + 7)$

5. $(w - 8)(w - 8)$

6. $(g - 2)(g + 2)$

7. $(2a + 5)(a + 3)$

8. $(p + 1)(2p + 3)$

9. $(3x - 4)(x + 1)$

10. $(z + 5)(4z - 3)$

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Study Guide

Special Products

Recall x^2 means that x is used as a factor twice.
Thus, $x^2 = x \cdot x$. When a binomial is squared, it is also used as a factor twice.

Therefore $(x + 3)^2 = (x + 3)(x + 3)$.
$= x^2 + 3x + 3x + 9$ *The inner and outer products are equal.*
$= x^2 + 6x + 9$

Similarly, $(x - 3)^2 = (x - 3)(x - 3)$.
$= x^2 - 3x - 3x + 9$ *The inner and outer products are equal.*
$= x^2 - 6x + 9$

Look at this special product of two binomials.

$(x + 3)(x - 3) = x^2 + 3x - 3x - 9$ *The inner and outer products are opposites.*
$= x^2 - 9$

Square of a Sum	Square of a Difference	Product of a Sum and a Difference
$(a + b)^2 = a^2 + 2ab + b^2$	$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$

Example 1: Find $(r - 7)^2$.

$(a - b)^2 = a^2 - 2ab + b^2$ *Square of a difference*
$(r - 7)^2 = r^2 - 2(r)(7) + 7^2$ *Replace a with r and b with 7.*
$= r^2 - 14r + 49$

Example 2: Find $(6y - 5)(6y + 5)$.

$(a + b)(a - b) = a^2 - b^2$ *Product of a sum and a difference*
$(6y - 5)(6y + 5) = (6y)^2 - 5^2$ *Replace a with 6y and b with 5.*
$= 36y^2 - 25$

Find each product.

1. $(y + 8)^2$ **2.** $(z - 4)^2$ **3.** $(9 + a)^2$

4. $(5b + 1)^2$ **5.** $(3d - e)^2$ **6.** $(1 + 5j)^2$

7. $(x + 5)(x - 5)$ **8.** $(q + 7)(q - 7)$ **9.** $(m + 10)(m - 10)$

10. $(k + 2)(k - 2)$ **11.** $(6x + 1)(6x - 1)$ **12.** $(2s + 3)(2s - 3)$

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Study Guide

Factors

Because $3 \times 4 = 12$, we say that 3 and 4 are **factors** of 12. In other words, factors are the numbers you multiply to get a product. Since $2 \times 6 = 12$, 2 and 6 are also factors of 12. The only factors of 5 are 1 and 5.

Numbers like 5 that have have exactly two factors, the number itself and 1, are called **prime numbers**.

Prime Number	2	3	29
Factors	1, 2	1, 3	1, 29
Products of Factors	1×2	1×3	1×29

Numbers like 12 that have more than two factors are called **composite numbers.**

Composite Number	4	12	33
Factors	1, 2, 4	1, 2, 3, 4, 6, 12	1, 3, 11, 33
Products of Factors	1×4 2×2	1×12 2×6 3×4	1×33 3×11

When two numbers are written as the product of their prime factors, they are in factored form.

Example 1: Write 45 in factored form.

$$\begin{aligned} 45 &= 9 \cdot 5 \\ &= 3 \cdot 3 \cdot 5 \end{aligned}$$ *Keep factoring until all factors are prime numbers.*

The factored form of 45 is $3 \cdot 3 \cdot 5$.

Example 2: Write $12x^2y$ in factored form.

$$\begin{aligned} 12x^2y &= 3 \cdot 4 \cdot x \cdot x \cdot y \\ &= 3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \end{aligned}$$

The factored form of $12x^2y$ is $3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$.

Find the factors of each number. Then classify each number as prime or composite.

1. 10 **2.** 7

3. 15 **4.** 21

5. 31 **6.** 49

7. 47 **8.** 39

Factor each monomial.

9. $18a$ **10.** $35xy$ **11.** c^3

12. $20r^2$ **13.** $6y^2z$

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Study Guide

Factoring Using the Distributive Property

When you use the Distributive Property to multiply a monomial by a polynomial, you show two factors and a product.

Distributive Property	Factors	Product
$3a(5a + 4) = 15a^2 + 12a$	$3a$ and $5a + 4$	$15a^2 + 12a$
$-2x(x^2 + 6x - 1) = -2x^3 - 12x^2 + 2x$	$-2x$ and $x^2 + 6x - 1$	$-2x^3 - 12x^2 + 2x$
$5rs(4r + 2s) = 20r^2s + 10rs^2$	$5rs$ and $4r + 2s$	$20r^2s + 10rs^2$

When you reverse the Distributive Property to identify the factors of the product, the polynomial is said to be in factored form. This is called **factoring** the polynomial.

Example: Factor $15ab^2 + 12a^2b^2$.

$15ab^2 = 3 \cdot 5 \cdot a \cdot b \cdot b$
$12a^2b^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b$

Begin by factoring each monomial. Then identify the factors both monomials have in common.

The common factors of $15ab^2$ and $12a^2b^2$ are $3 \cdot a \cdot b \cdot b$, so the greatest common factor is $3ab^2$.

$15ab^2 = 3ab^2(5)$
$12a^2b^2 = 3ab^2(4a)$

Now write each monomial as a product of $3ab^2$ and its other factors.

The factored form for $15ab^2 + 12a^2b^2$ is $3ab^2(5 + 4a)$.

Use the Distributive Property to check that the factored form is equivalent to the given polynomial.

Check: $3ab^2(5 + 4a) = 3ab^2(5) + 3ab^2(4a)$ or $15ab^2 + 12a^2b^2$ ✓

***Factor each polynomial. If the polynomial cannot be factored, write* prime.**

1. $12a + 3b$

2. $8w + 6$

3. $15d^2 - 18d$

4. $5c^4 + 2c^2$

5. $12mn^3 - 5n$

6. $4g + 13h^3$

7. $2 - 16x^2$

8. $35xy^3 + 7x^2y$

9. $27pw - 25q$

10. $48c^2d^2 + 36c^2d$

11. $ad + 3x^2 + 9$

12. $7gh^2 + 7g + 14gh$

13. $35a^2 + 15a - 20ab^2$

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Study Guide

Factoring Trinomials: $x^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $x^2 + 5x + 6$.

The first term in the trinomial is x^2. Since $x \cdot x = x^2$, the first term of each binomial is x.

$$x^2 + 5x + 6 = (x + \blacksquare)(x + \blacksquare)$$

To find the last terms, find a number pair whose product is 6 and whose sum is 5.

Product	Factors	Sum
6	1, 6	$1 + 6 = 7$
6	2, 3	$2 + 3 = 5$ ✓

Therefore, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Example 2: Factor $x^2 - 8x + 12$.

The first terms are both x. To find the last terms, find a number pair whose product is 12 and whose sum is -8.

Product	Factors	Sum
12	$-1, -12$	$-1 + (-12) = -13$
12	$-2, -6$	$-2 + (-6) = -8$ ✓
12	$-3, -4$	

Once the correct sum is found, it is not necessary to check any more factors. Therefore, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Example 3: Factor $x^2 - 2x - 15$.

The first terms are both x. To find the last terms, find a number pair whose product is -15 and whose sum is -2.

Product	Factors	Sum
-15	$1, -15$	$1 + (-15) = -14$
-15	$-1, 15$	$-1 + 15 = 14$
-15	$3, -5$	$3 + (-5) = -2$ ✓

Therefore, $x^2 - 2x - 15 = (x + 3)(x - 5)$.

Factor each trinomial.

1. $x^2 + 3x + 2$

2. $w^2 + 6w + 9$

3. $r^2 + 14r + 24$

4. $z^2 - 6z + 5$

5. $f^2 - 6f + 8$

6. $x^2 - 15x + 56$

7. $v^2 + 15v + 36$

8. $k^2 - 23k + 42$

9. $y^2 - 20y + 100$

10. $a^2 + 4a - 45$

11. $x^2 + 7x - 18$

12. $m^2 - 21m - 22$

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Study Guide

Factoring Trinomials: $ax^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $5x^2 + 37x + 14$.

The first term in the trinomial is $5x^2$. The only factors of 5 are 5 and 1, so the first terms of the binomials are $5x$ and x.

$$5x^2 + 37x + 14 = (5x + \blacksquare)(x + \blacksquare)$$

The last term in the trinomial is 14, which has two pairs of factors, 1 and 14, and 2 and 7. Try the factor pairs until you find the one that gives a middle term of $37x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$5x, x$	1, 14	$(5x + 1)(x + 14)$	$x + 70x = 71x$	$5x^2 + 71x + 14$
$5x, x$	14, 1	$(5x + 14)(x + 1)$	$14x + 5x = 19x$	$5x^2 + 19x + 14$
$5x, x$	2, 7	$(5x + 2)(x + 7)$	$2x + 35x = 37x$	$5x^2 + 37x + 14$ ✓

Therefore, $5x^2 + 37x + 14 = (5x + 2)(x + 7)$.

Example 2: Factor $6x^2 - 23x + 7$.

There are two possible factor pairs of the first term, $2x$ and $3x$, and $6x$ and x. The last term is positive. The sum of the inside and outside terms is negative. So, the factors of 7 are -1 and -7. Try the factor pairs until you find the one that gives a middle term of $-23x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$2x, 3x$	$-1, -7$	$(2x - 1)(3x - 7)$	$-3x - 14x = -17x$	$6x^2 - 17x + 7$
$3x, 2x$	$-1, -7$	$(3x - 1)(2x - 7)$	$-2x - 21x = -23x$	$6x^2 - 23x + 7$ ✓

Therefore, $6x^2 - 23x + 7 = (3x - 1)(2x - 7)$.

Factor each trinomial.

1. $3x^2 + 4x + 1$

2. $2w^2 + 3w + 1$

3. $2r^2 + 5r + 3$

4. $8z^2 + 14z + 5$

5. $5f^2 + 27f + 10$

6. $2x^2 - 3x + 1$

7. $7v^2 - 10v + 3$

8. $9k^2 - 9k + 2$

9. $4y^2 + 3y - 1$

10. $5a^2 + 6a - 8$

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Study Guide

Special Factors

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Trinomials like the one above that have two equal binomial factors are called **perfect square trinomials**. Recall that when a number is multiplied by itself, the result is a perfect square. For example, $4 \cdot 4 = 16$, so 16 is a perfect square.

Factoring Perfect Square Trinomials	**Symbols:** $a^2 + 2ab + b^2 = (a + b)(a + b)$ $a^2 - 2ab + b^2 = (a - b)(a - b)$ **Example:** $x^2 - 2x + 1 = (x - 1)(x - 1)$

Studying these properties of $x^2 + 10x + 25$ will help you factor other perfect square trinomials.

1. The first term, x^2, is a perfect square. $x \cdot x = x^2$
2. The last term, 25, is a perfect square. $5 \cdot 5 = 25$
3. The middle term, $10x$, is twice the product of 5 and x. $2(5x) = 10x$

Example 1: Factor $y^2 - 14y + 49$.

The first term is a perfect square. The last term is a perfect square. The middle term is twice the product of the first and last terms.

So $y^2 - 14y + 49$ is a perfect square trinomial.

$y^2 - 14y + 49 = (y - 7)(y - 7)$ or $(y - 7)^2$

When two perfect squares are subtracted, the polynomial is called the **difference of two squares**. The difference of two squares also has a special pair of factors.

Factoring Differences of Squares	**Symbols:** $a^2 - b^2 = (a - b)(a + b)$ **Example:** $x^2 - 25 = (x - 5)(x + 5)$

Example 2: Factor $g^2 - 36$.

$g^2 - 36$ is the difference of two squares.

$g^2 - 36 = (g - 6)(g + 6)$

Factor each perfect square trinomial.

1. $y^2 + 16y + 64$

2. $a^2 + 14a + 49$

3. $25s^2 + 10s + 1$

4. $r^2 - 8r + 16$

5. $p^2 - 20p + 100$

6. $36h^2 - 12h + 1$

7. $4a^2 + 12a + 9$

8. $9v^2 + 24v + 16$

Factor each difference of squares.

9. $b^2 - 64$

10. $k^2 - 4$

11. $81 - x^2$

12. $36m^2 - 1$

13. $4y^2 - 9$

14. $4 - 25d^2$

Study Guide

Graphing Quadratic Functions

Graphing **quadratic functions** of the form $y = ax^2 + bx + c$, where $a \neq 0$, can be simplified if you know some common characteristics.

Characteristic	Effect
sign of a: $a > 0$ $a < 0$	graph opens upward graph opens downward
axis of symmetry	vertical line at $x = -\frac{b}{2a}$
vertex	maximum or minimum point of the graph; x-coordinate is $-\frac{b}{2a}$

Example: Use characteristics of quadratic functions to graph $y = x^2 - 2x - 1$.

Step 1 First identify a, b, and c in $y = ax^2 + bx + c$: $a = 1$, $b = -2$, and $c = -1$. Since a > 0, the graph opens upwards.

Step 2 Find the axis of symmetry.

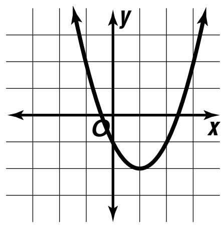
$x = -\frac{b}{2a}$ *Equation of the axis of symmetry*

$x = -\frac{-2}{2(1)}$ or 1 *$a = 1$ and $b = -2$*

Step 3 Find the vertex. Since the equation of the axis of symmetry is $x = 1$, the x-coordinate of the vertex is 1. Substitute 1 into the equation $y = x^2 - 2x - 1$ to get $y = (1)^2 - 2(1) - 1$ or -2. The vertex is at $(1, -2)$.

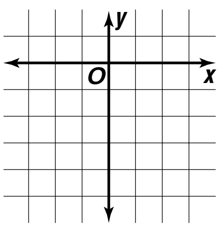
Step 4 Construct a table using values for x that will be on both sides of the axis of symmetry. Choose x-values less than 1 and x-values greater than 1. Graph the points.

x	-1	0	1	2	3
y	2	-1	-2	-1	2

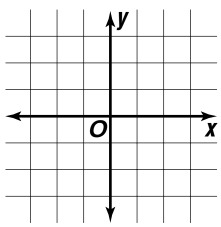


Graph each quadratic equation. Then give the coordinates of the vertex.

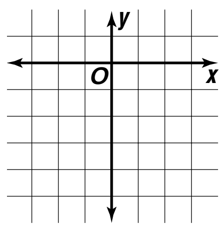
1. $y = -3x^2$



2. $y = x^2 + 2x$



3. $y = -x^2 + 2x - 2$



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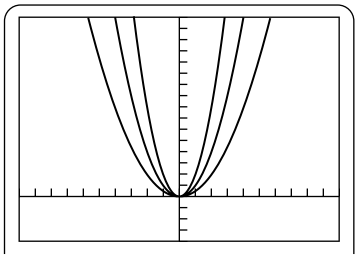
Study Guide

Families of Quadratic Functions

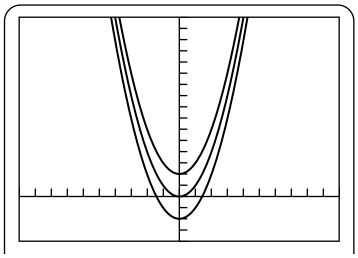
Families of parabolas are parabolas that share the same shape, vertex, or axis of symmetry. Graphing calculators make it easy to study families of graphs. Graph each of the following groups of equations on the same screen to compare and contrast the graphs.

$y = x^2$, $y = 2x^2$, $y = 0.5x^2$



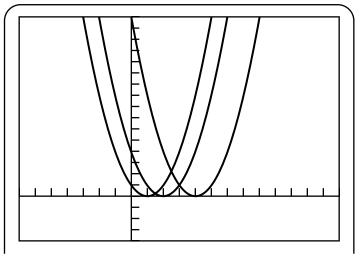
same vertex as $y = x^2$; open upward; different shapes

$y = x^2$, $y = x^2 - 2$, $y = x^2 + 2$



same shape as $y = x^2$; open upward; different vertices

$y = (x - 1)^2$, $y = (x - 2)^2$, $y = (x - 4)^2$



same shape; open upward; shift right

Example: Describe how the graph of $y = (x + 2)^2$ changes from the parent graph of $y = x^2$. Then name the vertex of each graph.

The constant -2 will make the value of the term $(x + 2)^2$ equal to 0. Therefore this graph will shift 2 units to the left. The vertex of $y = x^2$ is at (0, 0), while the vertex of $y = (x + 2)^2$ is at $(-2, 0)$.

Graph each group of equations on the same screen. Compare and contrast the graphs.

1. $y = -x^2$
$y = -4x^2$
$y = -5x^2$

2. $y = (x + 2)^2$
$y = (x + 4)^2$
$y = (x + 6)^2$

3. $y = x^2 - 1$
$y = x^2 - 2$
$y = x^2 - 3$

Describe how each graph changes from the parent graph of $y = x^2$. Then name the vertex of each graph.

4. $y = 4x^2$

5. $y = x^2 - 2$

6. $y = -x^2 - 1$

7. $y = (x + 1)^2$

8. $y = (x - 4)^2$

9. $y = 0.4x^2$

10. $y = x^2 + 3$

11. $y = (x + 3)^2$

12. $y = -x^2 - 5$

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Study Guide

Solving Quadratic Equations by Graphing

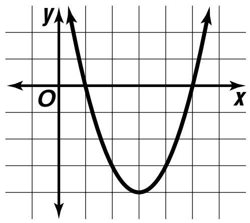
A **quadratic equation** is the equation you get if you set the related quadratic function equal to 0. For example, suppose $h(t) = -16t^2 + 40t + 4$ is the quadratic function representing the height h of a baseball at any time t. A solution to the quadratic equation $0 = -16t^2 + 40t + 4$ represents the time it takes for the ball to hit the ground. The solutions of a quadratic equation are called the **roots** of the equation. They are also the x-intercepts of the related quadratic function.

Example: Find the roots of $x^2 - 6x + 5 = 0$ by graphing the related quadratic function.

Before making a table of values, find the axis of symmetry.

$x = -\frac{b}{2a} = -\frac{-6}{2(1)}$ or 3 $\qquad$ *a = 1 and b = −6*

The equation of the axis of symmetry is $x = 3$. Now make a table using x-values around 3. Graph each point on the coordinate plane.



x	$x^2 - 6x + 5$	*f*(*x*)
1	$1^2 - 6(1) + 5$	0
2	$2^2 - 6(2) + 5$	−3
3	$3^2 - 6(3) + 5$	−4
4	$4^2 - 6(4) + 5$	−3
5	$5^2 - 6(5) + 5$	0

The x-intercepts of the function are 1 and 5. So the roots are 1 and 5.

Check: Substitute 1 and 5 for x in the equation $x^2 - 6x + 5 = 0$.

$$\begin{aligned} x^2 - 6x + 5 &\stackrel{?}{=} 0 \\ 1^2 - 6(1) + 5 &\stackrel{?}{=} 0 \\ 1 - 6 + 5 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned} \qquad \begin{aligned} x^2 - 6x + 5 &\stackrel{?}{=} 0 \\ 5^2 - 6(5) + 5 &\stackrel{?}{=} 0 \\ 25 - 30 + 5 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Solve each equation by graphing the related function.

1. $x^2 - 4x + 3 = 0$ **2.** $x^2 - 2x + 1 = 0$ **3.** $x^2 + 7x + 6 = 0$

4. $x^2 + 5x - 14 = 0$ **5.** $x^2 + 10x + 25 = 0$ **6.** $x^2 - 8x - 9 = 0$

7. $x^2 + 6x = 0$ **8.** $x^2 + x + 1 = 0$ **9.** $x^2 + 3x + 2 = 0$

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Study Guide

Solving Quadratic Equations by Factoring

The roots of a quadratic equation can be found by factoring trinomial expressions. Using the Zero Product Property shown below, set each factor equal to 0, and then solve each equation.

Zero Product Property	For all numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$, or both a and $b = 0$.

Example: Solve $x^2 + 7x + 6 = 0$ by factoring.

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$ *Factor.*

$x + 6 = 0$ or $x + 1 = 0$ *Zero Product Property*

$x = -6$ or $x = -1$ *Solve each equation.*

The solutions are -6 and -1.

Check: Substitute -6 and -1 for x in the equation $x^2 + 7x + 6 = 0$.

$$x^2 + 7x + 6 = 0 \qquad x^2 + 7x + 6 = 0$$
$$(-6)^2 + 7(-6) + 6 \stackrel{?}{=} 0 \qquad (-1)^2 + 7(-1) + 6 \stackrel{?}{=} 0$$
$$36 - 42 + 6 \stackrel{?}{=} 0 \qquad 1 - 7 + 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

Solve each equation by factoring. Check your solution.

1. $x^2 + 4x + 3 = 0$ **2.** $t^2 - 2t + 1 = 0$ **3.** $x^2 + 5x = 0$

4. $y^2 + 5y - 6 = 0$ **5.** $p^2 + 12p + 36 = 0$ **6.** $(k + 2)(k - 3) = 0$

7. $4m(m + 3) = 0$ **8.** $(g + 2)(g + 7) = 0$ **9.** $h^2 - h - 2 = 0$

10. $n(n - 3) = 0$ **11.** $(2g + 2)(g + 4) = 0$ **12.** $x^2 - 12x = 0$

13. $s^2 - 4s - 12 = 0$ **14.** $x^2 + 7x + 10 = 0$ **15.** $y^2 + 16y + 64 = 0$

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Study Guide

Solving Quadratic Equations by Completing the Square

Quadratic equations can be solved by **completing the square**. You complete the square for a quadratic expression of the form $x^2 + bx$, according to the steps shown below.

Completing the Square for a Quadratic $x^2 + bx$	**Step 1** Take $\frac{1}{2}$ of b and square it. **Step 2** Add the result to $x^2 + bx$. The perfect square is $x^2 + bx + \left(\frac{b}{2}\right)^2$.	**Example:** For $x^2 + 6x$, $b = 6$. $\frac{1}{2} \times 6 = 3$ $3^2 = 9$ The perfect square is $x^2 + 6x + 9$.

Once you determine $\left(\frac{b}{2}\right)^2$, you must add this number to each side of the equation. Finally, you find the square root of each side to solve the resulting equation.

Example: Solve $x^2 + 8x - 9 = 0$ by completing the square.

$x^2 + 8x - 9 = 0$	
$x^2 + 8x = 9$	*Add 9 to each side.*
$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 9 + \left(\frac{8}{2}\right)^2$	*Add the square of $\frac{1}{2}b$ to each side.*
$x^2 + 8x + 16 = 25$	
$(x + 4)^2 = 25$	*Factor $x^2 + 8x + 16$.*
$x + 4 = \pm\sqrt{25}$	*Take the square root of each side.*
$x = \pm 5 - 4$	*Subtract 4 from each side.*
$x = 5 - 4$ or $x = -5 - 4$	
$x = 1$ or $x = -9$	The solutions are 1 and -9.

Solve each equation by completing the square.

1. $x^2 + 6x + 9 = 0$

2. $m^2 - 7m - 8 = 0$

3. $y^2 + 4y = 6$

4. $y^2 + 14y = 0$

5. $t^2 - 12t - 7 = 0$

6. $z^2 - 8z - 9 = 0$

7. $n^2 + 5n = 0$

8. $x^2 + 4x = 1$

9. $k^2 + k - 2 = 0$

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Study Guide

The Quadratic Formula

A general formula for solving *any* quadratic equation is the **Quadratic Formula**. Begin with a quadratic equation in the general form $ax^2 + bx + c = 0$, where $a \neq 0$. Identify the values of a, b, and c. Then substitute the values into the Quadratic Formula. If the value of $b^2 - 4ac$ is negative, the equation has no real solutions.

The Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Example: Use the Quadratic Formula to solve $2x^2 + 3x - 8 = 0$.

$$2x^2 + 3x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *$a = 2; b = 3; c = -8$*

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-8)}}{2(2)}$$ *Substitute the values of a, b, and c into the Quadratic Formula.*

$$x = \frac{-3 \pm \sqrt{73}}{4}$$

The solutions are $\dfrac{-3 + \sqrt{73}}{4}$ and $\dfrac{-3 - \sqrt{73}}{4}$.

Use the Quadratic Formula to solve each equation.

1. $x^2 + 4x + 4 = 0$

2. $n^2 - 8n - 9 = 0$

3. $x^2 + 5x = 8$

4. $m^2 + 18m = 0$

5. $t^2 - 8t + 7 = 0$

6. $k^2 - 10k + 25 = 0$

7. $n^2 - 9n = 0$

8. $2a^2 + a = -6$

9. $m^2 + m - 6 = 0$

10. $n^2 - 6n + 9 = 0$

11. $2y^2 + y = 5$

12. $-x^2 + 9x - 7 = 0$

11–7

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Study Guide

Exponential Functions

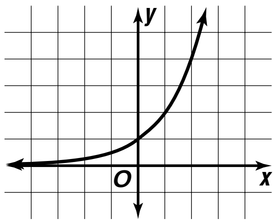
An **exponential function** is a function of the form $y = a^x$, where $a > 0$ and $a \neq 1$. If a is a value greater than one, then the function represents *exponential growth*. This can be very rapid growth, as in the case of $y = 2^x$, or less rapid growth as in the case of $y = 1.05^x$. To graph exponential functions, first make a table of ordered pairs.

x	2^x	y
-1	2^{-1}	0.5
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8

x	1.05^x	y
-1	1.05^{-1}	0.95
0	1.05^0	1
1	1.05^1	1.05
2	1.05^2	1.1
3	1.05^3	1.16

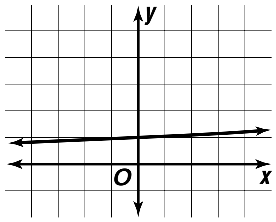
x	2^{2x}	y
-1	2^{-2}	0.25
0	2^0	1
1	2^2	4
2	2^4	16
3	2^6	64

$y = 2^x$



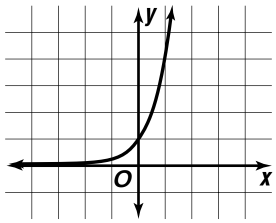
y-intercept = 1

$y = 1.05^x$



y-intercept = 1

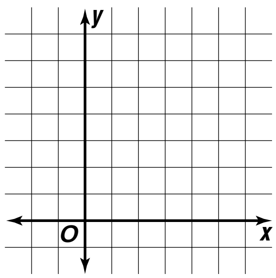
$y = 2^{2x}$



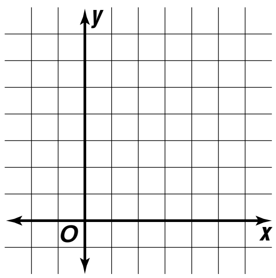
y-intercept = 1

Graph each exponential function.

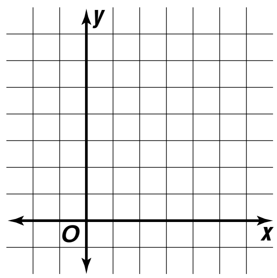
1. $y = 3^x - 1$



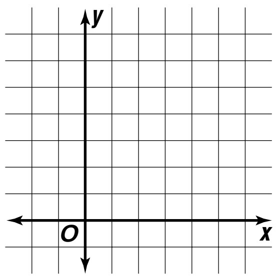
2. $y = 4^x + 2$



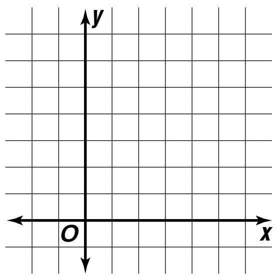
3. $y = 2^x - 1$



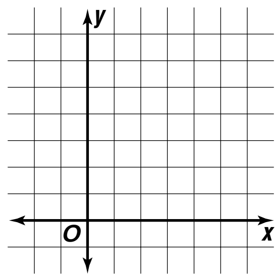
4. $y = 3^{2x}$



5. $y = 4^{2x}$



6. $y = 1.08^x$



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Study Guide

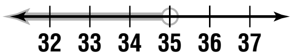
Inequalities and Their Graphs

Many mathematical relationships can be expressed with inequalities. For example, the President of the United States must be at least 35 years old. If a represents his or her age, this can be expressed with the inequality $a \geq 35$. Some verbal phrases that can be used for inequalities are listed in the chart below.

$<$	$\leq$	$>$	$\geq$
• less than • fewer than	• less than or equal to • at most • no more than • a maximum of	• greater than • more than	• greater than or equal to • at least • no less than • a minimum of

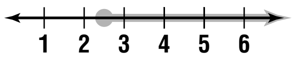
Example 1: Write and graph an inequality to describe the age of people who *cannot* be President of the United States.

Let a represent the age of a person who is less than 35 years old. Then $a < 35$. Since a cannot equal 35, graph a *circle* at 35. Then graph all numbers less than 35 by drawing a line and an arrow to the left.



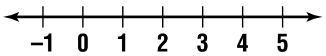
Example 2: Graph $n \geq 2.5$ on a number line.

Since n can equal 2.5, graph a *bullet* at 2.5. Then graph all numbers greater than 2.5 by drawing a line and an arrow to the right.

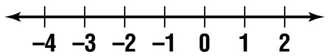


Write an inequality to describe each number. Then graph the inequality on a number line.

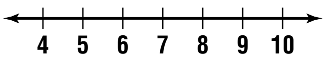
1. a number less than 3



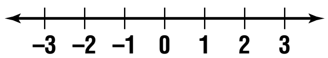
2. a number more than -2



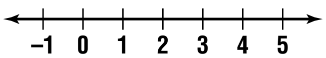
3. a maximum number of 8



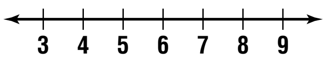
4. a number that is at least -1



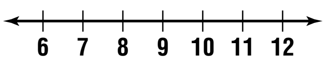
5. a number greater than or equal to $1\frac{1}{2}$



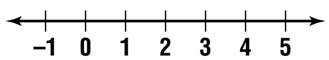
6. a minimum number of $6\frac{1}{5}$



7. a number that is at most 10.2



8. a number less than 2.4



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Study Guide

Solving Addition and Subtraction Inequalities

Suppose you already have \$50 and want to earn at least enough money to buy a DVD player for \$325. Let m = the amount of money you earn. You can represent this situation with the inequality $m + 50 \geq 325$. Then the solution to $m + 50 \geq 325$ is the amount of money you must earn. You can use the Addition and Subtraction Properties for Inequalities to solve inequalities involving addition or subtraction. The properties are summarized below.

Addition and Subtraction Properties for Inequalities
For all numbers a, b, and c,
1. if $a > \text{b}$, then $a + c > b + \text{c}$, and $a - c > b - \text{c}$.
2. if $a \geq \text{b}$, then $a + c \geq b + \text{c}$, and $a - c \geq b - \text{c}$.
3. if $a < \text{b}$, then $a + c < b + \text{c}$, and $a - c < b - \text{c}$.
4. if $a \leq \text{b}$, then $a + c \leq b + \text{c}$, and $a - c \leq b - \text{c}$.

Example: Solve $m + 50 \geq 325$. Check your solution.

$$m + 50 \geq 325$$
$$m + 50 - 50 \geq 325 - 50 \quad \textit{Subtract 50 from each side.}$$
$$m \geq 275$$

Check: Substitute a number less than 275, the number 275, and a number greater than 275 into the inequality.

Let $m = 200$.	Let $m = 275$.	Let $m = 300$.
$m + 50 \geq 325$	$m + 50 \geq 325$	$m + 50 \geq 325$
$200 + 50 \stackrel{?}{\geq} 325$	$275 + 50 \stackrel{?}{\geq} 325$	$300 + 50 \stackrel{?}{\geq} 325$
$250 \geq 325$; *false*	$325 \geq 325$; *true*	$350 \geq 325$; *true*

In *set-builder notation* the solution is {all numbers greater than or equal to 275}, or $\{m \mid m \geq 275\}$.

Solve each inequality. Check your solution.

1. $n + 3 > 6$

2. $x - 6 > -2$

3. $-2 + y \leq 8$

4. $x - 4 \leq 12$

5. $-3 \leq t + 2$

6. $1 + p > -1$

7. $y + 1.2 < 3.4$

8. $-2.6 + x > 1.9$

9. $-1.8 + y \leq 0$

10. $x - \frac{1}{2} > \frac{3}{4}$

11. $1 \leq y - \frac{2}{3}$

12. $p - \frac{1}{8} \geq 1\frac{1}{2}$

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Study Guide

Solving Multiplication and Division Inequalities

Suppose the family car gets 25 miles to a gallon of gasoline and you want to calculate how many gallons of gasoline you will need for a trip that is more than 300 miles long. Let g = the number of gallons of gasoline you will need. You can represent this situation with the inequality $25g > 300$. You can use the Multiplication and Division Properties for Inequalities to solve inequalities involving multiplication or division. The properties are summarized below. They are also true for $\geq$ and $\leq$.

Multiplication and Division Properties for Inequalities
For all numbers a, b, and c,
1. if c is positive and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
2. if c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
3. if c is negative and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
4. if c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Example 1: Solve $25g \geq 300$.

$$25g \geq 300$$
$$\frac{25g}{25} \geq \frac{300}{25} \quad \textit{Divide by 25.}$$
$$g \geq 12$$
$$\{g \mid g \geq 12\}$$

Example 2: Solve $\frac{y}{-2} \leq 8$.

$$\frac{y}{-2} \leq 8$$
$$-2\left(\frac{y}{-2}\right) \geq -2(8) \quad \textit{Multiply by } -2 \textit{ and reverse the symbol.}$$
$$y \geq -16$$
$$\{y \mid y \geq -16\}$$

Solve each inequality. Check your solution.

1. $3n > 6$

2. $\frac{x}{4} < 18$

3. $-2y \leq 8$

4. $3x > -9$

5. $-8 \leq \frac{t}{-2}$

6. $-2p > -1$

7. $2.4y < -4.8$

8. $-1.5x > 7.2$

9. $6.2y \leq 3.1$

10. $\frac{x}{12} > -3$

11. $\frac{n}{-3} \geq 1.4$

12. $7p > -7$

12–4

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Study Guide

Solving Multi-Step Inequalities

Solving inequalities may require more than one operation. The best strategy to use is to undo the operations in reverse order. In other words, first undo addition or subtraction and then undo multiplication or division, just as you did in solving equations with more than one operation. Remember that multiplying or dividing by a negative number reverses the inequality symbol.

Example 1: Solve $6 + 4x \geq 18$.

$$\begin{aligned} 6 + 4x &\geq 18 \\ 6 + 4x - 6 &\geq 18 - 6 && \textit{Subtract 6 from each side.} \\ 4x &\geq 12 \\ \frac{4x}{4} &\geq \frac{12}{4} && \textit{Divide each side by 4.} \\ x &\geq 3 \text{ or } \{x \mid x \geq 3\} \end{aligned}$$

Example 2: Solve $4 - 3x < -8 + x$.

$$\begin{aligned} 4 - 3x &< -8 + x \\ 4 - 3x - 4 &< -8 + x - 4 && \textit{Subtract 4 from each side.} \\ -3x &< -12 + x \\ -3x - x &< -12 + x - x && \textit{Subtract x from each side.} \\ -4x &< -12 \\ \frac{-4x}{-4} &> \frac{-12}{-4} && \textit{Divide each side by } -4\textit{. Reverse the symbol.} \\ x &> 3 \text{ or } \{x \mid x > 3\} \end{aligned}$$

Solve each inequality. Check your solution.

1. $2n + 8 > 26$

2. $6x - 12 \leq 48$

3. $-12 - 4y \leq 16$

4. $3x - 1 > -9 - x$

5. $-8 \leq \frac{t}{-2} + 2$

6. $3 \leq 3p + 2$

7. $2 - y < -1.6$

8. $-2x - 8 > 4.2$

9. $y - 3 \leq 2y - 3.1$

10. $3.2x - 16 > -3.2$

11. $6y - 8.2 \leq 36.8$

12. $-1 - 2x < 2$

12–5

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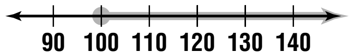
Study Guide

Solving Compound Inequalities

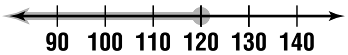
In a doctor's office, you may see a sign that displays the normal weight range for a person of your age and height. For example, if you are a 14-year-old girl who is 5 foot 2 inches tall, it may say that your normal weight is between 100 and 120 pounds, inclusive. Another way to write this information is to use an inequality. If w represents weight, then $100 \leq w \leq 120$ is a **compound inequality** that represents this situation. Another way to write the inequality is to write two inequalities using the word *and*: $100 \leq w$ and $w \leq 120$. A compound inequality using *and* is true if and only if *both* inequalities are true. The graph of a compound inequality using *and* is the **intersection** of the graphs of the inequalities, as shown below.

Example 1: Graph $100 \leq w$ and $w \leq 120$.

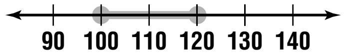
Step 1 Graph $100 \leq$ w.



Step 2 Graph $w \leq 120$



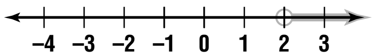
Step 3 Find the intersection of the graphs.



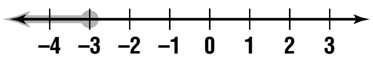
Another type of compound inequality uses the word *or*. A compound inequality using *or* is true if and only if either or both inequalities are true. Its graph is the **union** of the graphs of the inequalities, as shown below.

Example 2: Graph the solution of $x > 2$ or $x \leq -3$.

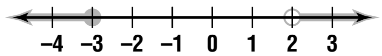
Step 1 Graph $x > 2$.



Step 2 Graph $x \leq -3$.

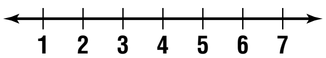


Step 3 Find the union of the graphs.

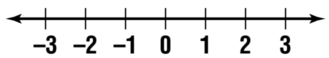


Graph the solution of each compound inequality.

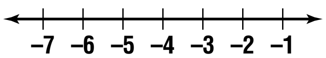
1. $n > 2$ and $n < 6$



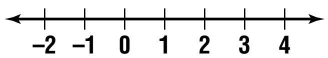
2. $x \leq -2$ or $x > 1$



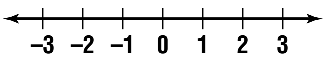
3. $y \leq -2$ and $y \geq -6$



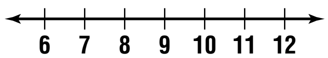
4. $1 \geq p$ and $p > 0$



5. $2 \leq y$ or $y < -1$



6. $h > 8$ and $h \leq 10$



12–6

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Study Guide

Solving Inequalities Involving Absolute Value

You have already studied equations of the form $|x| = n$ involving absolute value, where n is a nonnegative number. Inequalities involving absolute value are similar. They are of the form $|x| > n$ or $|x| < n$, where n is a nonnegative number.

To solve both equations and inequalities involving absolute value, there are two cases to consider.

Case 1 The value of the expression within the absolute value symbol is positive.

Case 2 The value of the expression within the absolute value symbol is negative.

Example 1: Solve $|x - 4| < 2$. Graph the solution.

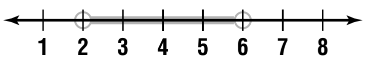
Case 1 $x - 4$ is positive.

$$x - 4 < 2$$
$$x - 4 + 4 < 2 + 4$$
$$x < 6$$

Case 2 $x - 4$ is negative.

$$-(x - 4) < 2$$
$$-(x - 4)(-1) > 2(-1) \quad \textit{Reverse the symbol.}$$
$$x - 4 > -2$$
$$x - 4 + 4 > -2 + 4$$
$$x > 2$$

The solution is $\{x | 2 < x < 6\}$.



Example 2: Solve $|x + 1| \geq 4$. Graph the solution.

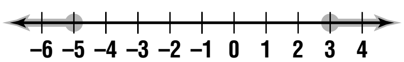
Case 1 $x + 1$ is positive.

$$x + 1 \geq 4$$
$$x + 1 - 1 \geq 4 - 1$$
$$x \geq 3$$

Case 2 $x + 1$ is negative.

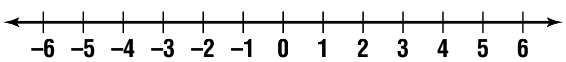
$$-(x + 1) \geq 4$$
$$-(x + 1)(-1) \leq 4(-1) \quad \textit{Reverse the symbol.}$$
$$x + 1 \leq -4$$
$$x + 1 - 1 \leq -4 - 1$$
$$x \leq -5$$

The solution is $\{x | x \geq 3 \text{ or } x \leq -5\}$.

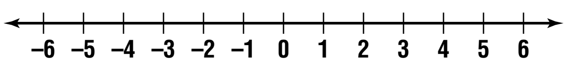


Solve each inequality. Graph the solution.

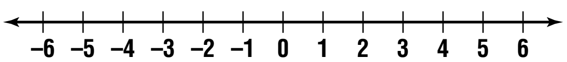
1. $|n| \leq 5$



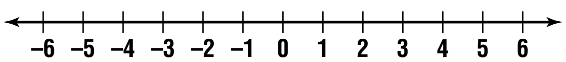
2. $|4x| \leq 12$



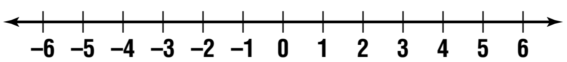
3. $|y + 1| > 2$



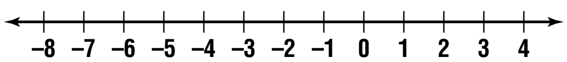
4. $|6p| > 2.4$



5. $|t + 3| \geq 2$



6. $|h + 2| < 6$



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Study Guide

Graphing Inequalities In Two Variables

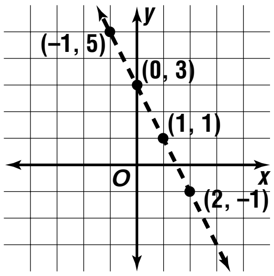
Inequalities, like equations, may have two variables instead of one. The solution of an inequality having two variables contains many ordered pairs. The graph of these ordered pairs fills an area of the coordinate plane called a **half-plane.** The graph of the related equation defines the **boundary** or edge for each half-plane.

Example: Graph $y < -2x + 3$.

Step 1 Determine the boundary by graphing the related equation, $y = -2x + 3$.

Make a table of values.

x	$-2x + 3$	y
-2	$-2(-2) + 3$	7
-1	$-2(-1) + 3$	5
0	$-2(0) + 3$	3
1	$-2(1) + 3$	1
2	$-2(2) + 3$	-1

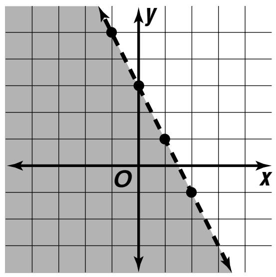


Step 2 Draw a dashed line because the boundary is not included. *Note:* If the inequality involved $\leq$ or $\geq$, the boundary would be included, and you would make the boundary a solid line.

Step 3 Use a point not on the boundary to find which half-plane is the solution. Use (0, 0).

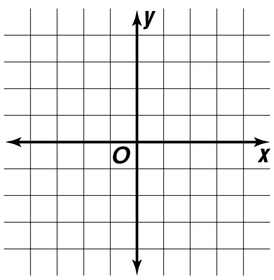
$y < -2x + 3$
$0 \overset{?}{<} -2(0) + 3$
$0 < 3$ *true*

Since $0 < 3$ is true, shade the half-plane containing (0, 0). *Note:* If the result were false, you would shade the other half-plane.

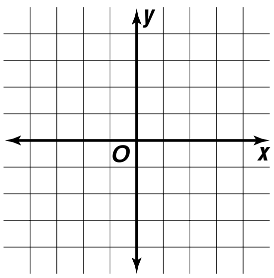


Graph each inequality.

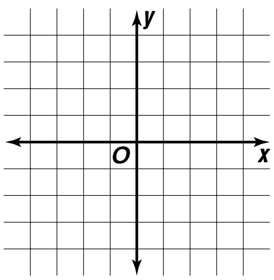
1. $y > x + 2$



2. $y \leq x - 2$



3. $y \leq -2x + 2$



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Study Guide

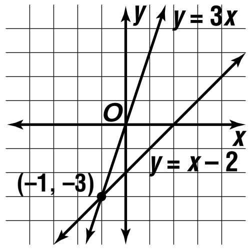
Graphing Systems of Equations

The ordered pair $(-1, -3)$ is the solution of the system of equations

$$y = x - 2$$
$$y = 3x$$

because when -1 is substituted for x and -3 is substituted for y, both equations are true.

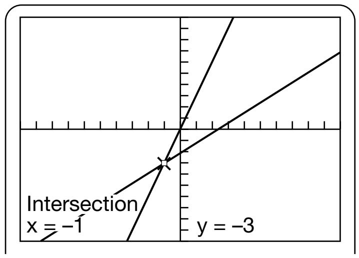
$y = x - 2$	$y = 3x$
$-3 \stackrel{?}{=} -1 - 2$	$-3 \stackrel{?}{=} 3(-1)$
$-3 = -3$ ✓	$-3 = -3$ ✓



You can also graph both equations to show that $(-1, -3)$ is the solution of the system.

The graphs appear to intersect at $(-1, -3)$. Since $(-1, -3)$ is the solution of each equation, it is the solution of the system of equations.

You can also use a graphing calculator to solve the system of equations.



Step 1 Enter these keystrokes in the Y= screen:

$\boxed{\text{X, T, }\theta\text{, }n}$ $\boxed{-}$ 2 $\boxed{\text{ENTER}}$

3 $\boxed{\text{X, T, }\theta\text{, }n}$ $\boxed{\text{ENTER}}$ $\boxed{\text{GRAPH}}$

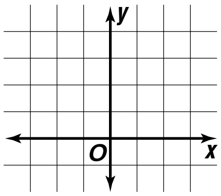
Step 2 Use the INTERSECT feature to find the intersection point.

$\boxed{\text{2nd}}$ [CALC] 5 $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$

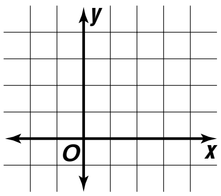
The solution is $(-1, -3)$.

Solve each system of equations by graphing.

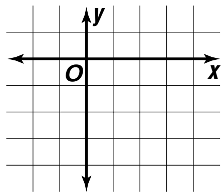
1. $x = -1$
$y = 3$



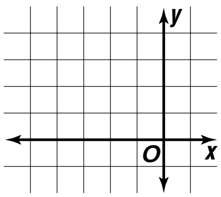
2. $y = 2$
$y = x$



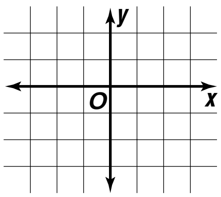
3. $y = -3$
$x = 2$



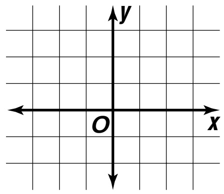
4. $y = -3x$
$y = x + 4$



5. $y = 1 - x$
$y = 2x - 5$



6. $y = -x - 2$
$y = 3x + 2$



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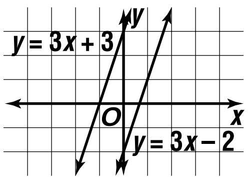
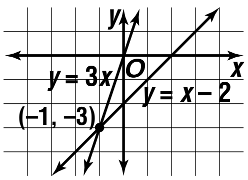
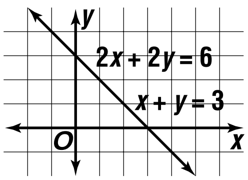
Study Guide

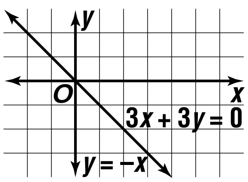
Solutions of Systems of Equations

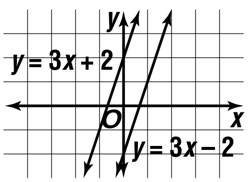
A system of linear equations is:

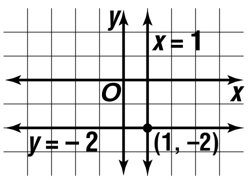
- **inconsistent** if the graphs of the equations are parallel. An inconsistent system has no solution.
- **consistent and independent** if the graphs of the equations intersect at one point. A consistent and independent system has one solution.
- **consistent and dependent** if the equations have the same graph. A consistent and dependent system has infinitely many solutions.

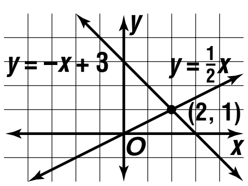
Graph	Description of Graph	Number of Solutions	Type of System
	parallel lines	0	inconsistent
	intersecting lines	1	consistent and independent
	same line	infinitely many	consistent and dependent

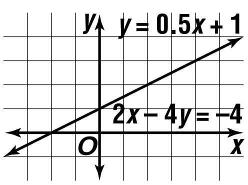
***State whether each system is* consistent and independent, consistent and dependent, *or* inconsistent.**

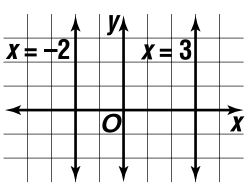
1. 

2. 

3. 

4. 

5. 

6. 

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Study Guide

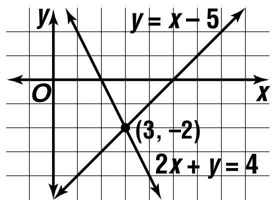
Substitution

You can use a method called *substitution* to solve a system of linear equations.

Example: Use substitution to solve the system of equations.

$y = x - 5$
$2x + y = 4$

The first equation tells you that y is equal to $x - 5$, so substitute $x - 5$ for y in the second equation. Then solve for x.



$2x + y = 4$	
$2x + x - 5 = 4$	*Replace y with x − 5.*
$3x - 5 = 4$	
$3x - 5 + 5 = 4 + 5$	*Add 5 to each side.*
$3x = 9$	
$\frac{3x}{3} = \frac{9}{3}$	*Divide each side by 3.*
$x = 3$	

Now choose one of the original equations. Substitute 3 for x in the equation you have chosen. Then solve for y.

$y = x - 5$	*Choose one of the original equations.*
$y = 3 - 5$	*Substitute 3 for x.*
$y = -2$	

The solution of the system of equations is $(3, -2)$.

Use substitution to solve each system of equations.

1. $y = x$
$x + y = 6$

2. $x + y = 0$
$y = 3$

3. $y = -3x$
$x + y = 8$

4. $y = x - 3$
$x + 2y = 6$

5. $x + y = 0$
$x - y = 4$

6. $y = 2x + 3$
$y - x = 10$

7. $x = -1$
$x + y = 5$

8. $y = 3 - 2x$
$4x + y = 5$

9. $x = y - 10$
$3x = y$

10. $2y = x + 6$
$y = 2x + 3$

11. $3x = y + 5$
$y = 2x - 5$

12. $y = 4x - 6$
$y = x - 3$

13. $x = 5y - 12$
$x - y = 0$

14. $3y = 2x - 3$
$y = -\frac{1}{3}x + 2$

15. $y = x - 6$
$5x - y = 6$

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Study Guide

Elimination Using Addition and Subtraction

In Lesson 13–3 you used substitution to solve systems of linear equations. You can also use the *elimination method* to solve systems of linear equations. When you use the elimination method, you eliminate one of the variables by adding or subtracting the equations. Add the equations to eliminate the variable whose coefficients are additive inverses. Subtract the equations to eliminate the variable whose coefficients are the same.

Example: Use elimination to solve the system of equations.

$2x - y = -3$
$2x + y = -9$

Step 1 The coefficients of y are -1 and 1, so add the equations to eliminate the y terms. Then solve for x.

$$\begin{array}{rl} 2x - y = -3 & \\ (+)2x + y = -9 & \textit{Add the equations.} \\ \hline 4x + 0 = -12 & \textit{The } y \textit{ terms are eliminated.} \\ 4x = -12 & \textit{Divide each side by 4.} \\ x = -3 & \textit{The value of } x \textit{ is } -3. \end{array}$$

Step 2 Replace x in one of the original equations with -3. Then solve for y.

$$\begin{array}{rl} 2x + y = -9 & \textit{Choose an equation.} \\ 2(-3) + y = -9 & \textit{Replace } x \textit{ with } -3. \\ -6 + y = -9 & \\ -6 + y + 6 = -9 + 6 & \textit{Add 6 to each side.} \\ y = -3 & \textit{The value of } y \textit{ is } -3. \end{array}$$

The solution of the system of equations is $(-3, -3)$.

You could also use subtraction to eliminate the x terms in the example.

Step 1 The coefficients of x are both 2, so subtract to eliminate the x terms.

$$\begin{array}{rl} 2x - y = -3 & \\ (-)2x + y = -9 & \textit{Subtract the equations.} \\ \hline 0 - 2y = 6 & \textit{The } x \textit{ terms are eliminated.} \\ -2y = 6 & \\ y = -3 & \end{array}$$

Step 2 Solve for x.

$$\begin{array}{rl} 2x + y = -9 & \\ 2x + (-3) = -9 & \textit{Replace } y \textit{ with } -3. \\ 2x = -6 & \\ x = -3 & \end{array}$$

The solution is $(-3, -3)$.

Use elimination to solve the system of equations.

1. $x - y = 2$
$x + y = 0$

2. $3x + 2y = 6$
$-3x + y = 0$

3. $2x - y = -4$
$-3x - y = 6$

4. $2x + y = 6$
$3x + y = 5$

5. $3x - 4y = 11$
$3x + 5y = -7$

6. $x + y = 6$
$-2x + y = -3$

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Study Guide

Elimination Using Multiplication

In some systems of equations, adding or subtracting the equations will not eliminate one of the variables. When this is true, you can eliminate by first multiplying one or both of the equations by a number, and then adding or subtracting the equations.

Example: Use elimination to solve the system of equations.

$$2x + y = 6$$
$$3x + 3y = 9$$

Adding or subtracting the equations will not eliminate the x terms or the y terms. If you multiply the first equation by -3, however, the y terms will be additive inverses. Then you can add the equations to eliminate the y terms.

$$\begin{array}{r} 2x + y = 6 \\ \underline{3x + 3y = 9} \end{array} \quad \boxed{\text{Multiply by } -3.} \Rightarrow \quad \begin{array}{r} -3(2x + y = 6) \\ \underline{3x + 3y = 9} \end{array} \quad \rightarrow \quad \begin{array}{r} -6x - 3y = -18 \\ \underline{(+)\ 3x + 3y = 9} \\ -3x + 0 = -9 \\ -3x = -9 \\ x = 3 \end{array}$$

Now find the value of y.

$2x + y = 6$	*Choose either of the original equations.*
$2(3) + y = 6$	*Replace x with 3.*
$6 + y = 6$	*Simplify. Then subtract 6 from each side.*
$y = 0$	

Check:

$2x + y = 6$	$3x + 3y = 9$
$2(3) + 0 \stackrel{?}{=} 6$	$3(3) + 3(0) \stackrel{?}{=} 9$
$6 = 6$ ✓	$9 = 9$ ✓

The solution of this system of equations is (3, 0).

Use elimination to solve each system of equations.

1. $x + 2y = 1$
$3x + y = 8$

2. $x + 11y = -6$
$2x + y = 9$

3. $8x - 3y = -32$
$x - y = 1$

4. $3x - 5y = 8$
$x + 2y = -1$

5. $3x - 4y = 5$
$x + 7y = 10$

6. $2x - y = 2$
$3x - 2y = 3$

7. $3x + 5y = 9$
$9x + 2y = -12$

8. $8x + 9y = -45$
$x + 6y = 9$

9. $12x - 10y = 30$
$2x + 5y = 15$

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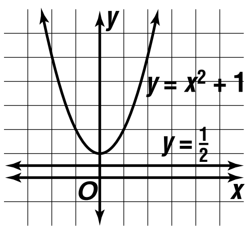
Study Guide

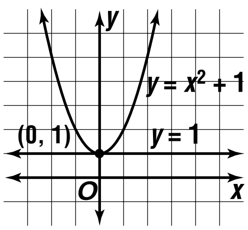
Solving Quadratic-Linear Systems of Equations

A system of equations that contains both a quadratic equation and a linear equation is called a **quadratic-linear** system of equations.

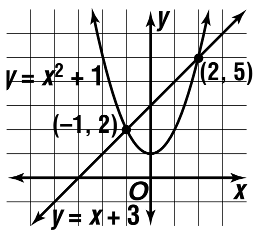
One way to solve quadratic-linear systems is to graph them and identify any ordered pairs that satisfy both equations. Another way is to use the substitution method.



The graphs do not intersect, so the system has no solution.



The graphs intersect at one point, so the system has one solution.



The graphs intersect at two points, so the system has two solutions.

Example: Use substitution to solve the system of equations.

$y = -2$
$y = x^2 - 11$

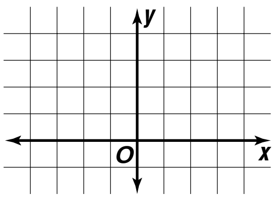
Since $y = -2$, substitute -2 for y in the second equation. Then solve for x.

$y = x^2 - 11$	
$-2 = x^2 - 11$	*Replace y with −2.*
$9 = x^2$	*Add 11 to each side.*
$3 = x$ or $-3 = x$	*Take the square root of each side.*

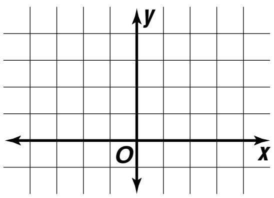
The solutions of the system of equations are $(-3, -2)$ and $(3, -2)$. Graphing the equations will show that they intersect at $(-3, -2)$ and $(3, -2)$.

Solve each system of equations by graphing.

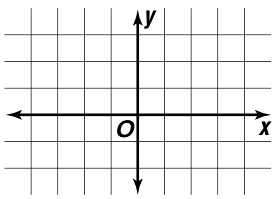
1. $x = -1$
$y = 2x^2$



2. $y = -1$
$y = x^2$



3. $y = x$
$y = x^2 - 2$



Use substitution to solve each system of equations.

4. $y = 4$
$y = x^2$

5. $x = -1$
$y = x^2$

6. $y = 5$
$y = x^2 - 4$

13–7

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Study Guide

Graphing Systems of Inequalities

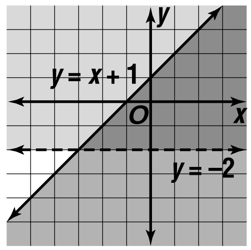
To graph a system of linear inequalities, first graph a boundary line for each inequality. Then shade on one side of each boundary line. The region where the shaded areas overlap contains the solutions of the system of inequalities.

Example: Solve the system of inequalities by graphing.

$y > -2$
$y \leq x + 1$

Step 1 Graph the boundary lines $y = -2$ and $y = x + 1$. Since y is *greater than* -2, make the line $y = -2$ dashed. The line $y = x + 1$ is solid because y is *less than or equal to* $x + 1$.

Step 2 Because $y > -2$ has a *greater than* symbol, shade above the boundary line. Shade below the boundary line for $y \leq x + 1$ because this inequality contains a *less than* symbol.



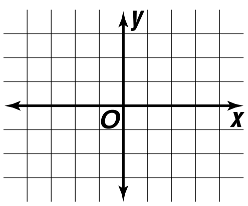
The region where the shaded areas intersect is the solution of the system of inequalities.

Use the following rules to help you graph inequalities.

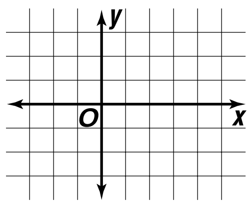
Inequality	Example	Boundary Line	Where to Shade
$y < ax + b$	$y < 2x$	dashed	below the line
$y \leq ax + b$	$y \leq -x + 1$	solid	below the line
$y > ax + b$	$y > 5x - 2$	dashed	above the line
$y \geq ax + b$	$y \geq -4x$	solid	above the line

***Solve each system of inequalities by graphing. If the system does not have a solution, write* no solution.**

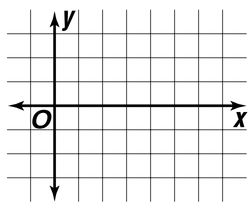
1. $y < 1$
$y > -3$



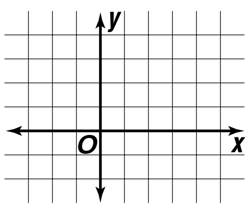
2. $x > 4$
$x < -1$



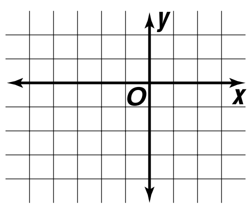
3. $y \geq -2$
$x \leq 5$



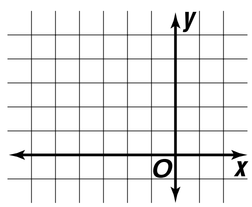
4. $x \leq 2$
$y > x$



5. $y < x$
$x \leq -1$



6. $y \geq 0$
$y \leq x + 3$



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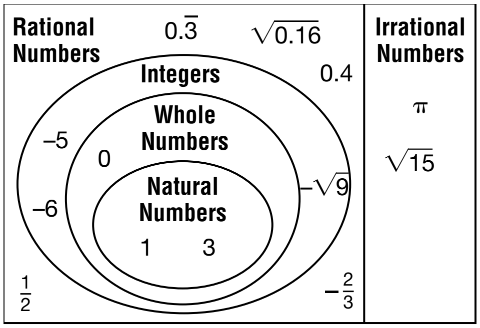
Study Guide

The Real Numbers

The numbers that we use every day belong to the set of real numbers. The real numbers can be divided into rational numbers and irrational numbers. Rational numbers can be expressed as fractions. Irrational numbers cannot be expressed as fractions. The set of rational numbers contains the natural numbers, the whole numbers, and the integers.

Natural Numbers: $\{1, 2, 3, 4, \ldots\}$
Whole Numbers: $\{0, 1, 2, 3, \ldots\}$
Integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

The Venn diagram shows how subsets of the real numbers are related.



- 3 is a natural number, a whole number, an integer, and a rational number.
- 0 is a whole number, an integer, and a rational number.
- $-\sqrt{9}$ or 3 is an integer and a rational number.
- $\sqrt{15}$ or 3.872983346… is an irrational number.
- $0.\overline{3}$ or $\frac{1}{3}$ is a rational number.

Example: Find an approximation to the nearest tenth of $\sqrt{12}$.

On a graphing calculator, press [2nd] [$\sqrt{\ }$] 12 [ENTER].

The result is 3.464101615. So an approximate value for $\sqrt{12}$ is 3.5.

Name the set or sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. -4 **2.** $\frac{2}{5}$ **3.** $-\sqrt{25}$ **4.** 10

5. 2.3 6 $\sqrt{3}$ **7.** $-4.324781\ldots$ **8.** $-\frac{24}{8}$

9. $\sqrt{100}$ **10.** $\frac{1}{9}$ **11.** -0.25 **12.** $\sqrt{15}$

Find an approximation to the nearest tenth for each square root.

13. $\sqrt{2}$ **14.** $\sqrt{14}$ **15.** $-\sqrt{20}$ **16.** $\sqrt{55}$

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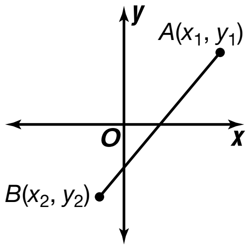
Study Guide

The Distance Formula

You can use the Distance Formula to find the distance between two points on the coordinate plane.

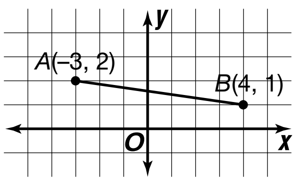
The Distance Formula

The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example: Find the distance between $A(-3, 2)$ and $B(4, 1)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-3))^2 + (1 - 2)^2} \\ &= \sqrt{7^2 + (-1)^2} \\ &= \sqrt{49 + 1} \\ &= \sqrt{50} \text{ or about 7.1 units} \end{aligned}$$



Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $P(-4, 0)$, $Q(5, 0)$

2. $X(0, 0)$, $Y(6, 8)$

3. $J(-5, 1)$, $K(-2, 5)$

4. $M(-4, -14)$, $N(3, 10)$

5. $R(-7, 4)$, $S(2, -1)$

6. $C(0, -5)$, $D(3, 2)$

7. $X(5, 9)$, $Y(2, 3)$

8. $A(-9, 1)$, $B(-8, 3)$

9. $G(5, -4)$, $H(10, 6)$

10. $U(7, -3)$, $V(-3, -2)$

11. $M(1, 3)$, $N(-1, 4)$

12. $X(12, -3)$, $Y(7, -15)$

13. $A(1, 20)$, $B(12, -4)$

14. $E(-3, -3)$, $F(-8, -8)$

15. $A(5, 4)$, $B(0, 6)$

16. $S(-6, -6)$, $T(-5, -1)$

14–3

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Study Guide

Simplifying Radical Expressions

To simplify a radical expression such as $\sqrt{54}$, first identify any perfect square factors of the radicand. Then apply the Product Property of Square Roots.

$\sqrt{54} = \sqrt{6 \cdot 9}$	*$3^2 = 9$, so 9 is a perfect square factor of 54.*
$= \sqrt{6} \cdot \sqrt{9}$	*Product Property of Square Roots*
$= \sqrt{6} \cdot 3$ or $3\sqrt{6}$	*Simplify $\sqrt{9}$.*

To simplify radical expressions such as $\frac{\sqrt{24}}{\sqrt{2}}$ and $\frac{\sqrt{6}}{\sqrt{5}}$ that involve division, you must eliminate the radicals in the denominator. To do so, you can use the Quotient Property of Square Roots and a method called **rationalizing the denominator**. Rationalizing the denominator involves multiplying the fraction by a special form of 1.

Examples: Simplify each expression.

a. $\frac{\sqrt{24}}{\sqrt{2}}$

$\frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}}$	*Quotient Property*
$= \sqrt{12}$	*$24 \div 2 = 12$*
$= \sqrt{3 \cdot 2^2}$	*$3 \cdot 4 = 3 \cdot 2^2$*
$= 2\sqrt{3}$	*$\sqrt{2^2} = 2$*

b. $\frac{\sqrt{6}}{\sqrt{5}}$

$\frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$	*$\frac{\sqrt{5}}{\sqrt{5}} = 1$*
$= \frac{\sqrt{6 \cdot 5}}{\sqrt{5 \cdot 5}}$	*Product Property*
$= \frac{\sqrt{30}}{\sqrt{25}}$	*Quotient Property*
$= \frac{\sqrt{30}}{5}$	*Simplify.*

Simplify each expression. Leave in radical form.

1. $\sqrt{28}$ **2.** $\sqrt{48}$ **3.** $\sqrt{50}$ **4.** $\sqrt{8}$

5. $\sqrt{99}$ **6.** $\sqrt{12} \cdot \sqrt{3}$ **7.** $\sqrt{8} \cdot \sqrt{6}$ **8.** $2\sqrt{3} \cdot \sqrt{3}$

9. $\frac{\sqrt{18}}{\sqrt{6}}$ **10.** $\frac{\sqrt{75}}{\sqrt{3}}$ **11.** $\frac{\sqrt{49}}{\sqrt{7}}$ **12.** $\frac{\sqrt{72}}{\sqrt{36}}$

13. $\frac{\sqrt{3}}{\sqrt{4}}$ **14.** $\frac{\sqrt{5}}{\sqrt{3}}$ **15.** $\frac{\sqrt{12}}{\sqrt{5}}$ **16.** $\frac{\sqrt{6}}{\sqrt{8}}$

14–4

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Study Guide

Adding and Subtracting Radical Expressions

You can use the Distributive Property to add and subtract radical expressions with the same radicand.

Example 1: Simplify each expression.

a. $2\sqrt{3} + 8\sqrt{3}$

$2\sqrt{3} + 8\sqrt{3} = (2 + 8)\sqrt{3}$ *Distributive Property*

$= 10\sqrt{3}$ *Simplify.*

b. $15\sqrt{10} - 2\sqrt{10}$

$15\sqrt{10} - 2\sqrt{10} = (15 - 2)\sqrt{10}$ *Distributive Property*

$= 13\sqrt{10}$ *Simplify.*

Recall that when you add monomials, only like terms can be combined. The same is true when you add or subtract radical expressions. Radical expressions are like terms if they have the same radicand when they are in simplest form.

Example 2: Simplify $4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7}$.

$4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7} = 4\sqrt{2} + \left(6\sqrt{7} - 11\sqrt{7}\right)$ *Group like terms.*

$= 4\sqrt{2} + (6 - 11)\sqrt{7}$ *Distributive Property*

$= 4\sqrt{2} - 5\sqrt{7}$ *Simplify.*

Simplify each expression.

1. $8\sqrt{5} + 8\sqrt{5}$

2. $5\sqrt{11} - 3\sqrt{11}$

3. $-9\sqrt{2} + \sqrt{2}$

4. $-3\sqrt{3} - 10\sqrt{3}$

5. $4\sqrt{6} + \sqrt{6}$

6. $\sqrt{10} - 6\sqrt{10} + 7\sqrt{10}$

7. $2\sqrt{2} - 5\sqrt{2} + 4\sqrt{5}$

8. $\sqrt{11} - 15\sqrt{3} - 10\sqrt{3}$

9. $8\sqrt{13} + 3\sqrt{13} - 4\sqrt{7} - 3\sqrt{7}$

10. $-4\sqrt{6} + 2\sqrt{6} + \sqrt{3} + \sqrt{3}$

11. $-3\sqrt{5} + 9\sqrt{2} + 5\sqrt{2} + 5\sqrt{5}$

12. $7\sqrt{2} + \sqrt{8}$

13. $3\sqrt{3} + \sqrt{27}$

14. $5\sqrt{32} - 6\sqrt{2}$

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Study Guide

Solving Radical Equations

Equations that contain radicals are called **radical equations**.

Steps for Solving Radical Equations
1. Isolate the radical on one side of the equation. **2.** Square each side of the equation to eliminate the radical. **3.** Check all solutions. Reject any solutions that do not satisfy the original equation.

Examples: Solve each equation. Check your solution.

a. $\sqrt{x} - 2 = 7$

$$\sqrt{x} - 2 = 7$$
$$\sqrt{x} - 2 + 2 = 7 + 2 \quad \textit{Add 2 to each side.}$$
$$\sqrt{x} = 9$$
$$\left(\sqrt{x}\right)^2 = 9^2 \quad \textit{Square each side.}$$
$$x = 81$$

Check: $\sqrt{x} - 2 = 7$

$$\sqrt{81} - 2 \stackrel{?}{=} 7 \quad \textit{Replace x with 81.}$$
$$9 - 2 \stackrel{?}{=} 7$$
$$7 = 7 \checkmark$$

The solution is 81.

b. $\sqrt{x + 1} + 5 = 4$

$$\sqrt{x + 1} + 5 = 4$$
$$\sqrt{x + 1} = -1 \quad \textit{Subtract 5.}$$
$$\left(\sqrt{x + 1}\right)^2 = (-1)^2 \quad \textit{Square each side.}$$
$$x + 1 = 1$$
$$x = 0$$

Check: $\sqrt{x + 1} + 5 = 4$

$$\sqrt{0 + 1} + 5 \stackrel{?}{=} 4$$
$$1 + 5 \stackrel{?}{=} 4$$
$$6 \neq 4$$

There is no solution.

Some radical equations have no solution when the domain is the set of real numbers. Example b has no solution because the square root of $x + 1$ cannot be negative.

Solve each equation. Check your solution.

1. $\sqrt{x} = 7$

2. $\sqrt{x} = 2$

3. $\sqrt{x} = -9$

4. $\sqrt{x} - 9 = -1$

5. $\sqrt{x} + 12 = 6$

6. $\sqrt{x} + 2 = 4$

7. $\sqrt{x} - 8 = -3$

8. $\sqrt{x} - 1 = -5$

9. $\sqrt{x + 4} = 3$

10. $2 = \sqrt{x - 5}$

11. $\sqrt{x - 2} + 1 = 6$

12. $\sqrt{x + 3} - 7 = 0$

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Study Guide

Simplifying Rational Expressions

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. For example, $\frac{3}{x}$, $\frac{3}{x-2}$, $\frac{x-2}{3}$, and $\frac{x}{x-2}$ are all rational expressions because 3, x, and $x - 2$ are all polynomials.

To simplify a rational expression, use the same steps that you use to simplify any fraction.

- First factor the numerator and denominator. The factors may be polynomials.
- Then divide the numerator and denominator by the greatest common factor. The greatest common factor may be a polynomial.

Example 1: Simplify $\frac{16a^4b^2}{20a^2b^5}$.

$$\frac{16a^4b^2}{20a^2b^5} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b}{2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b}$$

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 5 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot b \cdot b}$$ *The GCF is $4a^2b^2$.*

$$= \frac{4a^2}{5b^3}$$

Example 2: Simplify $\frac{m(m+1)}{m^2 - 5m - 6}$.

$$\frac{m(m+1)}{m^2 - 5m - 6} = \frac{m(m+1)}{(m-6)(m+1)}$$ *Factor $m^2 - 5m - 6$.*

$$= \frac{m\overset{1}{\cancel{(m+1)}}}{(m-6)\underset{1}{\cancel{(m+1)}}}$$ *The GCF is $(m + 1)$.*

$$= \frac{m}{m-6}$$

Simplify each rational expression.

1. $\frac{10}{25}$

2. $\frac{3m}{6n}$

3. $\frac{12xy}{6x}$

4. $\frac{-8abc}{16ac}$

5. $\frac{6a^2b^3}{-2a^4b^3}$

6. $\frac{3(x+2)}{6(x+2)}$

7. $\frac{m-4}{m(m-4)}$

8. $\frac{d(d-4)}{c(d-4)}$

9. $\frac{(c-2)(d+3)}{(c-2)(d+6)}$

10. $\frac{x^2 - 2x}{x-2}$

11. $\frac{y^2 + 3y}{y(y-3)}$

12. $\frac{x+3}{x^2 + x - 6}$

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Study Guide

Multiplying and Dividing Rational Expressions

To multiply rational expressions, simplify the expressions first, then multiply.

Example 1: Find $\frac{12a^2}{18b^2} \cdot \frac{9ab}{12a^4}$.

$$\frac{12a^2}{18b^2} \cdot \frac{9ab}{12a^4} = \frac{\overset{1}{\cancel{12}}\,\overset{1}{\cancel{a^2}}}{\underset{2}{\cancel{18}}\,\underset{b}{\cancel{b^2}}} \cdot \frac{\overset{1}{\cancel{9}}\,\overset{1}{\cancel{a}}\,\overset{1}{\cancel{b}}}{\underset{1}{\cancel{12}}\,\underset{a}{\cancel{a^4}}}$$

$$= \frac{1}{2ab}$$

To divide a rational expression by a nonzero rational expression, multiply by its reciprocal. Simplify the expressions, if necessary.

Example 2: Find $\frac{3m + 6}{m - 4} \div (m + 2)$.

$$\frac{3m + 6}{m - 4} \div (m + 2) = \frac{3m + 6}{m - 4} \cdot \frac{1}{m + 2} \qquad \textit{The reciprocal of } (m + 2) \textit{ is } \frac{1}{m + 2}.$$

$$= \frac{3(m + 2)}{\cancel{m - 4}} \cdot \frac{1}{m + 2}$$

$$= \frac{3(m + 2)}{m - 4} \cdot \cancel{\frac{1}{m + 2}}$$

$$= \frac{3}{m - 4}$$

Find each product or quotient.

1. $\frac{4x}{3y} \cdot \frac{y^3}{8}$

2. $\frac{8a^2}{12b^2} \cdot \frac{18b}{4a}$

3. $\frac{2(x - y)}{x} \cdot \frac{x^2}{x - y}$

4. $\frac{6(m - n)}{7} \cdot \frac{7}{12(m - n)}$

5. $\frac{x^2}{y^2} \div \frac{x}{y}$

6. $\frac{m + n}{4} \div \frac{m + n}{6}$

7. $\frac{x - 4}{x} \div \frac{1}{x^2}$

8. $\frac{7x}{x + 3} \div \frac{21}{2x + 6}$

9. $\frac{2m + 4}{m - 3} \div (m + 2)$

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Study Guide

Dividing Polynomials

To divide 864 by 16, you can use long division and find that the remainder is 0. Therefore, 16 is said to be a **factor** of 864. Likewise, to divide polynomials, you can use long division. Each term may be an algebraic expression, and if the remainder is 0, the divisor is a factor of the dividend. If the divisor is not a factor of the dividend, the remainder will not be zero.

Examples: Find each quotient.

a. $(4x + 2) \div (2x + 1)$

$$\begin{array}{r|l} \begin{array}{r} 2 \\ 2x + 1 \overline{)\,4x + 2} \\ \underline{4x + 2} \\ 0 \end{array} & \begin{array}{l} \textit{4x} \div \textit{2x} = \textit{2} \\ \\ \textit{Multiply 2 and 2x + 1.} \\ \textit{Subtract.} \end{array} \end{array}$$

Therefore, $(4x + 2) \div (2x + 1) = 2$.

b. $(x^2 - 4x + 3) \div (x - 1)$

$$\begin{array}{r|l} \begin{array}{r} x - 3 \\ x - 1 \overline{)\,x^2 - 4x + 3} \\ \underline{x^2 - x} \quad\; \\ -3x + 3 \\ \underline{-3x + 3} \\ 0 \end{array} & \begin{array}{l} x^2 \div x = x \\ \\ \textit{Multiply } x \textit{ and } x - 1. \\ \textit{Subtract; bring down 3.} \\ \textit{Multiply } -3 \textit{ and } x - 1. \\ \textit{Subtract.} \end{array} \end{array}$$

Therefore, $(x^2 - 4x + 3) \div (x - 1) = x - 3$.

c. $(2x^2 + 9x + 8) \div (x + 3)$

$$\begin{array}{r|l} \begin{array}{r} 2x + 3 \\ x + 3 \overline{)\,2x^2 + 9x + 8} \\ \underline{2x^2 + 6x} \quad\; \\ 3x + 8 \\ \underline{3x + 9} \\ -1 \end{array} & \begin{array}{l} 2x^2 \div x = 2x \\ \\ \textit{Multiply } 2x \textit{ and } x + 3. \\ \textit{Subtract; bring down 8.} \\ \textit{Multiply 3 and } x + 3. \\ \textit{Subtract. The remainder is } -1. \end{array} \end{array}$$

Therefore, $(2x^2 + 9x + 8) \div (x + 3) = 2x + 3 + \frac{-1}{x + 3}$.

Find each quotient.

1. $(6x - 3) \div (2x - 1)$

2. $(x^2 - 2x + 1) \div (x - 1)$

3. $(x^2 + 5x + 4) \div (x + 4)$

4. $(2x^2 - 4x) \div (x - 2)$

5. $(5r^3 - 15r^2) \div (r - 3)$

6. $(a^2 + 6a + 5) \div (a + 5)$

7. $(2a^2 - 5a - 3) \div (2a + 1)$

8. $(6x^2 - 2x + 5) \div (x + 1)$

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Study Guide

Combining Rational Expressions with Like Denominators

You know that to add or subtract fractions with like denominators, you add or subtract the numerators and then write the sum or difference over the common denominator. For example, the sum of $\frac{1}{5}$ and $\frac{3}{5}$ is $\frac{4}{5}$. Use this same method to add or subtract rational expressions with like denominators.

Examples: Find each sum or difference. Express the answer in simplest form.

a. $\frac{a}{5} + \frac{3a}{5} = \frac{a + 3a}{5}$ *The common denominator is 5. Add the numerators.*

$= \frac{4a}{5}$

b. $\frac{8}{5m} - \frac{3}{5m} = \frac{5}{5m}$ *The common denominator is 5m. Subtract the numerators.*

$= \frac{1}{m}$ *Divide by the GCF, 5.*

c. $\frac{c}{c-1} + \frac{c-2}{c-1} = \frac{c + c - 2}{c-1}$ *The common denominator is c − 1. Add the numerators.*

$= \frac{2c-2}{c-1}$ *Factor the numerator*

$= \frac{2(c-1)}{c-1}$ *Divide by the GCF, c − 1.*

$= 2$

Find each sum or difference. Write in simplest form.

1. $\frac{6}{m} - \frac{2}{m}$

2. $\frac{4}{3x} + \frac{2}{3x}$

3. $\frac{f}{8} + \frac{7f}{8}$

4. $\frac{6t}{13} - \frac{5t}{13}$

5. $\frac{18}{25r} - \frac{3}{25r}$

6. $\frac{9x}{10} + \frac{x}{10}$

7. $\frac{8}{6n} + \frac{-2}{6n}$

8. $\frac{1}{x-2} + \frac{3}{x-2}$

9. $\frac{8}{y+1} - \frac{1}{y+1}$

10. $\frac{b}{b-1} - \frac{1}{b-1}$

11. $\frac{v}{v+2} - \frac{v}{v+2}$

12. $\frac{h}{h+4} + \frac{h+8}{h+4}$

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Study Guide

Combining Rational Expressions with Unlike Denominators

You add $\frac{1}{5}$ and $\frac{3}{4}$ by first finding the common denominator, 20.

Likewise, to add or subtract rational expressions with unlike denominators, first rename the expressions so the denominators are alike. Then add or subtract the numerators and write the sum or difference over the common denominator. Simplify if necessary. The least common denominator (LCD) may make the computations easier.

Example: Find $\frac{3}{4a^2} + \frac{5}{2a}$.

Step 1 First find the LCD.

$4a^2 = 2 \cdot 2 \cdot a \cdot a$

$2a = 2 \cdot a$

The LCD is $4a^2$.

Step 2 Rename each expression with the LCD as denominator.

The denominator of $\frac{3}{4a^2}$ is already $4a^2$, so only $\frac{5}{2a}$ needs to be renamed.

$$\frac{5}{2a} = \frac{5}{2a} \cdot \frac{2a}{2a} = \frac{10a}{4a^2}$$

Step 3 Add.

$$\frac{3}{4a^2} + \frac{5}{2a} = \frac{3}{4a^2} + \frac{10a}{4a^2}$$

$$= \frac{3 + 10a}{4a^2}$$ *The expression is in simplest form.*

Find each sum or difference. Write in simplest form.

1. $\frac{x}{6} + \frac{x}{12}$

2. $\frac{f}{8} - \frac{f}{16}$

3. $\frac{d}{6} + \frac{d}{3}$

4. $\frac{4}{x} - \frac{6}{2x}$

5. $\frac{3d}{4} - \frac{d}{8}$

6. $\frac{3x}{m} - \frac{1}{2m}$

7. $\frac{3}{m} - \frac{4}{m^2}$

8. $\frac{6}{x} + \frac{4}{x^3}$

9. $\frac{3}{4y^2} + \frac{1}{8y}$

10. $\frac{1}{x} + \frac{1}{y}$

11. $\frac{2c}{a} - \frac{4}{b^2}$

12. $\frac{3}{2n} + \frac{2t}{4n^2}$

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Study Guide

Solving Rational Equations

A rational equation is an equation that contains at least one rational expression. There are three steps in solving rational equations.

Step 1 Find the LCD of all terms.

Step 2 Multiply each side of the equation by the LCD.

Step 3 Use the Distributive Property to simplify.

Example: Solve $\frac{3}{5x} + \frac{2}{x} = \frac{1}{5}$.

$$\frac{3}{5x} + \frac{2}{x} = \frac{1}{5} \qquad \textit{The LCD is 5x.}$$

$$5x\left(\frac{3}{5x} + \frac{2}{x}\right) = 5x\left(\frac{1}{5}\right) \qquad \textit{Multiply each side by the LCD.}$$

$$5x\left(\frac{3}{5x}\right) + 5x\left(\frac{2}{x}\right) = 5x\left(\frac{1}{5}\right) \qquad \textit{Distributive Property}$$

$$\overset{1}{\cancel{5x}}\left(\frac{3}{\underset{1}{\cancel{5x}}}\right) + \overset{5}{5\cancel{x}}\left(\frac{2}{\underset{1}{\cancel{x}}}\right) = \overset{x}{\cancel{5}}x\left(\frac{1}{\underset{1}{\cancel{5}}}\right)$$

$$3 + 10 = x \qquad \textit{Simplify.}$$

$$13 = x$$

Check:

$$\frac{3}{5x} + \frac{2}{x} = \frac{1}{5}$$

$$\frac{3}{5(13)} + \frac{2}{13} \stackrel{?}{=} \frac{1}{5}$$

$$\frac{3}{65} + \frac{2}{13} \stackrel{?}{=} \frac{1}{5} \qquad \textit{The LCD is 65.}$$

$$\frac{3}{65} + \frac{10}{65} \stackrel{?}{=} \frac{13}{65}$$

$$\frac{13}{65} = \frac{13}{65} \checkmark$$

Solve each equation. Check your solution.

1. $\frac{x}{6} + \frac{x}{12} = \frac{1}{2}$

2. $\frac{3x}{8} - \frac{x}{4} = \frac{1}{4}$

3. $\frac{y}{3} + \frac{2y}{5} = \frac{11}{3}$

4. $\frac{9y}{10} - \frac{y}{2} = \frac{2}{5}$

5. $\frac{3d}{4} - \frac{d}{8} = \frac{5}{16}$

6. $\frac{x}{4} = \frac{x+2}{8}$

7. $\frac{t-4}{6} = \frac{t+1}{8}$

8. $\frac{x+1}{x} + \frac{x-2}{x} = 4$

9. $\frac{8}{4-s} - \frac{s}{4-s} = 2$

10. $\frac{x}{8} - \frac{x}{4} = \frac{x-1}{2}$

11. $\frac{m+2}{m} - \frac{m-1}{m} = 3$

12. $\frac{1}{4s} - \frac{3}{2s} = \frac{1}{8}$