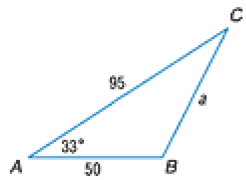
Lesson 13–5

Example 1 Solve a Triangle Given Two Sides and Included Angle

Solve ΔABC for $b = 95$, $c = 50$, and $A = 33^\circ$.

You are given the measure of two sides and the included angle. Begin by drawing a diagram and using the Law of Cosines to determine a.



$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 95^2 + 50^2 - 2(95)(50) \cos 33^\circ$	$b = 95$, $c = 50$, and $A = 33^\circ$
$a^2 \approx 3557.6$	Simplify using a calculator.
$a \approx 59.6$	Take the square root of each side.

Next, you can use the Law of Sines to find the measure of angle B.

$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 33^\circ}{59.6} = \frac{\sin B}{95}$	$a = 59.6$, $b = 95$, and $A = 33^\circ$
$\sin B = \frac{95 \sin 33^\circ}{59.6}$	Multiply each side by 95.
$\sin B \approx 0.8681$	Use a calculator.
$B \approx 60.2$	Use the $\sin^{-1}$ function.

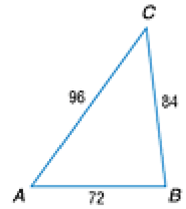
If you use fairly accurate measures for b, c, and A in your diagram, you can see that B must be an obtuse angle. The measure of B is approximately $180^\circ - 60.2^\circ$ or 119.8°. Then the measure of C is approximately $180^\circ - (33^\circ + 119.8^\circ)$ or 27.2°. You can check the reasonableness of this result by noting that $B > A > C$ and $b > a > c$.

Therefore, $a \approx 59.6$, $B \approx 119.8^\circ$, and $C \approx 27.2^\circ$.

Example 2 **Solve a Triangle Given Three Sides**

Solve $\triangle ABC$ for $a = 84$, $b = 96$, and $c = 72$.

You are given the measures of three sides. Draw a diagram and use the Law of Cosines to find the measure of angle *A*.



$$a^2 = b^2 + c^2 - 2bc \cos A$$ Law of Cosines

$$84^2 = 96^2 + 72^2 - 2(96)(72) \cos A$$ $a = 84$, $b = 96$, and $c = 72$

$$84^2 - 96^2 - 72^2 = -2(96)(72) \cos A$$ Subtract 96^2 and 72^2 from each side.

$$\frac{84^2 - 96^2 - 72^2}{-2(96)(72)} = \cos A$$ Divide each side by $-2(96)(72)$.

$$0.5313 \approx \cos A$$ Use a calculator.

$$57.9° \approx A$$ Use the $\cos^{-1}$ function.

You can use the Law of Sines to find the measure of angle *B*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ Law of Sines

$$\frac{\sin 57.9°}{84} = \frac{\sin B}{96}$$ $A = 57.9°$, $a = 84$, and $b = 96$

$$\sin B = \frac{96 \sin 57.9°}{84}$$ Multiply each side by 96.

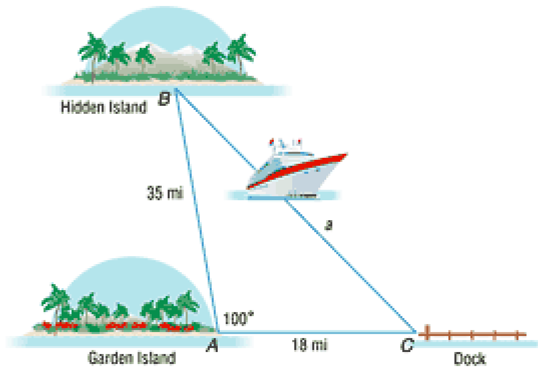
$$\sin B \approx 0.9681$$ Use a calculator.

$$B \approx 75.5°$$ Use the $\sin^{-1}$ function.

The measure of angle *C* is approximately $180° - (57.9° + 75.5°)$ or $46.6°$.

Therefore, $A \approx 57.9°$, $B \approx 75.5°$, and $C \approx 46.6°$.

Example 3 **Apply the Law of Cosines**
RECREATION A tour boat leaves the dock and cruises to Garden Island and then on to Hidden Island as shown in the diagram. How long will it take the boat to return to the dock if it takes a straight course and travels at 12 miles per hour?



In order to find the time it will take for the tour boat to return to the dock, you must find the distance *a* in the diagram.

You are given the measures of two sides and their included angle, so use the Law of Cosines to find *a*.

$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 18^2 + 31.5^2 - 2(18)(31.5) \cos 100°$	$A = 100°$, $b = 18$, and $c = 31.5$
$a^2 = 1513.2$	Use a calculator to simplify.
$a \approx 38.9$	Take the square root of each side.

The distance back to the dock is approximately 39 miles. At 12 miles per hour the trip will take 39 ÷ 12 or 3.25 hours.